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Forced Vibrations and Sound Emission of an Elastic Hemisphere Fixed in an Endless Plane Screen

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The axisymmetric low frequency vibration and the sound emission of a hemisphere clamped to an infinite plane screen are analyzed. The boundary value problem for an integro-differential system is formulated and the asymptotic expansion of the exact solution is constructed. The acoustic pressure far away from the hemisphere is determined. The polar pattern of emission is constructed and its dependence on the frequency of excitation is analyzed.

1 Vibration Equations of a Thin Spherical Shell

We consider a thin spherical shell of radius R and thickness h. We write the differential equations of axisymmetric vibration with angular frequency ω in non-dimensional form

$$\begin{aligned} &\Delta U + (2+\lambda)U - \lambda(2+\nu)w = p(\theta) + f(\theta) \\ &\mu^4[\Delta\Delta w + 2\Delta w] + \left(1-\lambda-\nu^2\right)w + (1+\nu)U = p(\theta) + f(\theta) \end{aligned} \tag{1}$$

where Δ is the Laplace operator in spherical coordinates.

$$\Delta = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\right]$$

In equations (1) f is the harmonic external force, p is the acoustic pressure at the surface of the shell, $\lambda = \dfrac{\rho\omega^2R^2\left(1-\nu^2\right)}{E}$ is a non-dimensional frequency parameter, E is Young's modulus, ν is Poisson's ratio and ρ is the shell density. The system contains a small geometrical parameter $\mu^4 = \dfrac{h^2}{12R^2}$, which is the relative shell thickness. We introduce non-dimensional variables as

$$u^* = Ru \qquad w^* = Rw \qquad p^* = \frac{Eh}{\left(1-\nu^2\right)}p \qquad f^* = \frac{Eh}{\left(1-\nu^2\right)}f$$

The auxiliary function U depends on the tangent displacement u and the deflection w and is determined by the expression

$$U = \frac{1}{\sin\theta}\left[\frac{\partial}{\partial\theta}(u\sin\theta)\right] + (1+\nu)w \tag{2}$$

The polar angle θ satisfies the condition $0 \le \theta \le \dfrac{\pi}{2}$.

We solve problem with clamped boundary conditions at the shell equator.

$$u\left(\frac{\pi}{2}\right) = 0 \qquad w\left(\frac{\pi}{2}\right) = 0 \qquad w'\left(\frac{\pi}{2}\right) = 0 \tag{3}$$

System (1) is based on the Kirchhoff-Love shell theory and for $\mu = 0$ it corresponds to the membrane theory, and in this case the order of the system reduces to two and we need keep only the first of conditions (3). Note that in this case an exact solution of the problem may be constructed.

2 Acoustic Medium

The shell is clamped to an infinite plane screen and is in contact with the acoustic medium that fills the upper half-space. The acoustic pressure p satisfies the Helmholtz equation

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial p}{\partial\theta}\right) + k^2 p = 0$$

where $k = \frac{wR}{c_0}$ is the wave parameter and c_0 is the sound velocity in the acoustic medium. We introduce the non-dimensional coordinate $r^* = Rr$.

We consider the boundary condition of the acoustic reflection at the screen. The acoustic pressure and the deflection satisfy the Euler equation (Skudrzyk, 1971) at the surface of a shell. Thus $\frac{\partial p}{\partial r}\Big|_{r=1} = \lambda\varepsilon w(\theta)$ at the shell, and $\frac{\partial p}{\partial\theta}\Big|_{\theta=0} = 0$ at the screen. Here $\varepsilon = \frac{\rho_0 R}{\rho h}$ is a non-dimensional density parameter and ρ_0 is the liquid density. The acoustic pressure also satisfies the Sommerfeld condition at infinity (Skudrzyk, 1971).

$$\lim_{r\to\infty} r\left(\frac{\partial p}{\partial r} - ikp\right) = 0$$

Let us examine the Neumann problem for the Helmholtz equation in the upper half-space. Then the acoustic pressure at a point with the polar coordinates (r,θ) may be represented as

$$p(r,\theta) = \lambda_\varepsilon \sum_{(n)} (2n+1)\frac{h_n^{(1)}(kr)}{nh_n^{(1)}(k) + kh_{n+1}^{(1)}(k)} P_n(\cos\theta)\int_0^{\pi/2} \omega(\theta_0)P_n(\cos\theta_0)\sin\theta_0 d\theta_0 \quad (4)$$

where $h_n^{(1)}(z)$ are Hankel spherical functions, $P_n(z)$ are Legendre polynomials of the nth order. The value of the deflection $w(\theta)$ contained in equation (4) is unknown.

