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Asymptotic Investigation of the Nonlinear Dynamic Boundary Value Problem for Rods

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An asymptotic procedure for a quasilinear dynamic boundary value problem is proposed. The method is based on the introduction of an artificial small parameter and its use for obtaining a simple approximate analytic solution.

1 Introduction

Asymptotic approaches for nonlinear dynamics of continuous systems are well developed for the infinite in spatial variables (Kevorkian and Cole, 1981; Nayfeh, 1981; Hinch, 1991). For systems of finite size we have an infinite number of resonances, and the Poincaré-Lighthill-Go method (Kevorkian and Cole, 1981; Nayfeh, 1981; Hinch, 1991) does not work. The use of an of averaging procedure (Mitrolol'sky et al., 1991) or the method of multiple scales (Lau et al., 1989) leads to infinite systems of nonlinear algebraic or ordinary differential equations, and a subsequent truncation method does not provide the possibility to obtain all important properties of the solutions (see below). The method of normal forms (Miloserdova and Potapov, 1983) is a very interesting approach for the two dimensional case. In this paper we use an asymtotic procedure which is based on the introduction of an artificial small parameter (Andrianov et al., 1994).

2 Asymptotic Procedure 1 - Using Natural Small Parameter

Let us assume a governing boundary value problem in the following form

$$\frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial \tau^2} = - \varepsilon U^3 \tag{1}$$

where all variables are nondimensional, and ε is a nondimensional small parameter $(\varepsilon << 1)$. From the physical point of view we have longitudinal vibrations of a rod with nonlinear drag. Let us introduce a change of variable

$$t = \omega \tau \tag{2}$$

We will now search for solutions using the ansatzes

$$\begin{aligned} U &= U_0 + \varepsilon U_1 + \varepsilon^2 U_2 + \cdots \\ \omega &= 1 + \varepsilon \omega_1 + \varepsilon^2 \omega_2 + \cdots \end{aligned} \tag{3}$$

After substituting expressions (2) and (3) into the governing boundary value problem (1) and splitting it with respect to ε one obtains

$$\frac{\partial^2 U_0}{\partial x^2} - \frac{\partial^2 U_0}{\partial t^2} = 0 \tag{4}$$

$$\frac{\partial^2 U_1}{\partial x^2} - \frac{\partial^2 U_1}{\partial t^2} = 2\omega_i \frac{\partial^2 U_0}{\partial t^2} - U_0^3 \tag{5}$$

The solution of equation (4) may be written in the form

$$U_0 = C_1 \sin x \sin t + C_2 \sin 2x \sin 2t + \cdots = \sum_{i=1}^{\infty} C_i \sin ix \sin it \tag{6}$$

Here C_1 is the amplitude of the fundamental oscillation, while the constants C_i for $i>1$ provide the next approximations. After routine but cumbersome transformations we arrive at the following infinite nonlinear algebraic equations

$$\begin{aligned} -32\omega_1 C_1 \sin x \sin t &= \sum_{a=1}^{\infty}\sum_{b=1}^{\infty} C_a C_b \left(3C_{a+b-1} + 3C_{a+b+1}\right) + 6C_1 \sum_{a=1}^{\infty} C_a^2 \\ -32\omega_1 i^2 C_i \sin ix \sin it &= \sum_{a=1}^{\infty}\sum_{b=1}^{\infty} C_a C_b \left(3C_{a+b-i} + 3C_{a+b+i} + C_{i-a-b}\right) + 6C_i \sum_{a=1}^{\infty} C_a^2 \end{aligned} \tag{7}$$

3 Asymptotic Procedure 2 - Introduction of Artificial Small Parameter

Systems like (7) may be obtained in various ways (Mitrolol'sky et al., 1991; Lau et al., 1989), and the main problem in this approach consists in its solution. Truncation of the infinite system (7) cannot give any information about resonances of higher order. We propose to introduce an artificial small parameter μ, writing it near all nondiagonal members of the system (7), and represent the unknown coefficients as expansions.

$$\begin{aligned} C_n &= C_n^{(0)} + C_n^{(1)}\mu + C_n^{(2)}\mu^2 + \cdots \qquad n = 2, 3, \cdots \\ \omega_1 &= \omega_1^{(0)} + \omega_1^{(1)}\mu + \omega_1^{(2)}\mu^2 + \cdots \end{aligned} \tag{8}$$

