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Asymptotic Solution for the Nonlinear Dynamic Problem of Mechanical Systems with Time Dependent Parameters

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A double asymptotic method for the effect of nonlinearity on vibration problems of some mechanical systems with time dependent characteristics is proposed. The mathematical tool is an asymptotic procedure developed with perturbation and WKB theories. The method illustrated for the some coefficient functions and results of calculations are compared with the purely numerical solutions. The approach with using of the „DOUBLE-1" program can be effected for the applied nonlinear dynamics problems for the mechanical systems with variable parameters.

1 Introduction

The problems of nonlinear dynamics have been one of the most fundamental subjects in the study of the behavior mechanical systems in modern aerospace, machinery and structural industry. In the absence of an exact analytical solution of the nonlinear differential equations in the general form the engineering and applied mechanics problems are often solved by numerical, approximate analytical or analytical-numerical methods.

We shall not attempt a review of the pertinent literature, but refer to the approximate methods of solution of the nonlinear dynamic problems of bars and thin walled plate and shell structures by Bolotin (1961), Young (1962), Elishakoff (1976), Leissa (1973), Timoshenko, Young and Weaver (1974), Huseyin (1978), Goldenveizer, Lidsky and Tovstik (1979), Gorman (1982). In addition, we mention a number of more recent papers by Steele (1989), Gristchak (1990), Kobajashi and Sonoda (1991), Andrianov and Krizhevsky (1993), Andrianov, Gristchak and Ivankov (1994), Smirnov and Rimrott (1994).

This paper focusses on an approximate analytical method using hybrid (perturbation and phase-integral or WKB methods) asymptotic expansions for some nonlinear dynamic problems of the mechanical systems with time dependent parameters.

2 Description of the Method

Consider the nonlinear dynamics problem for the mechanical system with time dependent characteristics of mass or density. The corresponding differential equation that describes the process can be taken in the form

$$f'' + \omega^2(t)f + \alpha P(t)f^2 + \alpha^2 Q(t)f^3 = 0 \tag{1}$$

where

$$\omega^2(t) = \omega_0^2\varphi(t)$$

with $\varphi(t)$ a given function of time.

The parameter of natural frequency of vibration $\omega_0 > 0$ we take into account as a large parameter. The function $\varphi(t) > 0$ is a continuous function, and $0 < \alpha << 1$. For example, for the nonlinear vibration problem of the shallow cylindrical shell the parameter $\alpha = \frac{a_1}{h}$, where a_1 is the longitudinal dimension and h is the shell thickness. We suppose here that the $\varphi(t)$, $P(t)$ and $Q(t)$ are continuously differentiable functions.

In order to obtain an approximate analytical solution of the initial nonlinear differential equation we will use the double asymptotic expansion method that includes two steps of solution (Gristchak and Golovan, 1995). On the first step (*outer perturbation expansion*) the solution of the equation (1) is presented as the expansion on the small parameter α.

$$f = f_0 + \alpha f_1 + \alpha^2 f_2 + \ldots \tag{2}$$

We take into account three terms of expansion (2) and after the substitution of equation (2) into equation (1) and comparing the coefficients with equal orders of parameter α, we obtain a system of equations for the unknown functions $f_0, f_1, f_2 \ldots$

$$f_0'' + \omega^2(t) f_0 = 0 \qquad \text{in } \alpha^0 \tag{3}$$

$$f_1'' + \omega^2(t) f_1 = -P(t) f_0^2 \qquad \text{in } \alpha^1 \tag{4}$$

$$f_2'' + \omega^2(t) f_2 = -2P(t) f_0 f_1 - Q(t) f_0^3 \qquad \text{in } \alpha^2 \tag{5}$$

Taking into account that ω_0^2 is a large parameter, the solution of the first homogeneous equation of the system we look for on the basis of the two terms phase-integral or WKB-method (Gristchak and Golovan, 1995) (*inner asymptotic expansion*). Omitting the details of simple calculations we obtain the function f_0 in the form

$$f_0 = \frac{A}{\varphi^{1/4}(\tau)} \cos\left(\int_0^t \omega_0 \varphi^{1/2}(\tau) d\tau\right) + \frac{B}{\varphi^{1/4}(\tau)} \sin\left(\int_0^t \omega_0 \varphi^{1/2}(\tau)\, d\tau\right) \tag{6}$$