The contact pressure at the surface of the shell is given by

$$p(\theta) = \lambda_\varepsilon \int_0^{\pi/2} G(\theta,\theta_0)\, w(\theta_0)\sin\theta_0\, d\theta_0 \quad (5)$$

Here

$$G(\theta,\theta_0) = \sum_{(n)} (2n+1)K_n(k)P_n(\cos\theta)P_n(\cos\theta_0)$$

where

$$K_n(k) = \frac{h_n^{(1)}(k)}{nh_n^{(1)}(k) + kh_{n+1}^{(1)}(k)}$$

The summation is taken for all even n.

Equations (1), (2), (5) and boundary conditions (3) form the boundary value problem for an integro-differential system of sixth order from which we can determine the amplitude of the tangent displacement u and the deflection w. Then we obtain the acoustic pressure $p(r,\theta)$ at an arbitrary point as a result of substitution of w into equation (4).

3 Recursive Algorithm

We assume that the non-dimensional parameters satisfy the following conditions $k,\lambda \sim 1$. Thus, we limit our consideration to the low frequency case. The solution of the boundary value problem is based on the asymptotic expansion in powers of the small geometrical parameter μ. The solution may be constructed by means of an algorithm similar to that of Vishik-Lyusternik (1957) and Wasow (1965). The applications of an asymptotic method to the problem of shell vibrations without a liquid may be found in Goldenveizer et al. (1979). For the integro-differential systems with a small parameter at the derivatives and a smooth kernel a similar asymptotic approach was developed in Lomov (1992).

We seek the asymptotic solution in the form

$$\begin{aligned} u(\theta,\eta,\mu) &= \bar{u}(\theta,\mu) + \mu\tilde{u}(\eta,\mu) \\ w(\theta,\eta,\mu) &= \bar{w}(\theta,\mu) + \tilde{w}(\eta,\mu) \\ U(\theta,\eta,\mu) &= \bar{U}(\theta,\mu) + \mu^2\tilde{U}(\eta,\mu) \end{aligned}$$

where $\eta = \mu^{-1}\left(\theta - \frac{\pi}{2}\right)$ and the functions $\bar{w}, \bar{u}$ and $\bar{U}$ keep their orders under differentiation. These functions may be found in the form

$$\begin{aligned} \bar{u}(\theta,\mu) &= \bar{u}_0(\theta,\mu) + \mu\bar{u}_1(\theta,\mu) + \cdots \\ \bar{w}(\theta,\mu) &= \bar{w}_0(\theta,\mu) + \mu\bar{w}_1(\theta,\mu) + \cdots \\ \bar{U}(\theta,\mu) &= \bar{U}_0(\theta,\mu) + \mu\bar{U}_1(\theta,\mu) + \cdots \end{aligned} \tag{6}$$

The functions $\bar{w}_k, \bar{u}_k$ and $\bar{U}_k$ are determined recursively from nonhomogeneous boundary value problems with the right hand sides depending on the previously determined functions.

The usual procedure to obtain these auxiliary problems is to substitute expansions (6) into equations (1) to (3) and (5) and equate the coefficients of μ^k. This is the so-called first or main iteration process (Wishik and Lyusternik, 1957). This algorithm must be adapted to the problem under consideration, since the kernel in equation (5) is a singular function. The functions $\tilde{w}, \tilde{u}$ and $\tilde{U}$ are the integrals of the edge effect in the neighbourhood of the edge $\theta = \frac{\pi}{2}$. They may be found in the form

$$\begin{aligned} \tilde{w}(\eta,\mu) &= \tilde{w}_0(\eta) + \mu\,\tilde{w}_1(\eta) + \cdots \\ \tilde{u}(\eta,\mu) &= \tilde{u}_0(\eta) + \mu\,\tilde{u}_1(\eta) + \cdots \\ \tilde{U}(\eta,\mu) &= \tilde{U}_0(\theta,\mu) + \mu\,\tilde{U}_1(\eta) + \cdots \end{aligned} \tag{7}$$

The approximations $\tilde{w}_k, \tilde{u}_k$ and $\tilde{U}_k$ are determined from auxiliary nonhomogeneous boundary value problems with the right hand sides obtained by means of equating of the coefficients of μ^k, depending on η, after the substitution of equations (7) into equations (1) to (3) and (5). The construction of the functions $\tilde{w}, \tilde{u}$ and $\tilde{U}$ is called the second or the additional iteration process. This process is similar to that for boundary value problems for ordinary differential equations. The functions $\tilde{w}_k, \tilde{u}_k$ and $\tilde{U}_k$ satisfy the following conditions:

$$\tilde{w}_k(\eta) \to 0 \qquad \tilde{u}_k(\eta) \to 0 \qquad \text{as} \qquad \eta \to -\infty \tag{8}$$

with $k = 0, 1, 2 \cdots$.

