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Global Dynamical Behavior of a Two-Body Satellite with Flexible Connection in the Gravitational Field

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The planar libration of a two-body satellite with flexible connection on a circular orbit under application of the gravitational torque is discussed in the present paper. All possible equilibrium states relative to the orbital coordinate frame are calculated, and Liapunov's direct method is used in the analysis of the stability of each state. It is shown that a bifurcation of equilibrium states and their stability can occur when the stiffness of the connection is small enough. The global dynamical behavior of the system is described qualitatively in the of the parameter space.

Introduction

The attitude dynamics of a rigid body in the gravitational field is not only a subject of classical mechanics, but serves as a theoretical basis for aerospace engineering. The libration of the moon was studied in the last century by Lagrange (1870). The so-called Lagrange and DeBra-Delp regions in the stability diagram of the parameter plane are regarded as an elegant description of the dynamical behavior of a rigid satellite in an orbital coordinate frame. As the construction of modern spacecraft becomes more complicated, multibody models were developed instead of the single rigid body. A linear formulation of a two-body satellite with flexible connection was proposed is by Robe and Kane (1967), and some nonlinear problems of the same model were discussed by Wittenburg (1974), Liu (1989) and Rimrott (1992). In the present paper the planar attitude motion of a two-body satellite with flexible connection on a circular orbit under application of the gravitational torque is discussed. All possible equilibrium states of the satellite relative to the orbital coordinate frame are determined, and Liapunov's direct method is used in the analysis of the stability of each state. It is shown that a bifurcation of the equilibrium state and its stability can occur when the stiffness of the connection is small enough.

Dynamical Equations

Consider a system composed of two rigid bodies B_i $(i = 1, 2)$ and a flexible axis, modelled as a sperical joint with spring. The principal axes x_i of bodies intersect at the center 0 of the joint. We introduce an orbital coordinate frame $(0-XYZ)$ with the origin 0 and X- and Z-axes along the position vector of the satellite and the normal of the orbital plane respectively. We assume that the principal axes x_i, y_i of the bodies are restricted in the orbital plane (X, Y), and inclined by angles ϕ_i with respect to the X- and Y-axes (Figure 1). We denote by m_i, A_{io}, B_{io}, C_{io} the mass and central principal moments of inertia of each body, by l_i the distance between the body's center of mass 0_i and the joint 0 and by K the stiffness coefficient of the spring. The center of mass 0_c of the system moves on a circular orbit with an angular velocity ω_c.

The Euler equations of each body about the point 0 are derived as follows

$$C_i \ddot{\phi}_i + \mu \ddot{\phi}_j \cos(\phi_i - \phi_j) + \mu(\omega_c + \dot{\phi}_j)^2 \sin(\phi_i - \phi_j) + (3/2)\omega_c^2 (B_2 - A_2) \sin 2\phi_2 + K(\phi_i - \phi_j) = 0 \tag{1}$$

$$(i = 1, 2; j = 2, 1)$$

where

$$A_i = A_{io} \qquad B_i = B_{io} \qquad C_i = C_{io} + \mu_i \tag{2}$$

$$\mu_i = m_1 m_2 l_i^2 / (m_1 + m_2) \qquad m = m_1 m_2 l_1 l_2 / (m_1 + m_2)$$

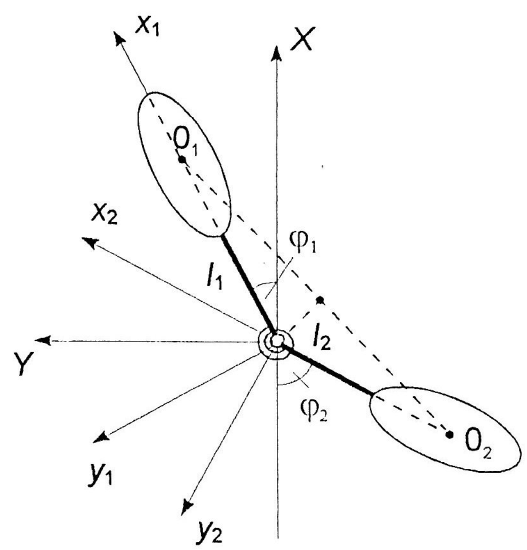

Figure 1. Two-body Satellite with Flexible Connection

Assuming that bodies B_i have the same mass and geometry, we delete the subscript i from the symbols m_i, A_i, B_i, C_i, l_i, μ_i and introduce dimensionsless parameters as follows:

$$\nu = \mu / C, \qquad \sigma = (B - A)/C \qquad k = K / C\omega_c^2 \qquad \tau = \omega_c t \tag{3}$$