Using the method of variation of arbitrary constants to obtain the solutions of equations (4) and (5) we will have

$$\begin{aligned} f_1 = & \left[\int_0^t \frac{P(\tau) f_0^2 \sin\left(\int_0^\tau \omega_0 \varphi^{h}(z) dz\right)}{\omega_0 \varphi^{1/4}(\tau)} d\tau\right] \frac{\cos\left(\int_0^t \omega_0 \varphi^{1/2}(\tau) d\tau\right)}{\varphi^{1/4}(t)} \\ & + \left[\int_0^t \frac{-P(\tau) f_0^2 \cos\left(\int_0^\tau \omega_0 \varphi^{1/2}(z) dz\right)}{\omega_0 \varphi^{1/4}(\tau)} d\tau\right] \frac{\sin\left(\int_0^t \omega_0 \varphi^{1/2}(\tau) d\tau\right)}{\varphi^{1/4}(t)} \end{aligned} \tag{7}$$

$$\begin{aligned} f_2 = & \left[\int_0^t \frac{\left(2P(\tau) f_0 f_1 + Q(\tau) f_0^3\right) \sin\left(\int_0^\tau \omega_0 \varphi^{1/2}(z) dz\right)}{\omega_0 \varphi^{1/4}(\tau)}\right] \frac{\cos\left(\int_0^t \omega_0 \varphi^{1/2}(\tau) d\tau\right)}{\varphi^{1/4}(t)} \\ & + \left[\int_0^t \frac{\left(-2P(\tau) f_0 f_1 - Q(\tau) f_0^3\right) \cos\left(\int_0^\tau \omega_0 \varphi^{1/2}(z) dz\right)}{\omega_0 \varphi^{1/4}(\tau)}\right] \frac{\sin\left(\int_0^t \omega_0 \varphi^{1/2}(\tau) d\tau\right)}{\varphi^{1/4}(t)} \end{aligned} \tag{8}$$

Substituting equations (6), (7) and (8) for functions f_0, f_1 and f_2 in expansion (2) we obtain the approximate analytical solution of equation (1) in the form

$$f(t) = AG_1(t) + BG_2(t) + \frac{A^2 \alpha}{\omega_0}\left(G_1(t) \int_0^t P(\tau) G_1^2(\tau) G_2(\tau) d\tau - G_2(t) \int_0^t P(\tau) G_1^3(\tau) d\tau\right)$$

$$+\frac{2AB\alpha}{\omega_0}\left(G_1(t)\int_0^t P(\tau)G_2^2(\tau)G_1(\tau)d\tau - G_1(t)\int_0^t P(\tau)G_1^2(\tau)G_2(\tau)d\tau\right)+\frac{B^2\alpha}{\omega_0}$$

$$-G_2(t)\int_0^t P(\tau)G_2^2(\tau)G_1(\tau)d\tau\Bigg)+\frac{A^3\alpha^2}{\omega_0}\left(G_1\left[\int_0^t Q(\tau)G_1^3(\tau)G_2(\tau)d\tau+\int_0^t\left(2P(\tau)G_1^2(\tau)G_2(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1^2(z)G_2(z)dz\right)d\tau\right.\right.$$

$$\left.-\int_0^t\left(2P(\tau)G_2^2(\tau)G_1(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_2^3(z)dz\right)d\tau\right]-G_2(t)\left[\int_0^t Q(\tau)G_1^4(\tau)d\tau+\int_0^t\left(2P(\tau)G_1^3(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1^2(z)G_2(z)dz\right)d\tau\right.$$

$$+\frac{A^2B\alpha^2}{\omega_0}\left(G_1(t)\left[\int_0^t 3Q(\tau)G_1^2(\tau)G_2(\tau)d\tau+\int_0^t\left(4P(\tau)G_1^2(\tau)G_2(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1(z)G_2^2(z)dz\right)d\tau\right.\right.$$

$$\left.-\int_0^t\left(2P(\tau)G_2^2(\tau)G_1(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1^2(z)G_2(z)dz\right)d\tau-\int_0^t\left(2P(\tau)G_2^3(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1^3(z)dz\right)d\tau\right]$$