When we consider singular boundary value problems for differential or integro-differential equations with smooth kernels, the first iteration process deals with the degenerate nonhomogeneous systems. In particular, if we examine the asymptotic approach to one-dimensional problems of free shell vibrations, the first iteration process deals with the boundary value problems of the membrane theory (Goldenveizer et al., 1979). In our case equation (5) is the improper integral where the kernel $G[\theta, \theta_0]$ has a logarithmic singularity at $\theta_0 = \theta$ (Brebbia et al., 1984). So, if we use the membrane approximation to construct the main iteration process, then the solutions of the auxiliary problems would have less derivatives than the exact solution. Therefore, we have to keep the higher derivatives in the first iteration process. Each auxiliary problem consists of the nondegenerate system similar to equation (1) and the symmetry conditions with respect to the plane screen.

$$\bar{u}_0\left(\frac{\pi}{2}\right) = 0 \qquad \bar{N}_0\left(\frac{\pi}{2}\right) = 0 \qquad \bar{w}_0'\left(\frac{\pi}{2}\right) = 0$$
$$\bar{u}_k\left(\frac{\pi}{2}\right) = -\tilde{u}_k 0 \qquad \bar{N}_k\left(\frac{\pi}{2}\right) = 0 \qquad \bar{w}_k'\left(\frac{\pi}{2}\right) = 0$$

where $N(\theta)$ is the shear force. If we consider the case of free vibrations without liquid, the solution of this problem differs from the solution in membrane approximation, but the difference is of order $O(\mu^4)$. At the same time the accuracy of the Kirchhoff-Love theory is only $O(\mu^2)$.

We estimate the components of the acoustic pressure, corresponding to the solutions $\tilde{w}^k(\eta)$ of the additional problems. Later we will prove that the values

$$I_k^n = \mu^{-1} \int_0^{\pi/2} \tilde{\omega}^k(\eta) P_n(\cos\theta) \sin\theta \, d\theta$$

are limited as $\mu \to 0$. Thus, we may consider these components as the perturbations of the right hand sides in the corresponding problems of the main process. We introduce

$$p(\theta,\mu) = \bar{p}(\theta,\mu) + \mu\tilde{p}(\theta,\mu)$$

where

$$\bar{p}(\theta,\mu) = \bar{p}_0(\theta,\mu) + \mu\bar{p}_1(\theta,\mu) + \cdots$$
$$\tilde{p}(\theta,\mu) = \tilde{p}_0(\theta,\mu) + \mu\tilde{p}_1(\theta,\mu) + \cdots$$

Here $\bar{p}_k(\theta,\mu)$ denotes equation (5) for $w(\theta) = \bar{w}_k(\theta,\mu)$ and $\tilde{p}_k(\theta,\mu)$ denotes equation (5) for $w(\theta) = \mu^{-1}\tilde{w}_k(\eta)$. Thus, we get the recursive algorithm. Note, that an interaction between the first and second iteration process occurs only through the boundary conditions.

4 Solution of the Initial Problem

Let us substitute expansions (6) into system (1). Then for an initial approximation we have the system

$$\begin{aligned} &\Delta\bar{U}_0 + (2+\lambda)\bar{U}_0 - \lambda(2+\nu)\bar{w}_0 = \bar{p}_0(\theta) + f(\theta) \\ &\mu^4[\Delta\Delta\bar{w}_0 + 2\Delta\bar{w}_0] + (1-\lambda-\nu^2)\bar{w}_0 + (1+\nu)\bar{U}_0 = \bar{p}_0(\theta) + f(\theta) \\ &\bar{u}_0\left(\frac{\pi}{2}\right) = 0 \qquad \bar{N}_0\left(\frac{\pi}{2}\right) = 0 \qquad \bar{w}_0'\left(\frac{\pi}{2}\right) = 0 \end{aligned} \tag{9}$$

Due to equation (2), the functions $\overline{U}_0$ and $\overline{w}_0$ depend on the tangent displacement $\overline{u}_0$.

The solution of equations (9) may be represented as Fourier series in Legendre polynomials.

$$\overline{w}_0 = \sum_{(n)} \overline{w}_0^n P_n(\cos\theta) \qquad \overline{U}_0 = \sum_{(n)} \overline{U}_0^n P_n(\cos\theta)$$

We write

$$\begin{aligned}\Delta^n &= \left[\mu^4\left(n(n+1)-1\right)^2 + \left(1-\nu^2-\lambda\right)\right]\left(2+\lambda-n(n+1)\right) + \lambda(1+\nu)(2+\nu) \\ &+ \lambda_\varepsilon K_n(k)\left[n(n+1) - (1-\nu+\lambda)\right]\end{aligned} \tag{10}$$

then

$$\begin{aligned}\overline{w}_0^n &= \left[1-\nu+\lambda-n(n+1)\right]\frac{f^n}{\Delta^n} \\ \overline{U}_0^n &= \left[\mu^4\left(n(n+1)-1\right)^2 + (1+\nu)(1-\nu+\lambda)\right]\frac{f^n}{\Delta^n}\end{aligned} \tag{11}$$

Here

$$f^n = (2n+1)\int_0^{\pi/2} f(\theta)P_n(\cos\theta)\sin\theta d\theta$$

For $\theta = \frac{\pi}{2}$ we get

$$\overline{w}_0\left(\frac{\pi}{2}\right) = \sum_{(n)} (-1)^{n/2}\, \overline{w}_0^n \frac{(n-1)}{n}$$

i.e. the initial solution does not satisfy the second of conditions (3). We use the initial approximation of the additional process to eliminate the residual in the boundary conditions.

