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# Asymptotic Methods in the Statics and Dynamics of Perforated Plates and Shells with Periodic Structures

I. V. Andrianov, V. V. Shevchenko, E. G. Kholod

*An analytical solution, describing homogenized coefficients for static and dynamic equations for periodically perforated domains, has been obtained by asymptotic methods and two-point Padé approximants.*

## 1 Introduction

The analysis of perforated plates and shells is of significant practical value: a lot of problems, arising in machine design, civil engineering etc., can be reduced to perforated plates and shells. The problems posed are often solved using numerical methods such as finite element procedures. Nevertheless, a numerical approach does not adequately fit the requirements of optimal structural design ideology. Then approximate analytical expressions, if they are accurate enough, will be of great practical advantage for these needs.

The presence in the solutions of slow and fast varying parts is the main obstacle on the way of numerical investigation of perforated structures. In many physical problems, some variables vary slowly, others fast. It is natural then to pose the question whether it might not be appropriate first to study a global structure, digressing from its local distinctive features, and then to investigate the system locally. It is the homogenization method that is aimed at a division into fast and slow components of the solution. Without going into detail - the more because the method has at present a lot of modifications - it will be noted only that it involves the introduction of „slow" (macroscopic) and the „fast" (microscopic) variables whose equations are separated and can be solved independently, or sequentially.

This method was developed for and gained wide use in solving problems in celestial mechanics and in the nonlinear oscillation theory, which are characterized by common differential equations. At present, the method is used with great advantage for solving variable coefficient partial differential equations in such disciplines as the theory of composites, or the design of reinforced, corrugated, perforated, etc. shells (Andrianov et al., 1983, 1985, 1988, 1991; Bakhvalov and Panasenko, 1989; Bensoussan et al., 1978; Berdichevsky, 1983; Bourgat, 1979; Caillarie, 1984; Cioranescu and Paulin, 1979; Duvaut, 1977; Kalamkarov, 1993; Lewinsky and Telega, 1988; Lions, 1980, 1982; Mignot et al., 1981; Nazarov and Paukshto, 1984; Oleynik et al., 1986; Sanchez-Palencia, 1980; Suquet, 1980; Vanninathan, 1981). An original nonhomogeneous medium or structure is reduced to a homogeneous one (generally anisotropic) with some effective characteristics. The homogenization method allows not only to obtain effective characteristics but also to investigate nonhomogeneous distributions of mechanical stresses in different materials and structures, which is of great significance for evaluating their strength. Then the main idea of the method is based on a separation of „fast" and „slow" variables. As a start, a certain periodic boundary problem is formulated („cell" or „local" problem) and its solution, assuming periodic continuation of boundary conditions, is obtained. For that purpose the local coordinates („fast" variables, in the case of the multiscaling method) are introduced. After that averaging upon local („fast") coordinates is performed. The approach presented fills the substantial gap between numerical methods of thin shell theory, which methods lack generality and the possibility to grasp the common features of behavior of the structures concerned, and approximate design schemes, based on heuristic hypotheses. Methods proposed are wideranging in applications and lead to simple and clear design formulae, useful for practical analyses. The aforesaid opens new prospects in the analysis of new important problems arising in modern engineering and not yet solved fully and effectively enough.

The theory of homogenization has been developed for perforated media by many authors in recent years (see above). Mathematical foundations of the method have already been established.

The main task in this field then is in solving the so-called cell (or local) problem. This problem has been usually treated by numerical methods. We have used asymptotic methods (perturbation of the domain size and perturbation of boundary conditions, singular perturbation) and two-point Padé approximants for solving the cell problem and have developed the approach in this paper.

The present paper covers the following problems:

- Bending of rectangular plates with periodic square perforations
- Eigenvalue problem of perforated plates
- Analytical approach for a large hole
- Matching of asymptotic expansions by means of two-point Padé approximants
- The plane theory of elasticity in the perforated domain
- Perforated shallow shells

## 2 Bending of Rectangular Plates with Periodic Square Perforations

We consider the biharmonic equation

$$D\nabla^4 W = P(x,y) \tag{1}$$

in domain $G$ which consists of a perforated medium with a large number of square holes which are arranged in a periodic manner with period $2a$ (Figure 1).

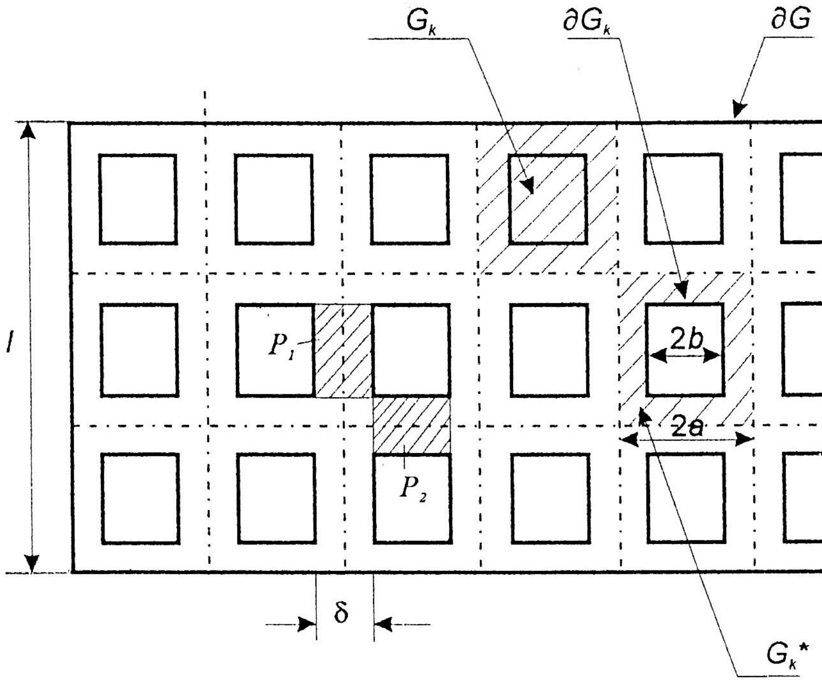


Figure 1. Perforated Element

Here $D$ denotes the plate stiffness, $D = Eh^3/\left(12\left(1-\nu^2\right)\right)$, and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Cartesian form of the Laplace operator.

The study of such problems is important from a theoretical as well as a numerical point of view. Because of the complicated structure of the perforated domain, any kind of calculation is difficult to perform. If we treat the boundary value problem we have to impose the boundary condition on the boundary of the holes which are many in number. So, we would like to approximate the given problem by a homogenized problem on a domain without holes. By the method of asymptotic development, a problem on a periodically perforated domain is reduced to solving problems in the basic cell and in the domain without holes. Let the boundary of holes $\partial G_k$ be free of stress, such that

$$\left(\nabla^2 W\right)_{nk} - (1-\nu)\left[0.5\left(W_{xx}-W_{yy}\right)\sin 2\theta - W_{xy}\cos 2\theta\right]_{s_k|\partial G_k} = 0 \tag{2}$$

$$\nabla^2 W + (1-\nu)\left(W_{xy}\sin 2\theta - W_{xx}\sin^2\theta - W_{yy}\cos^2\theta\right)_{|\partial G_k} = 0 \tag{3}$$

where $\theta$ is the angle between axis OX and normal $n_k$. Boundary conditions (without loss of generality) along the domain boundary $\partial G$ (Figure 1) may be formalized as follows:

$$W = W_{n_k n_k} = 0 \quad \text{on } \partial G \tag{4}$$

We denote

$$\varepsilon_1 = 2a/l \qquad (\varepsilon_1 << 1) \qquad \xi = \varepsilon_1^{-1}x \qquad \eta = \varepsilon_1^{-1}y$$

The method used here is a variant of multiscaling techniques used in Andrianov et al. (1983, 1985, 1988, 1991), Baker (1981), Bakhvalov and Panasenko (1989), and Bensoussan et al. (1978). Let us represent the solution in the form of a formal expansion

$$W(x,y) = W_0(x,y,\xi,\eta) + \varepsilon_1 W_1(x,y,\xi,\eta) + \varepsilon_1^2 W_2(x,y,\xi,\eta) + \cdots \tag{5}$$

where $x = x_1,\ y = y_1$.

