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The Effect of the Resistance of Lines of Electric Transmission on the Stability of the System "Synchronous Generator - Synchronous Motor"

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A formula for the calculation of the critical resistance of the lines of electric transmission between a synchronous generator and a synchronous motor is obtained. Above the critical resistance the system loses its stability. In real electric transmissions the resistance found by the formula is increased proportionally to the large coefficient for voltage transform. Therefore there is essentially less concern for the problem of the length of the lines of electric transmission. But if the resistance is counted as R-load, then the practical importance of the paper becomes obvious.

1 Introduction

In a paper by Rodyukov (1993) when splitting the correct equations for synchronous motors it is shown that in the equation for the moments there appears a destabilization term. This term is decisive in the study of the stability of synchronous motors without damping circuits and cannot be neglected. In the same paper it is concluded that for synchronous motors with damping circuits this term can be neglected. The results of our paper on the resistance of the line of electric transmission (or simply R-load) on the stability of the system one synchronous generator (SG) - one synchronous motor (SM), forces us to state that the destabilization term can be neglected in the correct split equations for synchronous machines only if the R-load is neglibly small. But in this case it is necessary to compare the real R-load with the critical value given by the formula which is the aim of this paper.

2 Synchronous Generator and Synchronous Motor

The equations coupling a system of SG and SM with commensurable power are taken from the paper by Eirola et al. (1996). The resistance ε_l of the line of electric transmission we include as a drop in the voltage on that line in the first two equations for the SM.

$$
\begin{aligned}
\dot{\psi}_{d1} &= \psi_{q1} - \alpha_s \psi_{d1} - u_d \\
\dot{\psi}_{q1} &= -\psi_{d1} - \alpha_s \psi_{q1} - u_q \\
\mu_1 \dot{i}_{d1} &= \dot{\psi}_{d1} + \varepsilon_{rf1}(\psi_{d1} - i_{d1} - u_{f1}) \\
\mu_1 \dot{i}_{q1} &= \dot{\psi}_{q1} + \varepsilon_{rf1}(\psi_{q1} - i_{q1}) \\
\dot{\theta}_1 &= s_1 \\
\dot{s}_1 &= -\delta_1[-k\varepsilon_s(b_1^2 + u_{f1}^2)s - (\psi_{d1} i_{q1} - \psi_{q1} i_{d1}) - M_r]
\end{aligned}
\qquad (1)
$$

$$
\begin{aligned}
&\dot{\psi}_{d2} = \psi_{q2} - \alpha_s\psi_{d2} - \varepsilon_l i_{d2} + u_d \\
&\dot{\psi}_{q2} = -\psi_{d2} - \alpha_s\psi_{q2} - \varepsilon_l i_{q2} + u_q \\
&\mu_2\dot{i}_{d2} = \dot{\psi}_{d2} - s(\psi_{q2} - \mu_2 i_{q2}) + \varepsilon_{rf2}(\psi_{d2} - i_{d2} - u_{f2}\cos\theta) \\
&\mu_2\dot{i}_{q2} = \dot{\psi}_{q2} + s(\psi_{d2} - \mu_2 i_{d2}) + \varepsilon_{rf2}(\psi_{q2} - i_{q2} + u_{f2}\sin\theta) \\
&\dot{\theta}_2 = s_2 \\
&\dot{s}_2 = \delta_2(\varepsilon_s(b_2^2 + u_{f2}^2)s - (\psi_{d2}i_{q2} - \psi_{q2}i_{d2}) - M_m)
\end{aligned}
\tag{2}
$$

where $s = s_1 - s_2$, $\theta = \theta_2 - \theta_1$, $k = L_{s1}/L_{s2}$. The power coefficient is included in the equation of the relations between the currents i_{d1}, i_{d2} and i_{q1}, i_{q2} by the first law of Kirchhoff, if the currents are in nondimensional form.

$$
i_{d1} = ki_{d2} \qquad i_{q1} = ki_{q2} \tag{3}
$$

To simplify cumbersome calculations we set $\mu_1 = \mu_2 = \mu$, $\varepsilon_{rf1} = \varepsilon_{rf2} = \varepsilon_{rf}$, $u_{f1} = u_{f2} = u_f$, $b_1 = b_2 = b$, $(b = (1-\mu)/\mu)$. Moreover we will neglect terms with coefficients α_s $(\alpha_s \sim 10^{-2} \ll 1)$. These simplifications do not affect the final conclusions but the calculations are considerably reduced.