Let us consider N harmonics in the Fourier expansion of the external force, where $N \sim \mu^{-1/2}$. If we omit in equations (10) and (11) the coeffcients of μ, the errors in the expressions for w_0^n and U_0^n will be of the order $O\left(\mu^2\right)$. Therefore, here we can use the membrane theory.

5 The Edge Effect

We substitute equations (7) into system (1), expression (2) and boundary conditions (3) and equate the coeff-cients of μ^0, depending on μ. Then we get the auxiliary boundary problem

$$\begin{aligned}&\frac{d^4\tilde{w}_0(\eta)}{d\eta^4} + 4\kappa^4\,\tilde{w}_0(\eta) = 0 \qquad \text{where} \qquad \kappa^4 = \frac{1-\nu^2-\lambda}{4} \\ &\tilde{w}_0(0) = -\overline{w}_0\left(\frac{\pi}{2}\right) \qquad \tilde{w}_0'(0) = 0\end{aligned} \tag{12}$$

We have the following expressions to determine $\tilde{u}_0$ and $\tilde{U}_0$:

$$\tilde{u}_0'(\eta) = -\frac{1+\nu}{\kappa}\tilde{w}_0(\eta) \qquad \tilde{U}_0''(\eta) = \frac{\lambda^2}{\kappa^2}(2+\nu)\tilde{w}_0(\eta) \tag{13}$$

The behavior of the solution of system (12) and functions (13) is determined by the sign of $1-\nu^2-\lambda$. The degeneracy is regular for $\lambda < 1-\nu^2$. Then the solution has the form of an edge effect, and it satisfies condition (8). In this case

$$\tilde{w}_0(\eta) = -\overline{w}_0\left(\frac{\pi}{2}\right)e^{\kappa\eta}(\cos\kappa\eta - \sin\kappa\eta) \quad \text{and} \quad \tilde{u}_0(\eta) = \overline{w}_0\left(\frac{\eta}{2}\right)\frac{1+\nu}{\kappa}e^{\kappa\eta}\cos\kappa\eta$$

therefore

$$\tilde{u}_0(0) = \frac{1+\nu}{\kappa}\overline{w}_0\left(\frac{\pi}{2}\right) \tag{14}$$

6 Perturbation of the Acoustic Pressure

Let us substitute the value of $\overline{w}_0$ obtained from equation (12) into equation (5). Evaluating the integrals

$$I_0^n = \mu^{-1}\int_0^{\pi/2} P_n(\cos\theta)\tilde{w}_0(\eta)\sin\theta d\theta$$

we should take into account only the contribution in the neighborhood of the point $\frac{\pi}{2}$.

For $n \sim 1$ integrals are evaluated by the Laplace method and equal

$$I_0^n = -\frac{1}{\kappa}\overline{w}_0\left(\frac{\pi}{2}\right)P_n(0) + O(\mu) \tag{15}$$

For $n >> 1$ we use the asymptotic expansion for Legendre polynomials

$$P_n(\cos\theta) = \left(\frac{2}{n\pi\sin\theta}\right)^{1/2}\sin\left\{\left(n+\frac{1}{2}\right)\theta + \frac{\pi}{4}\right\} + O\left(n^{-1}\right)$$

We have

$$\int_0^{\pi/2} Pn(\cos\theta)\exp\kappa\eta\sin\kappa\eta\sin\theta d\theta = \mu(-1)^{n/2}\sqrt{\frac{2}{n\pi}}\kappa\frac{n^2\mu^2-2\kappa^2}{4\kappa^4+n^4\mu^4} + O\left(\mu^2\right)$$

$$\int_0^{\pi/2} Pn(\cos\theta)\exp\kappa\eta\cos\kappa\eta\sin\theta d\theta = \mu(-1)^{n/2}\sqrt{\frac{2}{n\pi}}\frac{8\kappa^3}{4\kappa^4+n^4\mu^4} + O\left(\mu^2\right)$$

These expressions are obtained by integration by parts while the exponentially small terms are neglected. Then

$$I_0^n = -\frac{1}{\kappa}\overline{w}_0\left(\frac{\pi}{2}\right)(-1)^{n/2}\sqrt{\frac{2}{n\pi}}\frac{10-\left(\frac{\mu n}{\kappa}\right)^2}{4+\left(\frac{\mu n}{\kappa}\right)^4} + O(\mu) \tag{16}$$

Finally,

$$\tilde{p}_0(\theta,\mu) = \lambda_\varepsilon \sum_{(n)} (2n+1) K_n(k) I_0^n P_n(\cos\theta)$$

where I_0^n is calculated by equation (15) for $n \sim 1$ and by equation (16) for $n >> 1$. It is evident that the I_0^n are limited as $\mu \to 0$ in the both cases.

