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Laminar Boundary Layer Flow and Heat Transfer along a Moving Cylinder with Suction or Injection

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The steady boundary layer flow and heat transfer over an isothermal longitudinal circular cylinder which is moving in a viscous incompressible fluid at rest is studied. It is assumed that a uniform suction or injection of fluid can take place through the cylinder surface. The two-dimensional boundary layer equations are solved numerically using an efficient finite-difference method, and velocity and temperature profiles, as well as skin friction and heat transfer coefficients are calculated. It is shown that fluid suction/injection can affect the flow and heat transfer characteristics considerably.

1 Introduction

The problem of boundary layer flow and heat transfer over a moving or stretching surface is of great importance in view of its relevance to a wide variety of technical applications, particularly in the manufacture of fibres in glass and polymer industries. The investigation of drag and heat transfer in such situations belongs to a separate class of problems in boundary layer theory, distinguishing itself from the study of flows over static surfaces. The boundary layer behavior on moving surfaces in a viscous fluid at rest was first considered by Sakiadis (1961), whose work was subsequently extended by Rotte and Beek (1969), Bourne and Elliston (1970), Crane (1972), Karnis and Pechoc (1978), Lin and Shih (1980), Choi (1982), and Eswara and Nath (1992). On the other hand, Pop et al. (1990) have studied the problem of boundary layers past a moving longitudinal cylinder in a non-Newtonian power-law fluid at rest. However, none of the above papers has dealt with the possibility of mass transfer through the cylinder wall. If the wall of the cylinder is porous or perforated, fluid at a prescribed temperature can be blown into the boundary layer (injection) or fluid at the wall surface can be withdrawn (suction). These mass transfer processes may measurably alter the flow and heat transfer characteristics.

The aim of the present analysis is to extend the problem of boundary layer flow and heat transfer over a moving longitudinal circular cylinder in a viscous fluid to the case where fluid injection or suction can take place through the cylinder wall with the intention of controlling the boundary layer characteristics. The transverse curvature of the cylinder and the mass transfer bring nonsimilarity into the governing equations. The transformed partial differential equations involving two independent variables are approximated by nonlinear ordinary differential equations using a very efficient finite-difference method as described by Katagiri (1969). Solutions of the ordinary differential equations are expressed in a form of integral equations which are then solved numerically using Simpson's rule. This scheme proved to be stable, accurate and efficient compared with other methods (e. g. local similarity and perturbation solutions (Lin and Shih, 1980). The effects of the mass transfer parameter σ on the velocity and temperature profiles are studied and the skin friction and heat transfer coefficients are calculated. It is proved that fluid suction or injection at the wall affects the skin friction and heat transfer rate considerably. Finally, it is worth mentioning that our results compare very well with those of Lin and Shih (1980), who used a different method for zero mass transfer $(\sigma = 0)$.

2 Basic Equations

Consider the steady flow of a viscous and incompressible fluid along a longitudinal cylinder of radius r_0, which moves with the constant velocity U. It is assumed that the surface of the cylinder is at a uniform temperature T_w, which is greater than the ambient temperature T_∞. All over the cylinder's surface, fluid is sucked in or ejected with a constant radial velocity v_w. We use the coordinates x and r, where the x-axis is taken along the cylinder axis and the r-axis is in the radial direction, as shown in Figure 1.

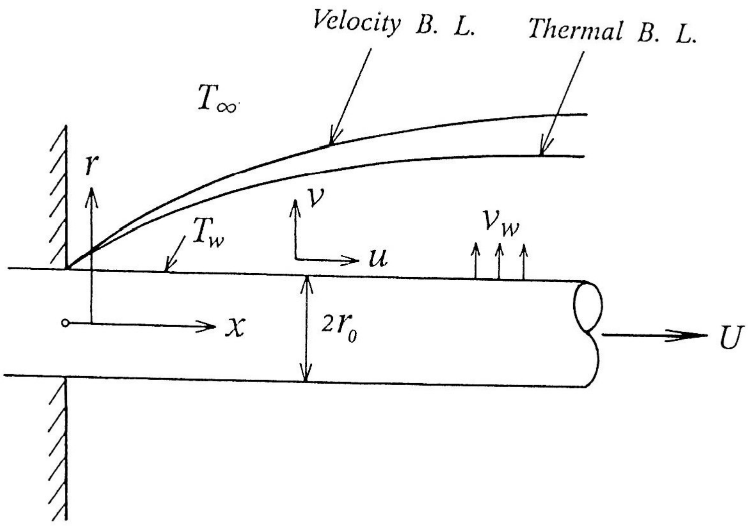


Figure 1. Continuous Moving Cylinder

Under the assumption of boundary layer theory, the governing equations are

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \qquad (3)$$

where u and v are the axial and radial velocity components, T is the fluid temperature, ν is the kinematic viscosity and α is the thermal diffusivity of the fluid. The boundary conditions of the problem are

for $r = r_0$ $\quad u = U \quad v = v_w \quad T = T_w$ (4)

and for $r \to \infty$ $\quad u = 0 \quad T = T_\infty$

where v_w is the velocity of suction or injection, when either $v_w < 0$ or $v_w > 0$, respectively. In order to facilitate a numerical solution, we introduce the following dimensionless boundary layer variables:

The axial coordinate parameter $\quad \xi = \frac{4}{r_0}(\nu x / U)^{1/2}$ (5a)

the similarity variable $\quad \eta = \frac{r^2 - r_0^2}{\xi r_0^2}$ (5b)

and the temperature parameter $\quad \theta(\xi,\eta) = (T - T_\infty)/(T_w - T_\infty)$ (5c)

Further there is the stream function

$$\psi = r_0(\nu x U)^{1/2} f(\xi,\eta) \tag{6a}$$

defined by

$$ru = \frac{\partial\psi}{\partial r} \quad \text{and} \quad rv = -\frac{\partial\psi}{\partial x} \tag{6b}$$

so that using equations (5), the velocity components become

$$u = \frac{1}{2}U\frac{\partial f}{\partial\eta}$$
$$v = -\frac{2\nu}{r_0\xi(1+\xi\eta)^{1/2}}\left(f+\xi\frac{\partial f}{\partial\xi}-\eta\frac{\partial f}{\partial\eta}\right) \tag{7}$$

Thus, the set of equations (1) and (2) is transformed to

$$(1+\xi\eta)\frac{\partial^3 f}{\partial\eta^3} + (\xi+f)\frac{\partial^2 f}{\partial\eta^2} = \xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta} - \frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right) \tag{8}$$

$$\frac{(1+\xi\eta)}{\Pr}\frac{\partial^2\theta}{\partial\eta^2} + \left(\frac{\xi}{\Pr}+f\right)\frac{\partial\theta}{\partial\eta} = \xi\left(\frac{\partial\theta}{\partial\xi}\frac{\partial f}{\partial\eta} - \frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right) \tag{9}$$

and the boundary conditions (4) become, for $\xi \geq 0$,

at $\quad \eta = 0 \quad \frac{\partial f}{\partial\eta} = 2 \quad f+\xi\frac{\partial f}{\partial\xi} = -2\sigma\xi \quad \theta = 1$ (10)

and at $\quad \eta\to\infty \quad \frac{\partial f}{\partial\eta} = 0$ and $\theta = 0$

where $\quad \sigma = r_0 v_w/4\nu$ (11)

is the fluid suction $(\sigma < 0)$ or fluid injection $(\sigma > 0)$ parameter. Using Gregory-Newton backward finite-differences, the solution of equations (8) and (9) subject to the boundary conditions (10) can be expressed as (see Katagiri, 1969)

$$\frac{df_i}{d\eta} = 2+\int_0^\eta E(\eta)\int_0^\eta\frac{F(\eta)}{E(\eta)}d\eta d\eta - \left\{2+\int_0^\infty E(\eta)\int_0^\eta\frac{F(\eta)}{E(\eta)}d\eta d\eta\right\}\frac{G(\eta)}{G(\infty)} \tag{12}$$

$$f_i = -(ih)\sigma + \int_0^\eta\frac{df_i}{d\eta}d\eta \tag{13}$$

$$\theta_i = 1 + \int_0^\eta\frac{Q(\eta)}{P(\eta)}d\eta d\eta - \left\{1+\int_0^\infty P(\eta)\int_0^\eta\frac{Q(\eta)}{P(\eta)}d\eta d\eta\right\}\frac{R(\eta)}{R(\infty)} \tag{14}$$

where

$$E(\eta) = \exp\left(-\int_0^\eta\left\{\frac{1}{1+ih\eta}\left(ih+f_i+\frac{i}{6}(11f_i-18f_{i-1}+9f_{i-2}-2f_{i-3})\right)\right\}d\eta\right) \tag{15}$$

$$F(\eta) = \frac{1}{1+ih\eta}\frac{i}{6}\left(11\frac{df_i}{d\eta} - 18\frac{df_{i-1}}{d\eta} + 9\frac{df_{i-2}}{d\eta} - 2\frac{df_{i-3}}{d\eta}\right)\frac{df_i}{d\eta} \tag{16}$$

$$G(\eta) = \int_0^{\eta} E(\eta)d\eta \tag{17}$$

$$P(\eta) = \exp\left[-\int_0^{\eta}\left\{\frac{1}{1+ih\eta}\left(ih + Pr\, f_i + \frac{i}{6}Pr\,(11f_i - 18f_{i-1} + 9f_{i-2} - 2f_{i-3})\right)\right\}d\eta\right] \tag{18}$$

$$Q(\eta) = \frac{Pr}{1+ih\eta}\frac{i}{6}(11\theta_i - 18\theta_{i-1} + 9\theta_{i-2} - 2\theta_{i-3})\frac{df_i}{d\eta}$$