Then equation (1) can be rewritten as

$$\ddot{\phi}_i + \nu\ddot{\phi}_j \cos(\phi_i - \phi_j) + \nu\left(1 + \dot{\phi}_j\right)^2 \sin(\phi_i - \phi_j) + (3/2)\sigma \sin 2\phi_i + k(\phi_i - \phi_j) = 0 \tag{4}$$

where the time variable t is replaced by τ. The conservative system (1) permits the Jacobi integral

$$H = \sum_{i=1}^{2}\left(\frac{1}{2}\dot{\phi}_i^2 + \frac{3}{2}\sigma \sin^2\phi_i\right) + \nu\left(\dot{\phi}_1\dot{\phi}_2 - 1\right)\cos(\phi_1 - \phi_2) + \frac{1}{2}k(\phi_1 - \phi_2)^2 = \text{const} \tag{5}$$

Relative Equilibrium States

In order to determine the relative equilibrium position ϕ_{io}, we let $\dot{\phi}_i$, $\ddot{\phi}_i$ of equation (4) be zero and obtain

$$2\nu \sin(\phi_{io} - \phi_{jo}) + 3\sigma \sin 2\phi_{io} + 2k(\phi_{io} - \phi_{jo}) = 0 \tag{6}$$

Two trivial solutions corresponding to equilibrium states S_1 and S_2 can be found from equation (6) directly.

$$S_1: \quad \phi_{10} = \phi_{20} = 0 \tag{7a}$$

$$S_2: \quad \phi_{10} = \phi_{20} = \pi/2 \tag{7b}$$

The following condition should be satisfied for the nontrivial solutions when $\phi_{10} \neq \phi_{20}$:

$$\sum_{i=1}^{2} \sin 2\phi_{io} = 0 \tag{8}$$

from which four constraint conditions corresponding to the remaining equilibrium states are derived as

$$S_3: \quad \phi_{20} = -\phi_{10} \tag{9a}$$

$$S_4: \quad \phi_{20} = \pi - \phi_{10} \tag{9b}$$

$$S_5: \quad \phi_{20} = -(\pi/2) + \phi_{10} \tag{9c}$$

$$S_6: \quad \phi_{20} = (\pi/2) + \phi_{10} \tag{9d}$$

The equilibrium positions ϕ_{io} can be obtained from equations (6) and (9) only when the following conditions are satisfied:

$$S_3: \quad \sigma < -2(v+k)/3 \tag{10a}$$

$$S_4: \quad \sigma > 2(v+k)/3 \tag{10b}$$

$$S_5: \quad \sigma < -(2v+k\pi)/3 \tag{10c}$$

$$S_6: \quad \sigma > (2v+k\pi)/3 \tag{10d}$$

The relationships between ϕ_{io} and parameter σ with $k = 0.1$ and $\nu = 0.2$ are shown in Figure 2.

Stability Analysis

The stability of each equilibrium state can be analysed by means of the Liapunov direct method with the Hamiltonian H as the Liapunov function. Since $\nu < 1$, the function H is positive definite with respect to $\dot{\phi}_i$ in the neighborhood of $\dot{\phi}_i = 0$. The condition of positive definitness of H with respect to ϕ_i at $\phi_i = \phi_{io}$ can be determined by use of the Hessian Matrix κ of H for each state.