7 The Correction for a Mode

The next term in expansion (6) must satisfy the nonhomogeneous boundary condition for $\bar{u}_1$, i.e.

$$\bar{u}_1\left(\frac{\pi}{2}\right) = -\tilde{u}_0\left(\frac{\pi}{2}\right)$$

For the tangent displacement u we get

$$u(\theta) = \frac{1}{\sin\theta}\int_0^\theta \left[U - (1+\nu)w\right]\sin\theta d\theta \tag{17}$$

We represent the following approximation in the form

$$\bar{u}_1 = \hat{u}_1 + \varphi(\theta) \qquad \bar{U}_1 = \hat{U}_1 + \psi(\theta) \qquad \bar{w}_1 = \hat{w}_1$$

where

$$\varphi(\theta) = \int_0^\theta \psi(\theta)\sin\theta d\theta$$

The function $\hat{u}_1(\theta)$ has to satisfy the homogeneous boundary condition $\hat{u}_1\left(\frac{\pi}{2}\right)=0$. Then due to equation (14) it follows that $\varphi\left(\frac{\pi}{2}\right)=-\tilde{u}_0\left(\frac{\pi}{2}\right)$ and the function ψ must satisfy the relation

$$\int_0^{\pi/2} \psi(\theta)\sin\theta d\theta = -\frac{1+\nu}{\kappa}\bar{w}_0\left(\frac{\pi}{2}\right)$$

Thus, for $\hat{U}_1$ and $\hat{w}_1$ we have

$$\Delta\hat{U}_1 + (2+\lambda)\hat{U}_1 - \lambda(2+\nu)\hat{w}_1 = \hat{p}_1(\theta) + f_1(\theta)$$

$$\mu^4[\Delta\Delta\hat{w}_1 + 2\Delta\hat{w}_1] + \left(1-\lambda-\nu^2\right)\hat{w}_1 + (1+\nu)\hat{U}_1 = \hat{p}_1(\theta) + g_1(\theta) \tag{18}$$

$$\hat{u}_1\left(\frac{\pi}{2}\right) = 0 \qquad \hat{N}_1\left(\frac{\pi}{2}\right) = 0 \qquad \hat{w}_1'\left(\frac{\pi}{2}\right) = 0$$

where

$$f_1(\theta) = \tilde{p}_0(\theta) - (2+\lambda)\psi(\theta) \qquad \text{and} \qquad g_1(\theta) = \tilde{p}_0(\theta) - (1+\nu)\psi(\theta)$$

The solution of equation (18) we seek in the form

$$\hat{w}_1(\theta) = \sum_{(n)} \hat{w}_1^n P_n(\cos\theta) \qquad \hat{U}_1(\theta) = \sum_{(n)} \hat{U}_1^n P_n(\cos\theta)$$

We take $\psi(\theta)$ such that

$$\psi(\theta) = -\frac{1+\nu}{\kappa}\overline{w}_0\left(\frac{\pi}{2}\right)P_0$$

Hence for $n=0$ we obtain

$$\overline{U}_1^0 = (1+\nu)(1-\nu+\lambda)\lambda_\varepsilon K_0(k)\frac{I_0^0}{\Delta^0} + \frac{1+\nu}{\kappa}\overline{w}_0\left(\frac{\pi}{2}\right) + \frac{1+\nu}{\kappa}\overline{w}_0\left(\frac{\pi}{2}\right)\frac{(2+\nu)\left(1-\nu^2+\lambda\nu\right)-\lambda_\varepsilon K_0(k)}{\Delta^0}$$

$$\overline{w}_1^0 = (1-\nu+\lambda)\lambda_\varepsilon K_0(k)\frac{I_0^0}{\Delta^0} + \frac{1}{\kappa}\overline{w}_0\left(\frac{\pi}{2}\right)\left(1+\nu^2\right)\frac{(\lambda-\nu)}{\Delta^0}$$

For $n = 2,4,\cdots$

$$\overline{U}_1^n = \left[\mu^4\left(n(n+1)-1\right)^2 + (1+\nu)(1-\nu+\lambda)\right](2n+1)\lambda_\varepsilon K_n(k)\frac{I_0^n}{\Delta^n}$$

$$\overline{\omega}_1^n = \left[1-\nu+\lambda-n(n+1)\right](2n+1)\lambda_\varepsilon K_n(k)\frac{I_0^n}{\Delta^n}$$

due to expressions (15) and (16) one can see that the coefficients $\overline{U}_1^n, \overline{w}_1^n$ are proportional to the residual $\overline{w}_0\left(\frac{\pi}{2}\right)$ in the second of the boundary conditions (3).

