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Stability of Mathematical Models for Systems of Synchronous and Asynchronous Machines

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The equivalentation principle for electric machines is formulated. It allows to examine analytically both the electromagnetic and electromechanic transient processes in a system consisting of n electric machines and also to obtain the condition of static and dynamic stability of such systems. The principle is illustrated by model examples of systems consisting of two electric machines. The mathematical formulations for the physical terms „the electrical network of infinite power“ and „the generator of infinite power“ are also obtained.

In a paper by Rodyukov (1993) there is obtained the final form for the equations, common both for non-salient-pole synchronous machines with structural damping circuits and without (in that case there are introduced so-called fictitious damping circuits taking into account the electromagnetic field of the eddy currents of the ferromagnetic rotor). These equations are more correct than the well-known Park-Gorev equations because they correspond more closely to the original idealized physical model of the synchronous machine (SM), (Rodyukov, 1990, 1991, 1992). The equations reflect more exactly the dynamic processes of SMs and therefore they are simpler than the Park-Gorev equations and symmetrical. Criteria of truth such as simplicity and symmetry are here shown intuitively.

The equations in the projections on the orthogonal coordinate axis, rotating with any angular velocity, have the following form:

$$\dot{\psi}_u = \dot{\gamma}_k \psi_\upsilon - \alpha_s \psi_u \pm u_u$$

$$\dot{\psi}_\upsilon = -\dot{\gamma}_k \psi_u - \alpha_s \psi_\upsilon \pm u_\upsilon$$

$$\mu \dot{i}_u = \dot{\psi}_u + (1 - s - \dot{\gamma}_k)(\psi_\upsilon - \mu i_\upsilon) + \varepsilon_{rf}\left[\psi_u - i_u - u_f \cos(\tau - \theta - \gamma_k)\right]$$

$$\mu \dot{i}_\upsilon = \dot{\psi}_\upsilon + (1 - s - \dot{\gamma}_k)(\psi_u - \mu i_u) + \varepsilon_{rf}\left[\psi_\upsilon - i_\upsilon - u_f \cos(\tau - \theta - \gamma_k)\right] \tag{1}$$

$$\dot{\theta} = s\gamma = \tau - \theta\nu = \gamma_k - \gamma = \gamma_k - \tau + \theta$$

$$\dot{s} = \delta\left[\varepsilon_s \left(b^2 + u_f^2\right)s - \left(\psi_u i_\upsilon - \psi_\upsilon i_u \mp M_m\right)\right]$$

$$i_f = \frac{\alpha_f}{\alpha_r + \alpha_f}\left\{u_f + \frac{\alpha_r}{\alpha_f}\left[(\psi_u - i_u)\cos(\tau - \theta - \gamma_k) + (\psi_\upsilon - i_\upsilon)\sin(\tau - \theta - \gamma_k)\right]\right\}$$

where

$$\psi_u = i_u + u_f \cos\nu + i_{ru}$$

$$\psi_\upsilon = i_\upsilon - u_f \sin\nu + i_{r\upsilon}$$

$$\psi_f = (1 - \mu)(i_u \cos\nu - i_\upsilon \sin\nu) + i_f + i_{ru}\cos\nu - i_{r\upsilon}\sin\nu$$