We use the equations (3) to find the unknown voltages u_d and u_q.

$$
\begin{aligned}
&(1+k)u_d = \psi_{q1} + \varepsilon_{rf}(\psi_{d1} - u_f) - k[\psi_{q2} - \varepsilon_l i_{d2} - s(\psi_{q2} - \mu i_{q2}) + \varepsilon_{rf}(\psi_{d2} - u_f\cos\theta)] \\
&(1+k)u_q = -\psi_{d1} + \varepsilon_{rf}\psi_{q1} - k[-\psi_{d2} - \varepsilon_l i_{q2} + s(\psi_{d2} - \mu i_{d2}) + \varepsilon_{rf}(\psi_{q2} + u_f\sin\theta)]
\end{aligned}
\tag{4}
$$

We substitute the expression for u_d and u_q into the system (1) and (2) and eliminate the equations for the currents for SG. Passing to the mechanical variables s and θ the order is again decreased by two.

Before we write the final form for the system (1) and (2) we find the expressions for M_r and M_m from the steady-state regime $(s_1 = 0, s_2 = 0, \theta = \theta_0)$.

$$
M_r = ku_f^2[\varepsilon_l(1+\cos\theta_0) - (1+k)\sin\theta_0]/[(1+k)^2 + \varepsilon_l^2] \tag{5}
$$

$$
M_m = -u_f^2[\varepsilon_l(1+\cos\theta_0) + (1+k)\sin\theta_0]/[(1+k)^2 + \varepsilon_l^2] \tag{6}
$$

From equations (5) and (6) we find the relation between M_r and M_m.

$$
M_r = kM_m + 2ku_f^2\varepsilon_l(1+\cos\theta_0)/[(1+k)^2 + \varepsilon_l^2] \tag{7}
$$

Because we are interested only in the static stability of the systems (1) and (2) we use equation (7) when we form the equation for the derivative of s.

Then the final form of the systems (1) and (2) will be

$$
\begin{aligned}
&\dot{\psi}_{d2} = \tfrac{1}{1+k}\{\dot{\psi}_{q1}+\dot{\psi}_{q2}+\varepsilon_{rf}(\psi_{d1}-u_f)-k[-s(\psi_{q2}-\mu i_{q2})+\varepsilon_{rf}(\psi_{d2}-u_f\cos\theta)]\} \\
&\dot{\psi}_{q2} = \tfrac{1}{1+k}\{-(\dot{\psi}_{d1}+\dot{\psi}_{d2})+\varepsilon_{rf}\psi_{q1}-k[s(\psi_{d2}-\mu i_{d2})+\varepsilon_{rf}(\psi_{q2}+u_f\sin\theta)]\} \\
&\dot{\psi}_{d1} = -\dot{\psi}_{d2}+\psi_{q1}+\psi_{q2}-\varepsilon_l i_{d2} \\
&\dot{\psi}_{q1} = -\dot{\psi}_{q2}-(\psi_{d1}+\psi_{d2})-\varepsilon_l i_{q2} \\
&\mu\dot{i}_{d2} = \dot{\psi}_{d2}-s(\psi_{q2}-\mu i_{q2})+\varepsilon_{rf}(\psi_{d2}-i_{d2}-u_f\cos\theta) \\
&\mu\dot{i}_{q2} = \dot{\psi}_{q2}+s(\psi_{d2}-\mu i_{d2})+\varepsilon_{rf}(\psi_{q2}-i_{q2}+u_f\sin\theta) \\
&\dot{\theta} = s \\
&\dot{s} = \delta\varepsilon_s(b^2+u_f^2)s-[\delta_2(\psi_{d2}i_{q2}-\psi_{q2}i_{d2})-k\delta_1(\psi_{d1}i_{q2}-\psi_{q1}i_{d2})]+\delta M_m \\
&\qquad\qquad +2k\delta_1 u_f^2\varepsilon_l(1+\cos\theta)/[(1+k)^2+\varepsilon_l^2]
\end{aligned}
\tag{8}
$$

where $\delta = \delta_2 + k\delta_1$.