8 The Polar Pattern

Let us determine the acoustic pressure in the far zone by means of a construction of the polar pattern. The functions $h_n^{(1)}(z)$ have the asymptotic expansion

$$h_n^{(1)}(z) = \frac{-i^{n+1}}{z}e^{iz} \qquad \text{for} \qquad z \gg n + \frac{1}{2} \tag{19}$$

By virtue of equation (19) we get the following approximate expression for the acoustic pressure:

$$p(r,\theta) = \lambda_\varepsilon \frac{i\exp(ikr)}{kr}\Psi(\theta) \tag{20}$$

where $\Psi(\theta)$ is the polar pattern (Skudrzyk, 1971)

$$\Psi(\theta) = \sum_{(n)} \psi^n P_n(\cos\theta) \tag{21}$$

Here

$$\psi^n = \sum_{(n)} \frac{(-1)^{n/2+1}(2n+1)}{nh_n^{(1)}(k) + kh_{n+1}^{(1)}(k)} \int_0^{\pi/2} P_n(\cos\theta) w(\theta) \sin\theta d\theta \tag{22}$$

Note that series (4) includes components for which $r \sim n$. It may be proved that expression (19) may be used in the derivation of equation (20), despite the fact formally the condition $z >> n+\frac{1}{2}$ is violated.

Note that the expansion

$$\psi^n \sim ik^n \frac{\sqrt{2}}{3} \frac{(-1)^{n/2+1}}{2^n e^{n(\ln n-1)}} \int_0^{\pi/2} P_n(\cos\theta) w(\theta) \sin\theta d\theta$$

is valid for $n>>1$. Here we use the asymptotic value for $h_n^{(1)}(k)$ for $n>>1$. The integrals on the right hand side of the last expression go to nought as $n \to \infty$ due to the Riemann-Lebesgue lemma (Olver, 1974). Thus, one can retain only $N \sim \mu^{-1/2}$ terms in equation (21).

9 Numerical Results

The sound emission of the steel hemisphere submerged in water for harmonic external force such that $f^0 = 1$ and $f^n = 0$ for $n > 0$ is examined. We use the following values for physical and geometrical parameters: R is the shell radius, 1 m; h is the shell thickness, 0.01 m; E is Young's modulus, $19.6 \times 10^{10}\ \mathrm{N/m^2}$; ρ is the density of the shell, $7700\ \mathrm{kg/m^3}$; ν is Poisson's ratio, 0.35; c_0 is the sound velocity, $1500\ \mathrm{m/s}$; ρ_0 is the density of the liquid, $1000\ \mathrm{kg/m^3}$. Only the three lowest resonance frequencies satisfy the condition $\lambda < 1-\nu^2$.

The form changing of the polar pattern $|\Psi(\theta)|$, in the vicinity of resonance frequencies 2059.5 Hz, 2913 Hz and 3294.8 Hz is presented in Figures 1 to 3 (solid lines). On each drawing the appropriate frequency in Hz is indicated. For better representation we plot also the polar pattern for the initial approximation (dashed lines) on each drawing. The external force under consideration has the form of a semicircle. We notice, that the form of the polar pattern sharply changes at passage through resonance. The drawings show also the process of the occurence of additional petals in the characteristics. Twenty-five terms in series (21) was considered reasonable for the calculations.

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Literature

1. Brebbia. C. A.; Trelles, J. F. C.; Wrobel, L. C.: Boundary Element Techniques, Springer-Verlag Berlin, Heidelberg, (1984).

2. Goldenveizer, A. L.; Lidsky, V. B.; Tovstik, P. E.: Free Vibrations of Thin Elastic Shells, (in Russian), Nauka, Moscow, (1979), 383 p.

3. Lomov, S. A.: Introduction to the General Theory of Singular Perturbation, Amer. Math. Soc., Providence, R. I., (1992), 375 p.

