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Anisotropic Creep Damage Modeling of Single Crystal Superalloys

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According to the results of microscopic investigations, creep damage of single crystal superalloys is mainly caused by nucleation, growth, and coalescence of microscopic voids and cracks. Due to the specific geometry of these mechanisms, damage generally develops anisotropically. This paper presents a phenomenological anisotropic creep damage model for cubic single crystal superalloys based on the continuum damage mechanics theory. In this model, material damage in the form of microvoids and microcracks is represented by a second-order symmetric tensor, and the microcrack opening and closing mechanism is described by introducing an active damage tensor. Both the initial anisotropy of the material and the damage induced anisotropy are considered in the damage evolution law. The constitutive creep model coupled with the material damage description for single crystal superalloys is then derived by using the effective stress concept. It is applied to the description of the monotonous creep behavior of the single crystal superalloy SRR99 at 760°C and compared with experimental results.

1 Introduction

Because of their improved high temperature mechanical properties in comparison to the polycrystalline ones, nickel-based single crystal superalloys find widely increasing application, especially for turbine blades, where creep deformation and rupture are important issues. For realistic life predictions to ensure the integrity of these components throughout their lifetime, experimental and theoretical investigations of creep deformation and rupture of single crystal superalloys are of special interest.

The highly anisotropic mechanical properties of single crystal superalloys add another complication to the nonlinear, rate-dependent behavior of polycrystalline alloys at elevated temperature. Some phenomenological approaches for the modeling of the rate-dependent behavior of cubic single crystals have been developed. For instance, the one proposed by Choi and Krempl (1989), which was applied by Kunkel (1996) to the simulation of the anisotropic viscoplastic behavior of the single crystal SRR99. Starting from a rheological model, another anisotropic continuum mechanics model for the description of the creep behavior of single crystals at high temperatures was proposed by Bertram and Olschewski (1993), and has been applied to the single crystal superalloy SRR99 at 760°C (Bertram and Olschewski, 1996) and of CMSX-6 at 760°C (Bertram and Olschewski, in press). However, the progressive degradation of the mechanical properties of materials during the loading process has not been considered in these approaches. They are therefore restricted to the undamaged material behavior and converge to the steady-state creep behavior. Bertram et al. (1992) proposed a uniaxial model to describe the monotonous creep behavior in the whole range up to rupture by introducing a Kachanov-Rabotnov damage variable. This, however, was not sufficient, as the damage mechanisms generally exhibit a three-dimensional anisotropic character.

For the description of the material degradation in creep processes of metals, Kachanov (1958) introduced a scalar variable ω called *continuity* as a state variable which can be used to predict not only the time to rupture, but also the strain rates in the tertiary creep phase. This idea led to the development of continuum damage mechanics (CDM). By considering the progressive material damage process, CDM theory offers the possibility to simulate the mechanical behaviour of engineering structures made of history-dependent materials (irreversible, progressive degradation of mechanical properties) under mechanical loads (s. Altenbach et al., 1990). In this paper, we present a phenomenological approach to material damage of single crystal superalloys based on the CDM theory. Both the initial anisotropy of the material and the damage induced anisotropy are considered in the model. As these materials have limited ductility, we consider small deformations and rotations. Taking into account geometric nonlinearity would not lead to a higher precision of our predictions, as the scattering inherent to creep processes is large.

For simplicity and as a first step, isothermal conditions are assumed so that the effect of temperature changes enters the constitutive equations only through the material parameters, which are temperature dependent.

2 Damage Variable and Active Damage Tensor

The first step in developing a damage model concerns the definition of the damage variables. Damage is the deterioration which occurs in materials prior to failure (Lemaitre, 1992). Creep processes of metals are accompanied by the formation and growth of microvoids and microcracks. The definition of damage due to Kachanov-Rabotnov assumes that the nucleation and propagation of microvoids and microcracks is the only mechanism leading to material degradation, and that the ratio of the reduced cross section area to the total area of a the (initial) cross section serves as a measure of damage. Thus, a scalar field variable called *continuity* ω, or, alternatively, *damage* $D = 1-\omega$ can be used to represent the material damage in the uniaxial model (Kachanov, 1958; Rabotnov, 1968).