$$\psi_{ru} = (1 - \mu) i_u + u_f \cos\nu + i_{ru}$$

$$\psi_{r\upsilon} = (1 - \mu) i_\upsilon - u_f \sin\nu + i_{r\upsilon}$$

Equations (1) are written in nondimensional form. Here ψ_u, ψ_υ, i_u, i_υ are the flux quasilinkages and quasicurrents of the stator windings, i_f, u_f the current and voltage of the excitation windings, $\psi_{ru}, \psi_{r\upsilon}$, i_{ru}, $i_{r\upsilon}$ the flux quasilinkages and quasicurrents of the damping circuits, $\alpha_s = \varepsilon_s / \mu$, α_r, α_f resistance of the stator and rotor windings and the excitation windings, $\alpha_{rf} = \alpha_r \alpha_f / (\alpha_r + \alpha_f)$, $\mu = 1 - M_{sr}^2 / (L_s L_r)$ the coefficient of the electromagnetic scattering in the air gap of SMs (L_s, L_r are the dimensional inductances in the stator and rotor windings and M_{sr} the dimensional amplitude value of the mutual inductance between them), $\tau = \omega_s t$ is the synchronous time ($\omega_s = 314\ rad / sek$), γ_k the angle of the auxiliary orthogonal coordinate system onto which are projected the stator and rotor variables, γ is the rotor rotation angle, θ the load angle, s the slippage, $s = 1 - \dot{\gamma}$. The quantity δ is the electromechanic coefficient inversely proportional to the moment of inertia of the rotor, $b = (1 - \mu)/\mu$, M_m is the mechanical moment on the rotor shaft. A point above the variable means differentiation with respect to the synchronous time τ . The quantities u_u and u_υ are unknown voltages on the stator windings of the generator. Upper marks correspond to the motor mode and lower marks to the generator mode.

Equations (1) have elementary solutions for different modes of SMs, when at the same time the solutions of the Park-Gorev equations for these modes are very complicated. To examine the stability of these equations is also much simpler than that of the Park-Gorev equations.

But the greatest advantage of equations (1) compared with the Park-Gorev equations is obtained when using them to examine *systems* of electrical machines (EMs). In this case because of the great power of electroenergetical systems (EESs) and the gravity of the consequences of accidents it is necessary to have a complete mathematical model of EESs including the complete equations of the EMs. But the Park-Gorev equations because of their complexity and non-symmetry lead in this case to substantial mathematical difficulties. Therefore to avoid these difficulties the electrical processes in the Park-Gorev equations are taken as steady-state ones. Thus it is possible to examine the stability of EESs but not the electromagnetic processes.

Using equations (1) for the description of EESs allows to avoid the mathematical difficulties arising when using Park-Gorev equations for these purposes and thus allows to examine both the electromagnetic and the electromechanic transient processes and also the stability of systems consisting of EMs.

This we demonstrate on an example of a directly coupled synchronous generator (SG) and a synchronous motor (SM). We will suppose that both EMs have damping circuits. In this case the term $\varepsilon_s (b^2 + u_f^2) s$ can be neglected in equations (1).

It is not difficult to show that when connecting EMs to a system it is more convenient to write equations (1) for both machines in the axis rigidly tied to the SG (synchronous generator) rotor. The variables and parameters of the SG we give the index „1" and of the SM the index „2". Then $(\gamma_k = \gamma_1 \Rightarrow \dot{\gamma}_k = \dot{\gamma}_1 = 1 - \dot{\theta}_1 = 1 - s_1)$

$$\dot{\psi}_{d1} = (1 - s_1)\psi_{q1} - \alpha_{s1}\psi_{d1} - u_d$$

$$\dot{\psi}_{q1} = -(1 - s_1)\psi_{d1} - \alpha_{s1}\psi_{q1} - u_q$$

$$\mu_1 \dot{i}_{d1} = \dot{\psi}_{d1} + \varepsilon_{rf1}(\psi_{d1} - i_{d1} - u_{f1})$$

$$\mu_1 \dot{i}_{q1} = \dot{\psi}_{q1} + \varepsilon_{rf1}(\psi_{q1} - i_{q1}) \qquad (2)$$

$$\dot{\theta}_1 = s_1$$

$$\dot{s}_1 = -\delta_1(\psi_{d1} i_{q1} - \psi_{q1} i_{d1} + M_{rot})$$

$$\dot{\psi}_{d2} = (1 - s_1)\psi_{q2} - \alpha_{s2}\psi_{d2} + u_d$$

$$\dot{\psi}_{q2} = -(1 - s_1)\psi_{d2} - \alpha_{s2}\psi_{q2} + u_q$$

$$\mu_2 \dot{i}_{d2} = \dot{\psi}_{d2} - (s_2 - s_1)(\psi_{q2} - \mu_2 i_{q2}) + \varepsilon_{rf2}[\psi_{d2} - i_{d2} - u_{f2}\cos(\theta_2 - \theta_1)]$$

$$\mu_2 \dot{i}_{q2} = \dot{\psi}_{q2} + (s_2 - s_1)(\psi_{d2} - \mu_2 i_{d2}) + \varepsilon_{rf2}[\psi_{q2} - i_{q2} - u_{f2}\sin(\theta_2 - \theta_1)] \quad (3)$$

$$\dot{\theta}_2 = s_2$$

$$\dot{s}_2 = -\delta_2(\psi_{d2} i_{q2} - \psi_{q2} i_{d2} - M_m)$$

In (2) M_{rot} is the rotation moment in the rotor shaft of the SG.
We see that the equations (3) remain as equations with constant coefficients even if they are not written in the projections to their own *d-q*-axis (which the Park-Gorev equations require) but to the *d-q*-axis of the SG rotor.