$$\kappa = \begin{bmatrix} \left(\dfrac{\partial^2 H}{\partial \phi_1^2}\right)_0 & \left(\dfrac{\partial^2 H}{\partial \phi_1 \partial \phi_2}\right)_0 \\ \left(\dfrac{\partial^2 H}{\partial \phi_1 \partial \phi_2}\right)_0 & \left(\dfrac{\partial^2 H}{\partial \phi_2^2}\right)_0 \end{bmatrix} \tag{11}$$

where

$$\frac{\partial^2 H}{\partial \phi_i^2} = 3\sigma\cos 2\phi_i + v\cos(\phi_1 - \phi_2) + k$$

$$\frac{\partial^2 H}{\partial \phi_1 \partial \phi_2} = -v\cos(\phi_1 - \phi_2) - k \quad (12)$$

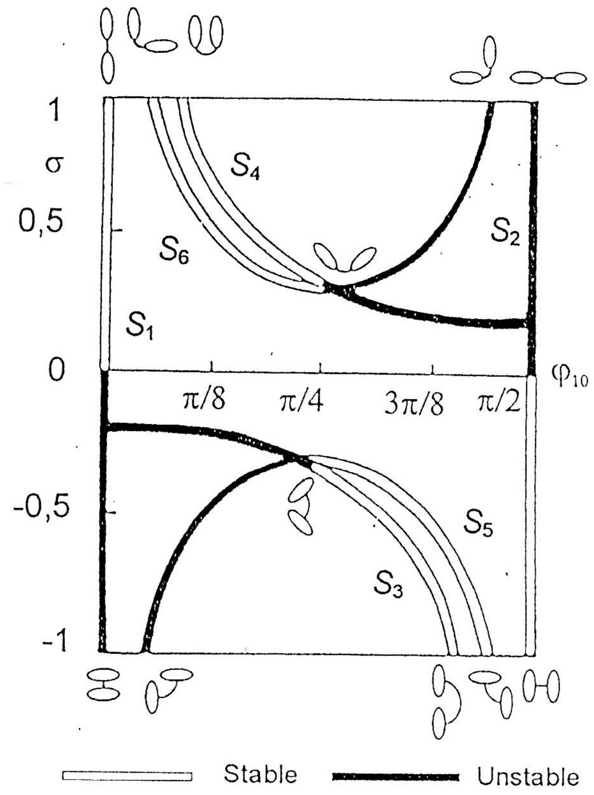

Figure 2. Equilibrium States and their Stability

The Hessian κ is the same as the stiffness matrix of equation (4) after linearization, hence the equilibrium state is stable if and only if the principal minor determinants of κ are positive, a stipulation from which the sufficient and necessary conditions of stability of each state are obtained (Figure 2).

Global Dynamical Behavior

The global dynamical behavior of a two-body system with flexible connection can be described qualitatively by means of a 3-dimensional space $(\phi_{10}, \phi_{20}, \sigma)$, as shown in Figure 3. The following conclusions are arrived at:

1. When the stiffness of connection increases indefinitely, $k \to \infty$, the system becomes single rigid body. There exist only two equilibrium states S_1 and S_2, the stability of which undergoes a change at $\sigma = 0$, and the well-known stability criterion for a rigid body is obtained.

$$B > A \qquad \text{stable } (S_1),\ \text{unstable } (S_2)$$

$$B < A \qquad \text{stable } (S_2),\ \text{unstable } (S_1) \quad (13)$$

2. When the stiffness of connection is small enough and satisfies $k + v < 3/2$, two nontrivial states S_3 and S_4 can be bifurcated from S_1 and S_2 at $|\sigma| = 2(k+v)/3$. The stability of S_3 and S_4 undergoes a change at $\phi_{10} = \pi/4$

$$\begin{array}{lll} \phi_{10} > \frac{\pi}{4} & & \text{stable } (S_3), \text{ unstable } (S_4) \\ \phi_{10} < \frac{\pi}{4} & & \text{stable } (S_4), \text{ unstable } (S_3) \end{array} \qquad (14)$$

3. Another two nontrivial states S_5 and S_6 are bifurcated from S_3 and S_4, and undergo a change of stability at $\phi_{10} = \pi/4$

$$\begin{array}{lll} \phi_{10} > \frac{\pi}{4} & & \text{stable } (S_5), \text{ unstable } (S_6) \\ \phi_{10} < \frac{\pi}{4} & & \text{stable } (S_6), \text{ unstable } (S_5) \end{array} \qquad (15)$$

4. When the stiffness of connection decreases to zero, $k = 0$, all above mentioned bifurcation phenomena vanish. The system is decomposed into two free rigid bodies, each of which follows the same stability criterion as (13) separately.