Changeability period 1 in respect to variables $\xi,\eta$ for the functions $W_j\ (j = 1,2,\cdots)$ is admitted. The operators $\partial/\partial x_1$ and $\partial/\partial y_1$ applied to a function $W_j$ become

$$\partial/\partial x_1 = \partial/\partial x_1 + \varepsilon_1^{-1}\,\partial/\partial\xi \qquad\qquad \partial/\partial y_1 = \partial/\partial y_1 + \varepsilon_1^{-1}\,\partial/\partial\eta \tag{6}$$

Substituting series (5) into boundary value problem (1) to (4), taking into account relations (6) and splitting it with respect to the power of $\varepsilon_1$, one obtains the following recurrent sequence of boundary value problems:

$$M_1[W_2] \equiv \left[W_{2\xi\xi}\left[1-(1-\nu)\sin^2\theta\right] + W_{2\eta\eta}\left[1-(1-\nu)\cos^2\theta\right] + (1-\nu)W_{2\xi\eta}\sin 2\theta\right]_{\partial G_k} = -M_2[W_0] \tag{7}$$

$$\nabla_1^4 W_3 = -4\left[\left(\nabla_1^2 W_2\right)_{x\xi} + \left(\nabla^2 W\right)\right] \tag{8}$$

$$\begin{aligned} L_1[W_3]_{\partial G_k} = & -L_2[W_0] - 3W_{2y\eta\eta}\sin\theta\left[1+(1-\nu)\cos^2\theta\right] \\ & - \left(2W_{2y\xi\eta} + W_{2x\eta\eta}\right)\cos\theta\left[1+(1-\nu)\left(1-3\sin^2\theta\right)\right] \\ & - 3W_{2x\xi\xi}\cos\theta\left[1+(1-\nu)\sin^2\theta\right] \\ & - \left(2W_{2x\xi\eta} + W_{2y\xi\xi}\right)\sin\theta\left[1+(1-\nu)\left(1-3\cos^2\theta\right)\right] \end{aligned} \tag{9}$$

$$M_1[W_3]_{\partial G_k} = -2W_{2x\xi}\left[1-(1-\nu)\sin^2\theta\right] - 2W_{2y\eta}\left[1-(1-\nu)\cos^2\theta\right] - (1-\nu)\left(W_{2x\eta} + W_{2y\xi}\right)\sin 2\theta \tag{10}$$

$$\nabla^4 W_0 + \nabla_1^4 W_4 + 4\left[\left(\nabla_1^2 W_3\right)_{x\xi} + \left(\nabla_1^2 W_3\right)_{y\eta}\right] + 2\left[\left(\nabla_1^2 W_2\right)_{xx} + \left(\nabla_1^2 W_2\right)_{yy} + 2\left(W_{2x\xi\xi} + 2W_{2xy\xi\eta} + W_{2yy\eta\eta}\right)\right]$$
$$= P(x,y)/D \tag{11}$$

$$L_1[W_4]_{\partial G_k} = -3\left(W_{3x\xi\xi}+W_{2xx\xi}\right)\cos\theta\left[1+(1-\nu)\sin^2\theta\right]-3\left(W_{3y\eta\eta}+W_{2xx\xi}\right)\sin\theta\left[1+(1-\nu)\left(1-3\cos^2\theta\right)\right]$$
$$-\left(W_{3x\eta\eta}+2W_{3y\eta\xi}+2W_{2yx\eta}+W_{2yy\xi}\right)\cos\theta\left[1+(1-\nu)\sin^2\theta\right] \qquad (12)$$
$$-\left(w_{3y\xi\xi}+2W_{3x\eta\xi}+2W_{2xy\xi}+W_{2xx\eta}\right)\sin\theta\left[1+(1-\nu)\cos^2\theta\right]$$

Let us introduce parameter $\varepsilon_2 = b/a$ and consider the case $\varepsilon_2 << 1$ (it means a small hole, case $\varepsilon_2 \simeq 1$ is considered in section 4).

Then we can use asymptotic methods of perturbation of domain size and boundary form perturbation (Nayfeh, 1981; Guz and Nemish, 1989). For this purpose we neglect in the first approximation the outer boundary of the cell and pass to polar coordinates. Then function $W_2$ is represented by an $\varepsilon_2$ - based expansion in polar coordinates.

$$W_2(\rho,\varphi) = W_{20}(\rho,\varphi) + \varepsilon_2 W_{21}(\rho,\varphi) + \varepsilon_2^2 W_{22}(\rho,\varphi) + \cdots \qquad (13)$$

Then we can translate the boundary conditions from line $\rho = R(1 + \varepsilon_2\cos 4\varphi)$ to circle $\rho = R$ by Taylor's formula (Nayfeh, 1981; Guz and Nemish, 1989). Substituting equation (13) into the boundary value problem (7) and splitting it into a recurrent system of similar boundary problems one obtains

$$\nabla_2^4 W_{20} = 0 \qquad (14)$$

$$L_3[W_{20}] \equiv \Big[W_{20\rho\rho\rho} + R^{-2}W_{20\rho\varphi\varphi} - R^{-1}W_{20\rho\rho} - 2R^{-3}W_{20\varphi\varphi}$$
$$-R^{-2}W_{20\rho} + (1-\nu)\left(R^{-2}W_{20\rho\varphi\varphi} - R^{-3}W_{20\varphi\varphi}\right)\Big]_{\rho=R} = 0 \qquad (15)$$

$$M_3[W_{20}] \equiv \left[W_{20\rho\rho} + \nu\left(R^{-2}W_{20\varphi\varphi} - R^{-1}W_{20\rho}\right)\right]_{\rho=R}$$
$$= -\nabla^2 W_0 + 0.5\,(1-\nu)\left[W_{0xx}(1-\cos 2\varphi) + W_{0yy}(1+\cos 2\varphi) - W_{0xy}\sin 2\varphi\right] \qquad (16)$$

Here

$$\nabla_2^2 \equiv \frac{\partial^2}{\partial\rho^2} + \rho^{-1}\frac{\partial}{\partial\rho} + \rho^{-2}\frac{\partial^2}{\partial\varphi^2}$$

The solution of this problem may be written in Cartesian coordinates in the following form:

$$W_2(x,y,\xi,\eta) = 0.5\left(C_{200}\ln N + D_{200}N\ln N\right) + N^{-2}\left(B_{202}M + 2B'_{202}Q\right) + N^{-1}\left(D_{202}M + 2D'_{202}Q\right)$$
$$+\varepsilon_2\Big[N^{-2}\left(B_{212}M + 2B'_{212}Q\right) + N^{-1}\left(D_{212}M + 2D'_{212}Q\right) + N^{-4}\left(B_{214}L + 2B'_{214}QM\right) \qquad (17)$$
$$+N^{-3}\left(D_{214}L + 2D'_{214}QM\right) + N^{-6}\left(B_{214}MK + 2B'_{214}QP\right) + N^{-5}\left(D_{214}MK + 2D'_{214}QP\right)\Big]$$

where

$N = \xi^2 + \eta^2$ $\quad$ $M = \xi^2 - \eta^2$ $\quad$ $Q = \xi\eta$

$L = \xi^4 - 6\xi^2\eta^2 + \eta^4$ $\quad$ $P = 3\xi^4 - 10\xi^2\eta^2 + 3\eta^4$ $\quad$ $K = \xi^4 - 14\xi^2\eta^2 + \eta^4$

$C_{200}$, $D_{200}$, $B_{202}$, $\cdots$ are very complicated coefficients, and are not given here.

Function $W_2$ does not satisfy conditions of periodicity (conditions on the outer boundary of the cell). For the correction term $W_2^H$ we obtain the following expressions, taking into account only the principal of the series

$$\left[W_2^H\right]_{\xi=-\bar a}^{\xi=\bar a} = f_1(\eta) \qquad \left[W_{2\xi}^H\right]_{\xi=-\bar a}^{\xi=\bar a} = f_2(\eta) \qquad \left[W_{2\xi\xi}^H\right]_{\xi=-\bar a}^{\xi=\bar a} = f_3(\eta) \qquad \left[W_{2\xi\xi\xi}^H\right]_{\xi=-\bar a}^{\xi=\bar a} = f_4(\eta) \tag{18}$$

$$\left[W_2^H\right]_{\eta=-\bar a}^{\eta=\bar a} = \psi_1(\xi) \qquad \left[W_{2\eta}^H\right]_{\eta=-\bar a}^{\eta=\bar a} = \psi_2(\xi) \qquad \left[W_{2\eta\eta}^H\right]_{\eta=-\bar a}^{\eta=\bar a} = \psi_3(\xi) \qquad \left[W_{2\eta\eta\eta}^H\right]_{\eta=-\bar a}^{\eta=\bar a} = \psi_4(\xi) \tag{19}$$

where

$$f_1(\eta) = -4\bar N^{-1}\bar Q\,\bar D \qquad f_2(\eta) = 4\bar N^{-2}\eta\,\bar M\bar D \qquad f_3(\eta) = -8\bar N^{-1}\bar Q\,\bar S\,\bar D \qquad f_4(\eta) = 24\bar N^{-4}\eta\,\bar L\,\bar D$$

$$\bar N = N \qquad \bar S = S \qquad \bar L = L(\xi=\bar a) \qquad \bar a = \varepsilon_1^{-1}a \qquad \bar D = D'_{202} + \varepsilon_2 D'_{212} \qquad \psi_1(\xi) = f_1(\eta)\ (\xi=\eta)$$

Now we consider problem equations (18) and (19) in the simply connected domain $\left(|\xi| \le \varepsilon_1^{-1}a,\ |\eta| \le \varepsilon_1^{-1}a\right)$, ignoring the hole. One easily obtains the solution of equations (18) and (19)

$$\begin{aligned} W_2^H = \sum_{n=1}^{\infty}\Bigg[&\left(A_n^{(21)}\sinh\frac{n\pi\eta}{a} + B_n^{(21)}\cosh\frac{n\pi\eta}{a} + C_n^{(21)}\eta\sinh\frac{n\pi\eta}{a}\right.\\ &\left.+ D_n^{(21)}\eta\cosh\frac{n\pi\eta}{a}\right)\cos\frac{n\pi\xi}{a} + \left(A_n^{(22)}\sinh\frac{n\pi\xi}{a} + B_n^{(22)}\cosh\frac{n\pi\xi}{a}\right.\\ &\left.+ C_n^{(22)}\xi\sinh\frac{n\pi\xi}{a} + D_n^{(22)}\xi\cosh\frac{n\pi\xi}{a}\right)\cos\frac{n\pi\eta}{a}\Bigg] \end{aligned} \tag{20}$$

Then one may satisfy boundary conditions on the boundary of the hole, ignoring conditions of periodicity, and so on. This is the main idea of domain size perturbation (Neuman-Schwarz alternating method) (Kantorovitch and Krylov, 1949).