3 Static Stability

To study the static stability of the equilibrium we proceed as in Eirola et al. (1996), that is, we set the derivatives of the electric variables equal to zero, solve these equations with respect to mechanical variables θ and s and substitute these solutions into the mechanical subsystem of the system (8).

To simplify the calculations we use the relations between the fluxes $\psi_{d2}, \psi_{q2}, \psi_{d1}$ and ψ_{q1} and the currents i_{d2}, i_{q2}. From the third and fourth of equations (8) we get expressions for ψ_{d1} and ψ_{q1}. Using these in the first and second equations we get expression in currents for ψ_{d2} and ψ_{q2}.

$$
\begin{aligned}
&\psi_{d2} = [(\varepsilon_{rf}^2+\mu s^2)i_{d2}+\varepsilon_{rf}s(1-\mu)i_{q2}+\varepsilon_{rf}u_f(\varepsilon_{rf}\cos\theta-s\sin\theta)]/(\varepsilon_{rf}^2+s^2) \\
&\psi_{q2} = [(-\varepsilon_{rf}s(1-\mu)i_{d2}+(\varepsilon_{rf}^2+\mu s^2)i_{q2}-\varepsilon_{rf}u_f(s\cos\theta-\varepsilon_{rf}\sin\theta)]/(\varepsilon_{rf}^2+s^2) \\
&\psi_{d1} = -\psi_{d2}-\varepsilon_l i_{q2} \\
&\psi_{q1} = -\psi_{q2}+\varepsilon_l i_{d2}
\end{aligned}
\tag{9}
$$

The currents i_{d2} and i_{q2} can then be solved from the fifth and sixth equation.

$$
\begin{aligned}
&i_{d2} = -u_f\frac{\varepsilon_{rf}^2(1+k)+s^2(k+\mu)+\varepsilon_{rf}\{[\varepsilon_{rf}(1+k)+\varepsilon_l s]\cos\theta-[s(k+\mu)-\varepsilon_l\varepsilon_{rf}]\sin\theta\}}{\varepsilon_{rf}^2[(1+k)^2+\varepsilon_l^2]+s^2[(k+\mu)^2+\varepsilon_l^2]+2\varepsilon_l\varepsilon_{rf}s(1-\mu)} \\
&i_{q2} = u_f\frac{-\varepsilon_{rf}s(1-\mu)-\varepsilon_l(\varepsilon_{rf}^2+s^2)+\varepsilon_{rf}\{[s(k+\mu)-\varepsilon_l\varepsilon_{rf}]\cos\theta+[\varepsilon_{rf}(1+k)+\varepsilon_l s]\sin\theta\}}{\varepsilon_{rf}^2[(1+k)^2+\varepsilon_l^2]+s^2[(k+\mu)^2+\varepsilon_l^2]+2\varepsilon_l\varepsilon_{rf}s(1-\mu)}
\end{aligned}
\tag{10}
$$

Substituting equations (9) and (10) into the last one of equations (8) we get the following equations for analyzing the stability of the electroenergetic system:

$$
\begin{aligned}
\dot{\theta} &= s \\
\dot{s} &= \delta\varepsilon_s(b^2 + u_f^2)s \\
&\quad - u_f^2 \frac{\delta\varepsilon_{rf}[s(1-\mu)-s(k+\mu)\cos\theta-\varepsilon_{rf}(1+k)\sin\theta]-(\delta_2-k\delta_1)\varepsilon_l\varepsilon_{rf}[\varepsilon_{rf}(1+\cos\theta)-s\sin\theta]+k\delta_1\varepsilon_l s^2}{\varepsilon_{rf}^2[(1+k)^2+\varepsilon_l^2]+s^2[(k+\mu)^2+\varepsilon_l^2]+2\varepsilon_l\varepsilon_{rf}s(1-\mu)} \\
&\quad + \delta M_m + 2k\delta_1 u_f^2 \frac{\varepsilon_l(1+\cos\theta)}{(1+k)^2+\varepsilon_l^2}
\end{aligned}
\tag{11}
$$

