TECHNISCHE MECHANIK, Band 15, Heft 3, (1995), 191-194
Manuskripteingang: 13. Juni 1995

Nonlinear Model of Delamination in Laminated Composites

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Von Karman's plate theory is made use of for the description of thin layer delamination. Such a model provides a way to obtain a scale effect in composites. This effect involves a change of fracture type when the layer thickness achieves some critical value.

1 Introduction

Delamination can occur during the manufacturing process of laminated composites as well as under load in service. In connection with the operation of the material with real defects some problems concerning the maximum safe size of defects and the decay carrying capacity of composites containing flaws may arise (Bolotin, 1984). A review of studies applying the methods of fracture mechanics is presented by Bolotin (1984) and Vasilchenko and Koshelev (1974).

In most cases the lamination stiffness changes very little when flaws grow. Nevertheless, lamination stiffness may have great significance for same biomechanics problems (Saulgozis et al., 1993) and for all problems where geometric nonlinearities should be taken into account.

2 Theory

In this paper, a half space with a penny-shaped crack M_h located at a small distance h from the free surface is considered. Let Ω be an area on surface R^2 limited by the circle ∂B. Let $M_h = \left\{\bar{x} \in R^3 : x_3 = h : x_1^2 + x_2^2 \leq R\right\}$, where h is small parameter and $R >> h$. The problem is being considered on the deformation of the elastic half space $\Omega = \left(\bar{x} : x_3 \geq 0\right)$, weakened by the crack M_h. Dead load is absent. The half space boundary is free of loading. The pressure q is applied to the crack surfaces $M_h \pm$. Let us consider this problem using the beam model (Slepyan, 1990) and Griffith's criterion (Kachanov, 1974). Let the penny-shaped crack be increased by ΔR, and its area be changed by $\delta S = 2\pi R \Delta R$. When the plate $H_h = \left\{\bar{x} : x_3 = h;\ x_1^2 + x_2^2 \leq R\right\}$ deflection is increased, the pressure q will do work δA, the strain energy of the plate will rise by δP and we will consider that the change of the strain energy of the rest of the half space is small. Some part of energy δW will be used for the rupture of the material on the space δS. According to the equation of energy balance (Kachanov, 1974)

$$\delta A = \delta P + \delta W \tag{1}$$

Delamination starts when

$$\frac{\partial}{\partial \delta S}(A - P) = \frac{\partial W}{\partial \delta S} \tag{2}$$

There is the possibility of large deflection of a thin plate. In this case $P \neq \frac{1}{2} A$. If h is small and the deflection w of the plate is comparable to h, then the strain energy P of bending of the plate is

$$P = \frac{D}{2}\int_{\Omega}\left[\left(\frac{\partial^2 w}{\partial x_1^2} + \frac{\partial^2 w}{\partial x_2^2}\right)^2 - 2(1-\nu)\left(\frac{\partial^2 w}{\partial x_1^2}\frac{\partial^2 w}{\partial x_2^2} - \frac{\partial^2 w}{\partial x_1^2 \partial x_2}\right)\right]d\Omega$$
$$+ \frac{h}{2E}\int_{\Omega}\left[\left(\frac{\partial^2 \Phi}{\partial x_1^2} + \frac{\partial^2 \Phi}{\partial x_2^2}\right)^2 - 2(1+\nu)\left(\frac{\partial^2 \Phi}{\partial x_1^2}\frac{\partial^2 \Phi}{x_2^2} - \frac{\partial^2 \Phi}{\partial x_1 \partial x_2}\right)\right]d\Omega \qquad (3)$$

where $D = \dfrac{Eh^3}{12(1-\nu^2)}$ is the plate stiffness, E is Young's modulus of elasticity, ν is Poisson's ratio, and Φ is Airy's stress function (Volmir, 1956).

The strain energy (3) corresponding to von Karman's plate theory (Volmir, 1956) is

$$A = 2\pi\int_0^R rqw\,dr \qquad (4)$$

i.e. the work of the pressure q. According to the Griffith theory (Kachanov, 1974)

$$\delta W = 2\gamma\delta S \qquad (5)$$

where γ is the fracture work per unit surface. As at a crack tip $\frac{\partial w}{\partial r}\big|_{r=R} = 0$ and $w\big|_{r=R} = 0$, we will consider the deflection of a circular flexible plate with a clamped edge. Existence and uniqueness theorems for a symmetric solution of von Karman's plate theory are being illustrated in Morozov (1978). Using the method of Bubnov-Galerkin (Volmir, 1956) and equations (2), (3), (4) and (5) we will find that

$$q = \frac{K(w_0)}{R^2}\sqrt{\frac{E\gamma h^3}{1-\nu^2}} \sim h^{3/2} \qquad (6)$$

where $K(w_0)$ is a function of the deflection w_0 in the center of the plate (Volmir, 1956). In the case of using only the first component in equation (3) $K_l = \frac{8}{\sqrt{3}} \approx 4{,}62$; for a large deflection $w_0 = h$ (nonlinear model in plate theory) $K_{nl} = 5{,}22$ $(\nu = 0{,}3)$.

The stress in a small domain at the edge crack corresponds to the state which is the local state of the plane strain (Kachanov, 1974). In this case the mode I stress intensity factor K_I is

$$K_I = \sqrt{\frac{2\gamma E}{1-\nu^2}} \qquad (7)$$

Substituting γ from equation (6) into equation (7), results in

$$K_I = \frac{qR^2}{K(w_0)}\sqrt{\frac{2}{h^3}} \qquad (8)$$

For small strains

$$K_{II} = \frac{qR^2}{4}\sqrt{\frac{3}{2h^3}} \qquad (9)$$

which coincides with the solution of this problem, by means of the asymptotic method (Nazarov and Polyakova, 1990). For von Karman's plate theory and $w_0 = h$

$$K_{Inl} = \frac{qR^2}{5.22}\sqrt{\frac{2}{h^3}} \tag{10}$$

An analysis of the expressions (9) and (10) shows that the ultimate load q_*, when the brittle fracture can take place, is a bit higher for the nonlinear model in plate theory. The thin plate M_h may not be delaminated, but be broken as result of bending. In this case the ultimate load q^* can be obtained by strength of material's methods

$$q^* = K'(w_0)\frac{\sigma_b h^2}{R^2} \tag{11}$$

here σ_b is the ultimate tensile strength.

For the linear model plate theory the calculations result in $K' = 4/3$, for the nonlinear model plate theory at $w_0 = h$ the calculations result in $K' = 1{,}064$ $(\nu = 0{,}3)$ (Morozov, 1978). Formulas (6) and (11) show the existence of a scale effect at delamination: when the crack is small enough and $q^* < q_*$ the plate is not delaminated, but is broken. A boundary value h_*, separating the types of fracture can be found from equations (7), (8) and (11) and amounts to

$$h_* = \frac{K^2(w_0)}{K'^2(w_0)}\,\frac{E\gamma\sigma_b^{-2}}{1-\nu^2} \tag{12}$$

For the linear theory the calculations result in $\frac{K^2(w_0)}{K'^2(w_0)} = 12$; for the von Karman plate at $w_0 = h$ the calculations result in $\frac{K^2(w_0)}{K'^2(w_0)} = 24{,}1$.

3 Conclusion

It has shown that one must always take into account the influence of nonlinear factors on the delamination in laminated composites.

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