The boundary value problems of equations (8) to (10) have been solved on the basis of the approach presented.

$$\begin{aligned} W_3(x,y,\eta,\xi) = {}& -0.5\left\{N\left(\ln N - \frac{11}{3}\right)\left(D_{200x}\xi + D_{200y}\eta\right) + N^{-1}\left[\left(D_{202x} - D'_{202y}\right)\xi S - \left(D_{202y} + D_{202x}\right)\eta S_1\right]\right\}\\ &+ N^{-1}\left[\left(C_{301}+\varepsilon_2 C_{311}\right)\xi + \left(C'_{301}+\varepsilon_2 C'_{311}\right)\eta\right] + 0.5\ln N\left[\left(D_{301}+\varepsilon_2 D_{311}\right)\xi + \left(D'_{301}+\varepsilon_2 D'_{311}\right)\eta\right]\\ &+ N^{-3}\left[\left(B_{301}+\varepsilon_2 B_{311}\right)\xi S + \left(B'_{301}+\varepsilon_2 B'_{311}\right)\eta S_1\right] + N^{-2}\left[\left(D_{301}+\varepsilon_2 D_{311}\right)\xi S + \left(D'_{301}+\varepsilon_2 D'_{311}\right)\eta S_1\right]\\ &- 0.5\varepsilon_2\left\{N^{-1}\left[\left(D_{212x} - D'_{212y}\right)\xi S - \left(D'_{212x} + D_{212y}\right)\eta S_1\right] + N^{-3}\left[\left(D_{214x} - D'_{214y}\right)\xi T - \left(D'_{214x} + D_{214y}\right)\eta T_1\right]\right.\\ &\left.+ N^{-5}\left[\left(D_{216x} - D'_{216y}\right)\xi H - \left(D'_{216x} + D_{216y}\right)\eta H_1\right]\right\} + \sum_{n=1}^{\infty}\Bigg[\left(A_n^{(31)}\sinh\frac{n\pi\eta}{a} + B_n^{(31)}\cosh\frac{n\pi\eta}{a} + C_n^{(31)}\eta\sinh\frac{n\pi\eta}{a}\right.\\ &\left.+ D_n^{(31)}\eta\cosh\frac{n\pi\eta}{a}\right)\cos\frac{n\pi\xi}{a} + \left(A_n^{(32)}\sinh\frac{n\pi\xi}{a} + B_n^{(32)}\cosh\frac{n\pi\xi}{a} + C_n^{(32)}\xi\sinh\frac{n\pi\xi}{a} + D_n^{(32)}\xi\cosh\frac{n\pi\xi}{a}\right)\cos\frac{n\pi\eta}{a}\Bigg] \end{aligned} \tag{21}$$

where

$$S = \xi^2 - 3\eta^2 \qquad T = \xi^4 - 10\xi^2\eta^2 + 5\eta^4 \qquad H = \xi^6 - 21\xi^4\eta^2 + 35\xi^2\eta^4 - 7\eta^6$$

$$S_1 = S \qquad T_1 = T \qquad H_1 = H\ (\xi=\eta)$$

Substituting solutions of cell boundary value problems (7) and (8) into equation (10) one obtains the homogenized equation

$$A\left(W_{0xxxx} + W_{0yyyy}\right) + 2BW_{0xxyy} = P(x,y)/D \tag{22}$$

where $A$ and $B$ are the very complicated coefficients, and not given here. The homogenized boundary conditions are

$$W_0 = W_{0nn} = 0 \qquad \text{on } \partial G \tag{23}$$

Coefficients $A$ (curve 1) and $B$ (curve 2) are calculated for $\varepsilon_1 = 0.125$, $\varepsilon_2 = -1/9$, $\nu = 0$ and $\nu = 0.3$ (see Figures 2 and 3).

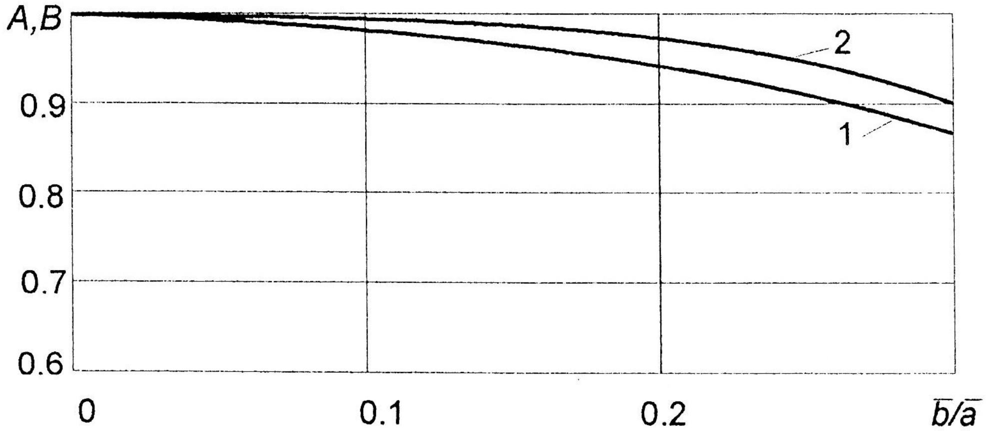


Figure 2. Homogenized Coefficients A and B versus Ratio $\bar{b}/\bar{a}$ for $\nu = 0$

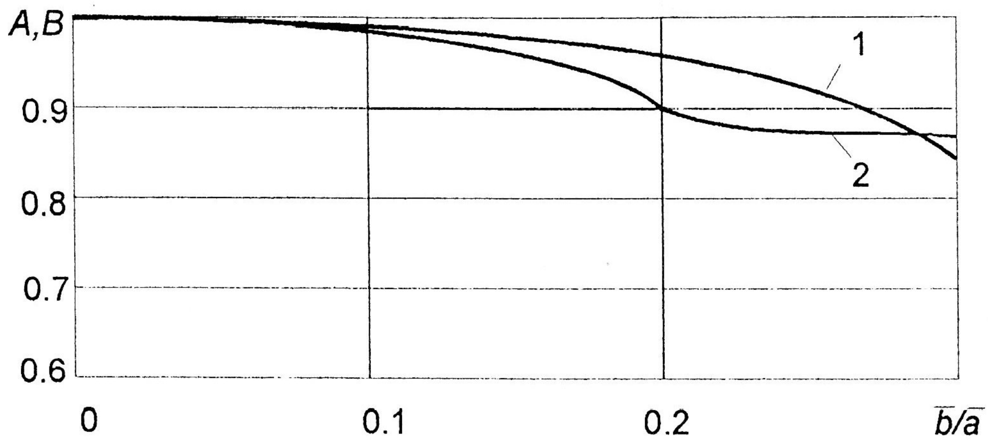


Figure 3. Homogenized Coefficients A and B versus Ratio $\bar{b}/\bar{a}$ for $\nu = 0{,}3$

Now we examine the accuracy of the homogenized coefficient computations. Let us consider simply supported square plates with stress-free circular and square holes, loaded by a uniformly distributed lateral pressure $P$. In this case $\varepsilon_1$ equals to 1, and it represents the worst case for our method. Calculated nondimensional deflections and bending moments $W^*$and $M^*\left(W^* = WD/Pa^4,\ M^* = M_y\left(Pa^2\right)\right)$ are shown for circular holes at a point $(b,0)$ for various values of parameter $b/a$ (see Figures 4 and 5, curve 2).