Because of its microscopic nature damage has, in general, an anisotropic character even if the material is originally isotropic (Betten, 1983). Therefore, tensor variables should be used for the three-dimensional representation of material damage (Leckie and Onat, 1981). Murakami and Ohno (1981) have demonstrated that the material damage in the form of microvoids and microcracks can be more appropriately described by a second-order symmetric tensor. Although such a variable cannot describe more complicated damage states than orthotropy, it has been often employed in the development of anisotropic damage theories (s. Murakami and Kamiya, 1997).

Microscopic investigations (Ai et al., 1990; Portella and Herzog, 1992; Rumi et al., 1994) show that the deterioration of nickel-based single crystal superalloys under creep loading conditions is directly caused by the growth and coalescence of initial microcracks, namely casting pores. Therefore a second-order symmetric damage tensor **D** is chosen as an internal variable to describe the anisotropic damage state for the present anisotropic damage model.

Damage effects characterized by microvoids and microcracks can be both activated or deactivated according to the loading conditions. The damage may still exist but eventually does not effect the stiffness of the material. The phenomenon of damage deactivation has been observed experimentally. The experimental results of Berthaud et al. (1990) for concrete show that Young´s modulus gains its undamaged value after the loading becomes compressive (s. Hansen and Schreyer, 1995). Johnson et al. (1956) have conducted uniaxial compression tests on a copper alloy for times beyond the uniaxial rupture time for tensile tests, without observing a deviation from the steady-state creep behavior. Material rupture under conditions of uniaxial compressive stress has not been observed in metals (Hayhurst, 1972). If the microcracks are open, the inner surfaces of them are expected to be stress free. The damage is *active* and the effective stress is larger than the usual stress. If the microcracks close under compression the effective stress is equal to the usual stress (without damage), and, thus, the damage becomes *inactive*. For the representation of this mechanism the following active damage tensor proposed by Hansen and Schreyer (1995) is used in the present model.

First we consider the spectral decomposition of the elastic strain tensor $\boldsymbol{\varepsilon}^e$ and the total strain tensor $\boldsymbol{\varepsilon}$

$$\boldsymbol{\varepsilon}^e = \sum_{i=1}^{3} \varepsilon_i^e \mathbf{n}_{ei}^{\varepsilon} \otimes \mathbf{n}_{ei}^{\varepsilon} \qquad \boldsymbol{\varepsilon} = \sum_{i=1}^{3} \varepsilon_i \mathbf{n}_i^{\varepsilon} \otimes \mathbf{n}_i^{\varepsilon} \tag{1a, b}$$

where ε_i^e and ε_i are the *i*th eigenvalues, $\mathbf{n}_{ei}^{\varepsilon}$ and $\mathbf{n}_i^{\varepsilon}$ are the corresponding *i*th eigenvectors of $\boldsymbol{\varepsilon}^e$ and $\boldsymbol{\varepsilon}$, respectively, and the symbol $\otimes$ denotes the tensor product. The crack opening/closing mechanism can be described by means of the Heaviside function $h(x)$ defining the tensors

$$\mathbf{H}_{\varepsilon e} = \sum_{i=1}^{3} h(\varepsilon_i^e) \mathbf{n}_{ei}^{\varepsilon} \otimes \mathbf{n}_{ei}^{\varepsilon} \tag{2}$$

$$\mathbf{H}_{\varepsilon} = \sum_{i=1}^{3} h(\varepsilon_i) \mathbf{n}_i^{\varepsilon} \otimes \mathbf{n}_i^{\varepsilon} \tag{3}$$

However, for numerical reasons it is advantageous to use a smooth version of h as suggested by Hansen and Schreyer (1995)

$$h(x)=\begin{cases}0 & \text{for} \quad x \le x_m \\ \dfrac{1}{2}\left\{1-\cos\left[\dfrac{\pi(x-x_m)}{x_p-x_m}\right]\right\} & \text{for} \quad x_m < x < x_p \\ 1 & \text{for} \quad x \ge x_p\end{cases} \tag{4}$$

with two parameters x_m and x_p. The positive spectral projection operators (fourth-order tensor) for the elastic and the total strains are defined as

