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Locally Coupled Analysis of Damage of Elastic-Perfectly Plastic Material with Crack Closure Effect

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In this paper, the basic equations describing the elastic-perfectly plastic damageable metal material are presented. The principle of locally coupled analysis of damage (connected with the critical material point) for cyclicly loaded material is described. The DAMAGE code is used for the evaluation of the damage of elastic-perfectly plastic material under 3D loading. The input data for this program are the tensor of deformation in the critical point of material as a function of loading history and material properties. The algorithm used in the program DAMAGE is also described.

1 Elements of Damage Mechanics

As a lot of papers dealing with the theory of continuous damage mechanics have already been published (Lemaitre, 1992), only the principal features used to build a model of ductile plastic and quasi brittle damage are summarised here.

1.1 Damage Variable

Consider a damaged body in which a volume element at macroscale level has been isolated, and let S be the overall section area of that element defined by its normal $\mathbf{n}$, S_D the total area of intersection of micro cracks and cavities in section S and $\tilde{S}$ the effective resisting area (Figure 1).

$$\tilde{S} < S - S_D$$

According to the concept of effective stress associated with the hypothesis of strain equivalence is described below. By definition, the damage variable D associated with the normal $\mathbf{n}$ is

$$D_n = \frac{S - \tilde{S}}{S}$$

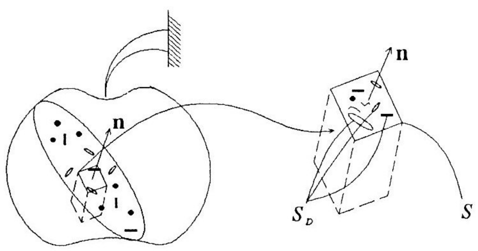

Figure 1. Damaged element

1.2 Hypothesis of Isotropy

In the general case, cracks and voids are oriented and D_n is a function of $\mathbf{n}$. This leads to an intrinsic variable of damage which can be a second order tensor (Cordebois and Sidloroff, 1982) or a fourth order tensor (Chaboche, 1978) depending upon the hypothesis made. In this paper we restrict ourselves to isotropic damage, the cracks and voids being equally distributed in all directions. Thus D_n does not depend upon $\mathbf{n}$ and the intrinsic damage variable is the scalar D.

1.3 Concept of Effective Stress

If $\mathbf{F}$ is the load acting on the section S of the element considered in Figure 1, $\mathbf{T} = \mathbf{F}/S$ is the usual stress vector which leads to the Cauchy stress tensor $(\sigma \cdot \mathbf{n} = \mathbf{T})$. The quantity $\tilde{S} = S(1-D)$ is the effective area which effectively carries the load $\mathbf{F}$. By definition the effective stress vector is

$$\tilde{\mathbf{T}} = \frac{\mathbf{F}}{\tilde{S}} = \frac{\mathbf{T}}{1-D}$$

which, since D is a scalar, leads to the effective stress tensor ($\tilde{\sigma} \cdot \mathbf{n} = \tilde{\mathbf{T}}$)

$$\tilde{\sigma} = \frac{\sigma}{1-D} \tag{1}$$

1.4 Hypothesis of Strain Equivalence

It is assumed that the strain behaviour is modified by damage only through the effective stress (Lemaitre, 1971). The strain behaviour of a damaged material is represented by constitutive equations of the virgin material (without any damage) in the potential of which the stress is simply replaced by the effective stress.