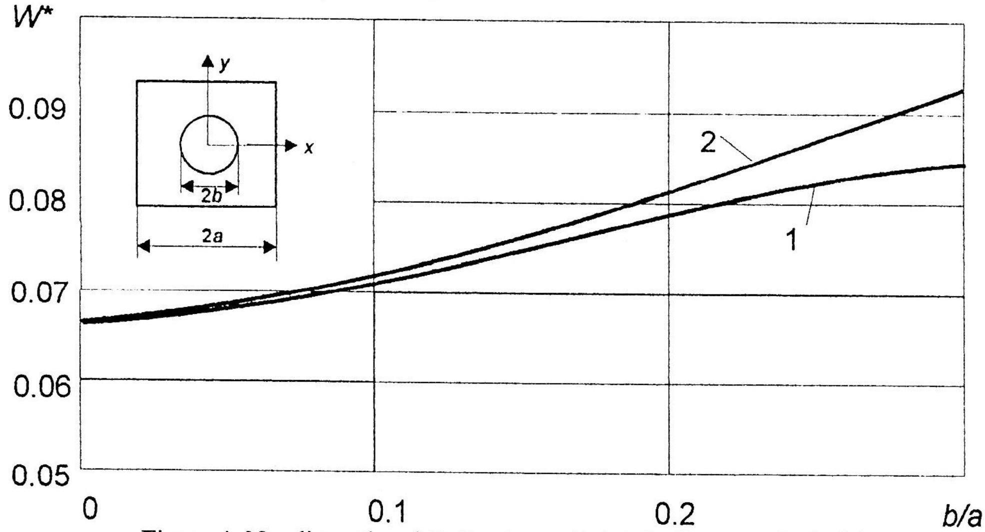


Figure 4. Nondimentional Deflection at Point (b, 0) versus Ratio b/a

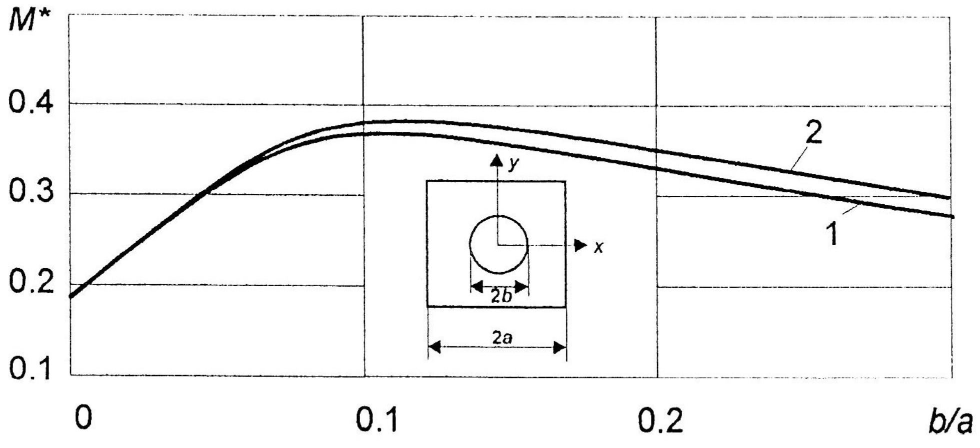


Figure 5. Nondimentional Bending Moment at Point (b, 0) versus Ratio b/a

Coefficients $W^*$and $M^*$ along the edges $x = a$ for $\nu = 0.3$ for square $(\varepsilon_2 = -1/9)$ holes are shown in Figures 6 and 7 (curve 2).

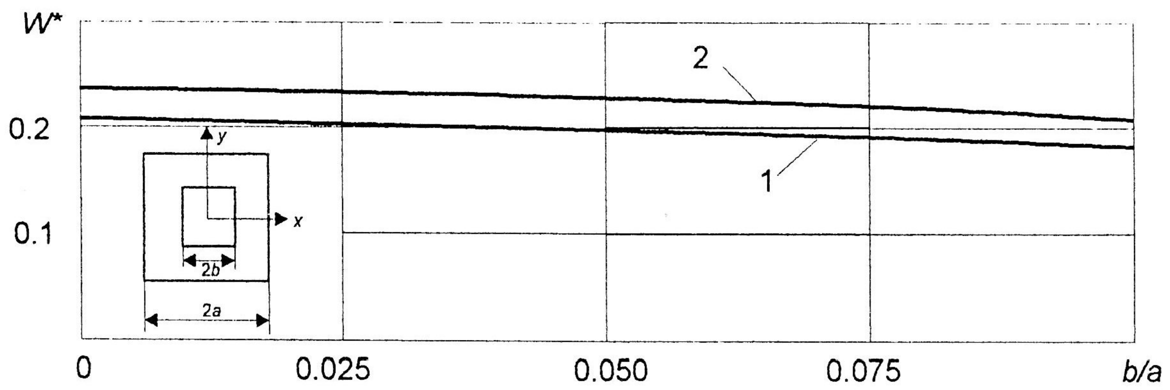


Figure 6. Nondimensional Deflection along Edge x = a for $\nu$ = 0 versus Ratio b/a

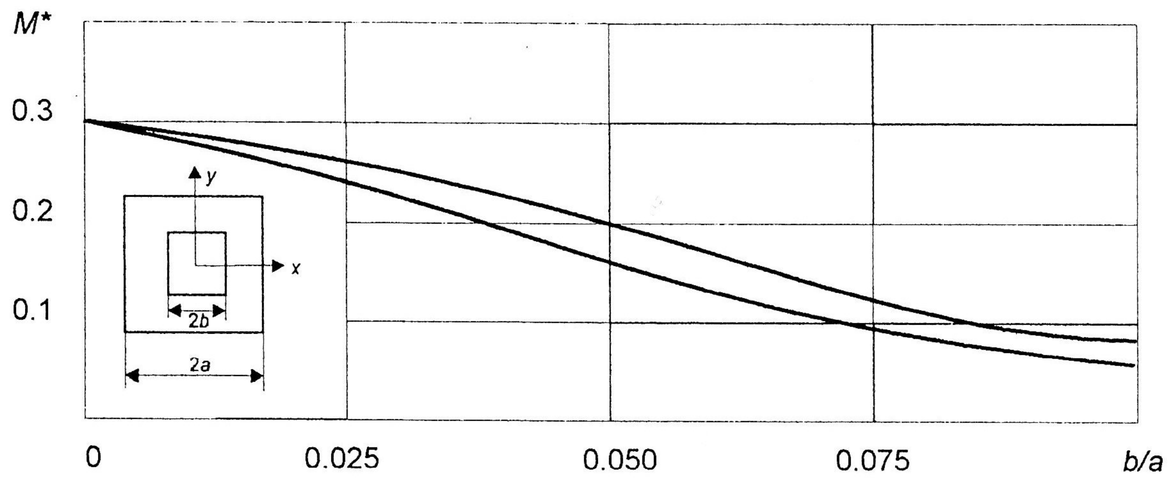


Figure 7. Nondimensional Deflection along Edge x = a for $\nu$ = 0,3 versus Ratio b/a

The results, obtained by the Fourier series method (Pickett, 1965), are presented in Figures 4 to 7 by curve 1. The discrepancy for deflections does not exceed 5 % (for bending moments 10 %), which confirms an acceptable accuracy of the method presented.

## 3 Eigenvalue Problem for Perforated Plate

Our aim is to describe the asymptotic behavior of the various eigenvalues when the number of holes in the domain increases to infinity. Using the notations introduced in chapter 1 we consider the following eigenvalue problem

$$\nabla^4 W - \lambda W = 0 \tag{24}$$

where $\lambda = \omega\mu h/D$, and $\omega$ is the natural frequency. For equation (24) we may formulate boundary conditions (2) to (4). We represent eigenvalue $\lambda$ and eigenfunction $W$ in the following forms:

$$W(x,y) = W_0(x,y) + \varepsilon_1^2 W_2(x,y,\xi,\eta) + \varepsilon_1^3 W_3(x,y,\xi,\eta) + \cdots \tag{25}$$

$$\lambda = \lambda_0 + \varepsilon_1\lambda_1 + \varepsilon_1^2\lambda_2 + \varepsilon_1^3\lambda_3 + \cdots \tag{26}$$

Substituting expansions (25) and (26) and boundary conditions (2) to (4) into equation (24) and splitting it into powers of $\varepsilon_1$, one obtains a recurrent system of boundary value problems. The first step in the solution process is the same as that above. One can obtain $W_1 = 0$, $W_2$ and $W_3$ (see formulas (19) - (20), (21)). It means that the boundary value problems (24) is quasi-static. The homogenized eigenvalue problem may be obtained by applying the averaging operator defined by equations (6)

$$A\left(W_{0xxxx} + W_{0yyyy}\right) + 2BW_{0xxyy} - \lambda_0 W_0 = 0 \tag{27}$$

This equation must be supplied with the homogenized boundary conditions (23).

## 4 Analytical Approach for a Large Hole

Let us now consider $\varepsilon_1 \simeq 1$ (case of large hole, Figure 8).