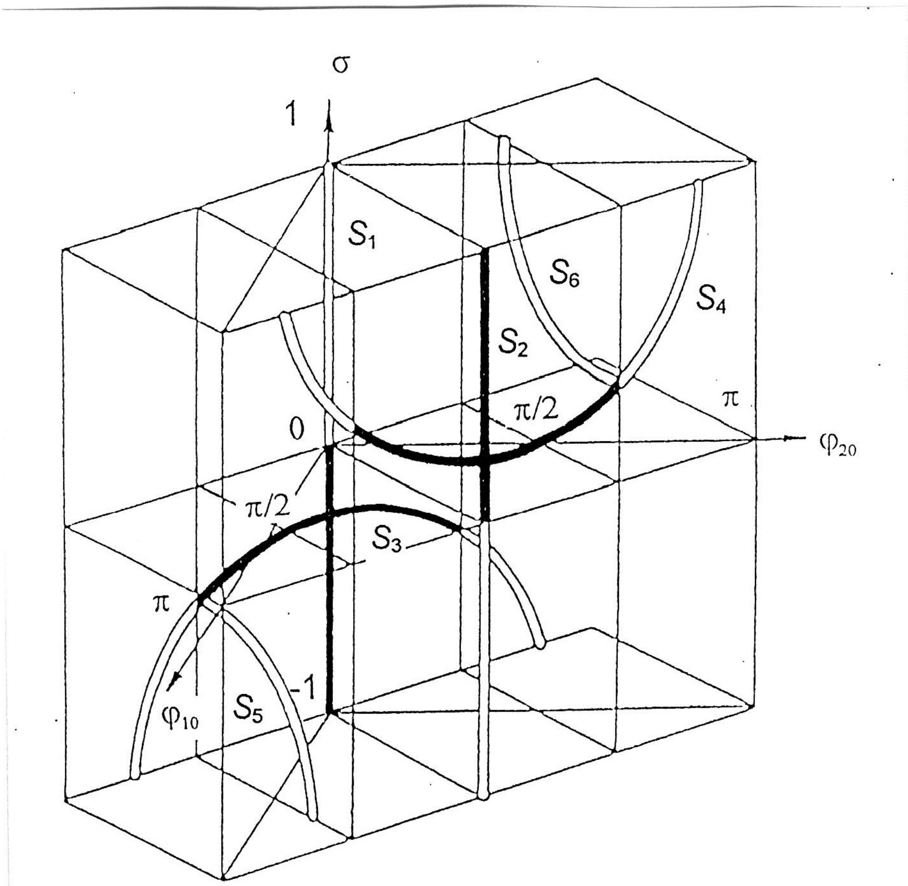

Figure 3. Global Dynamical Behavior

Literature

1. DeBra, D.B.; Delp, B.H.: Rigid Body Attitude Stability and Natural Frequencies in a Circular Orbit, J. Astronautical Sciences, 8, 1, (1961), 14-17.

2. Liu, Y.Z.: The Stability of the Permanent Rotation of a Free Multibody System, Acta Mechanica, 79, (1989), 43-51.

3. Liu, Y.Z.; Rimrott, F.P.J.: On the Permanent Rotation of a Torquefree Two-Body System with a Flexible Connection, Trans. ASME, J. Appl. Mechanics, (to be published).

4. Rimrott, F.P.J.: Introductory Attitude Dynamics, Springer-Verlag, (1988), 383 p.

5. Rimrott, F.P.J.; Janabi-Sharifi, F.: A Torque-Free Flexible Model Gyro, ASME J. Appl. Mechanics, 59, 1, (1992), 7-15.

6. Robe, T.R.; Kane, T.R.: Dynamics of an Elastic Satellite, Int. J. Solids and Structures, 3, (1967), 333-352.

7. Wittenburg, J.: Permanente Drehungen zweier durch Kugelgelenk gekoppelter starrer Körper, Acta Mechanica, 19, (1974), 215-226.

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