$$\overset{<4>}{\mathbf{P}}_{\varepsilon e} = \mathbf{H}_{\varepsilon e} \wedge \mathbf{H}_{\varepsilon e} \qquad\qquad \overset{<4>}{\mathbf{P}}_{\varepsilon} = \mathbf{H}_{\varepsilon} \wedge \mathbf{H}_{\varepsilon} \tag{5a, b}$$

respectively, where the composition of two second-order tensors, denoted by the wedge $\wedge$, is defined by $\mathbf{A} \wedge \mathbf{B} = a_{ij}b_{kl}(\mathbf{e}_i \otimes \mathbf{e}_k \otimes \mathbf{e}_j \otimes \mathbf{e}_l)$ for an orthonormal basis $\mathbf{e}_i$. The positive projection of the elastic and the total strain tensors are then given by

$$\boldsymbol{\varepsilon}^{e+} = \overset{<4>}{\mathbf{P}}_{ze} : \boldsymbol{\varepsilon}^{e} \tag{6}$$

$$\boldsymbol{\varepsilon}^{+} = \overset{<4>}{\mathbf{P}}_{z} : \boldsymbol{\varepsilon} \tag{7}$$

respectively, where the symbol : denotes the double contraction. A positive projection operator based on the total strain is defined as follows (Hansen and Schreyer, 1995)

$$\overset{<4>}{\mathbf{T}} = \overset{<4>}{\mathbf{I}} - \left(\overset{<4>}{\mathbf{I}} - \overset{<4>}{\mathbf{P}}_{\varepsilon e}\right) : \left(\overset{<4>}{\mathbf{I}} - \overset{<4>}{\mathbf{P}}_{\varepsilon}\right) \tag{8}$$

The *active damage tensor* is then given by

$$\mathbf{D}_a = \overset{<4>}{\mathbf{T}} : \mathbf{D} \tag{9}$$

3 Effective Stress and Damage Active Stress

According to the effective stress concept of CDM, the effect of damage on the deformation behavior can be represented by a magnification of the stress tensor called *effective stress tensor* $\tilde{\boldsymbol{\sigma}}$ (Lemaitre, 1971). Let a fourth-order tensor $\overset{<4>}{\mathbf{M}}(\mathbf{D})$ characterize the damage state. We assume the following general form of the transformation between the stress tensor $\boldsymbol{\sigma}$ and the effective stress tensor $\tilde{\boldsymbol{\sigma}}$ (Chaboche, 1981 and 1984)

$$\tilde{\boldsymbol{\sigma}} = \overset{<4>}{\mathbf{M}}(\mathbf{D}) : \boldsymbol{\sigma} \tag{10}$$

Following the suggestion of Cordebois and Sidoroff (1982), the previous transformation is taken in the particular form

$$\overset{<4>}{\mathbf{M}} = (\mathbf{I}-\mathbf{D})^{-\frac{1}{2}} \wedge (\mathbf{I}-\mathbf{D})^{-\frac{1}{2}} \tag{11}$$

In the principal coordinate system of damage tensor, the matrix form of equation (11) is given by

$$\left[\tilde{\sigma}_{11} \;\; \tilde{\sigma}_{22} \;\; \tilde{\sigma}_{33} \;\; \tilde{\sigma}_{23} \;\; \tilde{\sigma}_{31} \;\; \tilde{\sigma}_{12}\right]^T = \left[\overset{<4>}{\mathbf{M}}\right]\left[\sigma_{11} \;\; \sigma_{22} \;\; \sigma_{33} \;\; \sigma_{23} \;\; \sigma_{31} \;\; \sigma_{12}\right]^T \tag{12}$$

with

$$\left[\overset{<4>}{\mathbf{M}}\right] \triangleq \begin{bmatrix} \frac{1}{1-d_1} & & & & & \\ & \frac{1}{1-d_2} & & & & \\ & & \frac{1}{1-d_3} & & & \\ & & & \frac{1}{\sqrt{(1-d_2)(1-d_3)}} & & \\ & & & & \frac{1}{\sqrt{(1-d_3)(1-d_1)}} & \\ & & & & & \frac{1}{\sqrt{(1-d_1)(1-d_2)}} \end{bmatrix} \tag{13}$$

where σ_{ij} and $\tilde{\sigma}_{ij}$ (i, j = 1, 2, 3) are the components of $\boldsymbol{\sigma}$ and $\tilde{\boldsymbol{\sigma}}$, respectively, and d_1, d_2 and d_3 denote the eigenvalues of the damage tensor $\mathbf{D}$. This definition of $\overset{<4>}{\mathbf{M}}(\mathbf{D})$ is identical with the one of Chow and Wang (1987a, b) and has been applied to the approach of dynamic fracture of brittle anisotropic solids (Fahrenthold, 1990) and to the analytical prediction of the initiation and propagation of ductile fracture in metals (Jubran and Cofer, 1991).