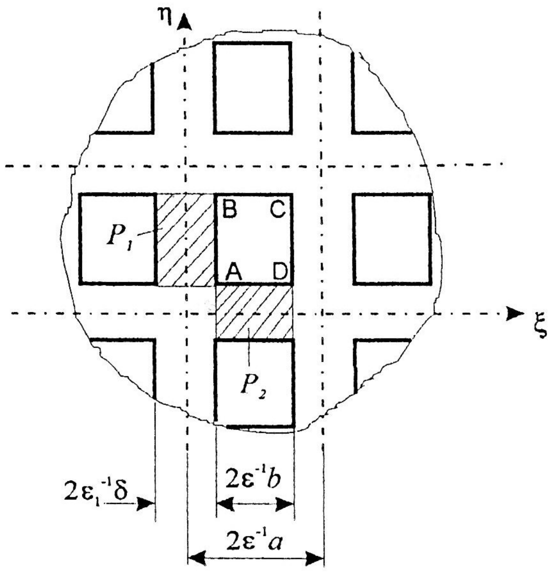


Figure 8. Parameters for Large Square Hole

In this case we can not use the previous approach, but the smallness of the parameter $\delta/b$, where $\delta$ is the thickness of the wall between two holes (see Figures 1, 8), must be taken into account. Then we may construct an asymptotic solution, using a singular perturbation technique, similar to that proposed in Christensen (1979) (this a variant of another singular perturbation technique, see Nayfeh, 1981). Let us denote $\overline{\xi} = \varepsilon_1^{-1}\delta\xi$, $\overline{\eta} = \varepsilon_1^{-1}\delta\eta$. Then cell problem (7) to (10) may be formulated as a set of four strips $\Pi_1 = \{|\overline{\xi}| < \varepsilon_3\}$, $\Pi_2 = \{|\overline{\eta}| < \varepsilon_3\}$, $\varepsilon_3 = \delta/b << 1$. Let us consider boundary value problem (7) for strip $\Pi_1$

$$W^{(1)}_{2_{\overline{\xi\xi\xi\xi}}} + 2\varepsilon_3^2 W^{(1)}_{2_{\overline{\xi\xi\eta\eta}}} + \varepsilon_3^4 W^{(1)}_{2_{\overline{\eta\eta\eta\eta}}} = 0 \qquad (28)$$

$$\left[W^{(1)}_{2_{\overline{\xi\xi\xi}}} + \varepsilon_3^2(2-\nu)W^{(1)}_{2_{\overline{\eta\eta\eta}}}\right]_{\overline{\xi}=\mp 1} = 0 \qquad (29)$$

$$\left[W^{(1)}_{2_{\overline{\xi\xi\xi}}} + \varepsilon_3^2 \nu W^{(1)}_{2_{\overline{\eta\eta}}}\right]_{\overline{\xi}=\mp 1} = -\varepsilon_1^{-2}\delta\left(W_{0xx} + \nu W_{0yy}\right) \qquad (30)$$

For function $W_2$ we use the following asymptotic expansion

$$W_2^{(1)} = W_2^{(10)} + \varepsilon_3^2 W_2^{(12)} + \varepsilon_3^3 W_2^{(13)} + \dots \qquad (31)$$

Substituting equation (31) into boundary value problems (28) to (30) and splitting it with respect to the powers of $\varepsilon_3$, one obtains a recurrent system of boundary value problems. Restricting oneself only to systems of first approximation, one has

$$W^{(10)}_{2_{\overline{\xi\xi\xi\xi}}} = 0 \qquad (32)$$

$$\left[W^{(10)}_{2_{\overline{\xi\xi\xi}}}\right]_{\overline{\xi}=\mp 1} = 0 \qquad (33)$$

$$\left[W^{(10)}_{2_{\overline{\xi\xi\xi}}}\right]_{\overline{\xi}=\mp 1} = -\varepsilon_1^{-2}\delta\left(W_{0xx} + \nu W_{0yy}\right) \qquad (34)$$

Solving the one-dimensional boundary value problem (32) to (34), one obtains

$$W_2^{(10)} = C_{21} + C_{22}\overline{\xi} + C_{23}\overline{\xi}^2 + C_{24}\overline{\xi}^3 \qquad (35)$$

Solutions of boundary value problem (7) for a strip have been constructed in the same way and have been obtained from equation (35) by a change of variables

$$W_2^{(20)} = D_{21} + D_{22}\overline{\eta} + D_{23}\overline{\eta}^2 + D_{24}\overline{\eta}^3 \qquad (36)$$

Constants $C_{12}$, $D_{12}$ have no influence on the homogenized coefficients of either bending moment, so we neglect them. Constants $C_{22}$, $D_{22}$ have been obtained from boundary conditions and additional conditions at points $A$, $B$, $C$, $D$ (see Figure 8). Thus we have the following expressions:

$$W_2^{(10)} = C_{21} - 0.5\left(W_{0xx} + \nu W_{0yy}\right)\xi^2 \qquad (37)$$

$$W_2^{(20)} = D_{21} - 0.5\left(W_{0yy} + \nu W_{0xx}\right)\eta^2 \qquad (38)$$

We have solved the cell boundary value problem (8) to (10) on the basis of the approach presented. The governing boundary value problem may be written in the form

$$W^{(10)}_{3_{\overline{\xi\xi\xi\xi}}} = 0 \qquad (39)$$

$$\left[W^{(10)}_{3_{\bar{\xi}\bar{\xi}\bar{\xi}}}\right]_{\bar{\xi}=\pm 1} = 2\varepsilon_1^{-3}\delta^3\left[W_{0xxx} - (1-2\nu)W_{0xyy}\right] \quad (40)$$

$$\left[W^{(10)}_{3_{\bar{\xi}\bar{\xi}}}\right]_{\bar{\xi}=\pm 1} = \underline{\pm\, 2\varepsilon_1^{-3}\delta^3\left[W_{0xxx} - (1-2\nu)W_{0xyy}\right]} \quad (41)$$

The underlined term has been added into boundary conditions (41) for the sake of solvability. Solving the one-dimensional boundary value problem (39) to (41), one obtains

$$W^{(10)}_3 = C_{31} + C_{32}\varepsilon_1\delta^{-1}\xi + \frac{1}{3}\left[W_{0xxx} - (1-2\nu)W_{0xyy}\right]\xi^3 \quad (42)$$

Solving for the strip $\Pi_2$ one obtains from equation (42) by setting $x = y,\ \xi = \eta,\ C_{31} = D_{31}$

$$W^{(20)}_3 = D_{31} + D_{32}\varepsilon_1\delta^{-1}\eta + \frac{1}{3}\left[W_{0yyy} - (1-2\nu)W_{0yxx}\right]\eta^3 \quad (43)$$

Solving for term $\mp\, 2\varepsilon_1^{-3}\delta^3\left[W_{0xxx} - (1-2\nu)W_{0xyy}\right]$ in the boundary conditions (41) one obtains the following boundary problems:

$$W^{1H}_{3_{\xi\xi\xi\xi}} + 2W^{1H}_{3_{\xi\xi\eta\eta}} + W^{1H}_{3_{\eta\eta\eta\eta}} = 0 \quad (44)$$

$$\left[W^{1H}_3\right]_{\eta=\varepsilon_1^{-1}\delta} = \left[W^{1H}_3\right]_{\eta=\varepsilon_1^{-1}(\delta+2b)} = 0 \quad (45)$$

$$\left[W^{1H}_{3\eta}\right]_{\eta=\varepsilon_1^{-1}\delta} = \left[W^{1H}_{3\eta}\right]_{\eta=\varepsilon_1^{-1}(\delta+2b)} = 0 \quad (46)$$

$$\left[W^{1H}_{3_{\xi\xi\xi}}\right]_{\xi=\mp\varepsilon_1^{-1}\delta} = 0 \quad (47)$$

$$\left[W^{1H}_{3_{\xi\xi}}\right]_{\xi=\mp\varepsilon^{-1}\delta} = \mp\, \varepsilon_1^{-1}\delta(1-\nu)W_{0xyy} \quad (48)$$

Boundary value problems (44) to (48) cannot be solved exactly, but we can use a variational Kantorovitch method (Christansen, 1979). Let us briefly present this method. First of all we must represent the solution for equation (44) in a form satisfying boundary conditions (45) and (46).

$$W^{1H}_3 = \chi(\xi)\varphi(\eta) \qquad \varphi(\eta) = \left(\eta - \varepsilon_1^{-1}a\right)^2 - \left(\varepsilon_1^{-1}b\right)^2$$