(19)

$$R(\eta) = \int_0^{\eta} P(\eta)d\eta \tag{20}$$

and where h is the step size. The dimensionless skin friction coefficient and the local Nusselt number are defined as

$$C_f = \frac{\tau_w}{pU^2} \quad \text{and} \quad Nu = \frac{Q}{k(T_w - T_\infty)} \tag{21}$$

where the skin friction τ_w and the local heat transfer Q per unit length of the cylinder are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial r}\right)_{r=r_0} \quad \text{and} \quad Q = -2\pi r_0 k\left(\frac{\partial T}{\partial r}\right)_{r=r_0} \tag{22}$$

ξ	Present results	Lin and Shih (1980)
0.0	- 1.77499	- 1.77497
0.05	- 1.79401	- 1.79077
0.1	- 1.81289	- 1.80622
0.5	- 1.96076	- 1.93135
1.0	- 2.13911	- 2.10325
1.5	- 2.31154	- 2.24323
2.0	- 2.47914	- 2.41258

Table 1. Comparison of the Skin Friction Coefficient *SFP* for $\sigma = 0$

After some calculations, we obtain the following skin friction and heat transfer parameters

$$SFP = 4C_f Re_x^{1/2} = \frac{\partial^2 f}{\partial \eta^2}(\xi, 0) = -\left\{1+\int_0^\infty E(\eta)\int_0^\eta \frac{F(\eta)}{E(\eta)}d\eta d\eta\right\}\frac{1}{G(\infty)} \tag{23}$$

$$HTP = \left(\frac{\xi}{4\pi}\right)Nu = -\frac{\partial\theta}{\partial\eta}(\xi,0) = \left\{1+\int_0^\infty P(\eta)\int_0^\eta \frac{Q(\eta)}{P(\eta)}d\eta d\eta\right\}\frac{1}{R(\infty)} \tag{24}$$

where $Re_x = U_x / \nu$ is the local Reynolds number.

ξ	Present results $Pr = 0.72$	1.0	10	Lin and Shih (1980) 0.72	1.0	10
0.0	0.71216	0.88749	3.36058	0.71217	0.88749	3.36059
0.05	0.72134	0.89700	3.37064	0.71948	0.89526	3.36934
0.1	0.73043	0.90644	3.38069	0.72692	0.90314	3.37813
0.5	0.80208	0.98034	3.46079	0.78852	0.96724	3.44792
1.0	0.88990	1.06955	3.55996	0.86611	1.04736	3.53217
1.5	0.97585	1.15577	3.65804	0.95695	1.13854	3.61983
2.0	1.06009	1.23957	3.75500	1.03830	1.22279	3.70241

Table 2. Comparison of Heat Transfer Coefficient *HTP* for $\sigma = 0$

3 Results and Discussion

The numerical scheme used for the solution of equations (12) to (14) consists of applying Simpson's rule. Since this method is described in detail in the papers by Katagiri (1969), Pop and Watanabe (1992, 1994) and Watanabe and Pop (1995) its description is omitted here. Results are obtained for various values of the axial coordinate ξ with the suction or injection parameter σ ranging from -2.0 to 2.0 and for the Prandtl number Pr equal to 0.72 (air), 1.0 and 10, respectively. In order to assess the accuracy of our method, the particular case of zero mass transfer $(\sigma = 0)$ of our results for the skin friction and heat transfer coefficients has been compared in Tables 1 and 2 with those of Lin and Shih (1980). It is seen from these tables that the present results are in good agreement. However, some differences are noted, which can be attributed to the use by Lin and Shih of the local similarity method, i.e. deleting the terms containing partial derivatives with respect to ξ in equations (8) and (9), and considering ξ as a parameter.