$$-G_2(t)\left[\int_0^t 3Q(\tau)G_1^3(\tau)G_2(\tau)d\tau+\int_0^t\left(4P(\tau)G_1^3(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1(z)G_2^2(z)dz\right)d\tau\right.$$

$$-\int_0^t\left(2P(\tau)G_1^2(\tau)G_2(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1^2(z)G_2(z)dz\right)d\tau \tag{9}$$

$$\left.\left.-\int_0^t\left(2P(\tau)G_1^2(\tau)G_2(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1^2(z)G_2(z)dz\right)d\tau-\int_0^t\left(2P(\tau)G_2^2(\tau)G_1(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1^3(z)dz\right)d\tau\right]\right)$$

$$+\frac{AB^2\alpha^2}{\omega_0}\left(G_1(t)\left[\int_0^t 3\frac{Q(\tau)}{\varphi(\tau)}G_1(\tau)G_2^3(\tau)d\tau+\int_0^t\left(\frac{2P(\tau)}{\varphi(\tau)^{3/4}}G_1^2(\tau)G_2(\tau)\int_0^\tau\frac{P(z)}{\omega_0\varphi(\tau)^{3/4}}G_2^3(z)dz\right)d\tau\right.\right.$$

$$\left.+\int_0^t\left(2P(\tau)G_2^2(\tau)G_1(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1(z)G_2^2(z)dz\right)d\tau-\int_0^t\left(4P(\tau)G_2^3(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1^2(z)G_2(z)dz\right)d\tau\right]$$

$$-G_2(t)\left[\int_0^t 3Q(\tau)G_1^2(\tau)G_2^2(\tau)d\tau+\int_0^t\left(2P(\tau)G_1^3(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_2^3(z)dz\right)d\tau\right.$$

$$\left.\left.+\int_0^t\left(2P(\tau)G_1^2(\tau)G_2(\tau)\int_0^\tau\frac{P(z)}{\omega_0}G_1(z)G_2^2(z)dz\right)d\tau-\int_0^t\left(4P(\tau)G_2^2G_1\int_0^\tau\frac{P(z)}{\omega_0}G_1^2G_2dz\right)d\tau\right]\right)$$

$$+\frac{B^3\alpha^2}{\omega_0}\left(G_1\left[\int_0^t Q(\tau)G_2^4d\tau+\int_0^1\left(2P(\tau)G_2^2G_1\int_0^\tau\frac{P(z)}{\omega_0}G_1^3dz\right)d\tau-\int_0^t\left(2P(\tau)G_2^3\int_0^\tau\frac{P(z)}{\omega_0}G_1G_2^2dz\right)d\tau\right]\right.$$

$$\left.-G_2\left[\int_0^t Q(\tau)G_1G_2^3d\tau+\int_0^t\left(2P(\tau)G_1^2G_2\int_0^\tau\frac{P(z)}{\omega_0}G_2^3dz\right)d\tau-\int_0^t\left(2P(\tau)G_2^2G_1\int_0^\tau\frac{P(z)}{\omega_0}G_1G_2^2dz\right)d\tau\right]\right)$$

Solution (9) we can rewrite in the form

$$f(t)=AG_1(t)+BG_1(t)+\frac{\alpha}{\omega_0}\left(A^2f_{11}+2ABf_{12}+B^2f_{22}\right)+\frac{\alpha^2}{\omega_0}\left(A^3f_{33}+A^2Bf_{34}+AB^2f_{43}+B^3f_{44}\right) \tag{9a}$$

where we use some new functions.