1.5 Thermodynamic Potential

Taking the free-energy ψ as thermodynamic potential, it is assumed that it is a convex function of all observable and internal variables. Using the hypothesis that the elasticity and plasticity behaviours are uncoupled gives (Lemaitre, 1992) gives

$$\psi_e = \frac{1}{2\rho}\mathbf{a}:\varepsilon^e:\varepsilon^e(1-D) \tag{2}$$

Where $\mathbf{a}$ is the linear elasticity tensor and ε^e is the elastic strain tensor. The damaged elasticity law is

$$\sigma = \rho\frac{\partial \psi_e}{\partial \varepsilon^e} = \mathbf{a}:\varepsilon^e(1-D) \tag{3}$$

and the variable Y associated with D, by the power dissipated $(Y\dot{D})$ in the phenomenon of damage, is defined by

$$Y = -\rho\frac{\partial \psi_e}{\partial D} = \frac{1}{2}\mathbf{a}:\varepsilon^e:\varepsilon^e \tag{4}$$

1.6 Damage Criterion

The density of elastic strain energy dw_e being defined as

$$dw_e = \sigma : d\varepsilon^e$$

If we replace $d\varepsilon^e$ by its value taken from the damage elasticity law written for $d\sigma = 0$ at constant temperature, one can see that Y is one half of the variation of dw_e due to an infinitesimal increase of damage at constant stress and temperature. This gives for Y the name of " damage strain energy release rate" (Chaboche, 1978):

$$Y = -\frac{1}{2}\frac{dw_e}{dD}\bigg|_{\sigma,T} \tag{5}$$

1.7 Crack Closure Effect

For certain materials and certain conditions of loading, the defects may close in compression. This is often the case for very brittle materials. If the defects close completely, the area which effectively carries load in compression is no longer $\widetilde{S}$, but S . In fact, the real defects of complicated shapes do not close completely. The effective area in compression $\widetilde{S}^-$ is such that

$$S - S_D < \widetilde{S}^- < S$$

Let as write this expression as $\widetilde{S}^- = S - hS_D = S(1-Dh)$, where h $(0 \le h \le 1)$ is a crack closure parameter which depends upon the material and the loading (Lemaitre, 1992).

In this case is more convenient to work with dual transformation of the state potential specific free energy, i. e. the Gibbs specific enthalpy ψ_e^*, which is given by

$$\psi_e^* = \frac{1}{2\rho(1-D)}\sigma:\mathbf{a}^{-1}:\sigma = \frac{1}{2E\rho(1-D)}\left[(1+\nu)\sigma_{ij}\sigma_{ij} - \nu\sigma_{kk}^2\right]$$

where E and ν are Young's modulus and Poisson's ratio of the material.
The Gibbs specific enthalpy written as a function of $\langle\sigma_{ij}\rangle$ or $\langle-\sigma_{ij}\rangle$ and crack closure parameter h is:

$$\psi_e^* = \frac{1}{2\rho E(1-D)}\left[(1+\nu)\langle\sigma_{ij}\rangle\langle\sigma_{ij}\rangle - \nu\langle\sigma_{kk}\rangle^2\right] + \frac{1}{2\rho E(1-Dh)}\left[(1+\nu)\langle-\sigma_{ij}\rangle\langle-\sigma_{ij}\rangle - \nu\langle-\sigma_{kk}\rangle^2\right]$$

where $\langle\sigma_{ij}\rangle = \sigma_{ij}$ if $\sigma_{ij} \ge 0$,

$\langle\sigma_{ij}\rangle = 0$ if $\sigma_{ij} < 0$.

Then the damage strain energy release rate is written as

$$Y = \frac{(1+\nu)}{2E}\left[\frac{\langle\sigma_{ij}\rangle\langle\sigma_{ij}\rangle}{(1-D)^2} + \frac{h\langle-\sigma_{ij}\rangle\langle-\sigma_{ij}\rangle}{(1-Dh)^2}\right] - \frac{\nu}{2E}\left[\frac{\langle-\sigma_{kk}\rangle^2}{(1-D)^2} + \frac{h\langle-\sigma_{kk}\rangle^2}{(1-Dh)^2}\right] \tag{6}$$