To the equations (2) and (3) it is necessary to add relations on the currents according to the first law of Kirchhoff. In dimensional form they look like

$$i_{d1} = i_{d2} \qquad i_{q1} = i_{q2}$$

Passing to nondimensional form we get

$$\frac{u_m}{\omega_s L_{s1}}\bar{i}_{d1} = \frac{u_m}{\omega_s L_{s2}}\bar{i}_{d2} \qquad and \qquad \frac{u_m}{\omega_s L_{s1}}\bar{i}_{q1} = \frac{u_m}{\omega_s L_{s2}}\bar{i}_{q2}$$

(u_m is the amplitude value of the grid voltage), from which

$$\bar{i}_{d1} = k\,\bar{i}_{d2} \qquad \bar{i}_{q1} = k\,\bar{i}_{q2} \qquad (k = L_{s1} / L_{s2}) \quad (4)$$

Below in equations (4) (as in (1) to (3)) we will omit the lines over the nondimensional variables.

The systems (2) to (4) are simple enough (compared with the system obtained by the use of Park-Gorev equations). However, because the electromechanic parameters in it are determined with a large error (more than 10%), but the order of the parameters are small, it is necessary to make some simplifications in the system.

It is elementary to show that the steady-state mode in practice is not dependent on the value of the parameters $\alpha_s, \mu, \varepsilon_{rf}$ if only the condition

$$\alpha_{s1}^2 << 1 \qquad and \qquad \alpha_{s2}^2 << 1$$

is fulfilled (the parameters $\mu_1, \mu_2, \varepsilon_{rf1}, \varepsilon_{rf2}$ are not present in the steady-state solution). That is, in such a mode they can be taken correspondingly equal.
We extend this principle to the transient processes of the systems (2) to (4) and call in the equivalentation principle for EMs.
Thus the equivalentation principle for EMs means that the corresponding parameters in EMs are taken equal, but in the equations (4) of the connections the parameter *k* is kept, leaving the possibility of taking into account the effect of the relations of the powers included in EES EMs on the mode of these systems.
It can be shown that the slippage s_1 in the equations for the flux quasilinkages is not essentially affecting the transient processes, and we will therefore neglect it in these equations.
Equations (2) to (4) can now be written in the form

$$\dot{\psi}_{d1}+\dot{\psi}_{d2}=\psi_{q1}+\psi_{q2}-\alpha_s(\psi_{d1}+\psi_{d2})$$

$$\dot{\psi}_{q1}+\dot{\psi}_{q2}=-(\psi_{d1}+\psi_{d2})-\alpha_s(\psi_{q1}+\psi_{q2})$$

$$\dot{\psi}_{d1}=\psi_{q1}-\alpha_s\psi_{d1}-u_d$$

$$\dot{\psi}_{q1}=\psi_{d1}-\alpha_s\psi_{q1}-u_q$$

$$\mu k\dot{i}_{d2}=\dot{\psi}_{d1}+\varepsilon_{rf}(\psi_{d1}-ki_{d2}-u_{f1}) \qquad (5)$$

$$\mu k\dot{i}_{q2}=\dot{\psi}_{q1}+\varepsilon_{rf}(\psi_{q1}-ki_{q2})$$

$$\mu\dot{i}_{d2}=\dot{\psi}_{d2}-(s_s-s_1)(\psi_{q2}-\mu i_{q2})+\varepsilon_{rf}\left[\psi_{d2}-i_{d2}-u_{f2}\cos(\theta_2-\theta_1)\right]$$

$$\mu\dot{i}_{q2}=\dot{\psi}_{q2}+(s_s-s_1)(\psi_{d2}-\mu i_{d2})+\varepsilon_{rf}\left[\psi_{q2}-i_{q2}+u_{f2}\sin(\theta_2-\theta_1)\right]$$

$$\dot{\theta}_2-\dot{\theta}_1=s_2-s_1$$

$$\dot{s}_2-\dot{s}_1=-\delta_2(\psi_{d2}i_{q2}-\psi_{q2}i_{d2}-M_m)+\delta_1\left[k(\psi_{d1}i_{q2}-\psi_{q1}i_{d2})+M_{rot}\right]$$