$$g_{11}=\int_0^t P(\tau)G_1^3(\tau)d\tau \qquad g_{12}=\int_0^t P(\tau)G_1^2(\tau)G_2(\tau)$$

$$g_{21}=\int_0^t P(\tau)G_2^2(\tau)G_1(\tau)d\tau \qquad g_{22}=\int_0^t P(\tau)G_2^3(\tau)d\tau \tag{9b}$$

$$\psi_{11} = \int_0^t Q(\tau)G_1^4(\tau)d\tau \qquad \psi_{12} = \int_0^t Q(\tau)G_1^2(\tau)G_2^2(\tau)d\tau$$

$$\psi_{22} = \int_0^t P(\tau)G_2^4(\tau)d\tau \qquad \psi_{13} = \int_0^t Q(\tau)G_1(\tau)G_2^3(\tau)d\tau \tag{9c}$$

$$\psi_{31} = \int_0^t Q(\tau)G_1^3(\tau)G_2(\tau)d\tau$$

$$f_{33} = G_1(t)\left[\psi_{31} + \frac{2}{\omega_0}G_{1212}(t) - \frac{2}{\omega_0}G_{2111}(t)\right] - G_2(t)\left[\psi_{11} + \frac{2}{\omega_0}G_{1112}(t) - \frac{2}{\omega_0}G_{1211}(t)\right] \tag{9d}$$

$$f_{34} = G_1(t)\left[3\psi_{12} + \frac{4}{\omega_0}G_{1221}(t) - \frac{2}{\omega_0}G_{2112}(t) - \frac{2}{\omega_0}G_{2211}(t)\right] - G_2(t)\left[3\psi_{31} + \frac{4}{\omega_0}G_{1121}(t) - \frac{2}{\omega_0}G_{1212}(t) - \frac{2}{\omega_0}G_{2111}(t)\right]$$

$$f_{43} = G_1(t)\left[3\psi_{13} + \frac{2}{\omega_0}G_{1222}(t) + \frac{2}{\omega_0}G_{2121}(t) - \frac{4}{\omega_0}G_{2212}(t)\right] - G_2(t)\left[3\psi_{12} + \frac{2}{\omega_0}G_{1122}(t) + \frac{2}{\omega_0}G_{1221}(t) - \frac{4}{\omega_0}G_{2112}(t)\right]$$

$$f_{44} = G_1(t)\left[\psi_{22} + \frac{2}{\omega_0}G_{2122}(t) - \frac{2}{\omega_0}G_{1121}(t)\right] - G_2(t)\left[\psi_{13} + \frac{2}{\omega_0}G_{1222}(t) - \frac{2}{\omega_0}G_{2121}(t)\right]$$

Here we denoted

$$G_{1212}(t) = \int_0^t P(\tau)G_1^2(\tau)G_2(\tau)g_{12}(\tau)d\tau \qquad G_{2111}(t) = \int_0^t P(\tau)G_2^2(\tau)G_1(\tau)g_{11}(\tau)d\tau$$

$$G_{1211}(t) = \int_0^t P(\tau)G_1^2(\tau)G_2(\tau)g_{21}(\tau)d\tau \qquad G_{1221}(t) = \int_0^t P(\tau)G_1^2(\tau)G_2(\tau)g_{21}(\tau)d\tau$$

$$G_{2111}(t) = \int_0^t P(\tau)G_2^2(\tau)G_1(\tau)g_{11}(\tau)d\tau \qquad G_{1222}(t) = \int_0^t P(\tau)G_1^2(\tau)G_2(\tau)g_{22}(\tau)d\tau$$

$$G_{2122}(t) = \int_0^t P(\tau)G_2^2(\tau)G_1(\tau)g_{22}(\tau)d\tau \qquad G_{2121}(t) = \int_0^t P(\tau)G_2^2(\tau)G_1(\tau)g_{21}(\tau)d\tau \tag{9e}$$

$$G_{1112}(t) = \int_0^t P(\tau)G_1^3(\tau)g_{12}(\tau)d\tau \qquad G_{2211}(t) = \int_0^t P(\tau)G_2^3(\tau)g_{11}(\tau)d\tau$$

$$G_{1121}(t) = \int_0^t P(\tau)G_1^3(\tau)g_{21}(\tau)d\tau \qquad G_{2211}(t) = \int_0^t P(\tau)G_2^3(\tau)g_{11}(\tau)d\tau$$

$$G_{1121}(t) = \int_0^t P(\tau)G_1^3(\tau)g_{21}(\tau)d\tau \qquad G_{2212}(t) = \int_0^t P(\tau)G_2^3(\tau)g_{12}(\tau)d\tau$$

$$G_{1122}(t) = \int_0^t P(\tau)G_1^3(\tau)g_{22}(\tau)d\tau$$

where $G_i(i = 1, 2)$ we will call G functions of the first kind

$$G_1 = \frac{\cos\left(\int_0^t \omega_0 \varphi^{1/2}(\tau) d\tau\right)}{\varphi^{1/4}(\tau)} \qquad G_2 = \frac{\sin\left(\int_0^t \omega_0 \varphi^{1/2}(\tau) d\tau\right)}{\varphi^{1/4}(\tau)} \tag{10}$$

and G_{ijkl} we will call G functions of the second kind.