1.8 Potential of Dissipation

Restricting ourselves to isotropic plasticity and isotropic damage, ductile plastic damage, as plasticity, is a phenomenon which does not depend explicitly upon time. Within these hypothesis the main features of ductile plastic damage can be described by a potential of dissipation restricted to the three variables (Lemaitre, 1992; Germain et al, 1983)

$$F_D(Y;(\dot{p},D)),$$

from which the damage rate is derived

$$\dot{D} = \frac{\partial F_D}{\partial Y} \dot{p}(1-D)$$

The variable $\dot{p}$, which governs the damage evolution, also gives the irreversible nature of the damage, as $\dot{p}$ is always positive or zero.

$$\dot{p} = \left(\frac{2}{3} \dot{e}_{ij}^{p} \dot{e}_{ij}^{p}\right)^{½} \tag{7}$$

In our work we used following damage potential (Lemaitre, 1992):

$$F_D\,(Y;(\dot{p},D)) = \frac{Y^2}{2S(1-D)} H_{(p-p_D)} \tag{8}$$

Then

$$\dot{D} = \frac{Y}{S} \dot{p} H_{(p-p_D)} \tag{9}$$

with the rupture condition for crack initiation,

$$D = D_C \tag{10}$$

S → energy strength of damage
p_D → damage threshold function the material and loading,

$$H_{(p-p_D)} = \left\{ \begin{array}{l} 1 \;\; if \;\; p \geq p_D \\ 0 \;\; if \;\; p < p_D \end{array} \right. \tag{11}$$

1.9 Damage Threshold

The damage threshold p_D (or ε_{p_D} in one dimension) corresponds to a nucleation of microcracks which does not produce any change in the material properties. The formula, which gives the value of the damage threshold p_D for any kind of loading and for elastic-perfectly plastic material is (Lemaitre, 1992)

$$p_D = e_{p_D} \frac{\sigma_u - \sigma_f}{\sigma_{eq} - \sigma_f} \tag{12}$$

with

$$\sigma_{eq} = \left(\frac{3}{2} \sigma_{ij}^{D} \sigma_{ij}^{D}\right)^{½}$$

where

σ_{ij}^{D} → deviatoric part of σ_{ij}
ε_{pD} → plastic threshold
σ_u → ultimate stress
σ_f → fatigue limit

1.10 Rupture Criterion

The rupture criterion in pure tension is

$$\frac{\sigma_R}{1-D_{1c}}=\sigma_u$$

σ_R → radial stress at rupture at the pure tension test

D_{1c} → the critical value of the damage for the case of reference under tension

In three dimensions, for the case of perfect plasticity in proportional loading we obtain a function for the critical damage D_c like (Lemaitre, 1992)

$$D_c=D_{1c}\left(\frac{\sigma_u}{\tilde{\sigma}_{eq}}\right)^2\frac{1}{R_v}\leq 1 \tag{13}$$

where

$$\tilde{\sigma}_{eq}=\frac{\sigma_{eq}}{1-D}$$

$$R_v=\frac{2}{3}(1+v)+3(1-2v)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2 \tag{14}$$

v → Poisson's ratio

σ_H → hydrostatic stress

2 Quasi-Brittle Damage

When the behaviour is brittle at the mesoscale but localised damage growth occurs at the microscale, we are dealing with quasi-brittle damage. Consider a mesovolume element exhibiting elastic behaviour everywhere except in a small microvolume μ representing a weak defect subjected to elasticity, plasticity and damage (Figure 2).

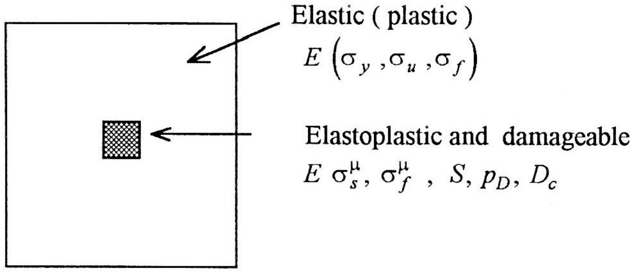

Figure 2. Two-scale volume element

The matrix is elastic with a yield stress σ_y and a fatigue limit σ_f . The inclusion has the same properties as the matrix except that it is perfectly plastic with a plastic threshold σ_s^{μ} and a fatigue limit σ_f^{μ} . Its weakness comes from the value of the plastic threshold, which may be taken equal to the fatigue limit of the material, as it is the lowest stress giving rise to possible damage $\sigma_s^{\mu}=\sigma_f$.