The first two of equations (5) are integrated independently and therefore we get the relations between ψ_{d1} and ψ_{d2} and ψ_{q1} and ψ_{q2}.

$$\psi_{d2}=-\psi_{d1}+A \qquad \textit{and} \qquad \psi_{q2}=-\psi_{q1}+B$$

where

$$A=e^{-\alpha_s\tau}\left\{\left[\psi_{d1}(0)+\psi_{d2}(0)\right]\cos\tau+\left[\psi_{q1}(0)+\psi_{q2}(0)\right]\sin\tau\right\}$$

$$B=e^{-\alpha_s\tau}\left\{-\left[\psi_{d1}(0)+\psi_{d2}(0)\right]\sin\tau+\left[\psi_{q1}(0)+\psi_{q2}(0)\right]\cos\tau\right\}$$

and $\psi_{d1}(0)$, $\psi_{d2}(0)$, $\psi_{q1}(0)$ and $\psi_{q2}(0)$ are initial conditions.
Substituting that form of ψ_{d2}, ψ_{q2} in the last of equations (5) under the sign of the small $\delta_2 << 1$ we do averaging on an infinite interval of time. That is, in these equations we assume $\psi_{d2}=-\psi_{d1}$ and $\psi_{q2}=-\psi_{q1}$.
In the steady-state mode we get $M_{rot}=k\,M_m$. We introduce the concept of *ideal regulator* of the primary motor SG, which immediately realizes the dependence. In a neighbourhood of the steady state of the system SG-SM we will assume $M_{rot}=k\,M_m$.
The four equations for the currents in equations (5) we use to determine the unknowns u_d and u_q.
When we have done all the operations mentioned, substituted the expressions for u_d and u_q in the equations for ψ_{d1} and ψ_{q1} respectively and introduced the simplifying notations

$$\psi_{d1}=\psi_d \qquad \psi_{q1}=\psi_q \qquad i_{d2}=i_d \qquad i_{q2}=i_q$$

$$\theta_2-\theta_1=\theta \qquad s_2-s_1=s \qquad \varepsilon_{rf}=\varepsilon_r \qquad \delta_2+k\delta_1=\delta$$

the system (5) takes the final form

$$\dot{\psi}_d = -\varepsilon_r \psi_d + \frac{1}{1+k}\left\{k(B-\alpha_s A) + \varepsilon_r\left[u_{f1} + k\left(A - u_{f2}\cos\theta\right)\right] + k s\left(\psi_q + \mu \dot{i}_q - B\right)\right\}$$

$$\dot{\psi}_q = -\varepsilon_r \psi_q + \frac{1}{1+k}\left\{-(A+\alpha_s B) + \varepsilon_r\left(u_{f2}\sin\theta + B\right) - s\left(\psi_d + \mu \dot{i}_d - A\right)\right\}$$

$$k\mu \dot{i}_d = \dot{\psi}_d + \varepsilon_r\left(\psi_d - k i_d - u_{f1}\right)$$

$$k\mu \dot{i}_q = \dot{\psi}_q + \varepsilon_r\left(\psi_q - k i_q\right)$$

$$\dot{\theta} = s \tag{6}$$

$$\dot{s} = \delta\left(\psi_d i_q - \psi_q i_d + M_m\right)$$

$$u_d = \psi_q - \alpha_s \psi_d + \varepsilon_r \psi_d - \frac{1}{1+k}\left\{\varepsilon_r\left(u_{f1} - k u_{f2}\cos\theta\right) + k\left[B - \alpha_s A + \varepsilon_r A + s\left(\psi_q + \mu \dot{i}_q - B\right)\right]\right\}$$

$$u_q = -\psi_d - \alpha_s \psi_q + \varepsilon_r \psi_q - \frac{1}{1+k}\left[-A - \alpha_s B + \varepsilon_r\left(u_{f2}\sin\theta + B\right) - s\left(\psi_d + \mu \dot{i}_d - A\right)\right]$$