After splitting with respect to μ solutions may be obtained routinely. It may be easily shown that for even n

$$C_n^{(k)} = 0$$

and

$$\omega_1^{(0)} = -0.281250\, C_1^2 \qquad \omega_1^{(1)} = -0.001438\, C_1^2$$

$$C_3^{(0)} = 0.0144927\, C_1 \qquad C_5^{(0)} = 0.0002071\, C_1 \qquad C_7^{(0)} = 0.0000030\, C_1$$

4 Numerical Example

Numerical results (dependencies of fundamental nondimensional frequency ω upon nondimensional amplitude C_1) are displayed in Figure 1 for various values of the small parameter ε.

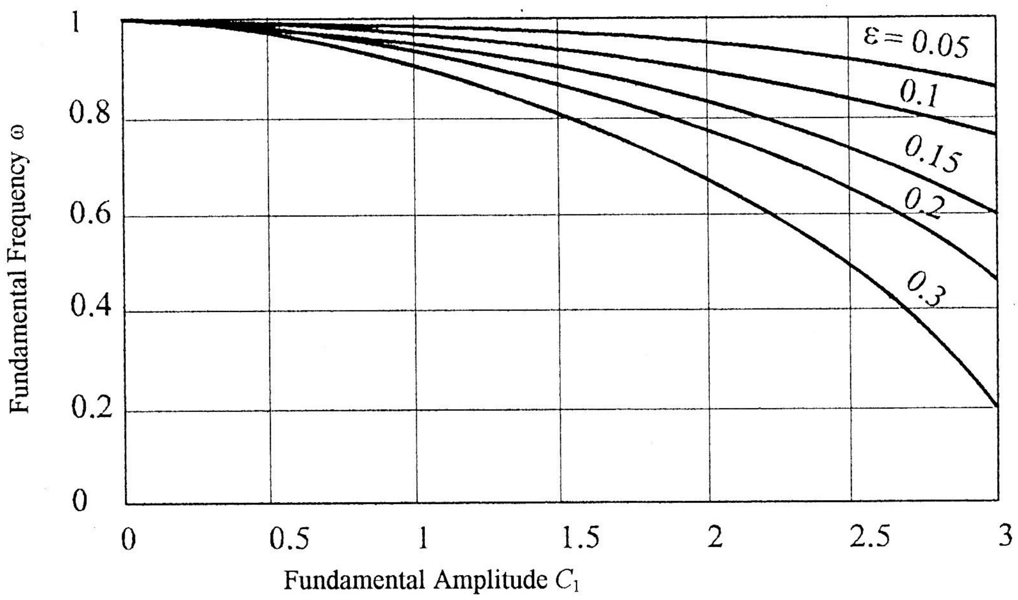

Figure 1. Amplitude-frequency dependencies for fundamental oscillation for various values of small parameter ε

5 Concluding Remarks

Various problems for nonlinear dynamic boundary value problems for continuous systems, such as rods, beams, plates, shells, may be solved effectively on the basis of the approach presented. The introduction of an artificial small parameter may also be useful for perturbed eigenvalue problems with multiple roots (Hinch, 1991).

Literature

1. Andrianov, I. V.; Gristchak, V. Z.; Ivankov, A. O.: New asymptotic method for the natural, free and forced oscillations of rectangular plates with mixed boundary conditions, Technische Mechanik 14, 3/4, (1994), 185-192.

2. Hinch, E. J.: Perturbation Methods, Cambridge University Press, (1991).

3. Kevorkian, J.; Cole, J. D.: Perturbation Methods in Applied Mathematics, New York, Springer-Verlag, (1981).

4. Lau, S. L.; Cheung, Y. K.; Chen, Chuhui: An alternative perturbation procedure of multiple scales for nonlinear dynamics systems, J. Appl. Mech. 56, N 3, (1989), 587-675.

5. Miloserdova, I. V.; Potapov, A. I.: Nonlinear standing waves in the rod of finite length, Journal of Acoustics 29, N 4, (1983), 515-520, (in Russian).

6. Mitrolol'sky, Yu. A.; Homa, G. P.; Gromak, M. I.: Asymptotic methods of investigation of quasi wave equations of hyperbolic type, Kiev, Naukova Dumka, (1991), (in Russian).

7. Nayfeh, A. H.: Introduction to Perturbation Techniques, New York, Wiley & Sons, (1981).

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