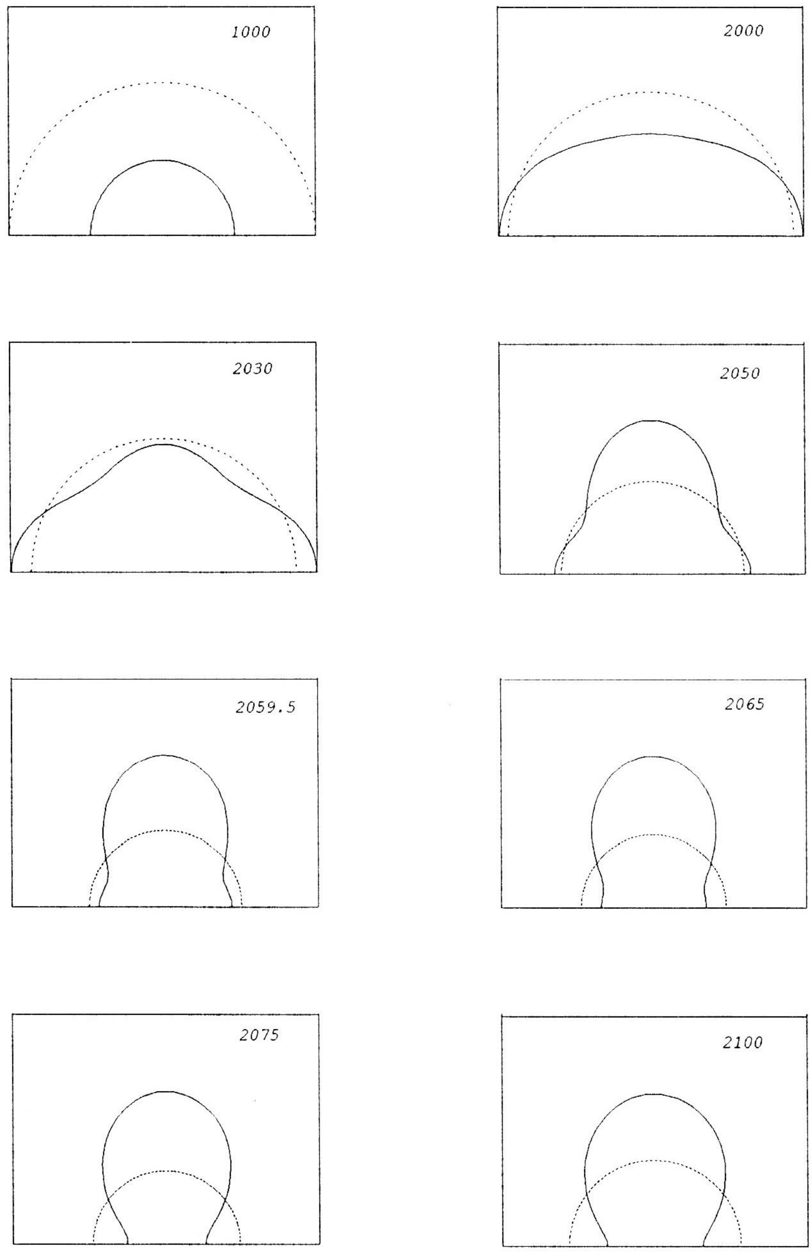

Figure 1. The Polar Patterns (2059.5 Hz)