From the boundary conditions

$$f(0) = 1 \qquad f'(0) = 0 \tag{11}$$

we obtain the constants A and B in the form

$$A = \varphi^{1/4}(0) \qquad B = \frac{\varphi'(0)}{4\omega_0 \varphi^{5/4}(0)} \tag{12}$$

3 Asymptotic Estimation of the Integrals

The integrals that are included in the approximate solution (9) may be calculated numerically. However, in order to obtain an approximate analytical solution in closed form we have estimated asymptotically all integrals that are in the solution. For example, for an asymptotic estimation of the integrals of type

$$\int_0^t \frac{P(\tau)\sin\left(\int_0^\tau k\omega_0 \varphi^{1/2}(z) dz\right)}{\varphi^{3/4}(\tau)} d\tau \tag{13}$$

we let

$$f(\tau) = \frac{P(\tau)}{\varphi^{3/4}(\tau)} \tag{14}$$

Integral (13) we represent now in the form

$$\int_0^t f(\tau)\sin(k\omega_0 h(\tau)) d\tau \tag{15}$$

The functions $h(\tau)$ and $f(\tau)$ are continuously differentiable functions and derivative $h'(\tau) = \varphi^{1/2}(\tau)$ does not equal to zero in all the points of the interval $[0,t]$. So, we can use integration by parts.

As a result we obtain

$$\int_0^t f(\tau)\sin(k\omega_0 h(\tau)) d\tau = -\frac{f(\tau)}{k\omega_0 h'(\tau)}\cos(k\omega_0 h(\tau))\Bigg|_0^t + \frac{1}{k\omega_0}\int_0^t \cos(k\omega_0 h(\tau))\left[\frac{f(\tau)}{h'(\tau)}\right]' d\tau \tag{16}$$

The integral term on the right hand of the equation (16) can be estimated in the same way. So, all integrals in the solution (9) were estimated asymptotically by this approach and we obtained an approximate analytical solution in closed form.

4 Numerical Analysis of an Approximate Solution

In to obtain numerical results in according with solution (9) and for the comparison purposes with the purely numerical approach for the problem we solve, the FORTRAN-program „DOUBLE-1“ was written.

The results of calculations for the functions

$$\varphi(t) = 0.0002t + 0.02 \qquad P(t) = 0.01t + 0.3 \qquad Q(t) = 0.1t + 0.05$$

and for the parameters

$$\omega_0 = 10 \qquad \alpha = 0.002$$

are given in Figure 1.