In the steady-state regime ($s = 0$ and $\theta = \theta_0$) the system (11) gives the following equation for determining the equilibrium of the original system (8):

$$
M_m = -u_f^2 \frac{(1+k)\sin\theta_0 + \varepsilon_l(1+\cos\theta_0)}{(1+k)^2+\varepsilon_l^2} \tag{12}
$$

We write equation (12) in the form

$$
M_m + u_f^2 \frac{\varepsilon_l}{(1+k)^2+\varepsilon_l^2} = -u_f^2 \frac{\sin(\theta_0+\kappa)}{\sqrt{(1+k)^2+\varepsilon_l^2}} \tag{13}
$$

where

$$
\sin\kappa = \varepsilon_l / \sqrt{(1+k)^2+\varepsilon_l^2} \tag{14}
$$

The first necessary condition for stability follows from equation (13).

$$
\sin(\theta_0+\kappa) \le 0 \tag{15}
$$

Two main equilibria correspond to the condition (15) for $\pi \le \theta_0 + \kappa \le 2\pi$. To determine the stable one the system (11) is linearized in the neighbourhood of the equilibrium ($s = 0, \theta = \theta_0$) and the stability conditions supplementary to equation (14) on coefficients of the second order characteristic equation are analyzed.

System (11) linearized around the equilibrium becomes

$$
\begin{aligned}
\dot{\tilde{\theta}} &= \tilde{s} \\
\dot{\tilde{s}} &= \delta\varepsilon_s(b^2 + u_f^2)\tilde{s} \\
&\quad - u_f^2 \frac{[(1+k)^2+\varepsilon_l^2]\{\delta[1-\mu-(k-\mu)\cos\theta_0]+(\delta_2-k\delta_1)\varepsilon_l\sin\theta_0\}+2\varepsilon_l(1-\mu)[\delta(1+k)\sin\theta_0+(\delta_2-k\delta_1)\varepsilon_l(1+\cos\theta_0)]}{\varepsilon_{rf}[(1+k)^2+\varepsilon_l^2]^2}\tilde{s} \\
&\quad + u_f^2 \frac{\delta(1+k)\cos\theta_0-(\delta_2-k\delta_1)\varepsilon_l\sin\theta_0}{(1+k)^2+\varepsilon_l^2}\tilde{\theta}
\end{aligned}
\tag{16}
$$

The characteristic equation of equations (16) can be written

$$
\lambda^2 + a_1\lambda + a_2 = 0
$$

The free term a_2 has the following form:

$$
a_2 = -u_f^2 \frac{\sqrt{\delta^2(1+k)^2+\varepsilon_l^2(\delta_2-k\delta_1)^2}}{(1+k)^2+\varepsilon_l^2}\cos(\theta_0+\kappa_1) \tag{17}
$$

where

$$\sin\kappa_1 = \frac{\varepsilon_l(\delta_2 - k\delta_1)}{\sqrt{\delta^2(1+k)^2 + \varepsilon_l^2(\delta_2 - k\delta_1)^2}} \tag{18}$$

From equation (17) using the necessary stability condition $a_2 > 0$ it follows that

$$\cos(\theta_0 + \kappa_1) < 0 \tag{19}$$

The coefficient a_1 we represent in the form

$$\begin{aligned} a_1 = & -\delta\varepsilon_s(b^2 + u_f^2) - u_f^2\cos(\theta_0 + \kappa_2) \\ & \times \frac{\sqrt{\{\delta(k+\mu)[(1+k)^2+\varepsilon_l^2]-2\varepsilon_l^2(1-\mu)(\delta_2-k\delta_1)\}^2+\{(\delta_2-k\delta_1)[(1+k)^2+\varepsilon_l^2]+2(1-\mu)\delta(1+k)\}^2\varepsilon_l^2}}{\varepsilon_{rf}[(1+k)^2+\varepsilon_l^2]^2} \\ & + u_f^2(1-\mu)\frac{\delta(1+k)^2 + \varepsilon_l^2(3\delta_2 - k\delta_1)}{\varepsilon_{rf}[(1+k)^2 + \varepsilon_l^2]^2} \end{aligned} \tag{20}$$