Sample results presented here consist of velocity and temperature profiles, as well as of skin friction and heat transfer coefficients for various combinations of the parameters ξ, σ and Pr. However, the results presented in Figures 2 to 11 refer to air ($Pr = 0.72$) only. We notice that the effect of suction is to reduce the velocity and temperature profiles. Fluid injection, on the other hand, increases these profiles. Further, Table 1 and Figure 10 show that the skin friction coefficient decreases as suction increases while it increases with the increase of injection. The contrary happens for the heat transfer coefficient, see Figure 11. It should also be noted that when $\sigma = 0$ (zero mass transfer) our results compare excellently with those of Crane (1972).

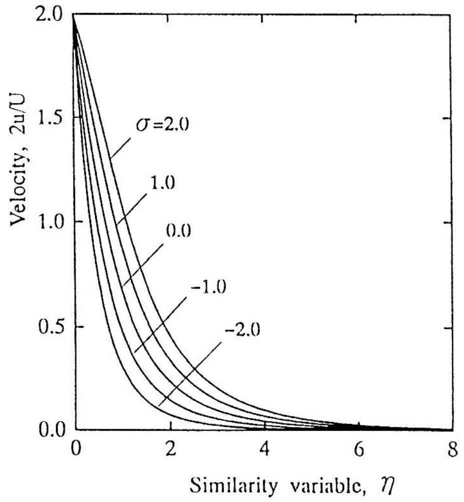


Figure 2. Velocity Profiles for $\xi = 0.5$

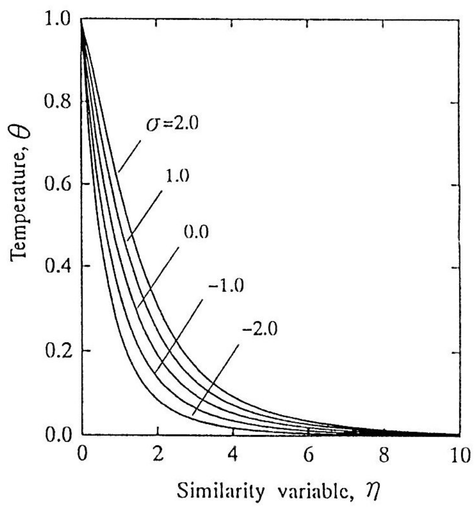