Then we substitute this expression into equation (44), multiply with $\varphi(\eta)$ and integrate over $\eta$. Then we obtain an ordinary differential equation with respect to $\chi(\xi)$, and after solving it and satisfying boundary conditions (47) and (48) we may write

$$W^{1H}_3 = \left[\left(\eta - \varepsilon_1^{-1}a\right)^2 - \left(\varepsilon_1^{-1}b\right)^2\right]\left(A_{12}\cosh(\alpha\varepsilon_1\xi/b)\sin(\beta\varepsilon_1\xi/b) + B_{12}\sinh(\alpha\varepsilon_1\xi/b)\cos(\beta\varepsilon_1\xi/b)\right) \quad (49)$$

where

$$A_{12} = \frac{21\varepsilon_1\delta(1-\nu)W_{0xyy}\left(\alpha^3\cosh\frac{\alpha\delta}{b}\cos\frac{\beta\delta}{b} - 3\alpha^2\beta\sinh\frac{\alpha\delta}{b}\sin\frac{\beta\delta}{b}\right)}{4b\left(\alpha^2+\beta^2\right)\left(\alpha\sin\frac{2\beta\delta}{b} - \beta\sinh\frac{2\alpha\delta}{b}\right)} - \frac{21\varepsilon_1\delta(1-\nu)W_{0xyy}\left(3\alpha\beta^2\cosh\frac{\alpha\delta}{b}\cos\frac{\beta\delta}{b} - \beta^3\sinh\frac{\alpha\delta}{b}\sin\frac{\beta\delta}{b}\right)}{4b\left(\alpha^2+\beta^2\right)\left(\alpha\sin\frac{2\beta\delta}{b} - \beta\sinh\frac{2\alpha\delta}{b}\right)}$$

$$B_{12} = \frac{21\varepsilon_1\delta(1-\nu)W_{0xyy}\left(\alpha^3\cosh\frac{\alpha\delta}{b}\cos\frac{\beta\delta}{b} + 3\alpha^2\beta\sinh\frac{\alpha\delta}{b}\sin\frac{\beta\delta}{b}\right)}{4b\left(\alpha^2+\beta^2\right)\left(\alpha\sin\frac{2\beta\delta}{b} - \beta\sinh\frac{2\alpha\delta}{b}\right)} - \frac{21\varepsilon_1\delta(1-\nu)W_{0xyy}\left(3\alpha\beta^2\cosh\frac{\alpha\delta}{b}\cos\frac{\beta\delta}{b} + \beta^3\sinh\frac{\alpha\delta}{b}\sin\frac{\beta\delta}{b}\right)}{4b\left(\alpha^2+\beta^2\right)\left(\alpha\sin\frac{2\beta\delta}{b} - \beta\sinh\frac{2\alpha\delta}{b}\right)}$$

The solution for strip $\Pi_1$ has been constructed similarly and can obtained from equation(49).

$$W_3^{2H} = \left[\left(\eta-\varepsilon_1^{-1}a\right)^2 - \left(\varepsilon_1^{-1}b\right)^2\right]\left(A_{22}\cosh(\alpha\varepsilon_1\eta/b)\sin(\beta\varepsilon_1\eta/b) + B_{22}\sinh(\alpha\varepsilon_1\eta/b)\cos(\beta\varepsilon_1\eta/b)\right) \tag{50}$$

$$A_{12} = A_{22} \qquad B_{12} = B_{22}(x=y)$$

Substituting the solution obtained into equation (10), one obtains the coefficients of the homogenized equation

$$A = 0.5(1-\nu)^2 \qquad B = 0.7(1-\nu) \tag{51}$$

For $\nu = 0.3$ we have $A = 0.455$, $B = 0.49$. Approximately we may assume $B \simeq 0.455$, then the plate bending equation may be written as follows

$$0.4555\left(W_{0xxxx} + 2W_{0xx} + W_{0yyyy}\right) = P(x,y)/D \tag{52}$$

For large circular holes we obtain

$$\overline{A} = kA \qquad \overline{B} = kB \qquad k = \left(1-\varepsilon^2\right)\left(1-\frac{\pi}{4}\varepsilon^2\right)^{-1} \tag{53}$$

The homogenized eigenvalue problem may be obtained in the following form:

$$A\left(W_{0xxxx} + W_{0yyyy}\right) + 2BW_{0xxyy} - \lambda_0 W_0 = 0$$

where coefficients $A$, $B$ are defined by expressions (51) (for large square holes) or (53) (for large circular holes).

## 5 Matching of Asymptotic Solutions by Means of Two-Point Padé Approximants

Practically any physical or mechanical problem, whose parameters include the variable parameter $\varepsilon$, can be approximately solved as it approaches zero, or infinity. How can this „limiting“ information be used in the study of a system at intermediate values of $\varepsilon$ ? This problem is one of the most complicated in asymptotic analysis. As yet there is no general answer, but in many instances it is alleviated by two-point Padé approximants (Baker and Graves-Morris, 1981; Andrianov, 1991).

The notion of two-point Padé approximants is defined by Baker and Graves-Morris (1981). Let

$$F(\varepsilon) = \sum_{i=0}^{\infty} a_i \varepsilon^i \qquad \text{when} \quad \varepsilon \to 0 \tag{54}$$

$$F(\varepsilon) = \sum_{i=0}^{\infty} b_i \varepsilon^i \qquad \text{when} \quad \varepsilon \to 1 \tag{55}$$

The two-point Padé approximant is represented by the function

$$F(\varepsilon) = \left(\sum_{k=0}^{m} \alpha_k \varepsilon^k\right)\left(1+\sum_{k=1}^{m} \beta_k \varepsilon^k\right)^{-1}$$

in which $m+1$ coefficients of expansion in the Taylor series when $\varepsilon \to 0$ and $m$ coefficients of expansion in the Taylor series when $\varepsilon \to 1$ coincide with the corresponding coefficients of the series (54) and (55).
In our case we have the following expressions $(\nu = 0.3)$ for a square hole:

$$A = (1 - 0.6504\varepsilon_2)(1 - 0.2317\varepsilon_2)^{-1} \qquad B = (1 - 0.7466\varepsilon_2)(1 - 0.4432\varepsilon_2)^{-1}$$

and for a circular hole:

$$A = (1 - \varepsilon_2)(1 - 0.5785\varepsilon_2)^{-1} \qquad B = (1 - \varepsilon_2)(1 - 0.6701\varepsilon_2)^{-1}$$

Figure 9 shows the numerical results for $A$ and $B$ for square holes for $\varepsilon_2 = 0.125$ and $\nu = 0.3$.

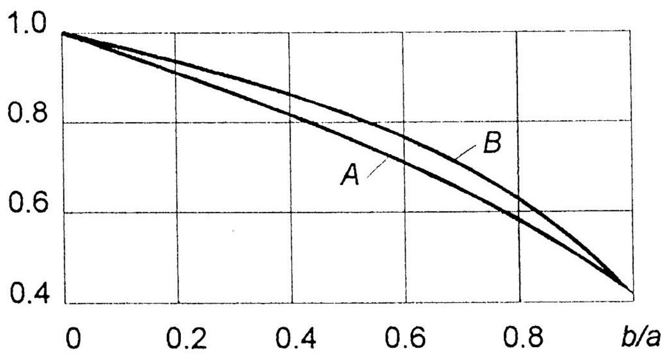


Figure 9. Homogenized Coefficients A and B for Square Hole and $\nu = 0{,}3$

The values of coefficients $A$ and $B$ for circular holes are compared to theoretical results, obtained by a two-periodic elliptic functions method (Grigoluk and Phylshtinsky, 1970) (curve 1 for coefficient $A$ and curve 2 for coefficient $B$ in Figure 10).

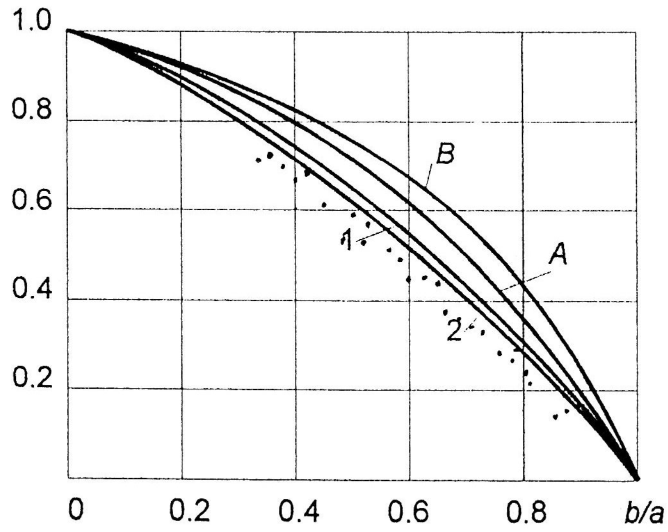


Figure 10. Homogenized Coefficients A and B

Experimental results for coefficient $A$ (Grigoluk and Phylshtinsky, 1970) are displayed in Figure 10 by dots. The accuracy of the method proposed is apparent.

