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A Frequency-Domain Criterion for Global Stability of Systems with Angular Coordinates

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An integro-differential Volterra equation with a periodic nonlinear function which has a zero mean value over a period is considered and the asymptotic behaviour of its solution is investigated. With this aim a frequency-domain stability condition is proved. Finally, this condition is applied to a special pendulum-like system.

1 Introduction

In this paper we consider an integro-differential Volterra equation with a periodic nonlinear function. Equations of this type describe the dynamics of certain mechanical and other engineering systems. They describe for example the dynamics of phase-locked loops (Lindsey, 1972; Vitterbi, 1966) and the helical movements of the tip of growing plants (Israelsson and Johnsson, 1967; Somolinos, 1978). They can also be used for the description of the motion of a mathematical pendulum and the dynamics of synchronous machines (Adkins, 1962; Yanko-Trinitskij, 1958). The most important problem connected with these equations is the problem of asymptotic behaviour of its solutions as the argument time tends to infinity. The desired mode for the mechanical system corresponds as a rule to the situation when every solution of the Volterra equation tends to a certain constant. We shall regard this case as a stable one in the sense of gradient-like behaviour (Leonov and Smirnova, 1996). A number of theorems with sufficient conditions of stability and instability of integro-differential Volterra equations with periodic nonlinear functions is contained in the books (Leonov et al., 1992; Leonov et al., 1996). All the theorems are formulated in terms of transfer functions of the linear part of the system, in the form of frequency-domain inequalities.

This paper deals with a particular case of such an equation, namely with the case when the nonlinear function has a zero mean value over one period. The aim of the paper is to simplify frequency-domain conditions of stability in this case. We are also going to show how a frequency-domain criterion can be applied to a concrete mechanical system.

2 Frequency-Domain Criterion for Global Stability

We consider the integro-differential equation

$$\dot{\sigma}(t) = \sigma_0(t) - \int_0^t \gamma(t-\tau)\,\varphi\big(\sigma(\tau)\big)\,d\tau \tag{1}$$

where $\varphi : \mathbf{R} \to \mathbf{R}$ is a Lipschitz continuous, Δ-periodic function with $\int_0^\Delta \varphi(\sigma)\,d\sigma = 0$, $\sigma_0 : \mathbf{R}^+ \to \mathbf{R}$ is continuous and bounded, $\gamma : \mathbf{R}^+ \to \mathbf{R}$ is $L_1[0,+\infty)$. Under these requirements for σ_0, γ, φ equation (1) has a unique solution on $[0,+\infty)$ for any initial data

$$\sigma(0) = \sigma^0 \tag{2}$$

Let us introduce a transfer function

$$K(p) = \int_0^\infty \gamma(t)\,e^{-pt}dt \qquad (p = \tau + i\omega,\ \ i^2 = -1)$$

Let us also make additional assumptions on σ_0 and γ.

p1) $\sigma_0(t) \to 0$ as $t \to \infty$ and there is a number $r_1 > 0$ such that $t \mapsto \sigma_0(t)e^{r_1 t}$ is $L_2(0, +\infty)$

p2) there is a number $r_2 > 0$ such that the function $t \mapsto \gamma(t)e^{r_2 t}$ is $L_2(0, +\infty)$

p3) φ is $\mathbf{C^1}$ and has a finite number of zeros on $[0, \Delta)$ and $[\varphi'(\sigma)]^2 + [\varphi(\sigma)]^2 \neq 0$

Theorem. *Suppose there exist numbers $\varepsilon > 0$, $\kappa \geq 0$, $\mu_1 \leq \inf\limits_{\sigma\in[0,\Delta)} \varphi'(\sigma)$, $\mu_2 \geq \sup\limits_{\sigma\in[0,\Delta)} \varphi'(\sigma)$ with $\mu_1 < 0$ and $\mu_2 > 0$ such that for all $\omega \in \mathbf{R}$ the following inequality is fulfilled:*

$$Re\left\{K(i\omega) - \kappa[K(i\omega) + \mu_1^{-1}i\omega]^*[K(i\omega) + \mu_2^{-1}i\omega]\right\} - \varepsilon\left|K(i\omega)\right|^2 \geq 0 \tag{3}$$

