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Flow and Heat Transfer on a Continuous Porous Surface Moving in Parallel with or Reversely to a Free Stream

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The forced flow and heat transfer problem over an isothermal porous flat plate in which the plate and the ambient fluid are both in motion is investigated. A very efficient numerical method has been used to solve non-similar boundary layer equations. The effects of the relative velocity parameter, Prandtl number and the suction or injection parameter on the flow field, temperature field, skin friction coefficient and local Nusselt number are shown and discussed for a plate moving in parallel with or reversely to the free stream.

1 Introduction

The problem concerning the flow field created by a moving wall in a quiescent fluid has copious applications in many engineering processes. For example, aerodynamic extrusion of plastic sheets, the boundary layer along material-handling conveyers, the cooling of an infinite metallic plate in a bath, or the glass blowing process possess the characteristics of a moving continuous surface. In all these cases, a study of the flow field and the heat transfer can be of significant importance since the quality of the final product depends to a large extent on the skin friction coefficient and the surface heat transfer rate. In view of these applications Sakiadis (1961) initiated the study of boundary layer flow over a continuous solid surface moving at a constant speed through an otherwise quiescent fluid environment. Subsequently, several investigators (Tsou et al., 1967; Vleggaar, 1977; Banks, 1983; Jeng et al., 1986; Vajravelu, 1986; Char et al., 1990; Bühler et al., 1990; Takhar et al., 1991; Pop et al., 1992 a,b; Andersson, 1992; Lin et al., 1993) have considered various aspects of this problem, such as the effect of mass transfer, wall temperature, variable fluid properties, and magnetic field.

In many manufacturing processes, the flow and thermal fields are strongly affected by the external flow, the movement of the solid surface, and mass transfer. Thus, the present analysis aims to study flow and heat transfer over an isothermal porous flat plate, which moves in parallel with or reversely to a free stream. We analyse the problem by introducing an appropriate relative velocity parameter σ and other transformation variables. Numerical results showing the velocity and temperature fields, skin friction coefficient, and the local Nusselt number as a function of the Prandtl number, the relative velocity, and the suction or injection parameter are presented.

2 Basic Equations

Consider the boundary layer flow over a horizontal porous flat plate, which moves continuously from a slot at a constant velocity U_s in a viscous fluid, which is at rest. The plate moves either in parallel $(U_s \geq 0)$ or reversely $(U_s \leq 0)$ to a free stream of uniform velocity $U_\infty(> 0)$. The plate is maintained at a constant temperature T_w and the ambient fluid has the uniform temperature $T_\infty(< T_w)$. Figure 1 shows a diagram of this system, where the axes *x, y* are fixed in space.

The governing boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the velocity components along the x and y axes, T is the temperature, Pr is the Prandtl number and ν is the kinematic viscosity. The appropriate boundary conditions are

$$y = 0: \qquad u = U_s \qquad v = v_0 \qquad T = T_w$$

$$y \to \infty: \qquad u = U_\infty \qquad T = T_\infty \tag{4}$$

where v_0 is the velocity of suction or injection, depending on whether $v_0 < 0$ or $v_0 > 0$. We look for a solution of these equations of the form

$$\psi = (2U_\infty \nu x)^{1/2} f(x,\eta) \qquad \theta(x,\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5a}$$

$$\eta = y\left(\frac{U_\infty x}{2\nu}\right)^{1/2}\frac{y}{x} \tag{5b}$$

Here ψ is the stream function defined by

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

so that, using equation (5), the velocity components become

$$u = U_\infty \frac{\partial f}{\partial \eta} \tag{7a}$$

$$v = -\left(\frac{2U_\infty \nu}{x}\right)^{1/2}\left(\frac{1}{2}f + x\frac{\partial f}{\partial x} + x\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial x}\right) \tag{7b}$$

Substituting equation (7) into equations (2) and (3), the dimensionless stream function f and dimensionless temperature θ satisfy the following transformed momentum and energy equations

$$\frac{\partial^3 f}{\partial \eta^3} + f\frac{\partial^2 f}{\partial \eta^2} = 2x\left(\frac{\partial^2 f}{\partial x\,\partial \eta}\frac{\partial f}{\partial \eta} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2}\right) \tag{8}$$

$$\frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} + f\frac{\partial \theta}{\partial \eta} = 2x\left(\frac{\partial \theta}{\partial x}\frac{\partial f}{\partial \eta} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial \eta}\right) \tag{9}$$

The boundary conditions (4) transform into

$$\eta = 0: \qquad \frac{\partial f}{\partial \eta} = \sigma \qquad f + 2x\frac{\partial f}{\partial x} = 2s \qquad \theta = 1$$

$$\eta \to \infty: \qquad \frac{\partial f}{\partial \eta} = 1 \qquad \theta = 0 \tag{10}$$

where σ and s are the parameters of relative motion and of suction $(s > 0)$ or injection $(s < 0)$, which are defined by

$$\sigma = \frac{U_s}{U_\infty} \qquad s = kx^{1/2} = -v_0\left(\frac{x}{2\,U_\infty \nu}\right)^{1/2} \tag{11}$$

We shall further transform equations (8) and (9) by introducing the following variable:

$$x^* = k x^{1/2} \tag{12}$$

By using the variable (12), the transformed equations (8) and (9) can be written as

$$\frac{\partial^3 f}{\partial \eta^3} + \left(f + x^* \frac{\partial f}{\partial x^*}\right)\frac{\partial^2 f}{\partial \eta^2} - x^* \frac{\partial^2 f}{\partial x^* \partial \eta}\frac{\partial f}{\partial \eta} = 0 \tag{13}$$

$$\frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} + \left(f + x^* \frac{\partial f}{\partial x^*}\right)\frac{\partial \theta}{\partial \eta} - x^* \frac{\partial \theta}{\partial x^*}\frac{\partial f}{\partial \eta} = 0 \tag{14}$$

subject to the boundary conditions

$$\eta = 0: \quad \frac{\partial f}{\partial \eta} = \sigma \qquad f + x^* \frac{\partial f}{\partial x^*} = 2x^* \qquad \theta = 1$$

$$\eta \to \infty: \quad \frac{\partial f}{\partial \eta} = 1 \qquad \theta = 0 \tag{15}$$