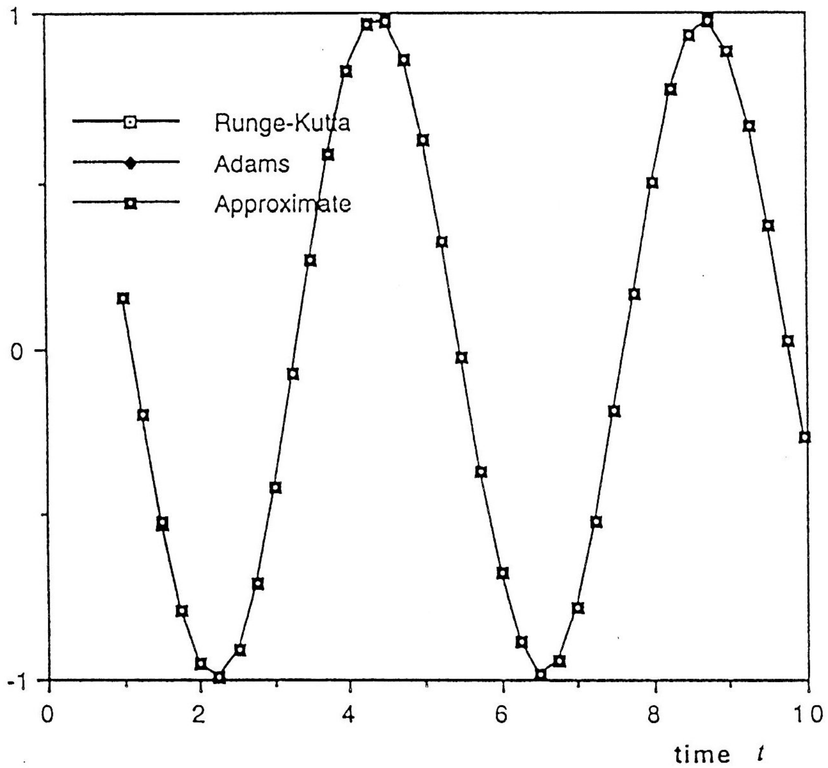

Figure 1. Comparison of the approximate analytical solution (9) with the direct numerical solutions

We compare our approximate results with the numerical approaches using Runge-Kutta and Adams methods. It is necessary to note that this result and the calculations for the other coefficientfunctions in the initial equation (1), (for example, exponential functions) gave to us the possibility to make a statement about the effectiveness of the proposed approximate analytical solution.

The estimation of the outer and inner expansions for this problem are given in Figure 2.

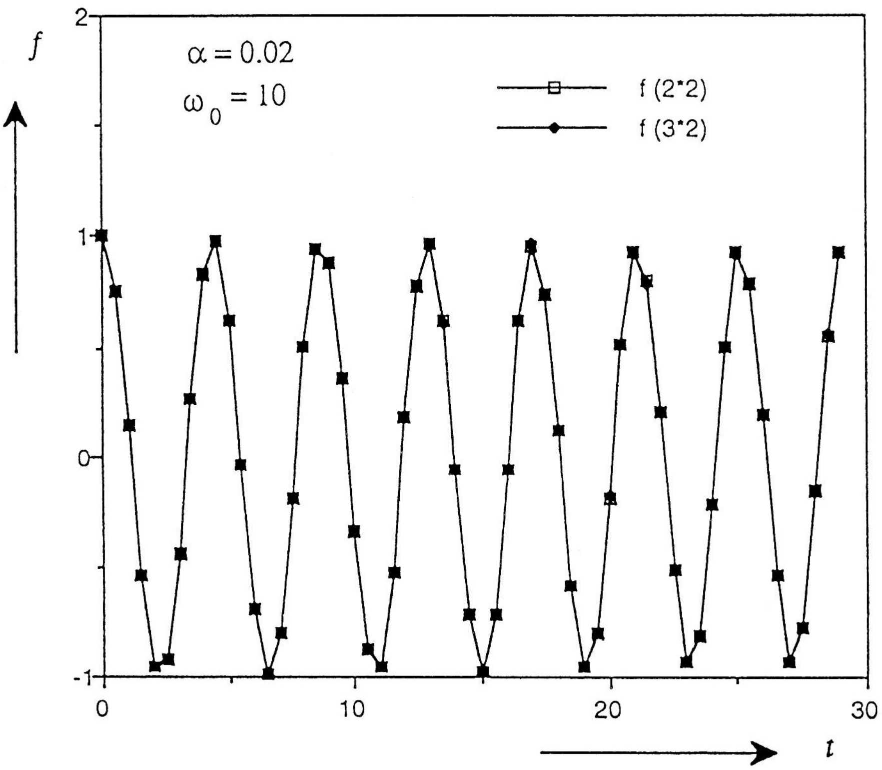

Figure 2. Results of calculations of the function $f(t)$ for two outer - two inner approximations and for three outer- two inner approximations

4 Concluding Remark

The ultimate goal of the proposed double expansion approach is to develop an algorithm for the approximate asymptotic solution of the some nonlinear dynamic problems for mechanical systems with time dependent characteristics. As shown in this paper, two approximations for inner and outer expansions for given coefficientfunctions are enough to obtain satisfactory results for practical purposes.

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