TECHNISCHE MECHANIK, Band 17, Heft 2, (1997), 113-120
Manuskripteingang: 02. Oktober 1996

Thermoparametric Vibrations of Noncircular Cylindrical Shell in Nonstationary Temperature Field

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Low-frequency vibrations of an elastic noncircular cylindrical shell in a nonstationary temperature field is investigated. By using the method of multiple scales, the solutions of the shell equations are constructed in the form of functions localized near "the weakest" generatrix. The equations for definition of the vibration amplitude are derived. The main region of instability of the shell equilibrium is established.

1 Introduction

Various papers concerned with parametric vibrations of cylindrical shells have been published (Yao, 1963 and 1965; Wenzke, 1963; Vijayaraghavan and Evan-Iwanowski, 1967; Grundmann, 1970) and in many of them the problem of dynamic instability of shells subjected to periodic axial or/and radial loads has been treated. However, taking into account the influence of the nonstationary temperature field can also lead to unstable forms of motion (Ogibalov and Gribanov, 1968). In particular, by using Galerkin's procedure, resonant thermoparametric vibrations of a circular cylindrical shell have been investigated by Kilichinskaya (1963).

The purpose of the present paper is to find the main region of instability of a noncircular cylindrical shell subjected to action of periodic temperature and to analyze the influence of a variable curvature of a shell on the dimensions of this region. The shell is supposed to have the "weakest" generatrix in a vicinity of which the modes of low-frequency vibrations are localized. We will examine the case of parametric resonance $\Omega^* \approx 2\omega^*$, where Ω^* is the frequency of the temperature fluctuation on the external surface of the shell, ω^* is one of the fundamental frequencies of the lower spectrum of free vibrations.

2 Governing Equations

We consider a thin elastic noncircular cylindrical shell of constant thickness h and length L. We introduce an orthogonal coordinate system x, y connected with the main curvatures lines, where x is a point coordinate on the generatrix ($0 \le x \le L$), and y is an arc length on the shell surface.

The distribution of variation of the temperature is assumed to be linear along the thickness and periodic in time.

$$T = T_2 + T_1 \left(\frac{1}{2} + \frac{z}{h}\right) \cos \Omega^* t^* \tag{1}$$

Here T_2 is the temperature of the internal surface of the shell, T_1 is the amplitude, t^* is the time, and z is the normal coordinate of a point.

According to theoretical and experimental data (Ogibalov and Gribanov, 1968), the coefficient of linear thermal extension α and Young's modulus E are assumed to be linear functions of T.

$$\alpha = \alpha_0 - \alpha' T \qquad E = E_0 - E' T \tag{2}$$

where α_0 and E_0 are the isothermal values of these parameters, and α' and E' are determined experimentally.

Suppose the shell edges are free, then initial temperature stresses in the shell are absent (Podstrigach and Schvets, 1978), and for an analysis of the lowest part of the spectrum of thermoparametric bending vibrations, the following governing equations can be used (Ogibalov and Gribanov, 1968):

$$\frac{E_1 E_3 - E_2^2}{(1-\nu^2)E_1} \Delta^* \Delta^* W^* - \frac{1}{R_2(y)} \frac{\partial^2 \Phi^*}{\partial x^2} + \rho h \frac{\partial^2 W^*}{\partial t^{*2}} = 0 \tag{3}$$

$$\Delta^* \Delta^* \Phi^* + \frac{E_1 h}{R_2(y)} \frac{\partial^2 W^*}{\partial x^2} = 0 \qquad \Delta^* = \partial^2/\partial x^2 + \partial^2/\partial y^2$$

$$E_j = \int_{-h/2}^{h/2} E z^{j-1} dz \qquad j = 1, 2, 3$$

where W^*, Φ^* are the normal deflection and the stress function, respectively, $R_1 = \infty$ and $R_2(y)$ are the main radii of a curvature, while ρ is the mass density.