Figure 3. Temperature Profiles for $\xi = 0.5$ when $Pr = 0.72$

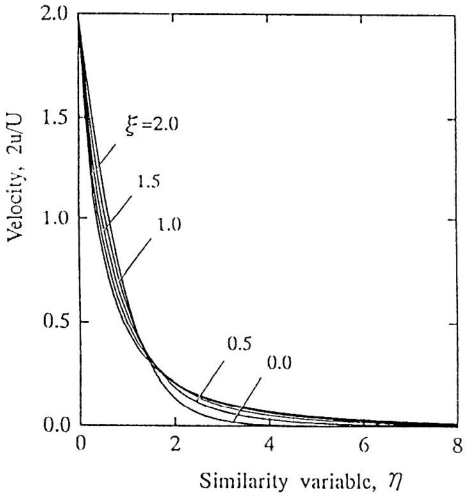


Figure 4. Velocity Profiles for $\sigma = -0.5$

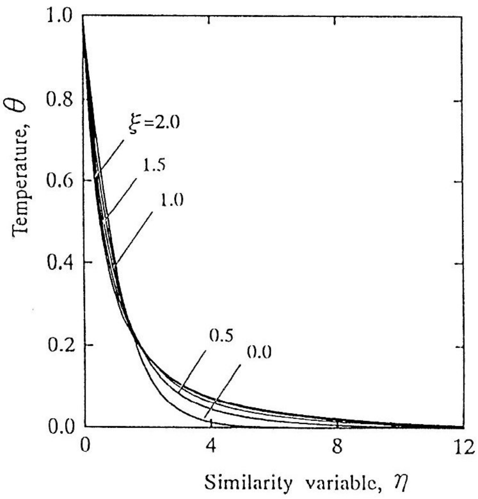


Figure 5. Temperature Profiles for $\sigma = -0.5$ when $Pr = 0.72$

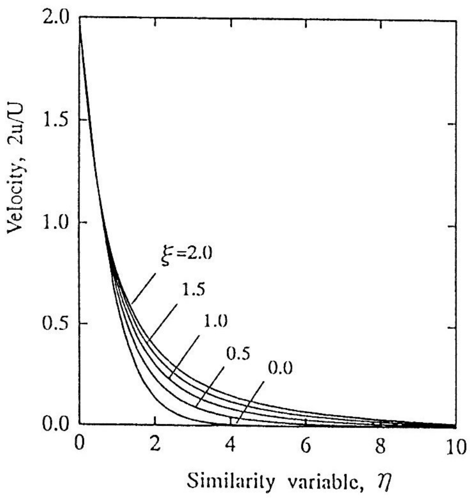


Figure 6. Velocity Profiles for $\sigma = 0$

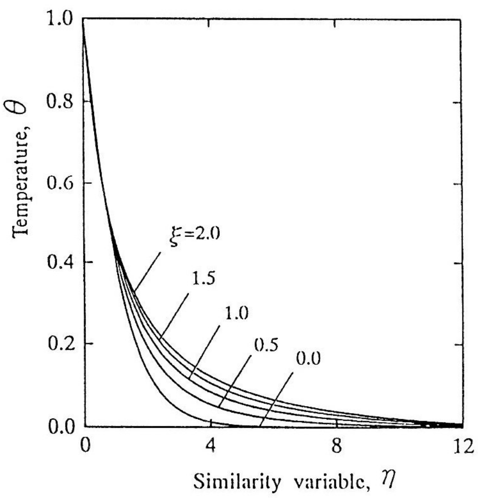


Figure 7. Temperature Profiles for $\sigma = 0$ when $Pr = 0.72$

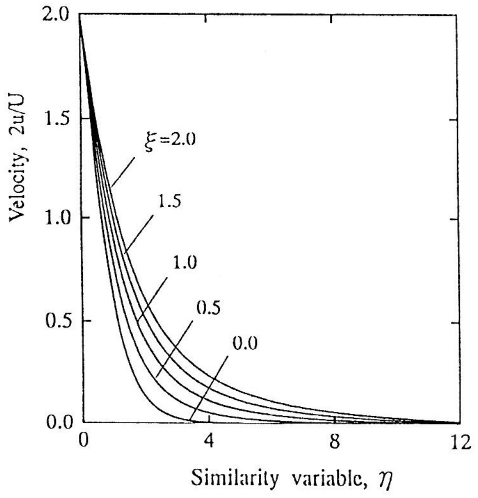


Figure 8. Velocity Profiles for σ = 0.5

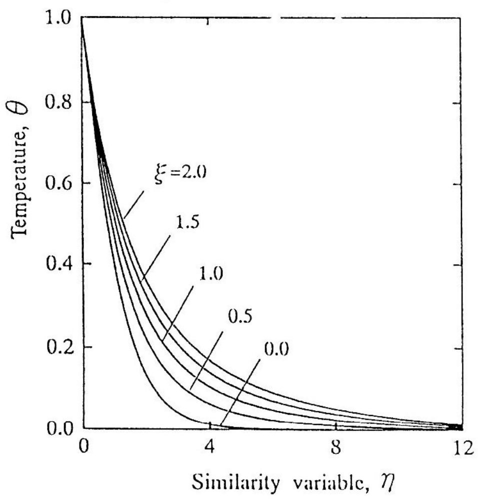


Figure 9. Temperature profiles for σ = 0.5 when $Pr = 0.72$

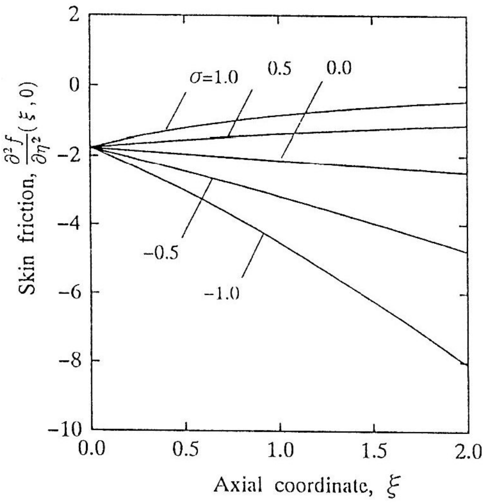
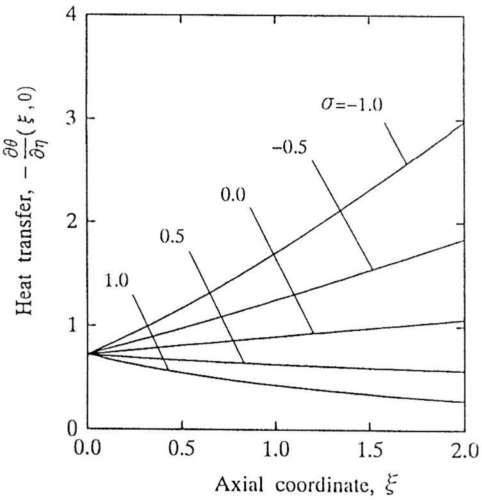


Figure 10. Skin Friction Coefficient versus ξ



11. Heat Transfer Coefficient versus ξ

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