Then the following relations take place

$$\dot\sigma(t) \in L_2[0, +\infty) \tag{4}$$

$$\dot\sigma(t) \to 0 \qquad as \qquad t \to +\infty \tag{5}$$

If in addition $K(0) \neq 0$ then

$$\sigma(t) \to q \qquad as \quad t \to +\infty, \qquad where\ \varphi(q) = 0 \tag{6}$$

Proof. Let $\sigma(t)$ be an arbitrary solution of equation (1). Let us define the following functions

$$\eta(t) := \varphi(\sigma(t))$$

$$\mu(t) := \begin{cases} 0, & t < 0 \\ t, & t \in [0,1] \\ 1, & t > 1 \end{cases}$$

$$\eta_\mu(t) := \mu(t)\,\eta(t) \qquad\qquad \zeta_\mu(t) := -\int_0^t \gamma(t-\tau)\eta_\mu(\tau)\,d\tau$$

$$\bar\sigma(t) := \sigma_0(t) - \int_0^t \gamma(t-\tau)\big(1-\mu(\tau)\big)\eta(\tau)\,d\tau$$

Note that

$$\dot\sigma(t) = \bar\sigma(t) + \zeta_\mu(t)$$

Furthermore for an arbitrary $T > 1$ we introduce the functions

$$\eta_T(t) := \begin{cases} \eta_\mu(t), & t \leq T \\ \eta(T)e^{-c(T-t)}\ (c > 0), & t > T \end{cases}$$

$$\zeta_T(t) := \begin{cases} 0, & t \leq 0 \\ -\int_0^t \gamma(t-\tau)\eta_T(\tau)\,d\tau, & t > 0 \end{cases}$$

It is clear that η_T, $\dot\eta_T$, ζ_T are $L_1(0,+\infty) \cap L_2(0,+\infty)$. So we can construct a functional

$$A_T := \int_0^T \left\{\zeta_T\eta_T + \varepsilon\zeta_T^2 + \kappa[\zeta_T - \mu_1^{-1}\dot\eta_T]\,[\zeta_T - \mu_2^{-1}\dot\eta_T]\right\} dt$$

The functions η_T, $\dot\eta_T$, ζ_T have the Fourier transforms $\widetilde{\eta}_T(i\omega)$, $\widetilde{\dot\eta}_T(i\omega)$, $\widetilde{\zeta}_T(i\omega)$ respectively which satisfy the relations

$$\widetilde{\zeta}_T(i\omega) = -K(i\omega)\widetilde{\eta}_T(i\omega) \qquad \textit{and} \qquad \widetilde{\dot\eta}_T(i\omega) = i\omega\widetilde{\eta}_T(i\omega) \tag{7}$$

By the Parseval equality we get

$$A_T = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \{ -\operatorname{Re} K(i\omega) + \varepsilon|K(i\omega)|^2 + \kappa \operatorname{Re}[K(i\omega) + \mu_1^{-1}i\omega]^* \cdot [K(i\omega)) + \mu_2^{-1}i\omega]\} \left|\widetilde{\eta}_T(i\omega)\right|^2 d\omega$$

It follows from equation (3) that

$$A_T \le 0 \qquad \text{for any} \qquad T > 1 \tag{8}$$

Note that

$$\dot\sigma(t) = \bar\sigma(t) + \zeta_T(t) \qquad \text{for} \quad t \in [0, T] \tag{9}$$

