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Softening Solids: Reality or Misinterpretation?

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The behaviour of a solid rod revealing a descending branch on the force-displacement diagram is discussed. It is shown that static equilibrium of a softening rate-insensitive elasto-plastic rod is unstable. Instability leads to strain localisation and a dynamic process of propagation of shock waves of unloading and reloading. Based on the elasto-visco-plastic model of a softening solid, the evolution of strain localisation is studied. To describe the behaviour of rock-type brittle "softening" materials, a new constitutive model with structural transformation is proposed and analysed.

1 Introduction

In some cases the force-displacement diagram recorded in the uniaxial test of solids possesses a descending part. It takes place in material tests of mild steels, of aluminium alloys, and most often of different kinds of rocks. The process of deformation becomes unstable since the equilibrium for increasing deformations is associated with a decreasing force. Analogous phenomena are observed in the so-called Van der Waals gas and in an electric circuit with negative resistance. However, existence of the descending part in the force-displacement diagram does not mean that the material itself is softening, i.e. the stress-strain diagram has a descending branch. Inspection of specimens enduring the postcritical deformation shows that in all cases substantional geometrical or structural transformations took place. In most cases, the stress and strain state of specimens becomes nonhomogeneous. In mild steels, a neck appears, in brittle aluminium alloys crack-type defects are formed, in rocks both cracks and shear bands emerge. As a result the descending part of the force-displacement diagram is not reproducible, in that it depends on the type and rate of loading. The specimen cannot be considered as an element of a material,and calculation of the stress-strain relation from the force-displacement diagram becomes questionable (Read and Hegemier, 1984). Only in the extreme case, when numerous defects or structural changes take place, the damaged solids may be considered as a continuum, and it is possible to perform homogenisation of stress and strain states. In this case, structural transformation takes place which results in the formation of a material with new mechanical properties and a natural stress-free state.

However, there exists a common opinion that in a rigid testing machine, i.e. in a test with displacement control, it is possible to record the descending part of the stress-strain diagram. Models of softening materials may be found in the literature, they are used for the solution of practical problems. For this reason, although there is no direct evidence of the existence of a softening material, we suggest that such a hypothetical solid does exist, and study how it would behave in the displacement control test. We consider also alternative models of materials which may lead to a descending part on the force-displacement diagram. The first, broadly used elasto-visco-plastic model (Sokolovsky, 1948), even when its limiting static stress-strain diagram has a discending branch, leads to a hyperbolic set of equations for which the considered problems are well-posed. The second is a newly suggested model with a discontinuous stress-strain diagram. It describes phase transformations in rocks due to the rupture of internal structure.

2 Softening Elasto-Plastic Solid

First we consider a hypothetic solid which is described by the rate-insensitive elasto-plastic model. For a uniaxial stress σ and corresponding strain ε the stress-strain relationship for active loading has the form

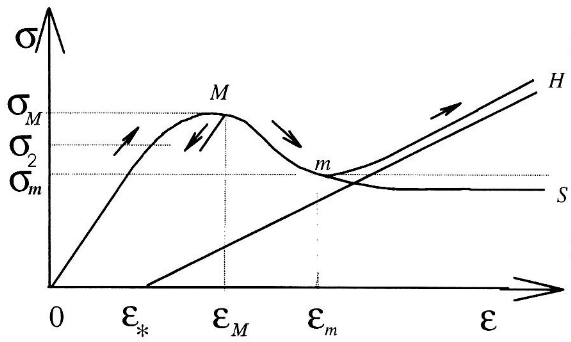
$$\sigma = f(\varepsilon) \qquad\qquad E(\varepsilon) = \frac{df(\varepsilon)}{d\varepsilon} \tag{1}$$

Here $f(\varepsilon)$ is the nonmonotonic function shown by the curve *OMmS* in Figure 1, ε_M is strain at the peak value of stress σ_M, $E(\varepsilon)$ is the tangent modulus, and an upper dot means differentiation with respect to time or any other parameter controlling the evolution of the process. For the active process $\dot{\varepsilon} > 0$, $E(\varepsilon) > 0$ when $\varepsilon < \varepsilon_M$, and $E(\varepsilon) < 0$ when $\varepsilon > \varepsilon_M$.