As mentioned above, the parameters α_s, ε_r and δ in equations (6) are small.
Their maximal order in SMs are

$$\alpha_s \sim 10^{-2} \qquad \varepsilon_r \sim 10^{-3} \qquad \delta \sim 10^{-4}$$

This is essential in the analysis of these equations, because the change in s is very small compared with the change in the electromagnetic quantities. Therefore on the analysis of the electromagnetic processes in the first four of equations (6) s can be considered as a constant, the value of which can be taken from the steady-state mode, that is $s = 0$. Then $\theta = \theta_0 =$ constant. Thus the electric equations are split from the whole system and integrated independently. Moreover they are integrated as four independent equations of the first order.

Integrating the electric equations and substituting the expressions for ψ_d, ψ_q, i_d and i_q as functions of time into the expressions for u_d and u_q, we get $u_d(\tau, \theta_0)$ and $u_q(\tau, \theta_0)$ for different electromagnetic transient processes.

The electric variables are rapidly damped compared with the oscillations of the rotor. If we introduce a new auxiliary time $\tau' = \delta\tau$ the small parameter δ will appear in front of the derivatives of the electric variables. It is easy to verify the five conditions of the Tihkonov theorem (Vasil'eva and Butuzov, 1973) for such systems and go over from the system (6) to the shortened system consisting of two differential equations with respect to θ and s and four algebraic equations (thereby A and B have to be set to zero)

$$0 = -\varepsilon_r \psi_d + \frac{1}{1+k}\left[\varepsilon_r\left(u_{f1} - k u_{f2}\cos\theta\right) + sk\left(\psi_q + \mu i_q\right)\right]$$

$$0 = -\varepsilon_r \psi_q + \frac{1}{1+k}\left[\varepsilon_r u_{f2}\sin\theta - s\left(\psi_d + \mu i_d\right)\right]$$

$$0 = \psi_d - k i_d - u_{f1}$$

$$0 = \psi_q - k i_q \tag{7}$$

$$\dot{\theta} = s$$

$$\dot{s} = \delta\left(\psi_d i_q - \psi_q i_d + M_m\right)$$

It is more convenient to express ψ_d and ψ_q in i_d and i_q using the third and fourth of equations (7) and substitute them into the other equations. After elementary calculations we get

$$0 = -(1+k)\varepsilon_r i_d + s(k+\mu)i_q - \varepsilon_r\left(u_{f1} + u_{f2}\cos\theta\right)$$

$$0 = -s(k+\mu)i_d - (1+k)\varepsilon_r i_q - s u_{f1} + \varepsilon_r u_{f2}\sin\theta \tag{8}$$

$$\dot{\theta} = s$$

$$\dot{s} = \delta\left(u_{f1} i_q + M_m\right)$$

From the first two of equations (8) we get

$$i_q = \frac{\varepsilon_r u_{f1}}{det}\left\{s\left[-(1-\mu)u_{f1} + (k+\mu)u_{f2}\cos\theta\right] + \varepsilon_r(1+k)u_{f2}\sin\theta\right\}$$

where $det = \varepsilon_r^2(1+k)^2 + s^2(k+\mu)^2$.

Substituting this expression for i_q into the fourth of equations (8) to examine the oscillations of the so-called equivalent synchronous machine, described by the system (6) we finally get the equations

$$\dot{\theta} = s$$

$$\dot{s} = \delta\left\{\frac{\varepsilon_r u_{f1}}{det}\left\{s\left[-(1-\mu)u_{f1} + (k+\mu)u_{f2}\cos\theta\right] + \varepsilon_r u_{f2}(1+k)\sin\theta\right\} + M_m\right\} \tag{9}$$

We examine the static stability of the equivalent synchronous machine, or, which is the same, of the system SG-SM.