where

$$\sin\kappa_2 = \frac{\varepsilon_l\{(\delta_2-k\delta_1)[(1+k)^2+\varepsilon_l^2]+2(1-\mu)\delta(1+k)\}}{\sqrt{\{\delta(k+\mu)[(1+k)^2+\varepsilon_l^2]-2\varepsilon_l^2(1-\mu)(\delta_2-k\delta_1)\}^2+\{(\delta_2-k\delta_1)[(1+k)^2+\varepsilon_l^2]+2(1-\mu)\delta(1+k)\}^2\varepsilon_l^2}} \tag{21}$$

The value of ε_l increases with the length of the line of electric transmission, but the third term in equation (20) giving the main contribution to the stability of the system, decreases, becoming for some value of ε_l equal to the destabilization term (20). Obviously, this value of ε_l is large enough. Assuming therefore that $\varepsilon_l^2 \gg 1$ we follow the role of the second term in (20) in the process considered.

For values $\varepsilon_l^2 \gg 1$ it follows from equations (14), (18) and (21) that

$$\sin\kappa_2 \to \sin\kappa_1 \to \sin\kappa \to 1$$

but

$$\kappa_2 \to \kappa_1 \to \kappa \to \pi/2$$

But according to equation (19) the term $\cos(\theta_0 + \kappa)$ must be negative. This means that the second term in equation (20) helps the third term to maintain stability. The critical situation arises when $\cos(\theta_0 + \kappa)$ approaches its maximal critical value, that is, zero. From this it is clear that the critical value of ε_l can be found from $\cos(\theta_0 + \kappa) = a_1 = 0$. Under the condition $\varepsilon_l^2 \gg 1$ this leads us to the following formula for calculation of $(\varepsilon_l)_{cr}$:

$$(\varepsilon_l)_{cr} = u_f\sqrt{\frac{(1-\mu)(3\delta_2 - k\delta_1)}{\varepsilon_s\varepsilon_{rf}(b^2 + u_f^2)(\delta_2 + k\delta_1)}} \tag{22}$$

If the power of the SM is much less than the power of the SG then $k \ll 1$. Thus when the SM works from a grid of infinite power ($k = 0$) we get from equation (22)

$$(\varepsilon_l)_{cr|_{k=0}} = u_f\sqrt{\frac{3(1-\mu)}{\varepsilon_s\varepsilon_{rf}(b^2 + u_f^2)}} \tag{23}$$

When the powers of SG and SM are comparable ($k \sim 1$), ε_l tends to the value

$$(\varepsilon_l)_{cr|_{k=1,\delta_1=\delta_2}} = u_f\sqrt{\frac{1-\mu}{\varepsilon_s\varepsilon_{rf}(b^2 + u_f^2)}} \tag{24}$$

Thus accounting for the parameters of the SG in the electroenergetic system decreases the critical value for the resistance ε_l up to $\sqrt{3}$ times.

The order of the parameters in modern synchronous machines are

$$\varepsilon_s \sim 10^{-3} \qquad \varepsilon_{rf} \sim 10^{-3}$$

Because $u_f \approx 1$ and $b^2 = (1-\mu)^2/\mu^2 \sim 10$ we get the order

$$(\varepsilon_l)_{cr} \sim 10^3$$

For the value of $(\varepsilon_l)_{cr}$ it is possible to find the corresponding maximal length of the line of electric transmission. But as was remarked at the beginning of the paper there is no transform of electroenergy when it is transmitted long distances and the formula (23) has no practical actuality. But if in series with an SM there is included an R-load then its effect on the stability of the SM must be taken into account either according to formula (23) (for autonomous energosystems) or according to formula (24).

Remark. $(\varepsilon_l)_{cr}$ corresponds to the case $M_m = 0$ in the work of the SM. This is obvious from equation (13) because when $\varepsilon_l \to \infty$ then $M_m \to 0$. The SM can work only for $\varepsilon_l < (\varepsilon_l)_{cr}$.

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