Unloading, which in the case under consideration is the process accompanied by a decreasing strain, is conventionally assumed to be elastic, with the initial modulus $E(0)$

$$\sigma - \sigma_u = E(0)(\varepsilon - \varepsilon_u) \qquad \dot{\varepsilon} < 0 \tag{2}$$

where σ_u and ε_u are stress and strain at the start of the unloading, respectively.



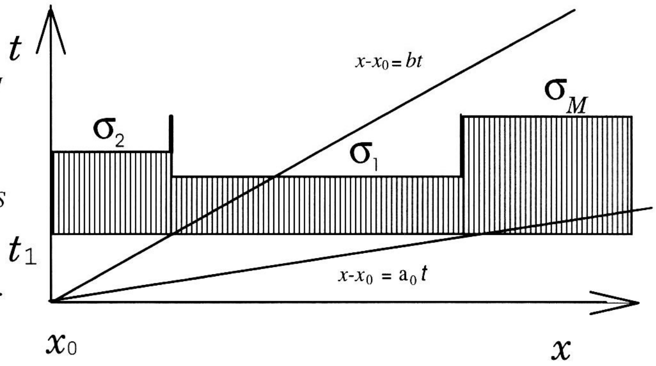


Figure 1. Stress-strain Diagram

Figure 2. Wave Fronts and Shape of Stress Pulse

$OMmS$ - for softening solid
$OMmH$ - for solid with secondary hardening
$OM + \varepsilon_* H$ - for solid with structural transformation

Consider a test of a rod specimen made of this solid. Let a specimen with initial length l be quasi-statically elongated in the rigid test machine up to the length $l + u_0$ such that $\varepsilon_0 = u_0/l > \varepsilon_M$. It is assumed that the stress-strain state in the specimen is homogeneous $\sigma = \sigma_0$, $\varepsilon = \varepsilon_0$. Let us investigate whether this stress-strain state is stable. Let us refer the rod to the axis x and take its origin at the end of the rod. Let us also assume that at some moment of time $t = 0$, the motion described by velocity $v_0(x)$ has begun in the rod. To study the stability of the suggested homogeneous stress-state we consider the evolution in time of the kinetic energy $K(t)$ of the rod. If the kinetic energy $K(0)$ given at the moment $t = 0$ decreases in time, then the state of the rod is stable, in the opposite case it is unstable.

The equation of motion of a rod has the form

$$\rho \frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial x} \tag{3}$$

where ρ is the material density, assumed to be constant. Using equation (1) and relation of displacement u with strain $\varepsilon = \partial u/\partial x$ and velocity $v = \partial u/\partial t$ we arrive at the next equation.

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad \text{with} \qquad a^2 = \frac{E(\varepsilon)}{\rho} \tag{4}$$

The initial conditions are

$$\sigma(x,0) = \sigma_0 \qquad \varepsilon(x,0) = \varepsilon_0 \qquad v(x,0) = v_0(x) \tag{5}$$

The boundary conditions are

$$u(0,t) = 0 \qquad u(l,t) = u_0 \tag{6}$$

We expand the kinetic energy $K(t)$ into a series with respect to time t.

$$K(t) = \frac{1}{2}\rho\int_0^l v^2(x)dx = K(0) + \dot{K}(0)t + \frac{1}{2}\ddot{K}(0)t^2 + \cdots \tag{7}$$