Furthermore, the weakness also comes from the fatigue limit σ_f^{μ} assumed to be reduced in the same proportion as the plastic threshold

$$\sigma_f^{\mu}=\sigma_f\frac{\sigma_s^{\mu}}{\sigma_y} \tag{15}$$

The complete inclusion problem will be solved numerically by the DAMAGE code using "locally coupled analysis". Here, however, some approximations allow us to derive the rupture conditions without resolution of the complete set of constitutive equations. Starting with the kinetic damage law for the inclusion,

$$\dot{D} = \frac{Y^{\mu}}{S}\dot{p}^{\mu}$$

we wish to express Y^{μ} and $\dot{p}^{\mu}$ as functions of macroscopic quantities such as the elastic strain ε and the stress σ. According to the Lin-Taylor hypothesis, we may assume that the inclusion is subjected to the state of strain (or strain rate) of the matrix, which is taken to be uniform

$$\dot{\varepsilon}^{\mu} = \dot{\varepsilon}$$

Neglecting the elastic strain $\varepsilon^{e\mu}$ in comparison to the plastic strain $\varepsilon^{p\mu}$ in the inclusion allows us to write

$$\dot{p}^{\mu} = \left(\frac{2}{3}\dot{\varepsilon}^{p\mu}_{ij}\,\dot{\varepsilon}^{p\mu}_{ij}\right)^{½} = \left(\frac{2}{3}\dot{\varepsilon}^{\mu D}_{ij}\,\dot{\varepsilon}^{\mu D}_{ij}\right)^{½} = \left(\frac{2}{3}\dot{\varepsilon}^{D}_{ij}\,\dot{\varepsilon}^{D}_{ij}\right)^{½} = \dot{\varepsilon}_{eq} \tag{16}$$

where ε^{D}_{ij} is the deviatoric part of ε_{ij} and the damage strain energy release rate is

$$Y^{\mu} = \frac{\sigma^{\mu^2}_{eq}\,R^{\mu}_{v}}{2E(1-D)^2}$$

The inclusion being perfectly plastic, then from the yield criterion,

$$\frac{\sigma^{\mu}_{eq}}{1-D} = \sigma^{\mu}_{s} \tag{17}$$

Finally,

$$\dot{D} = \frac{\sigma^{\mu^2}_{s}}{2ES}\left[\frac{2}{3}(1+\nu)+3(1-2\nu)\left(\frac{\sigma_H}{\sigma^{\mu}_{s}}\right)^2\right]\dot{\varepsilon}_{eq} \quad \text{if} \quad \varepsilon_{eq} \geq p_D \tag{18}$$

with

$$p_D = \varepsilon_{pD}\frac{\sigma_u - \sigma_f}{\sigma^{\mu}_{s} - \sigma^{\mu}_{f}} \qquad \sigma^{\mu}_{f} = \sigma_f\frac{\sigma^{\mu}_{s}}{\sigma_y} \qquad \sigma^{\mu}_{s} = \sigma_f \tag{19), (20), (21}$$

3 High Cycle Fatigue

If the amplitude of the loading is low, the amplitude of the plastic strain may be very small, even negligible at the mesoscale in comparison with the amplitude of the elastic strain. This corresponds to high values of the number of cycles to failure.For instance N_R >100 000.