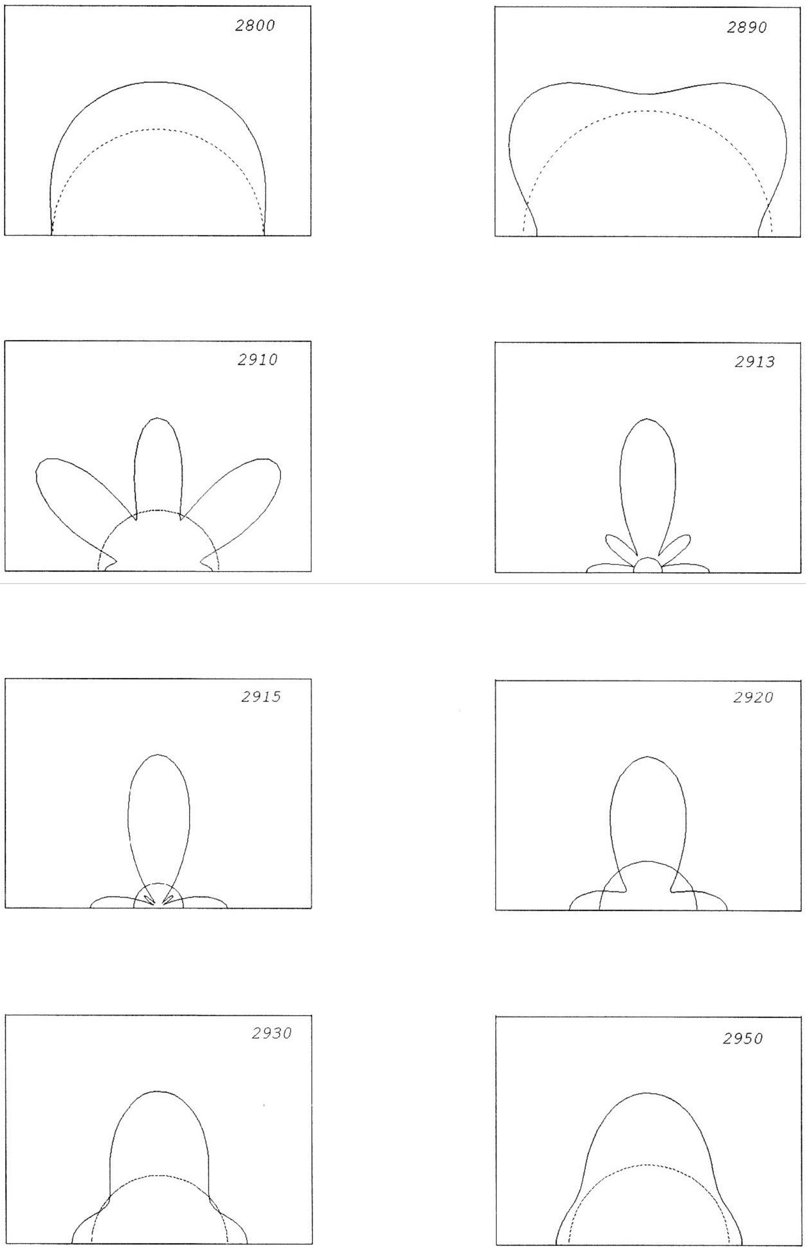

Figure 2. The Polar Patterns (2913 Hz)

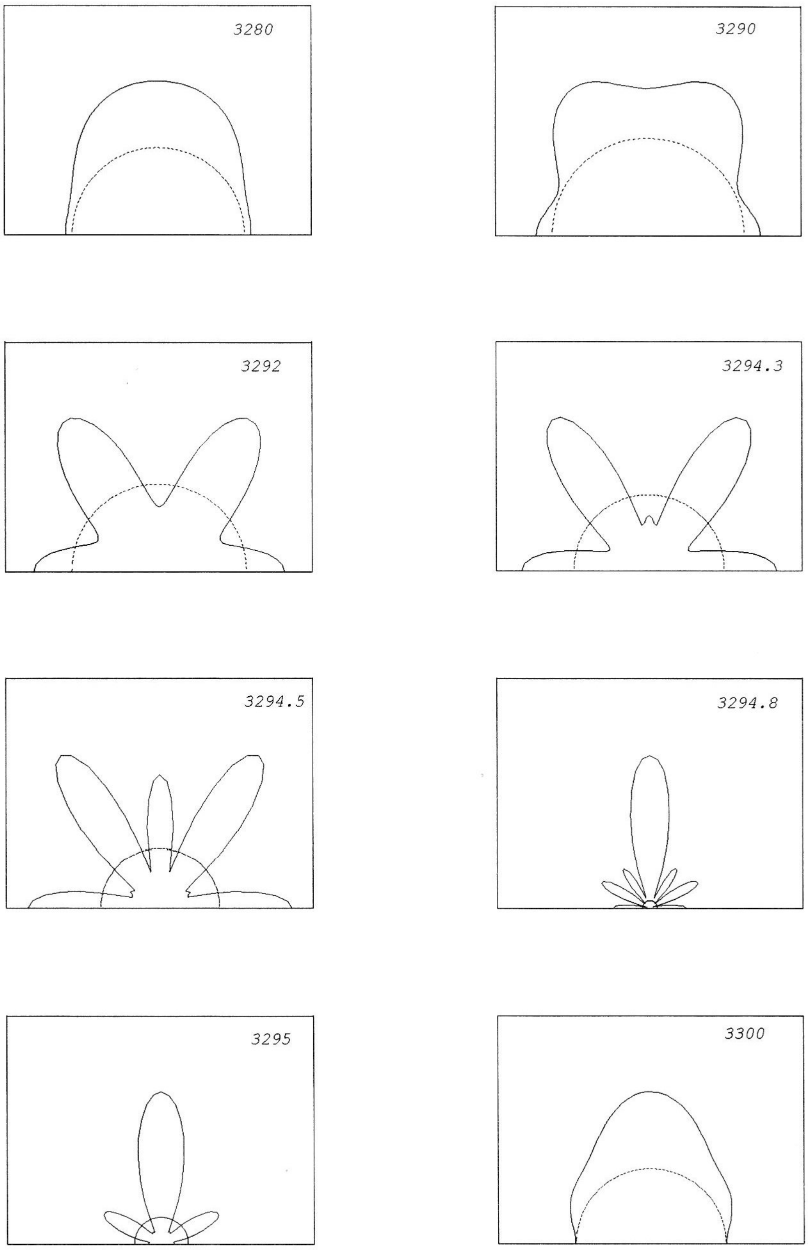

Figure 3. The Polar Patterns (3294.8 Hz)

4. Olver, F. W. J.: Asymptotic and Special Functions, Academic Press, New York, London, (1974), 584 p.

5. Skudrzyk. E.: The Foundation of Acoustics, Vol. 2, Springer-Verlag, Wien, (1971).

6. Wasow, W.: Asymptotic Expansions for Ordinary Differential Equations, Pure Appl. Math., Vol. 14, [R. Courant, L. Bars and J. J. Stoker, eds.], Interscience, New York, (1965), 362 p.

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