Let us split the integral A_T in the following way:

$$A_T = \sum_{k=1}^{5} A_{Tk}$$

where

$$A_{T1} := \int_T^{+\infty} \{\zeta_T\eta_T + \kappa\dot\eta_T^2\mu_1^{-1}\mu_2^{-1} - \kappa(\mu_1^{-1} + \mu_2^{-1})\zeta_T\dot\eta_T\}\, dt$$

$$A_{T2} := \int_T^{+\infty} (\varepsilon + \kappa)\zeta_T^2\, dt$$

$$A_{T3} := \int_0^T \{\bar\sigma\eta_\mu + (\varepsilon + \kappa)(\bar\sigma)^2 - 2\varepsilon\bar\sigma\dot\sigma + \kappa(\mu_1^{-1} + \mu_2^{-1})\dot\eta_\mu\bar\sigma\}\, dt$$

$$A_{T4} := \int_0^1 \{(\mu - 1)\dot\sigma\eta + \kappa\mu_1^{-1}\mu_2^{-1}(\dot\eta_\mu^2 - \eta^2) - \kappa(\mu_1^{-1} + \mu_2^{-1})\dot\sigma\,(\dot\eta - \dot\eta_\mu)\}\, dt$$

$$A_{T5} := \int_0^T \{\dot\sigma\eta + \varepsilon\dot\sigma^2 + \kappa(\dot\sigma - \mu_1^{-1}\dot\eta)(\dot\sigma - \mu_2^{-1}\dot\eta)\}\, dt$$

It is clear that $|A_{T1}|$ and $|A_{T2}|$ are bounded by constants which do not depend on T and that $A_{T2} > 0$. Taking into consideration that $\bar\sigma(t)e^{r_0t} \in L_2[0, +\infty)$ where $r_0 = \min\{r_1, r_2\}$ we get that $|A_{T3}|$ is bounded by a constant which does not depend on T. So it follows from inequalities (8) that

$$A_{T5} < c_0 \tag{10}$$

where c_0 does not depend on T.

Since $\mu_1 \le \frac{d\varphi}{d\sigma} \le \mu_2$ we have

$$\left[\dot\sigma(t) - \mu_1^{-1}\frac{d\varphi(\sigma(t))}{dt}\right]\left[\dot\sigma(t) - \mu_2^{-1}\frac{d\varphi(\sigma(t))}{dt}\right] \ge 0$$

and consequently,

$$\int_0^T \{\dot\sigma(t)\varphi(\sigma(t)) + \varepsilon\dot\sigma^2(t)\}\, dt < c_1 \tag{11}$$

where c_1 does not depend on T.

Since the integral

$$\int_0^T \dot\sigma(t)\varphi(\sigma(t))\,dt = \int_{\sigma(0)}^{\sigma(T)} \varphi(\sigma)\,d\sigma$$

and $\int_0^\Delta \varphi(\sigma)\,d\sigma = 0$ we can affirm that this integral is also bounded by a constant independent of T. Then it follows from inequality (11) that the inclusion (4) is true.

Let us now demonstrate that the relation (5) is also true. For this purpose we shall show at first that $\dot\sigma(t)$ is uniformly continuous on $[0,+\infty)$. For $\sigma_0(t)$ this property follows from p1). So let us consider the function

$$\zeta(t) := \int_0^t \gamma(t-\tau)\eta(\tau)\,d\tau = \int_0^t \eta(t-\tau)\gamma(\tau)\,d\tau$$

We have

$$\left|\zeta(t+\Delta t)-\zeta(t)\right| \le \left|\int_t^{t+\Delta t} \eta(t+\Delta t-\tau)\gamma(\tau)\,d\tau\right| + \int_0^t \left|\eta(t+\Delta t-\tau)-\eta(t-\tau)\right|\left|\gamma(\tau)\right|d\tau$$

The function $\eta(t)$ is bounded and uniformly continuous on $[0,+\infty)$. Hence and from property p2) it follows that $\zeta(t)$ is uniformly continuous on $[0,+\infty)$.