Here

$$K(0) = \frac{1}{2}\rho\int_0^l v_0^2(x)dx$$

For $\dot{K}(0)$ according to equation (3) we have

$$\dot{K}(0) = \rho\int_0^l v\dot{v}dx = 0$$

Thus stability of the homogeneous state is determined by the sign of $\ddot{K}(0)$, for which in view of equations (3) to (6),

$$\begin{aligned}\ddot{K}(0) &= \int_0^l \rho v\ddot{v}dx + \int_0^l \rho\dot{v}^2 dx = \int_0^l v\frac{\partial\sigma}{\partial x\partial t}dx = \int_0^l \frac{\partial(v\dot{\sigma})}{\partial x}dx - \int_0^l \dot{\sigma}\frac{\partial v_0}{\partial x}dx \\ &= v\dot{\sigma}\Big|_0^l - \int_0^l E(\varepsilon)\frac{\partial\varepsilon}{\partial t}\frac{\partial v_0}{\partial x}dx = -\int_0^l E(\varepsilon)\left(\frac{\partial v_0(x)}{\partial x}\right)^2 dx\end{aligned} \tag{8}$$

In view of equation (1) for $\varepsilon > \varepsilon_M$ there exists a $v_0(x)$ for which $\ddot{K} > 0$. For instance, it will take place if

$$v_0(x) = \alpha x \qquad 0 < x < x_* \qquad v_0(x) = \beta(l - x) \qquad x_* < x < l$$

$$x_* = \frac{\beta l}{\alpha+\beta} \qquad \frac{\alpha}{\beta} = -\frac{E(0)}{E(\varepsilon_0)}$$

This proves instability of the static equilibrium of the homogeneous stress-strain state at the descending part of the stress-strain diagram. Note that the criterion of stability used is in accordance with the Hill (1958) quasistatic criterion, which was applied earlier in similar problems (Nikitin and Ryzhak, 1986; Ryzhak, 1993).

Since the static equilibrium is unstable, the dynamic process has to start when the stress reaches its peak value σ_M. The dynamic process is governed by equation (4). When the material is hardening, $E(\varepsilon) > 0$, the velocity of propagation of small disturbances is a. When the material is softening, $E < 0$, equation (4) becomes elliptic and it seems that the problem with initial conditions becomes ill-posed. It has a homogeneous solution obeying the initial and boundary conditions (5) and (6), which is unstable as shown above. The velocity of propagation of disturbances vanishes when the strain reaches the value ε_M, while for $\varepsilon > \varepsilon_M$ propagation of disturbances does not exist. However, when the relative displacement of the rod ends is $u_0 > \varepsilon_M l$ the postcritical strains $\varepsilon > \varepsilon_M$ must unavoidably appear somewhere in the rod. The only possibility to accommodate the postcritical strains under conditions that they cannot be distributed along any distance and cannot propagate, is to suggest that they are stationary localised at one or a number of cross sections. The location of cross-sections of localisation cannot be found from this analysis. Stress drops at these cross-sections, a circumstance that produces waves of unloading propagating with elastic velocity $a_0 = (E(0)/\rho)^{1/2}$. Thus the ill-posedness of the problem turns out to be apparent. Regions of validity of the elliptic equation turn out to be a set of zero measure. Localisation of strains leads to the well-posed dynamic problem.

Consider now a dynamic process in the vicinity of one of these cross-sections $x = x_0$. For generality we assume that starting from $\sigma = \sigma_m$, $\varepsilon = \varepsilon_m$ the material again becomes hardening, according to curve *OMmH* in Figure 1.

$$E(\varepsilon) > 0 \qquad \varepsilon > \varepsilon_m \qquad \dot{\varepsilon} > 0 \tag{9}$$

At the cross-section $x = x_0$ softening takes place and therefore the stress drops there. For the rate-insensitive material the stress drop from σ_M down to σ_m occurs instantaneously. Of course it is possible to account for an influence of viscosity, as was done by Loret and Prevost (1990), Slemrod (1989) and Suliciu (1990), or involve a kinetic equation at the wave front as was postulated by Abeyaratne and Knowles (1991). We restrict our consideration to the simplest suggestion.

At the moment of localisation, we take as initial condition everywhere, except at the cross-section $x = x_0$, stresses at the peak value $\sigma = \sigma_M$ and velocity is absent. Due to symmetry, we restrict our consideration to cross-sections lying right next to x_0. Then

$$\begin{aligned} &\sigma(x,0) = \sigma_M \qquad v(x,0) = 0 \qquad \text{for } x > x_0 \\ &\sigma(x_0,0) = \sigma_m \end{aligned} \tag{10}$$

The problem under consideration does not contain any characteristic length or time, and therefore, is self-similar. Stress, velocity and strain between fronts of waves depend on the ratio x/at only, but in our case they are constant.