## 6 The Plane Theory of Elasticity in Perforated Domain

The governing boundary value problem for the perforated domain (see Figure 1) may be obtained as follows (boundaries of holes are free of stress):

$$\nabla^4 \psi = 0 \tag{56}$$

$$\left[\psi_{xx} \sin^2\theta + \psi_{yy} \cos^2\theta - \psi_{xy} \sin 2\theta\right]_{\partial G_k} = 0 \tag{57}$$

$$\left[0.5\left(\psi_{yy} - \psi_{xx}\right)\sin 2\theta + \psi_{xy} \cos 2\theta\right]_{\partial G_k} = 0 \tag{58}$$

where $\psi$ is the potential function. Let us introduce expansion

$$\psi(x,y) = \psi_0(x,y) + \varepsilon_1^2 \psi_2(x,y,\xi,\eta) + \varepsilon_1^3 \psi_3(x,y,\xi,\eta) + \cdots \tag{59}$$

Substituting this expansion into boundary value problem (56) to (58) and splitting with respect to powers of $\varepsilon_1$ one obtains cell boundary problems for the whole domain.

$$\nabla_1^4 \psi_2 \equiv \psi_{2\xi\xi\xi\xi} + 2\psi_{2\xi\xi\eta\eta} + \psi_{2\eta\eta\eta\eta} = 0 \tag{60}$$

$$L_3[\psi_2] \equiv \left[\psi_{2\xi\xi} \sin^2\theta + \psi_{2\eta\eta} \cos^2\theta - \psi_{2\xi\eta} \sin 2\theta\right]_{\partial G_k} = -\psi_{0xx} \sin^2\theta - \psi_{2yy} \cos^2\theta + \psi_{2xy} \sin 2\theta \tag{61}$$

$$M_3[\psi_2] \equiv \left[0.5\left(\psi_{2\eta\eta} - \psi_{2\xi\xi}\right)\sin 2\theta + \psi_{2\xi\eta} \cos 2\theta\right]_{\partial G_k} = 0.5\left(\psi_{0yy} - \psi_{0xx}\right)\sin 2\theta - \psi_{2xy} \cos 2\theta \tag{62}$$

$$\nabla_1^4 \psi_3 = -4\left[\left(\nabla_1^2 \psi_2\right)_{x\xi} + \left(\nabla_1^2 \psi_2\right)_{y\eta}\right] \tag{63}$$

$$\left[L_3[\psi_2]\right]_{\partial G_k} = -2\left[\psi_{2x\xi}\sin^2\theta + \psi_{2y\eta}\cos^2\theta - 0.5\left(\psi_{2x\eta}-\psi_{2y\xi}\right)\sin 2\theta\right] \quad (64)$$

$$\left[M_3[\psi_2]\right]_{\partial G_k} = -\left(\psi_{2y\eta}-\psi_{2x\xi}\right)\sin 2\theta - \left(\psi_{2x\eta}-\psi_{2y\xi}\right)\cos 2\theta \quad (65)$$

$$\nabla^4\psi_0+\nabla_1^4\psi_4+4\left[\left(\nabla_1^2\psi_3\right)_{x\xi}+\left(\nabla_1^2\psi_3\right)_{y\eta}\right]+2\left[\left(\nabla_1^2\psi_2\right)_{xx}+\left(\nabla_1^2\psi_2\right)_{yy}+2\left(\psi_{0xx\xi\xi}+\psi_{0yy\eta\eta}+2\psi_{0xy\xi\eta}\right)\right]=0 \quad (66)$$

where $\nabla_1 = \nabla(x=\xi,\ y=\eta)$. The homogenized equation can be easily obtained by applying the average operator defined by equation (6).

$$\nabla^4\psi_0 + 2\left|G_k^*\right|^{-1}\iint_{G_k^*}\left\{2\left[(\nabla_1^2\psi_3)_{x\xi}+\left(\nabla_1^2\psi_3\right)_{y\eta}\right]+\left(\nabla_1^2\psi_2\right)_{xx}+\left(\nabla_1^2\psi_2\right)_{yy}+2\left(\psi_{0xx\xi\xi}+\psi_{0yy\eta\eta}+2\psi_{0xy\xi\eta}\right)\right\}d\xi\, d\eta = 0$$

(67)

The cell problem has been solved on the basis of the approach presented in section 2. We obtain

$$\begin{aligned}
\psi_2(x,y,\xi,\eta) &= A_{200} + 0.5\,C_{200}\ln N + N^{-2}\left(B_{202}M + 2B'_{202}Q\right) + N^{-1}\left(D_{202}M + 2D'_{202}Q\right)\\
&+ \varepsilon^2\Big[N^{-2}\left(B_{212}M + 2B'_{212}Q\right) + N^{-1}\left(D_{212}M + 2D'_{212}Q\right) + N^{-4}\left(B_{214}L + 4B'_{214}QM\right) + N^{-3}\left(D_{214}L + 4D'_{214}QM\right)\\
&+ N^{-6}\left(B_{216}MK + 4B'_{216}QP\right) + N^{-6}\left(D_{216}MK + 4D'_{216}QP\right) + \sum_{n=1}^{\infty}\Big[\Big(A_n^{(21)}\sinh\frac{n\pi\eta}{a} + B_n^{(21)}\cosh\frac{n\pi\eta}{a} + C_n^{(21)}\eta\sinh\frac{n\pi\eta}{a}\\
&+ D_n^{(21)}\eta\cosh\frac{n\pi\eta}{a}\Big)\cos\frac{n\pi\xi}{a} + \Big(A_n^{(22)}\sinh\frac{n\pi\xi}{a} + B_n^{(22)}\cosh\frac{n\pi\xi}{a} + C_n^{(22)}\xi\sinh\frac{n\pi\xi}{a} + D_n^{(22)}\xi\cosh\Big)\cos\frac{n\pi\eta}{a}\Big]
\end{aligned}$$

(68)

$$\begin{aligned}
\psi_3(x,y,\xi,\eta) &= -0.5N^{-1}\left[\left(D_{202x} - D'_{202y}\right)\xi S - \left(D_{202y} + D'_{202x}\right)\eta S_1\right]\\
&+ N^{-1}\left(C_{301}\xi + C'_{301}\eta\right) + 0.5\ln N\left(D_{301}\xi + D'_{301}\eta\right) + N^{-3}\left(B_{301}\xi S + B'_{301}\eta S_1\right)\\
&+ N^{-2}\left(D_{301}\xi S + D'_{301}\eta S_1\right) + \varepsilon_2\Big\{-0.5N^{-1}\left[\left(D_{212x} - D'_{212y}\right)\xi S - \left(D_{212y} - D'_{212x}\right)\eta S_1\right]\\
&- 0.5N^{-3}\left[\left(D_{214x} + D'_{214y}\right)\xi T - \left(D_{214y} - D'_{214x}\right)\eta T_1\right] - 0.5N^{-5}\Big[\left(D_{216x} - D'_{216y}\right)\xi H\\
&- \left(D_{216y} - D'_{216x}\right)\eta H_1\Big] - N^{-1}\left(C_{311}\xi + C_{311}\eta\right) + 0.5\ln N\left(D_{311}\xi + D'_{311}\eta\right) \qquad (69)\\
&+ N^{-3}\left(B_{311}\xi S + B'_{311}\eta S_1\right) + N^{-2}\left(D_{311}\xi S + D'_{311}\eta S_1\right)\Big\}\\
&+ \sum_{n=1}^{\infty}\Big[\Big(A_n^{(31)}\sinh\frac{n\pi\eta}{a} + B_n^{(31)}\cosh\frac{n\pi\eta}{a} + C_n^{(31)}\eta\sinh\frac{n\pi\eta}{a} + D_n^{(31)}\eta\cosh\frac{n\pi\eta}{a}\Big)\cos\frac{n\pi\xi}{a}\\
&+ \Big(A_n^{(32)}\eta\sinh\frac{n\pi\xi}{a} + B_n^{(32)}\cosh\frac{n\pi\xi}{a} + C_n^{(32)}\xi\sinh\frac{n\pi\xi}{a} + D_n^{(32)}\xi\cosh\frac{n\pi\xi}{a}\Big)\cos\frac{n\pi\eta}{a}\Big]
\end{aligned}$$

where

$N=\xi^2+\eta^2$ $\quad M = \xi^2-\eta^2$ $\quad Q = \xi\eta$ $\quad L = \xi^4-6\,\xi^2\eta^2+\eta^4$ $\quad P=3\,\xi^4-10\,\xi^2\eta^2+3\eta^4$

$K=\xi^4-14\,\xi^2\eta^2+\eta^4$ $\quad S = \xi^2-3\eta^2$ $\quad P = \xi^4-10\xi^2\eta^2+5\eta^4$

$H=\xi^6-21\,\xi^4\eta^2+35\,\xi^2\eta^4-7\eta^6$ $\quad S_1 = S$ $\quad T_1 = T$ $\quad H_1 = H(\xi=\eta)$

$C_{301}$, $C'_{301}$, $\cdots$ are complicated coefficients,which are not given here.

Substituting cell problem solutions (68) and (69) into equation (67) one obtains

$$C\left(\psi_{0xxxx} + \psi_{0yyyy}\right) + 2F\psi_{0xxyy} = 0 \tag{70}$$

where $C$ and $F$ are very complicated coefficients,which are not given here. Figure 11 shows the homogenized coefficients $C$ (curve 1) and $F$ (curve 2) for $\varepsilon_1 = 0.125$, $\varepsilon_2 = -1/9$.