For the free edges $x = 0, L,$ the boundary conditions have the form

$$M_x \equiv \frac{E_1 E_3 - E_2^2}{(1-\nu^2)E_1} \left(\frac{\partial^2 W^*}{\partial x^2} + \frac{\partial^2 W^*}{\partial y^2} \right) + \frac{1}{1-\nu} \left(M_T - \frac{E_2}{E_1} N_T \right) = 0 \tag{4}$$

$$Q_x = N_x = N_{xy} = 0 \tag{5}$$

Here M_x, Q_x and N_x, N_{xy} are the bending moment, the shear force and the tangential forces in the median surface of the shell. In the last term in equation (4)

$$M_T = \int_{-h/2}^{h/2} \alpha E T z dz \qquad N_T = \int_{-h/2}^{h/2} \alpha E T dz$$

are the moment and the tangential force, due to the thermal expansion of the shell.

We introduce dimensionless quantities as follows:

$$x = R s \qquad (0 \le s \le l = L/R)$$

$$y = R \varphi \qquad (y_1/R = \varphi_1 \le \varphi \le \varphi_2 = y_2/R)$$

$$t^* = t_c t \qquad W^* = C R W \qquad \Phi^* = C \varepsilon^4 E_s h R^2 \Phi$$

where $R = R_2(0)$, $\varepsilon^8 = h^2 / [12(1-\nu^2) R^2]$ is a small parameter, ν is Poisson's ratio, $E_s = E_0 - E' T_2$ is the static value of Young's modulus, $t_c = \sqrt{\rho R^2 / (\varepsilon^4 E_s)}$ is the characteristic time, and C is an arbitrary constant.

Let us consider the case when the amplitude of temperature fluctuations is not too large, so that

$$\frac{E' T_1}{E_s} = 2 \eta \varepsilon \tag{6}$$

where $\eta \sim 1$ at $\varepsilon \to 0$.

Introducing dimensionless quantities into equations (3) and taking into account relations (1), (2), (6), the governing equations can be rewritten in nondimensional form as follows

$$\varepsilon^4 \tau_1(t)\, \Delta^2 W - \chi(\varphi) \frac{\partial^2 \Phi}{\partial s^2} + \frac{\partial^2 W}{\partial t^2} = 0 \qquad \varepsilon^4 \Delta^2 \Phi - \chi(\varphi)\, \tau_2(t) \frac{\partial^2 W}{\partial s^2} = 0 \tag{7}$$

Here

$$\tau_1(t) = 1 - \varepsilon\, \eta \cos \Omega\, t - \frac{1}{3} \varepsilon^2 \eta^2 \cos^2 \Omega\, t + O(\varepsilon^3)$$

$$\tau_2(t) = 1 - \varepsilon\, \eta \cos \Omega\, t \qquad \Omega = \Omega^* t_c \qquad \chi(\varphi) = R/R_2(\varphi)$$

It is supposed here that $\chi(\varphi)$ is an infinity differentiable function so that

$$\chi(0) = 1 \qquad \chi'(0) = 0 \qquad \chi''(0) > 0 \tag{8}$$

Then the generatrix $\varphi = 0$ is the "weakest" one, the modes of low-frequency free vibrations being localized in its neighbourhood (Tovstik, 1983; Mikhasev, 1992). We will examine thermoparametric vibrations being excited by a nonstationary temperature field (1) near the line $\varphi = 0$.

To find an approximate solution corresponding to the main stress-strain state of the shell, the main boundary conditions should be selected from conditions (4), (5). It is assumed here that $\alpha' T_1/\alpha_s \sim \varepsilon$, $\alpha_s T_2 \sim 1$ at $\varepsilon \to 0$, where $\alpha_s = \alpha_0 + \alpha' T$ is the stationary value of α. Then the last term in equation (4) may be disregarded, and the main boundary conditions, with an accuracy up to values on the order of ε^2, appear as (Tovstik, 1995)

$$\frac{\partial^2 W}{\partial s^2} = \frac{\partial^3 W}{\partial s^3} = 0 \qquad \text{for } s = 0, l \tag{9}$$

3 Asymptotic Solution

The method of multiple scales will be applied here. The uniformly valid solution of equations (7) is assumed to be of the form

$$W = \sum_{k=0}^{\infty} \varepsilon^{k/2}\, W_k\, (s; \xi_0, \xi_1, \ldots, \xi_j;\ t_0, t_1, \ldots, t_j) \tag{10}$$

$$\Phi = \sum_{k=0}^{\infty} \varepsilon^{k/2}\, \Phi_k\, (s; \xi_0, \xi_1, \ldots, \xi_j;\ t_0, t_1, \ldots, t_j)$$

where

$$\xi_j = \varepsilon^{(j-1)/2}\, \xi \qquad \xi = \varepsilon^{-1/2}\, \varphi \qquad t_j = \varepsilon^j\, t \tag{11}$$