An instantaneous stress drop at $x = x_0$ produces a wave of unloading, Figure 2, from the state $\sigma = \sigma_M$, $\varepsilon = \varepsilon_M$, which in accordance with equation (2) propagates at an elastic velocity $a = a_0 \equiv (E(0)/\rho)^{1/2}$. The law of momentum conservation yields at the front $x = x_0 + a_0 t$ of this wave

$$a\rho v_1 = \sigma_M - \sigma_m \tag{11}$$

Equation (11) determines velocity in the region behind the front $x = x_0 + a_0 t$. After the stress drops down to σ_m and the strain increases to ε_m at $x = x_0$, resistance to deformation in this cross-section is recovered. This produces a wave of loading, Figure 2, corresponding to the branch $\varepsilon > \varepsilon_m$ of the secondary hardening. Velocity b of the front $x = x_0 + bt$ of this wave as well as stress σ_2 behind this front are unknown, and are to be found from the solution. Due to symmetry, the cross-section $x = x_0$ is at rest, and therefore $v_2 = 0$. Unknowns b and σ_2 are found from the conditions of displacement discontinuity at the front $x = x_0 + bt$.

$$b(\varepsilon_1 - \varepsilon_2) + v_1 = 0 \qquad \sigma_m - \sigma_2 + b\rho v_1 = 0 \tag{12}$$

Equations (12) along with equation $\sigma = f(\varepsilon)$ from equation (1) and with equation (11) form the system of non-linear equations for the determination of $b, \sigma_2, \varepsilon_2$. Assume that equation $\sigma = f(\varepsilon)$ is piecewise linear

$$\sigma = E(0)\varepsilon \quad \text{for} \quad \varepsilon < \varepsilon_M \qquad \text{and} \qquad \sigma = E_s(\varepsilon - \varepsilon_s) \quad \text{for} \quad \varepsilon > \varepsilon_m \tag{13}$$

where E_s is the modulus of secondary hardening, and ε_s is a material parameter. Then equations (12) and (13) give

$$\frac{b}{a_0} = \frac{\left(\left((1-\alpha)\sigma_m + \sigma_s\right)^2 + 4\alpha(\sigma_M - \sigma_m)^2\right)^{\frac{1}{2}} - (1-\alpha)\sigma_m - \sigma_s}{2(\sigma_M - \sigma_m)}$$

$$\sigma_2 = \sigma_m + \frac{b}{a_0}(\sigma_M - \sigma_m) \tag{14}$$

Here $\alpha = E_s/E_0$, $\sigma_s = E_s\varepsilon_s$. The wave scheme adopted (Figure 2) is valid if only $b < a_0$. With the help of the first of equations (14) it may be shown that $b < a_0$ if $\sigma_M > \sigma_m$ and $0 < E_s < \infty$. The first of these conditions means existence of softening, the second-existence of the secondary hardening. Both these conditions are adopted in the formulation of the problem. For stability of the shock wave $x = x_0 + bt$ or its "evolutionarity" (Kulikovsky, 1988) it is necessary to obey the inequality

$$b < a_s = \sqrt{E_s/\rho} \tag{15}$$

Inequality (15) with the help of equations (14) may be reduced to inequality $\varepsilon_m E_0 > \sigma_m$, which for the softening material $\sigma_M > \sigma_m$ is always met. In Figure 2 stress distribution along the rod at some moment of time $t = t_1$ for a material with linear secondary hardening is shown. Material parameters are taken as follows: $\sigma_M = 2\sigma_m$, $\alpha = 1/4$, $\varepsilon_m = \frac{4}{3}\varepsilon_M$. In this case, from equations (14), $b/a_0 = 1/3$. It is worth noting that the solution does not depend on details of softening, so that from the test data it is possible to determine only the peak values of the corresponding stresses and strains. Transfer from the unloaded state $\sigma = \sigma_m$, $\varepsilon = \varepsilon_M - (\sigma_M - \sigma_m)/E(0)$ to the state $\sigma_2 < \sigma_M$ at the branch of secondary hardening has to pass through the peak value $\sigma = \sigma_m$. This takes place inside the shock front of the wave of secondary loading, and is shown in Figure 2.