The effective stress tensor with respect to damage deactivation for the present model is defined as

$$\tilde{\boldsymbol{\sigma}} = \left(\mathbf{1} - \mathbf{D}_a\right)^{-1/2} \cdot \boldsymbol{\sigma} \cdot \left(\mathbf{1} - \mathbf{D}_a^T\right)^{-1/2} \tag{14}$$

4 Creep Damage Model

4.1 Thermodynamic Consideration

Damage of materials is a progressive irreversible thermodynamic process. Following the theory of thermodynamics of irreversible processes, the damage evolution law may be given by (Krajcinovic, 1983)

$$\dot{\mathbf{D}} = \frac{\partial \phi}{\partial \mathbf{Y}_D} \tag{15}$$

where ϕ is the dual potential of the dissipation potential, and $\mathbf{Y}_D$ is the thermodynamic force associated with damage, called *damage driving force*. The potential ϕ must possess the following properties for the satisfaction of the second law of thermodynamics: ϕ must be a non-negative, convex function with zero at the origin.

Let us assume, as usual in classical continuum mechanics, a decoupling of intrinsic and thermal dissipation. For practical purposes, we furthermore postulate that the dissipation due to damage processes and the dissipation associated with the other mechanisms, such as the plastic strain and the hardening process, are independent. This postulate should be understood as excluding direct coupling between plasticity and damage (see Lu and Chow, 1990). It follows from equation (15) that

$$\dot{\mathbf{D}} = \frac{\partial \phi_D}{\partial \mathbf{Y}_D} \tag{16}$$

where ϕ_D denotes the *damage dissipation potential.* Using the standard model (Germain, 1983), the damage dissipation potential takes a quadratic, non-negative form

$$\phi_D = \frac{1}{2} \mathbf{Y}_D : \overset{<4>}{\mathbf{S}} : \mathbf{Y}_D \tag{17}$$

where $\overset{<4>}{\mathbf{S}}$ is a fourth-order tensor that must be symmetric and positive-definite. The damage evolution law is then given by

$$\dot{\mathbf{D}} = \overset{<4>}{\mathbf{S}} : \mathbf{Y}_D \tag{18}$$

The thermodynamic restrictions are automatically satisfied.

4.2 Damage Driving Force

Damage growth in a creep process generally depends on the current state of stress and damage, and for anisotropic materials also on the material symmetry. If the initial anisotropic properties of a given material can be characterized by a fourth-order tensor $\overset{<4>}{\mathbf{A}}$, the damage law can be expressed by (Betten, 1982)

$$\dot{\mathbf{D}} = \mathbf{G}\left(\boldsymbol{\sigma}, \mathbf{D}, \overset{<4>}{\mathbf{A}}\right) \tag{19}$$

In analogy to the effective stress concept for the creep equations, a *damage active stress* is assumed to represent the contributions of both stress and damage state to the damage evolution. It is introduced as

$$\hat{\boldsymbol{\sigma}} = (\mathbf{1} - \mathbf{D}_a)^{-p} \cdot \boldsymbol{\sigma} \cdot (\mathbf{1} - \mathbf{D}_a^T)^{-p} \tag{20}$$

where the material parameter p is used to distinguish the effect of damage on the damage growth from that on the creep rate. Experimental observations show that the creep rate is less sensitive to the damage state in comparison with the rate of void growth (Murakami and Ohno, 1981). Thus, the damage law (19) has the following reduced form

$$\dot{\mathbf{D}} = \mathbf{G}(\hat{\boldsymbol{\sigma}}, \overset{<4>}{\mathbf{A}}) \tag{21}$$