With equalities (8)and (11) in mind, the function $\chi(\varphi)$ may be represented as follows

$$\chi(\varphi) = 1 + \frac{1}{2}\varepsilon\chi''(0)\xi_1^2 + \ldots \tag{12}$$

The substitution of equations (10) to (12) into equation (7) produces a sequence of the boundary-value problems

$$\sum_{j=0}^{k} D_j W_{k-j} = 0 \qquad k = 0, 1, 2, \ldots \tag{13}$$

$$\frac{\partial^2 W_k}{\partial s^2} = \frac{\partial^3 W_k}{\partial s^3} = 0 \qquad \text{for } s = 0, l \tag{14}$$

where

$$D_0 = \frac{\partial^8}{\partial \xi_0^8} + \frac{\partial^6}{\partial \xi_0^4 \partial t_0^2} + \frac{\partial^4}{\partial s^4} \qquad D_1 = 8\frac{\partial^8}{\partial \xi_0^7 \partial \xi_1} + 4\frac{\partial^6}{\partial \xi_0^3 \partial \xi_1 \partial t_0^2} \tag{15}$$

$$D_2 = 28\frac{\partial^8}{\partial \xi_0^6 \partial \xi_1^2} + 8\frac{\partial^8}{\partial \xi_0^7 \partial \xi_2} + 2\frac{\partial^6}{\partial \xi_0^4 \partial t_0 \partial t_1} + 6\frac{\partial^6}{\partial \xi_0^2 \partial \xi_1^2 \partial t_0^2} + 4\frac{\partial^6}{\partial \xi_0^3 \partial \xi_2 \partial t_0^2}$$

$$-\frac{1}{2}\chi''(0)\xi_1^2\left(\frac{\partial^8}{\partial \xi_0^8} + \frac{\partial^6}{\partial \xi_0^4 \partial t_0^2} - \frac{\partial^4}{\partial s^4}\right) - \eta \cos \Omega t \left(\frac{\partial^8}{\partial \xi_0^8} + \frac{\partial^4}{\partial s^4}\right)$$

3.1 Zeroth- and First-order Approximations

In the zeroth-order approximation ($k = 0$) we have homogeneous differential equations. Their solution may be written in the form

$$W_{0,n} = \left[Z_{c,n}^{(0)}(\xi_j, t_j)\cos \omega_n t_0 + Z_{s,n}^{(0)}(\xi_j, t_j)\sin \omega_n t_0\right] e^{i p_n \xi_0} y_n(s) \tag{16}$$

under the condition

$$\omega_n = f(p_n) = \sqrt{p_n^4 + \frac{\lambda_n}{p_n^4}} \qquad n = 1, 2, \ldots \tag{17}$$

where $j \geq 1$, $i = \sqrt{-1}$, and λ_n and $y_n(s)$ are eigenvalues and eigenfunctions, respectively, of the boundary-value problem

$$y^{\mathrm{VI}} - \lambda y = 0 \tag{18}$$

$$y'' = y''' = 0 \qquad \text{for } s = 0, l$$

The minimization of the function $f(p_n)$ yields

$$\omega_n^0 = \min f(p_n) = f(p_n^0) = 2^{1/2}\lambda_n^{1/4} \qquad p_n^0 = \lambda_n^{1/8} \tag{19}$$

Here $\omega = \omega_n^0 + O(\varepsilon^2)$ is the approximate value of the dimensionless fundamental frequency of free vibrations of the circular cylindrical shell with radius $R_2 = R/\chi(0) = R$.

Taking into account equation (19), $D_1 W_{0,n} \equiv 0$, and for $k = 1$ we obtain the homogeneous equation again. The solution of this equation may be also found in form (16), where the functions $Z_{c,n}^{(0)}$, $Z_{s,n}^{(0)}$ are substituted by $Z_{c,n}^{(1)}$, $Z_{s,n}^{(1)}$, respectively.