In the steady-state mode

$$M_m = -\frac{u_{f1}u_{f2}}{1+k}\sin\theta_0 \tag{10}$$

The equation for small oscillations around the equilibrium assumes the form

$$\ddot{\tilde{\theta}} + \frac{\delta u_{f1}}{\varepsilon_r (1+k)^2} \left\{ \left[(1-\mu) u_{f1} - (k+\mu) u_{f2} \cos\theta_0 \right] \dot{\tilde{\theta}} - \varepsilon_r (1+k) u_{f2} \tilde{\theta} \cos\theta_0 \right\} = 0 \tag{11}$$

For the negativeness of the real parts of both roots of the characteristic equation corresponding to the differential equation, both the coefficients have to be greater than zero. But in our situation it is enough to require the condition $\cos\theta_0 < 0$.
This is enough for the positiveness of the coefficients.

The mechanical moment on the rotor shaft of the SM must be positive and that is why it is necessary to require

$$\sin\theta_0 \le 0$$

Finally for the condition of static stability of the system SG-SM it is necessary that both the conditions

$$\begin{cases} \sin\theta_0 \le 0 \\ \cos\theta_0 < 0 \end{cases}$$

are fulfilled. These two conditions are satisfied for an angle θ_0 in the third quadrant

$$\pi \le \theta_0 < 3\pi/2$$

The corresponding equilibrium for the system SG-SM will be asymptotically stable. From equations (6) it is possible to get a system of equations describing an asynchronous machine, equivalent to a system SG-AM (asynchronous motor). To do that we have to set $u_{f2} = 0$ and omit the fifth equation.

All the integrations above for the system (6) are valid also for the new system. Therefore in order to get the equation for the oscillations of the rotor of the equivalent asynchronous machine we set $u_{f2} = 0$ in equations (9) and omit the first equation

$$\dot{s} = \delta \left[-\frac{\varepsilon_r s (1-\mu)}{\varepsilon_r^2 (1+k)^2 + s^2 (k+\mu)^2} u_{f1}^2 + M_m \right] \tag{12}$$

In the steady-state mode

$$M_m = \frac{\varepsilon_r s (1-\mu)}{\varepsilon_r^2 (1+k)^2 + s^2 (k+\mu)^2} u_{f1}^2$$

The equation for small oscillations around the equilibrium will have the form

$$\dot{\tilde{s}} = -\delta \tilde{s} \frac{\varepsilon_r (1+k)^2 - s_0 (k+\mu)^2}{\varepsilon_r (1+k)^2 + s_0 (k+\mu)^2} \varepsilon_r (1-\mu) u_{f1}^2 \tag{13}$$

For the negativeness of the root of the corresponding characteristic equation it is necessary to require the condition

$$s_0^2 (k+\mu)^2 < \varepsilon_r^2 (1+k)^2 \qquad \text{or} \qquad s_0 < \frac{\varepsilon_r (1+k)}{k+\mu} \tag{14}$$

This condition is the condition for asymptotical stability of the corresponding static equilibrium for the system SG-AM.

As conclusion we give some remarks about the coefficient u_d and u_q and the possibility to consider the dynamics of a separate EM, that is outside the EES.

From equations (6) one can write

$$u_d \approx \psi_q \qquad \text{and} \qquad u_q \approx -\psi_d$$

From the steady-state mode of the system (6) we find

$$u_d \approx \psi_q = \frac{k}{1+k} u_{f2} \sin\theta$$

$$u_q \approx -\psi_d = -\frac{1}{1+k}\left(u_{f1} - k u_{f2} \cos\theta\right) \tag{15}$$

If k tends to zero then

$$u_d \to 0 \qquad u_q \to -u_{f1} \qquad \text{for } k \to 0$$

Thus for $k << 1$ the function of the motor does not affect the grid (the generator), that is we have a generator and a grid of infinite power with respect to the motor.
If we let $u_d = 0$ and $u_q = -u_{f1}$ in the original system (2) and (3) then from equations (2) we get $\theta_1 \equiv 0$ and $s_1 \equiv 0$ but equations (3) give the equations of an SM working from a grid (of the generator) of infinite power.

If we in the final conclusions about the equivalent SM and AM set $k = 0$, $\theta_1 = 0$, $s_1 = 0$ then these conclusions will be for SM and AM working from a grid of infinite power.

If k tends to infinity in the expressions (15) we get

$$u_d \to u_{f2} \sin\theta \qquad u_q \to u_{f2} \cos\theta \qquad \mathit{for} \quad k \to \infty$$

Substituting these expressions into equations (2) for the SG and setting $\theta_2 = 0$ we get an independent system of equations for an SG working on a grid of infinite power.

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