It is to see from equation (21) that for isotropic materials the damage evolution can be identified by the damage active stress. On the other hand, according to thermodynamics, the damage driving force is responsible for the damage development. Based on the results of the microscopic investigations it is therefore assumed that only the tensile damage active stresses are responsible for damage growth, and that for isotropic materials the anisotropy of damage development only depends on the principal directions of damage active stress tensor. Consequently, motivated by these considerations and the uniaxial damage model of Kachanov and Rabotnov, we postulate the following expression for the damage driving force for isotropic materials

$$\mathbf{Y}_D = \langle \hat{\boldsymbol{\sigma}} \rangle^n = \sum_{i=1}^{3} \langle \hat{\sigma}_i \rangle^n \hat{\mathbf{n}}_i^\sigma \otimes \hat{\mathbf{n}}_i^\sigma \tag{22}$$

where n is a material parameter, $\hat{\sigma}_i$ and $\hat{\mathbf{n}}_i^\sigma$ are the ith eigenvalue and eigenvector of $\hat{\boldsymbol{\sigma}}$, and $\langle . \rangle$ are the McCauley brackets.

For an anisotropic material, however, the damage will develop differently if the same load acts in different directions. In order to obtain the creep potential of anisotropic solids, Betten (1981) proposed to map the actual creep state of an anisotropic solid onto a fictitious isotropic state with equivalent creep rate by a suitable transformation. The anisotropic behavior is described by using a *mapped stress tensor* instead of the actual stress tensor in the isotropic creep potential. Following this idea, we introduce the *mapped damage active stress* $\hat{\boldsymbol{\sigma}}_m$ with help of an orientation function η_i

$$\hat{\boldsymbol{\sigma}}_m = \sum_{i=1}^{3} (\eta_i \hat{\sigma}_i) \hat{\mathbf{n}}_i^\sigma \otimes \hat{\mathbf{n}}_i^\sigma \tag{23}$$

The orientation function η_i modifies the effect of the ith principal damage active stress $\hat{\sigma}_i$ depending on its orientation related to the material structure. For single crystals with cubic symmetry we suggest the following orientation function

$$\eta_i = \left[\sum_{j=1}^{3}\left(\hat{\mathbf{n}}_i^\sigma \cdot \mathbf{e}_j^k\right)^{2m}\right]^q = \left[\left(\hat{\mathbf{n}}_i^\sigma \cdot \mathbf{e}_1^k\right)^{2m} + \left(\hat{\mathbf{n}}_i^\sigma \cdot \mathbf{e}_2^k\right)^{2m} + \left(\hat{\mathbf{n}}_i^\sigma \cdot \mathbf{e}_3^k\right)^{2m}\right]^q \tag{24}$$

where m and q are material parameters, and $\mathbf{e}_j^k$ (j = 1,2,3) are the lattice vectors. The damage driving force (with respect to material symmetry) is then given by

$$\mathbf{Y}_D = \sum_{i=1}^{3}\langle \eta_i \hat{\sigma}_i \rangle^n \hat{\mathbf{n}}_i^\sigma \otimes \hat{\mathbf{n}}_i^\sigma \tag{25}$$

4.3 Damage Evolution Law

In the damage law (20), the fourth-order tensor $\overset{<4>}{\mathbf{S}}$ may depend on the thermodynamic state variables. As the effect of damage is included in the damage active stress, it can be assumed that $\overset{<4>}{\mathbf{S}}$ characterizes the initial undamaged material structure and remains constant during the damage process. Therefore, $\overset{<4>}{\mathbf{S}}$ can be called *structure tensor*. Bertram and Olschewski (1993) used following structure tensor in their creep model for cubic single crystals

$$\overset{<4>}{\mathbf{S}} = \beta_1 \mathbf{I} \otimes \mathbf{I} + \beta_2 \overset{<4>}{\mathbf{I}} + \beta_3 \overset{<4>}{\mathbf{R}} \tag{26}$$

with

$$\overset{<4>}{\mathbf{R}} = \sum_{i=1}^{3} \mathbf{e}_i^k \otimes \mathbf{e}_i^k \otimes \mathbf{e}_i^k \otimes \mathbf{e}_i^k \tag{27}$$

where $\mathbf{e}_j^k$ (j = 1,2,3) are the lattice vectors. This structure tensor is also chosen for the present damage model.