According the Barbalat lemma (Popov, 1973) if a uniformly continuous function $f(t)$ on $[0,+\infty)$ is $L_2(0,+\infty)$ then it tends to 0 as $t\to+\infty$. So relation (4) implies relation (5).

Let us demonstrate now that relation (4) implies relation (6). We have from equation (1)

$$\begin{aligned}\dot\sigma(t)-\sigma_0(t) &= -\int_0^t \gamma(t-\tau)\eta(\tau)\,d\tau = -\int_0^t \left(\frac{d}{d\tau}\int_{t-\tau}^{\infty}\gamma(\lambda)\,d\lambda\right)\cdot\eta(\tau)\,d\tau \\ &= -\eta(t)\int_0^\infty \gamma(\lambda)\,d\lambda + \eta(0)\int_t^\infty \gamma(\lambda)\,d\lambda + \int_0^t\left\{\dot\eta(t)\int_{t-\tau}^{\infty}\gamma(\lambda)\,d\lambda\right\}d\tau\end{aligned} \tag{12}$$

The left side of equation (12) tends to 0 as $t\to+\infty$. Further, it follows from relation (4) that $\dot\eta\in L_2[0,+\infty)$ and it follows from p2) that $\int_t^{+\infty}\gamma(\lambda)\,d\lambda \in L_2[0,+\infty)$.

It is well known (Gelig, 1966) that a convolution of two functions which belong to $L_2[0,+\infty)$ tends to 0 as its argument goes to infinity. So

$$\int_0^t\left\{\dot\eta(\tau)\int_{t-\tau}^{+\infty}\gamma(\lambda)\,d\lambda\right\}dt \to 0 \qquad \text{as} \quad t\to+\infty$$

It follows also from p2) that $\int_t^{+\infty}\gamma(\lambda)\,d\lambda \to 0$ as $t\to+\infty$. Then since $\int_0^\infty \gamma(\lambda)\,d\lambda \ne 0$ we deduce from equation (12) that

$$\varphi(\sigma(t)) \to 0 \qquad \text{as} \quad t\to+\infty \tag{13}$$

Let now m_0 be the least distance between the zeros of φ on $[0,\Delta)$ and let $\delta\in\left(0,\frac{m_0}{2}\right)$. We introduce the set

$$M := \left\{\sigma \,|\, \sigma \notin \cup_k(\sigma_k-\delta,\sigma_k+\delta),\ \varphi(\sigma_k)=0\right\}$$

and the number

$$m := \min_{\sigma\in M}\left|\varphi(\sigma)\right|$$

Suppose $\varepsilon < m$. Then the inequality $|\varphi(\sigma)| < \varepsilon$ implies that σ belongs to the δ-neighborhood of a certain zero of the function φ. Let us denote it by q. It follows from equation (13) that there exists a number $T > 0$ such that for $t > T$ all values of $\sigma(t)$ belong to the δ-neighborhood of q. Since we can choose δ as small as desired it follows that relation (6) is true. The theorem is proved.

3 Example

Let us consider now the problem of self-synchronization of two rotors on a vibrator with one degree of freedom. The equations describing the change of the slowly variable components α_1, α_2 of the phases of the rotor motion are (Sperling et al., 1997)

$$\begin{aligned} I_1\ddot{\alpha}_1 + k_1\dot{\alpha}_1 + A\sin(\alpha_1 - \alpha_2) = 0 \\ I_2\ddot{\alpha}_2 + k_2\dot{\alpha}_2 - A\sin(\alpha_1 - \alpha_2) = 0 \end{aligned} \tag{14}$$

with

$$A = \frac{1}{2} f_1 f_2 A_{xx} \qquad A_{xx} = \frac{1}{M(\omega^2 - \Omega^2)}$$

where $I_i, k_i, f_i \quad (i = 1, 2)$ are the moments of inertia, the coefficients of viscosity, and the centrifugal forces of the rotors respectively, A_{xx} is the harmonic influence coefficient, M and ω are the mass and the natural frequency of the vibrator, and Ω is the synchronous angular velocity.