3 Elasto-visco-plastic Softening Solid

Dynamic process and sharp localisation are not observed for polycrystalline solids. It means that the constitutive equation (16) does not describe their behaviour. One of the reasons may be the rate sensitivity of the material. To account for rate sensitivity of real solids we adopt the elasto-visco-plastic model proposed by V. Sokolovsky (1948). For the infinitesimal deformations the deviator $\mathbf{e}$ of the total strain tensor may be expressed as the sum of elastic part $\mathbf{e}^e$ and plastic part $\mathbf{e}^p$, $\mathbf{e} = \mathbf{e}^e + \mathbf{e}^p$. The constitutive equations of the elasto-visco-plastic solids for the arbitrary stress-strain state may be written in the form (Nikitin, 1957)

$$2\mu\dot{\mathbf{e}}^e = \dot{\mathbf{S}}$$

$$\mu\dot{\varepsilon}^p = \kappa\frac{\mathbf{S}}{\tau}H(\tau - f(\gamma)) \tag{16}$$

where

$$H(z) = 0 \quad \text{for } z < 0 \qquad H(z) = z \quad \text{for } z > 0$$

Here $\mathbf{S}$ is the deviator of the stress tensor, τ and γ are the second invariants of the stress and strain tensors, respectively, $\tau = f(\gamma)$ is the asymptotic limit of the stress-strain diagram when $\dot{\gamma} \to 0$, and κ is a material constant.

For this solid, we consider stretching of a semi-infinite rod at the end of which constant velocity or constant stress are applied. Softening is assumed to be ideal, i.e. for the uniaxial positive stress the constitutive equation has a form

$$E\dot{\varepsilon} = \dot{\sigma} \qquad \text{for} \quad \sigma < \sigma_m \quad \text{or} \quad \varepsilon < \varepsilon_M$$
$$E\dot{\varepsilon} = \dot{\sigma} + \kappa(\sigma - \sigma_m) \qquad \text{for} \quad \sigma > \sigma_m \quad \text{or} \quad \varepsilon > \varepsilon_m \tag{17}$$

where $\sigma_m = const < \sigma_M$.

When stress at the end of the rod reaches the peak value σ_M, due to the softening even in the case of quasistatic loading a dynamic process starts which is governed by equation (3) of motion, the constitutive equation (17) and the equations of compatibility

$$\rho\frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial x} \qquad E\frac{\partial v}{\partial x} = \frac{\partial \sigma}{\partial t} + \kappa(\sigma - \sigma_m) \qquad \frac{\partial v}{\partial x} = \frac{\partial \varepsilon}{\partial t} \tag{18}$$

The system (3.3) preserves hyperbolicity for the softening material and has been solved by the standard method of characteristics for the cases $v(0,t) = v_0$ and $\sigma(0,t) = \sigma_0$. Shock waves do not arise. However, at some distance $x = x_*$ from the end, a stationary discontinuity arises. Plastic strains are localised near the end within a zone of width x_*. The magnitude x_* does not change in time and vanishes when $v_0 \to 0$. In the case when a constant stress σ_0 is applied at the end of the rod, localisation of deformation also takes place, but it is not severe, and the width of the plastic zone increases approximately linearly in time.

4 Phase or Structural Transformation

In some cases, due to internal rupture or shear band formation, the mesostructure of a solid is changing when the stress reaches some critical state. If after this change the material may be considered as continuous, the process may be interpreted as phase or structural transformation under the action of stress. One solid is transformed into another one which possesses different mechanical properties and natural free stress state, branches OM and $\varepsilon_* H$ in Figure 1. Both parental and damaged solids are hardening and rheologically stable.