Substituting equations (25) and (26) into equation (18), we obtain the damage evolution law

$$\dot{\mathbf{D}} = \overset{<4>}{\mathbf{S}} : \mathbf{Y}_D = \left(\beta_1 \mathbf{I} \otimes \mathbf{I} + \beta_2 \overset{<4>}{\mathbf{I}} + \beta_3 \overset{<4>}{\mathbf{R}}\right) : \sum_{i=1}^{3}\langle \eta_i \hat{\sigma}_i \rangle^n \hat{\mathbf{n}}_i^\sigma \otimes \hat{\mathbf{n}}_i^\sigma \tag{28}$$

or, alternatively

$$\dot{\mathbf{D}} = \left(\alpha_1 \mathbf{I} \otimes \mathbf{I} + \alpha_2 \overset{<4>}{\mathbf{I}} + \alpha_3 \overset{<4>}{\mathbf{R}}\right) : \sum_{i=1}^{3}\left\langle \frac{\eta_i \hat{\sigma}_i}{B} \right\rangle^n \hat{\mathbf{n}}_i^\sigma \otimes \hat{\mathbf{n}}_i^\sigma \tag{29}$$

with

$$\alpha_1 + \alpha_2 + \alpha_3 = 1 \tag{30}$$

5 Anisotropic Creep Model Coupled with Damage

According to the effective stress concept, any constitutive equation for the damaged material can be derived in the same way as for a virginal (undamaged) material if the stress tensor is replaced by the effective stress tensor. By implementing the derived damage model into the three-dimensional viscoplastic model proposed by Bertram and Olschewski (1993 and 1996), the viscoplastic constitutive equations coupled with the damage model for cubic single crystal superalloys results as

$$\dot{\varepsilon} = \overset{<4>}{\mathbf{A}}_1 : \dot{\tilde{\sigma}} + \overset{<4>}{\mathbf{A}}_2 : \tilde{\sigma} + \overset{<4>}{\mathbf{A}}_3 : \tau \tag{31}$$

$$\dot{\tau} = \overset{<4>}{\mathbf{A}}_4 : \dot{\tilde{\sigma}} + \overset{<4>}{\mathbf{A}}_5 : \left(\tilde{\sigma} - \tau\right) \tag{32}$$

with an internal variable τ and the following fourth-rank material tensors

$$\overset{<4>}{\mathbf{A}}_1 = \sum_{i=1}^{3} \frac{1}{C_i + K_i} \overset{<4>}{\mathbf{P}}_i \qquad \overset{<4>}{\mathbf{A}}_2 = \sum_{i=1}^{3} \frac{1}{C_i + K_i} \left(\frac{C_i}{D_i} + \frac{C_i}{L_i} + \frac{K_i}{L_i} \right) \overset{<4>}{\mathbf{P}}_i \tag{33a, b}$$

$$\overset{<4>}{\mathbf{A}}_3 = -\sum_{i=1}^{3} \frac{C_i}{C_i + K_i} \frac{1}{D_i} \overset{<4>}{\mathbf{P}}_i \qquad \overset{<4>}{\mathbf{A}}_4 = \sum_{i=1}^{3} \frac{K_i}{C_i + K_i} \overset{<4>}{\mathbf{P}}_i \tag{33c, d}$$

$$\overset{<4>}{\mathbf{A}}_5 = \sum_{i=1}^{3} \frac{K_i}{C_i + K_i} \frac{C_i}{D_i} \overset{<4>}{\mathbf{P}}_i \tag{33e}$$

and the three structure tensors

$$\overset{<4>}{\mathbf{P}}_1 = \frac{1}{3} \mathbf{I} \circ \mathbf{I} \qquad \overset{<4>}{\mathbf{P}}_2 = \overset{<4>}{\mathbf{R}} - \overset{<4>}{\mathbf{P}}_1 \qquad \overset{<4>}{\mathbf{P}}_3 = \overset{<4>}{\mathbf{I}} - \overset{<4>}{\mathbf{P}}_1 - \overset{<4>}{\mathbf{P}}_2 \tag{34a, b, c}$$

where C_i, K_i, D_i, L_i (i = 1, 2, 3) are material parameters, and $\mathbf{I}$ and $\overset{<4>}{\mathbf{I}}$ denote the identity tensor of rank two and four, respectively.