Our aim is to investigate the domain of attraction for the equilibria set of (14), which corresponds to all existing Lyapunov stable as well as Lyapunov unstable self-synchronized motions. Note that the equilibria set of (14) is as follows:

$$\{(\alpha_{1\,eq}, \alpha_{2\,eq}) : \alpha_{1\,eq} = \text{const}, \quad \alpha_{2\,eq} = \text{const}, \quad \alpha_{1\,eq} - \alpha_{2\,eq} = \pi n\}$$

Let us exchange each of equations (14) by the equivalent integro-differential equation

$$\dot{\alpha}_l(t) = \dot{\alpha}_l(0)e^{-\frac{k_l}{I_l}t} + \frac{(-1)^l A}{I_l} \int_0^t e^{-\frac{k_l}{I_l}(t-\tau)} \sin\big(\alpha_1(\tau) - \alpha_2(\tau)\big)\, d\tau \qquad (l = 1, 2) \tag{15}$$

Let us introduce the function $\sigma = \alpha_1 - \alpha_2$. It follows from equation (15) that $\sigma(t)$ satisfies

$$\dot{\sigma}(t) = \left(\dot{\alpha}_1(0)e^{-\frac{k_1}{I_1}t} - \dot{\alpha}_2(0)e^{-\frac{k_2}{I_2}t}\right) - A\int_0^t \left(\frac{1}{I_1}e^{-\frac{k_1}{I_1}(t-\tau)} + \frac{1}{I_2}e^{-\frac{k_2}{I_2}(t-\tau)}\right) \sin\big(\sigma(\tau)\big)\, d\tau \tag{16}$$

In order to investigate the asymptotics of a solution of equation (16) we apply the theorem of the preceding section. Note that all the requirements for the functions $\sigma_0(t)$, $\gamma(t)$ and $\varphi(\sigma)$ are fulfilled. So we must only verify that inequality (3) is true. We suppose that $A > 0$. In case $A < 0$ it is sufficient to replace in equation (16) A by $|A|$ and $\sin\big(\sigma(\tau)\big)$ by $\big(-\sin\big(\sigma(\tau)\big)\big)$, and all the conclusions of the section remain true.

In our case

$$K(p) = A\left(\frac{1}{I_1 p + k_1} + \frac{1}{I_2 p + k_2}\right)$$

Consequently

$$\operatorname{Re} K(i\omega) - \varepsilon|K(i\omega)|^2 = \frac{k_1(k_2^2 + I_2^2\omega^2) + k_2(k_1^2 + I_1^2\omega^2) - \varepsilon A[(k_1 + k_2)^2 + (I_1 + I_2)^2\omega^2]}{(k_1^2 + I_1^2\omega^2)(k_2^2 + I_2^2\omega^2)} A$$

It is evident that if $\kappa = 0$ and if for all ω

$$\varepsilon < \frac{1}{A}\frac{k_1(k_2^2 + I_2^2\omega^2) + k_2(k_1^2 + I_1^2\omega^2)}{(k_1 + k_2)^2 + (I_1 + I_2)^2\omega^2}$$

the frequency-domain inequality (3) is fulfilled. Consequently

$$\dot{\sigma}(t) \to 0 \qquad \text{as} \quad t \to +\infty \tag{17}$$

$$\sin\left(\sigma((t)\right) \to 0 \qquad \text{as} \quad t \to +\infty \tag{18}$$

$$\sigma(t) \to \pi n_0 \qquad \text{as} \quad t \to +\infty \tag{19}$$

where n_0 is a certain integer.