Another feature which makes the damage analysis of high cycle fatigue difficult is its high degree of localisation. Very often only a very small microelement damaged at the free surface of the body gives rise to one microcrack by slips which later propagates perpendicularly to the loading. Then, for high cycle fatigue, materials may be considered as quasi-brittle and modelled by a damageable microinclusion embedded in an elastic mesoelement. Its complete numerical analysis, including high cycle fatigue, is performed by the DAMAGE code

4 Locally Coupled Analysis

Quite often, the damage is so localised that the volume of the damaged material is small in comparison to the macroscale of the structural component. This allows us to perform an uncoupled analysis at the macroscale and to consider the coupling between strain and damage only on the small volume of the critical point as shown schematically in Figure 3. This is the case of smallscale damage. The method of locally coupled analysis may often be used with good accuracy for brittle and fatigue types of damage.

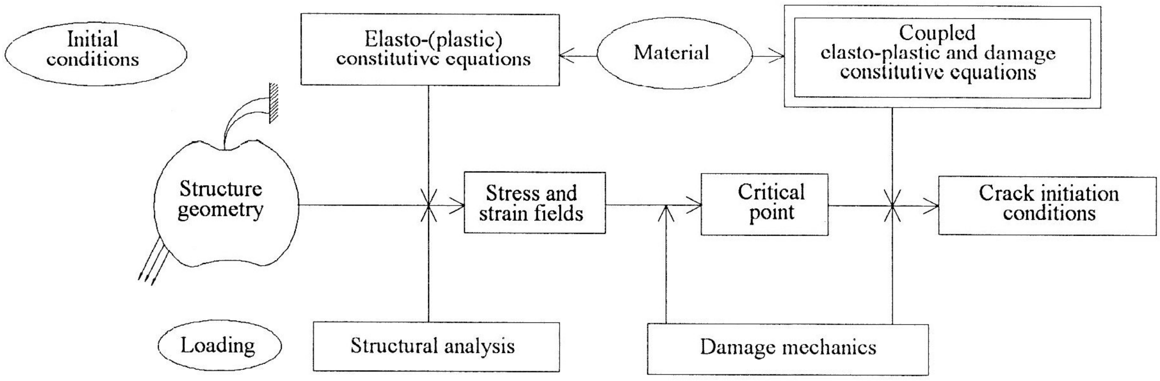

Figure 3. Locally coupled analysis of crack initiation

4.1 Localisation of Damage

Damage localisation results from stress concentration, of course, but also occurs because some weakness always exists at the microscale. Let us generalise, for any kind of damage, what has been said for quasi-brittle materials. The mechanical model was a two scale volume element, elastic or elastoplastic at the mesoscale and elastoplastic and damageable at the microscale (Figure 2). The only material characteristics which differ in the matrix and in the inclusion are the yield stress of the inclusion σ_s^μ and its fatigue limit σ_f^μ. This has taken into account the microinternal stresses and the weak defects always existing everywhere in all materials. The second assumption which simplifies calculations is the Lin-Taylor strain compatibility hypothesis $\varepsilon^\mu = \varepsilon$.

5 Description of the Postprocessor DAMAGE

The DAMAGE code was built on the base of code DAMAGE 90 (I. Doghri, 1990) published in (Lemaitre, 1992). We assume the material perfectly plastic at the microscale with considering the microcrack closure effect (material parameter h). In this code we may use for piecewise perfect plasticity, several values of plastic threshold σ_s^μ as the loading or the timelike parameter vary. This allows us to take some cases of high values of strainhardening and cyclic stress strain curves for multilevel fatigue processes into consideration. Then the material parameters must be considered as follows:

E and ν for elasticity,

σ_f , σ_y and σ_u for plasticity,

$\sigma_s^\mu = \sigma_f$ for pure plasticity,

$\sigma_s^\mu = \sigma_{s(t)}$ given as a input piecewise plasticity,

S, ε_{P_D}, D_{1c}, h for damage.

The input of the calculation is the time history of the strain components $\varepsilon_{ij\,(t)}$. We can use two loading cases:

— general loading history where the loading history is defined by the value of the strain components at the given timelike parameter values. DAMAGE interpolates linearly between these values.