3.2 Second-order Approximation

We will introduce a parameter σ of detuning for the frequency of temperature fluctuation as follows

$$\Omega = 2\omega_n^0 + \varepsilon\sigma \qquad \sigma \sim 1 \tag{20}$$

Taking into account equations (16), (19), (20), in the second-order approximation $(k = 2)$, equation (13) has the form

$$D_0 W_2 = -16\lambda_n^{3/4} [N_c \cos(\omega_n^0 t_0) + N_s \sin(\omega_n^0 t_0)] \, y_n(s) - \tag{21}$$

$$- \lambda_n [\cos(3\omega_n^0 t_0 + \sigma t_1) Z_{c,n}^{(0)} + \sin(3\omega_n^0 t_0 + \sigma t_1) Z_{s,n}^{(0)}] \, y_n(s)$$

where

$$N_c = \frac{\partial^2 Z_{c,n}^{(0)}}{\partial \xi_1^2} - c_1 \xi_1^2 Z_{c,n}^{(0)} + c_2 \left[\cos(\sigma t_1) Z_{c,n}^{(0)} - \sin(\sigma t_1) Z_{s,n}^{(0)} \right] - c_3 \frac{\partial Z_{s,n}^{(0)}}{\partial t_1}$$

$$N_s = \frac{\partial^2 Z_{s,n}^{(0)}}{\partial \xi_1^2} - c_1 \xi_1^2 Z_{s,n}^{(0)} - c_2 \left[\sin(\sigma t_1) Z_{c,n}^{(0)} + \cos(\sigma t_1) Z_{s,n}^{(0)} \right] + c_3 \frac{\partial Z_{c,n}^{(0)}}{\partial t_1}$$

$$c_1 = \lambda_n^{1/4} \chi''(0)/16 \qquad c_2 = \lambda_n^{1/4} \eta/16 \qquad c_3 = 2^{-5/2}$$

The first term on the right side of equation (21) generates secular terms. The absence condition of these terms gives the equations

$$N_c = 0 \qquad N_s = 0 \tag{22}$$

with respect to $Z_{c,n}^{(0)}$, $Z_{s,n}^{(0)}$. The solution of the foregoing equations may be found in the form

$$Z_{c,n}^{(0)} = P_c(\xi_1, \ldots; t_1, \ldots) \, e^{-\frac{1}{2} b(t_1) \xi_1^2} \qquad Z_{s,n}^{(0)} = P_s(\xi_1, \ldots; t_1, \ldots) \, e^{-\frac{1}{2} b(t_1) \xi_1^2} \tag{23}$$

where

$$P_c = \sum_{j=0}^{m} A_{c,j}(\xi_2, \ldots; t_1, \ldots) \, \xi_1^j \qquad P_s = \sum_{j=0}^{m} A_{s,j}(\xi_2, \ldots; t_1, \ldots) \, \xi_1^j \tag{24}$$

We substitute equations (23) and (24) into conditions (22) and equate coefficients at ξ_1^j. For $j = m + 2, m + 1$ a system of algebraic equations is obtained as follows

$$\left(c_1 - b^2\right) A_{c,j-2} - 2^{-7/2}\, \dot{b}\, A_{s,j-2} = 0 \tag{25}$$

$$2^{-7/2}\, \dot{b}\, A_{c,j-2} - \left(c_1 - b^2\right) A_{s,j-2} = 0$$

which has a nontrivial solution if

$$b = c_1^{1/2} = \tfrac{1}{4} \lambda_n^{1/8} \sqrt{\chi''(0)} \tag{26}$$

Coefficients at ξ_1^m and ξ_1^{m-1} yield two identical homogeneous systems of differential equations

$$\dot{\mathbf{Y}}_j - \mathbf{A}_j\,(t_1)\,\mathbf{Y}_j = 0 \qquad\qquad j = m,\, m - 1 \tag{27}$$

where

$$\mathbf{Y}_j = (\, A_{s,j},\, A_{c,j}\,)^{\mathrm{T}}$$

$$\mathbf{A}_j\,(t_1) = \begin{pmatrix} -a_1 \sin \sigma t_1 & a_1 \cos \sigma t_1 - a_{2,j} \\ a_1 \cos \sigma t_1 + a_{2,j} & a_1 \sin \sigma t_1 \end{pmatrix}$$

$$a_1 = c_2 / c_3 = 2^{-3/2}\, \lambda_n^{1/4}\, \eta \qquad\qquad a_{2,j} = (1 + 2j) b / c_3 = (1 + 2j)\, \lambda_n^{1/8} \sqrt{2\chi''(0)} \tag{28}$$

The mark "T" means a transposition.