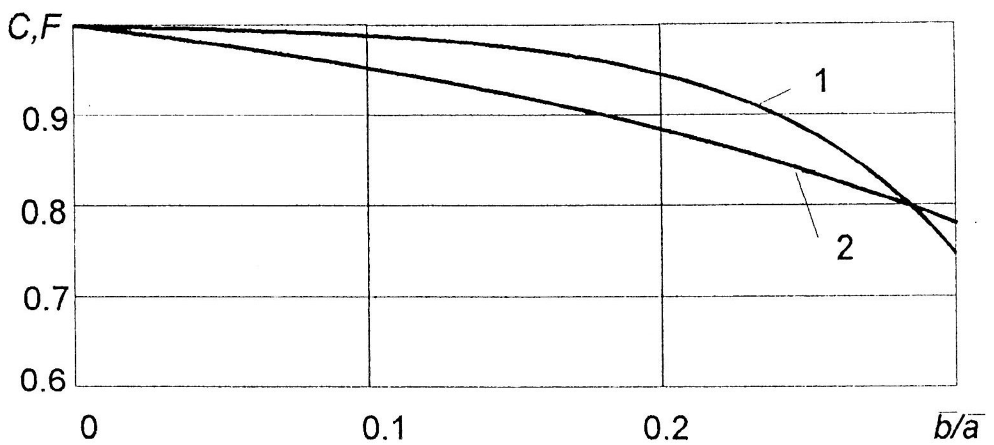


Figure 11. Homogenized Coefficients C and F versus Ratio $\bar{b}/\bar{a}$

## 7 Perforated Shallow Shells

For perforated shallow shells the governing equations are

$$\nabla_k^2\psi + D\nabla^4 W = P(x,y) \qquad (Eh)^{-1}\nabla^4\psi - \nabla_k^2 W = 0 \tag{71}$$

where $\nabla_k^2 = k_1\frac{\partial^2}{\partial y^2} + k_2\frac{\partial^2}{\partial x^2} \qquad k_1 = R_1^{-1} \qquad k_2 = R_2^{-1}$

and $R_1$, $R_2$ are principal curvature radii. If hole boundaries are free of stresses, we have Kirchhoff's boundary conditions for a free boundary

$$\left[\psi_{xx}\sin^2\theta + \psi_{yy}\cos^2\theta - \psi_{xy}\sin 2\theta\right]_{\partial G_k} = 0 \tag{72}$$

$$\left[05.\left(\psi_{yy} - \psi_{xx}\right)\sin 2\theta + \psi_{xy}\cos 2\theta\right]_{\partial G_k} = 0 \tag{73}$$

$$\left[\left(\nabla^2 W\right)_{n_k} - (1-\nu)\left[0.5\left(W_{xx} - W_{yy}\sin 2\theta - W_{xy}\cos 2\theta\right]_{S_k}\right]_{\partial G_k} = 0 \tag{74}$$

$$\left[\nabla^2 W - (1-\nu)\left(-W_{xx}\sin^2\theta - W_{yy}\cos^2\theta + W_{xy}\sin 2\theta\right)\right]_{\partial G_k} = 0 \tag{75}$$

We assume (without loss of generality) the boundaries of shells simply supported. We obtain the asymptotic expansions (5) and (59). Substituting this expansions into equations (71) and boundary conditions (72) to (74) and splitting with respect to the powers of $\varepsilon_1$ one obtains recurrent systems of cell boundary value problems and boundary value problems for the whole domain. Cell boundary value problems consist of boundary value problems of plate bending and plane theory of elasticity. Then we can use the solutions obtained above.

## 8 Conclusions

The asymptotic method proposed may be used for an analytical investigation of the static stress-strain state and the investigation of oscillations of perforated rectangular plates and shallow shells.

## Literature


1. Andrianov, I. V.: Application of Padé-Approximants in Perturbation Methods, Advances in Mechanics 14, N 2, (1991), 3-25.

2. Andrianov, I. V.; Lesnichaya, V. A.; Manevich, L. I..: Homogenization Methods in Statics and Dynamics of Reinforced Shells, (in Russian), Moscow, Nauka, (1985).

3. Andrianov, I. V.; Manevich, L. I.: Method of Homogenization in Application to the Theory of Shells, Advances in Mechanics 6, N 2/3, (in Russian), (1983), 3-29.

4. Andrianov, I. V.; Starushenko, G. A.: Application of the Averaging Method for the Calculation of Perforated Plates, Soviet Applied Mechanics 24, N 4, (1988), 410-415.

5. Andrianov, I. V.; Starushenko, G. A.: Solution of Dynamic Problems for Perforated Structures by the Method of Averaging, Soviet Math. 57, N 5, (1991), 3410-3412.

6. Baker, G. A.; Graves-Morris, P.: Padé Approximants, New York, Addison-Wesley Publ. Co., (1981).

7. Bakhvalov, N.; Panasenko, G.: Averaging Processes in Periodic Media, Mathematical Problems in Mechanics of Composite Materials, Kluwer Academic Publishers,Dordrecht (1989).

8. Bensoussan, A.; Lions, J.-L.; Papanicolaou, G.: Asymptotic Methods in Periodic Structures, New York, North-Holland Publ. Co., (1978).

9. Berdichevsky, V. L.: Variational Principles of the Continuum Mechanics, Moscow, Nauka (in Russian), (1983).

10. Bourgat, J. F.: Numerical Experiments of the Homogenization Method for Operators with Periodic Coefficients, Lecture Notes in Math., N 704, Berlin, Springer-Verlag, (1979), 330-356.

11. Caillerie, D.: Thin Elastic and Periodic Plates, Mathematical Methods in Applied Science, 6, (1984), 151-191.

12. Christensen, R. M.: Mechanics of Composite Materials, New York, J. Wiley and Sons, (1979).

13. Cioranescu, D.; Paulin, J. S. J.: Homogenization in Open Sets with Holes, J. Math. Anal. Appl. 71, (1979), 590-607.

14. Duvaut, G.: In Singular Perturbation and Boundary Layer Theory, Comportement macroscopique d'une plaque perforee, Lecture Notes in Math., N 594, Berlin, Springer-Verlag, (1977), 131-145.

15. Grigoluk, E. I.; Phylshtinsky, L. A.: Perforated Plates and Shells, Moscow, Nauka, (in Russian), (1970).

16. Guz, A. N.; Nemish, Yu. N.: Method of Boundary Form Perturbation in the Mechanics of Solids, Kiev, Visha shcola (in Russian), (1989).

17. Kalamkarov, A. L.: Composite and Reinforced Elements of Constructions, New York, London, Wiley and Sons, (1993).

18. Kantorovitch, L. V.; Krylov, V. I.: Approximate Methods of Higher Analysis, Moscow, Nauka, (in Russian), (1949).

19. Lewinsky, T.; Telega, J. J.: Asymptotic Method of Homogenization of Fissured Elastic Plates, J. of Elasticity 19, (1988), 37-62.

20. Lions, J.-L.: Asymptotic Expansions in Perforated Media with a Periodic Structure, Rocky Mountain J. of. Math. 10, N 1, (1980), 125-140.

21. Lions, J.-L.: On some Homogenization Problems, ZAMM 62, N 5, (1982), 251-262.

22. Mignot, F.: Puel, J.-P.; Suquet, P.-M.: Flambage des plaques elastiques multiperforees, Annales Fac. Sci., Toulouse 3, (1981), 1-57.

23. Nayfeh, A. H.: Introduction to Perturbation Techniques, New York, J. Wiley and Sons, (1981).

24. Nazarov, S. A.; Paukshto, M. V.: Discrete Models and Homogenization in the Theory of Elasticity Problems, Leningrad, L. University,(in Russian), (1984).

25. Oleynik, O. A.; Shamaev, A. S.; Yosif'yan, G. A.: Homogenization of Eigenvalues and Eigenfunctions of the Boundary Value Problems in Perforated Domains for Elliptic Equations with Non-Uniformly Oscillating Coefficients, Current Topics in Partial Differential Equations, Kinokunia Co., Tokyo, (1986), 87-216.

26. Pickett, G.: Bending, Buckling and Vibration of Plates with Holes, in Developments in Theoretical and Applied Mechanics 2, 1965, 9-22.

27. Sanchez-Palencia, E.: Non-Homogeneous Media and Vibration Theory, Berlin, Springer-Verlag, (1980).

28. Suquet, P.-M.: Methodes d' homogeneisation en mechanique des solides, Comportements Rheologiques et Structure des Materieux - CR 15eme Coll., GFR, Paris, (1980), 87-128.

29. Vanninathan, M.: Homogenization of Eigenvalue Problems in Perforated Domains, Proc. Indian Acad., Science (Math. Science) 90, N 3, (1981), 239-271.

*Addresses:* Professor Igor V. Andrianov and Professor E. G. Kholod, Department of Mathematics, Pridneprovye State Academy of Civil Engineering and Architecture, 24a Chernyshevskogo, UA-320092 Dnepropetrovsk.

## Nachruf

Professor Valeryi Schevtschenko (Shevchenko) ist am 2. September 1994 verstorben. Er wurde am 5. März 1949 in der Ukraine geboren, studierte von 1966 bis 1972 Elektrotechnik an der Staatsuniversität Dnepropetrovsk, von der er 1988 auch seinen Doktorgrad in Mathematik und Physik erhielt. Zuletzt war er außerordentlicher Professor für Mathematik an der Ukrainischen Metallurgischen Akademie in Dnepropetrovsk.
V. Schevtschenko ist Autor oder Koautor von 45 wissenschaftlichen Arbeiten über asymptotische Methoden, Padé-Näherungen und Festigkeitslehre.