— piecewise periodic loading for which the loading is a certain number of blocks of cycles defined by the

two consecutive maximum and minimum set of strain components and the number of cycles in each block. The strain interpolations are also linear. It is capable of accounting for initial values of damage D_0 and plastic strain p_0.

6 Applications

The DAMAGE code was applied for the computation of the number of cycles after which the damage of material reached the critical value D_c in the critical point of considered body (in a single Gauss integration point) subjected to loading. The results were compared with the data gained from the Manson-Coffin curve for low cycle fatigue and from the Wöhler curve for high cycle fatigue of material STN 12 060.1 (carbon steel for refinement) (Bodnáv, 1993). The following material properties were used for the computation:

E	=	200300 MPa	ε_{P_D}	=	0.3
ν	=	0.3	D_{1c}	=	0.4
σ_f	=	189 MPa	p_0	=	0
σ_y	=	250 MPa	D_0	=	0
σ_u	=	621.4 MPa	h	=	0.2

S = 12 MPa for low cycle fatigue and S= 34 MPa for high cycle fatigue

σ_s^μ was obtained from a cyclic strain curve (Bodnár, 1993).

The loading was 3-dimensional with

$\varepsilon_{max} = +\varepsilon_{11}$ and $\varepsilon_{min} = -\varepsilon_1$

$\varepsilon_{22} = \varepsilon_{33} = \pm\nu\varepsilon_{11}$

$\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0$

The comparison of the computed data with the experimentally given ones implies that the utilisation of damage mechanics for the solution of fatigue processes gives a good agreement between of theory and practice. The advantages of damage mechanics are displayed markedly when the possibility of combination of cycles with different nonsymmetry coefficients and different amplitudes under the 3-dimensional loading is taken into consideration.

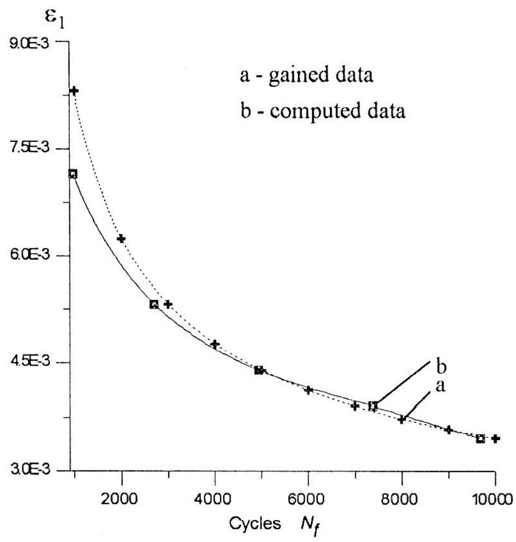

Figure 4. Comparison of Low Cycle Fatigue

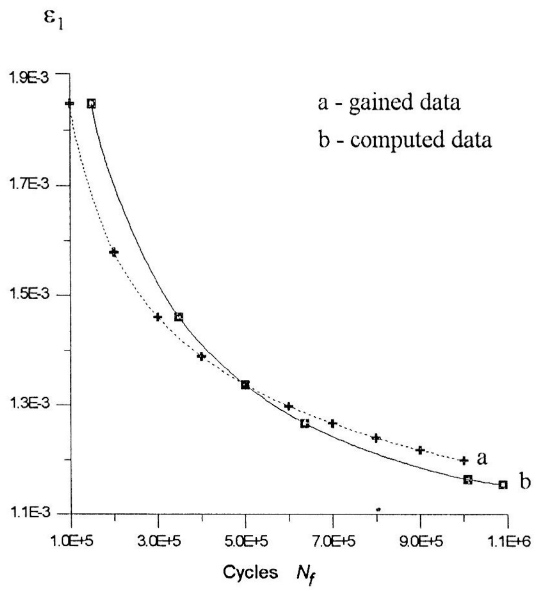

Figure 5. Comparison of High Cycle Fatigue

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