Consider again uniaxial stretching of a rod. As a reference configuration we choose the natural free-of-stress configuration of the parental material. In the simplest case, when both solids are elastic the free energy and stress-strain relation for the parental material have forms

$$A_p = E_p\varepsilon^2 \qquad \sigma = E_p\varepsilon \tag{19}$$

where E_p is Young's modulus of the parental solid. In the natural configuration of the parental solid the damaged solid is not stress free. Besides, the structural transformation is a process consuming some energy. Therefore, the free energy of the damaged material must contain zero and first order terms in terms of ε.

$$A_d = A_* - E_d\varepsilon_*\varepsilon + \frac{1}{2}\mu_d\varepsilon^2 \qquad \sigma = E_d(\varepsilon - \varepsilon_*) \tag{20}$$

Here E_d is the Young's modulus of the damaged material, A_* is the energy of the structural transformation and γ_* is the kinematic characteristic of the structural transformation. Structural transformation takes place when the stress reaches the critical value $\sigma = \sigma_{cr}$. Thus, the structural transformation is characterised by the 3 material parameters σ_{cr}, A_* and ε_*. The process of deformation becomes unstable when $\sigma = \sigma_{cr}$. The same reasonings as for the softening solid show that strains are localised at some cross-sections, and a dynamic process starts. Stress drops at the localisation at, say, $x = x_0$ to some level $\sigma_1 < \sigma_{cr}$ which is to be found in the process of solution. The wave pattern is similar to that for the softening solid. As above, the wave of structural transformation $x = x_0 + bt$ is preceded by the wave of elastic unloading of the parental solid $x = x_0 + a_p t$. At the front of the structural transformation, in addition to the law of momentum conservation and the condition of displacement discontinuity, the law of energy conservation is to be met. This results in the next set of simultaneous equations for the determination of unknowns σ_1, v_1, σ_2 and b

$$\sigma_1 - \sigma_{cr} + a_p \rho v_1 = 0 \qquad \sigma_2 - \sigma_1 - b v_1 = 0$$

$$b(\varepsilon_2 - \varepsilon_1) - v_1 = 0 \qquad b\left(A_p - A_d - \frac{1}{2}\rho v_1^2\right) - \sigma_1 v_1 = 0 \tag{21}$$

The shape of stress distribution along the distance is similar to that for the softening solid which is shown in Figure 2. Table 1 gives values of b/a_p, ε_1 and ε_2 for different $\alpha = E_d/E_p$, $\varepsilon_\alpha = E_d \varepsilon_* / \sigma_{cr}$ and, $A_0 = 2E_d A_* / \sigma_{cr}^2$, obtained from the solution of the simultaneous equations (21).

$\alpha/\varepsilon_\alpha$		$A_0 = 0$	$A_0 = 1$	$A_0 = 2$	$A_0 = 3$
$^{1/4}/_{-1/8}$	b/a_p ε_1 ε_2	0,492 0,183 1,842	0,146 0,731 2,580	0,005 0,987 3,450	
$^{1/4}/_{1/8}$	b/a_p ε_1 ε_2	0	0,297 0,223 2,601	0,143 0,628 3,225	0,055 0,832 3,866
$^{1/4}/_{1/4}$	b/a_p ε_1 ε_2	0	0,334 0,141 2,712	0,199 0,447 3,228	0,108 0,662 3,795

Table 1. Velocity of the Front of the Wave of Structural Transformation b and Strains Ahead (ε_1) and Behind (ε_2)

5 Conclusions

Common opinion that the descending branch of the stress-strain diagram of a softening solid (would such a solid exist) may be recorded in a displacement-controlled test is questionable. Structures made of softening rate-insensitive materials lose stability when loads reach some critical value. Postcritical behaviour does not lead to the ill-posed problem as it first seems but rather to a well-posed dynamic problem. Strain corresponding to the descending branch of the stress-strain diagram is localised at surfaces with one dimension less than the problem under consideration.

The model of the rate-sensitive softening solid permits the calculation of the evolution in time of the strain localisation.

A new constitutive model with structural transformation is suggested. This model describes apparent softening.

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