The dependence of the viscosities D_i and L_i on the applied stress is expressed as

$$D_i = D_{0i} \exp\left\{ -\sum_j Z_{ij} J_j \right\} \quad \text{and} \quad L_i = L_{0i} \exp\left\{ -\sum_j Z_{ij} J_j \right\} \tag{35a, b}$$

with the material parameters Z_{ij} (i = 1, 2, 3, 4; j = 1, 2, 3) and the following scalar invariants with cubic symmetry

$$J_1 = \sqrt{\tilde{\sigma}_{11}\tilde{\sigma}_{22} + \tilde{\sigma}_{22}\tilde{\sigma}_{33} + \tilde{\sigma}_{33}\tilde{\sigma}_{11}} \qquad J_2 = \tilde{\sigma}_{12}^2 + \tilde{\sigma}_{23}^2 + \tilde{\sigma}_{31}^2 \tag{36a, b}$$

$$J_3 = \tilde{\sigma}_{11}\tilde{\sigma}_{22}\tilde{\sigma}_{33} \qquad J_4 = \tilde{\sigma}_{11}\left(\tilde{\sigma}_{12}^2 + \tilde{\sigma}_{13}^2\right) + \tilde{\sigma}_{22}\left(\tilde{\sigma}_{23}^2 + \tilde{\sigma}_{21}^2\right) + \tilde{\sigma}_{33}\left(\tilde{\sigma}_{31}^2 + \tilde{\sigma}_{32}^2\right) \tag{36c, d}$$

From the assumption of incompressibility under creep, it follows

$$D_1^{-1} = 0, \; L_1^{-1} = 0 \quad \text{and} \quad Z_{i1} = 0 \; (i = 1, 2, 3, 4) \tag{37}$$

6 Comparison with Experimental Results

This viscoplastic model coupled with damage has been applied to the description of the uniaxial creep behavior in different orientations for the nickel-based superalloy SRR99 at 760 °C. The material parameters of the viscoplastic model are adopted from the work of Bertram and Olschewski (1996). The corresponding material parameters of the damage model are shown in Table 1. They have been calibrated by these tests using a nonlinear optimization procedure. The parameter x_m and x_p for the description of active/passive damage mechanisms could not be determined by experiments and have been appropriately chosen. Of course, there remains always a need for more and other types of experiments leading to a better identification of the model.

Figure 1 and Figure 2 show the comparison of the theoretical prediction and the experimental results, where *creep model* means uncoupled with damage and *damage model* means the coupled model. Phi = {φ_1, φ_2} denotes the first (φ_1) and the second (φ_2) Eulerian angle, determining the orientation of the crystal relative to the load axis. The orientation dependence of the creep strain response under constant uniaxial load (creep stress 680 MPa) is presented in Figure 1. In the same orientation, but under different applied loads, the creep behavior is strongly nonlinear. This character is shown in Figure 2 for the case φ_1 =16° and φ_2 = 45°. Note that the time scale of the different figures differs widely.

α_1	α_2	α_3	B [MPa]	n	p	m	q
0.0	0.5	0.5	1442.0	14.133	0.45489	51.852	−0.31326

Table 1. Material Parameters of Damage Model (SRR99 at 760 °C)