The coefficients at ξ_1^{m-2}, ξ_1^{m-3}, ... produce the nonhomogeneous system of differential equations

$$\dot{\mathbf{Y}}_j - \mathbf{A}_j\,(t_1)\,\mathbf{Y}_j = (1 + j)(2 + j)\; c_3^{-1}\, \mathbf{E}\, \mathbf{Y}_{j+2} \qquad\qquad j = m - 2,\, m - 3, \ldots, 0 \tag{29}$$

Here $\mathbf{E}$ is a matrix with the elements $e_{11} = e_{22} = 0$, $e_{12} = 1$, $e_{21} = -1$

Examining equation (13) for $k \geq 3$, one can analogously construct the functions $W_{1,n}$, $W_{2,n}$, ...

4 Regions of Stability and Instability

Thus, relations (6) and (20) being valid, the shell normal deflection is defined by the approximate formula

$$W^*_{nm} = CR \left\{ \sum_{j=0}^{m} [A_{c,j}(\varepsilon t)\cos(\omega_n^0 t) + A_{s,j}(\varepsilon t)\sin(\omega_n^0 t)] (\varepsilon^{-1/2}\varphi)^j \times \right.$$

$$\left. \times \exp[\varepsilon^{-1}(i p_n \varphi - 0{,}5\varepsilon^{-1/2} b \varphi^2)]\, y_n(s) + O(\varepsilon^{1/2}) \right\} \quad (30)$$

It should be noted that $b > 0$. Therefore, excited vibrations are concentrated in a small neighbourhood of the generatrix $\varphi = 0$.

If the temperature T is constant ($\eta = 0$), then systems (27) and (29) can be integrated in closed form. In this case, function (30) are the modes of free vibrations (Tovstik, 1983) with the fundamental frequency

$$\omega^*_{nj} = \sqrt{\frac{\varepsilon^4 E_s}{\rho R^2}} \left[\omega_n^0 + \varepsilon \omega_{nj}^{(1)} + O(\varepsilon^2)\right] \qquad j = 0, 1, \dots, m$$

where $\omega_{nj}^{(1)} = a_{2,j}$, and a_{2j} and ω_n^0 are calculated by formulas (19) and (28).

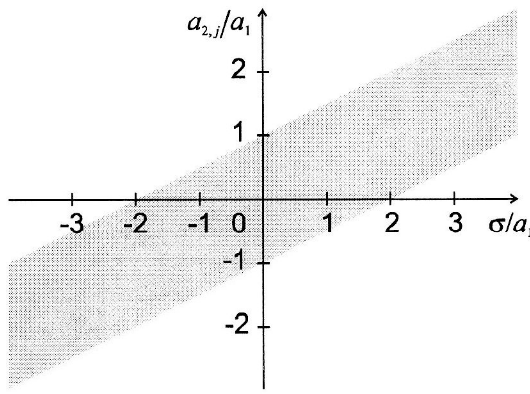

Figure 1. The Main Region of Instability

In the case of pulsating temperature ($\eta \neq 0$), formula (30) defines the unstable or stable vibrations. In Figure 1 the shadowed area is the main unstable region, i.e., points (σ/a_1, $a_{2,j}/a_1$) in this region yield unstable solutions for equations (27) and (29): $|\mathbf{Y}_j| \to \infty$ for $t \to \infty$. Otherwise, in the region outside the shadowed area, solutions of equations (27) and (29) are bounded. If the shell is circular ($a_{2,j} = 0$), vibrations are unstable for $-2 \le \sigma/a_1 \le 2$, however, a variable curvature displaces the interval of instability. It should be noted, that for a noncircular cylindrical shell, two the nearest modes W^*_{nm} and W^*_{nm+1} may be stable (or bounded) and unstable, simultaneously.

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