Let us now replace the integro-differential equation (16) by the equivalent integral equation

$$\begin{aligned}\sigma(t) = \sigma(0) + \int_0^t \Bigg\{ \dot\alpha_1(0) e^{-\frac{k_1}{I_1}\theta} - \dot\alpha_2(0) e^{-\frac{k_2}{I_2}\theta} \\ -A \int_0^\theta \left(\frac{1}{I_1} e^{-\frac{k_1}{I_1}(\theta-\tau)} + \frac{1}{I_2} e^{-\frac{k_2}{I_2}(\theta-\tau)} \right) \sin\sigma(\tau)\, d\tau \Bigg\} d\theta\end{aligned} \tag{20}$$

We can write equation (20) also in the following form

$$\sigma(t) = \widehat{\sigma}(t) - A \int_0^t \left\{ \sin\sigma(\tau) \int_\tau^t \left(\frac{1}{I_1} e^{-\frac{k_1}{I_1}(\theta-\tau)} + \frac{1}{I_2} e^{-\frac{k_2}{I_2}(\theta-\tau)} \right) d\theta \right\} d\tau \tag{21}$$

with

$$\widehat{\sigma}(t) = \sigma(0) + \frac{I_1\dot\alpha_1(0)}{k_1} - \frac{I_2\dot\alpha_2(0)}{k_2} + \frac{I_2\dot\alpha_2(0)}{k_2} e^{-\frac{k_2}{I_2}t} - \frac{I_1\dot\alpha_1(0)}{k_1} e^{-\frac{k_1}{I_1}t}$$

Finally let us rewrite equation (21) as

$$\sigma(t) = \widehat{\sigma}(t) - A\left(\frac{1}{k_1} + \frac{1}{k_2}\right) \int_0^t \sin\sigma(\tau)\, d\tau + A \int_0^t \left(\frac{1}{k_1} e^{-\frac{k_1}{I_1}(t-\tau)} + \frac{1}{k_2} e^{-\frac{k_2}{I_2}(t-\tau)} \right) \sin\sigma(\tau)\, d\tau \tag{22}$$

It is known (Leonov et al., 1996) that the convolution of a function from $L_1[0, +\infty)$ and a function which tends to 0 as its argument tends to infinity also tends to 0 as its argument tends to infinity. That is why the last term of the right side of equation (22) tends to 0 as $t \to +\infty$. Then since $\widehat{\sigma}(t)$ and $\sigma(t)$ have finite limits as $t \to +\infty$ it follows from equation (22) that

$$\lim_{t\to+\infty} \int_0^t \sin\sigma(\tau)\, d\tau = \text{const} \tag{23}$$

Let us revert to equation (15). It follows from equation (23) that

$$\dot\alpha_l(t) \to 0 \qquad \text{as} \quad t \to +\infty \qquad (l = 1, 2) \tag{24}$$

On the other hand we have from equation (15) that

$$\begin{aligned}\alpha_l(t) = \alpha_l(0) + \dot\alpha_l(0)\frac{I_l}{k_l}\left(1 - e^{-\frac{k_l}{I_l}t}\right) \\ + \frac{(-1)^l A}{k_l} \int_0^t \sin\sigma(\tau)\, d\tau - \frac{(-1)^l A}{k_l} \int_0^t \sin\left(\sigma(\tau)\right) e^{-\frac{k_l}{I_l}(t-\tau)}\, d\tau\end{aligned} \tag{25}$$

It follows from equations (23), (18) and (25) that

$$\alpha_l(t) \to \text{const} \qquad \text{as} \quad t \to +\infty \qquad (l = 1, 2) \tag{26}$$

The limit relations (26) and (19) mean that every solution of system (14) tends to a certain equilibrium as $t \to +\infty$. So the domain of attraction for the equilibria set (14) is the space $\mathbf{R}^2$. However, not every equilibrium is Lyapunov stable. The synchronized motion corresponding to the phase differences πn is stable in this sense, if A_{xx} is positive and n is an even number as well as if A_{xx} is negative and n is an odd number. Otherwise the motion is Lyapunov unstable (Sperling et al., 1997).

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