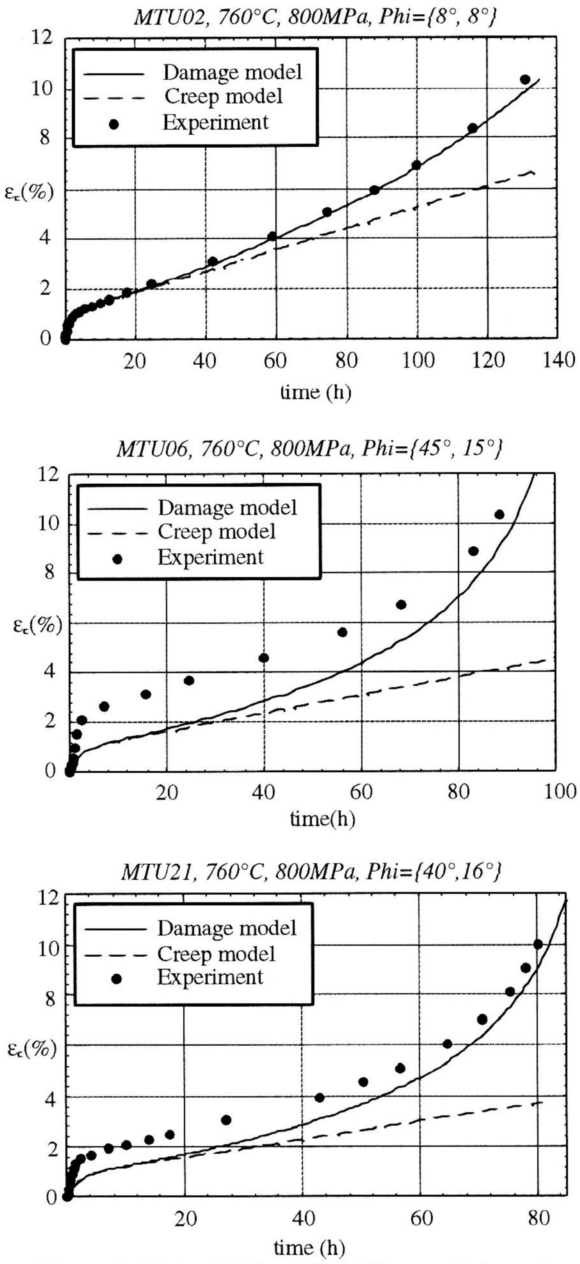

Figure 1. Uniaxial Creep in Different Orientations under the Same Load (σ_c = 800 MPa)

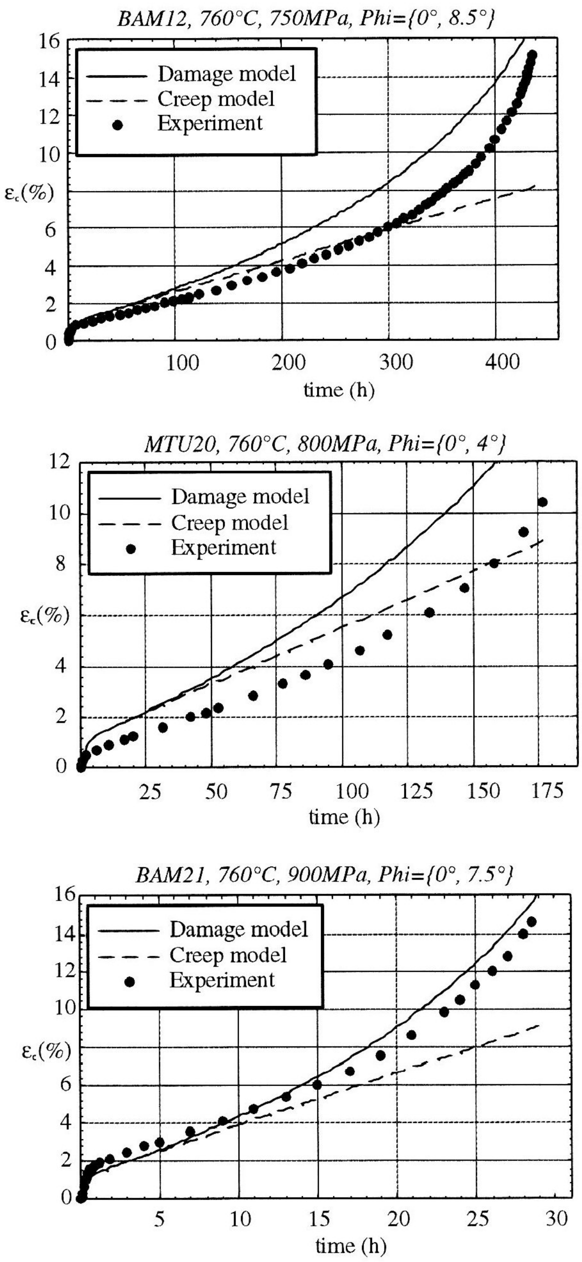

Figure 2. Uniaxial Creep under Different Loads in the Same Orientation

The comparison shows that the developed model provides satisfying prediction of the damage process in the range of the uniaxial creep experiments. Both the influence of the material symmetry on the creep behavior and the nonlinearity of the material response with respect to the applied loads are reproduced by the present model.

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