Equations (13) and (14) are solved employing the difference-differential method described by Katagiri (1969). The partial derivatives with respect to x^* are approximated by finite differences using a backward difference four point formula of Gregory-Newton with a uniform step size h. The solution of the resulting ordinary differential equations at the ith station of $x_i^* = ih$ can be expressed in the form of integral equations as

$$\frac{df_i}{d\eta} = \sigma + \int_0^\eta E(\eta) \int_0^\eta \frac{P(\eta)}{E(\eta)} d\eta\, d\eta + \left\{1 - \sigma - \int_0^\infty E(\eta) \int_0^\eta \frac{P(\eta)}{E(\eta)} d\eta\, d\eta\right\}\frac{G(\eta)}{G(\infty)} \tag{16}$$

$$f_i = \pm ih + \int_0^\eta \frac{df_i}{d\eta} d\eta \tag{17}$$

$$\theta_i = 1 + \int_0^\eta F(\eta) \int_0^\eta \frac{Q(\eta)}{F(\eta)} d\eta\, d\eta - \left\{1 + \int_0^\infty F(\eta) \int_0^\eta \frac{Q(\eta)}{F(\eta)} d\eta\, d\eta\right\}\frac{H(\eta)}{H(\infty)} \tag{18}$$

where plus or minus signs in equation (17) denote suction or injection respectively, and

$$E(\eta) = \exp\left[\int_0^\eta \left\{-f_i - \frac{i}{6}(11 f_i - 18 f_{i-1} + 9 f_{i-2} - 2 f_{i-3})\right\} d\eta\right] \tag{19}$$

$$G(\eta) = \int_0^\eta E(\eta) d\eta \tag{20}$$

$$P(\eta) = \frac{i}{6}\left(11\frac{df_i}{d\eta} - 18\frac{df_{i-1}}{d\eta} + 9\frac{df_{i-2}}{d\eta} - 2\frac{df_{i-3}}{d\eta}\right)\frac{df_i}{d\eta} \tag{21}$$

$$F(\eta) = \exp\left[\int_0^\eta \left\{-Pr\, f_i - Pr\frac{i}{6}(11 f_i - 18 f_{i-1} + 9 f_{i-2} - 2 f_{i-3})\right\} d\eta\right] \tag{22}$$

$$H(\eta) = \int_0^\eta F(\eta) d\eta \tag{23}$$

$$Q(\eta) = Pr\frac{i}{6}(11\theta_i - 18\theta_{i-1} + 9\theta_{i-2} - 2\theta_{i-3})\frac{df_i}{d\eta} \tag{24}$$

The physical quantities of interest include the skin friction τ_w and the heat flux q_w at the plate, which are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad q_w = -\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{25}$$

where μ and λ are the dynamic viscosity and thermal conductivity respectively. With the definition of the local skin friction coefficient C_f and the local Nusselt number Nu as

$$C_f = \frac{\tau_w}{\rho U_\infty^2} \qquad Nu = \frac{x q_w}{\lambda(T_w - T_\infty)} \tag{26}$$

we obtain

$$C_f \cdot Re_x^{1/2} = \frac{1}{\sqrt{2}}\left(\frac{\partial^2 f}{\partial \eta^2}\right)_{\eta=0}$$

$$Nu \cdot Re_x^{-1/2} = -\frac{1}{\sqrt{2}}\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} \tag{27}$$

where $Re_x = xU_\infty / \nu$ is the local Reynolds number. Using equations (17) and (18), we have

$$C_f \cdot Re_x^{1/2} = \frac{1}{\sqrt{2}}\left\{1 - \sigma - \int_0^\infty E(\eta) \int_0^\eta \frac{P(\eta)}{E(\eta)} d\eta d\eta\right\}\frac{1}{G(\infty)}$$

$$Nu \cdot Re_x^{-1/2} = \frac{1}{\sqrt{2}}\left\{1 + \int_0^\infty F(\eta) \int_0^\eta \frac{Q(\eta)}{F(\eta)} d\eta d\eta\right\}\frac{1}{H(\infty)} \tag{28}$$

3 Results and Discussion

The method chosen for the numerical solution of equations (16) to (18) is essentially that used in our previous papers (Pop et al., 1992 b,c; Watanabe et al., 1993). The numerical integration starts at $x^* = 0$, where equations (13) and (14) reduce to ordinary differential equations. The results were obtained for the suction or injection parameter s ranging from -0.5 to 0.5, for relative velocity parameters $\sigma = -0.3, -0.2, 0.0, 0.5, 1., 1.5, 2$ and for the Prandl numbers $Pr = 0.3, 0.5, 0.73, 1, 2, 3, 6.7, 10, 15$. Tables 1 and 2 list results for the skin friction coefficient and the local Nusselt number, respectively. To verify the proper treatment of the present problem, the results have been compared with those of the static non-porous plate case by setting $\sigma = 0.0$ and $s = 0.0$, which is the well-known Blasius problem (see Schlichting, 1960). For this case, one finds from Schlichting (1960) that $C_f \cdot Re_x^{1/2} = 0.332$. Our result is $C_f \cdot Re_x^{1/2} = 0.33206$ for $\sigma = 0.0$ and $s = 0.0$, see Table 1. Thus, these values are in very good agreement.

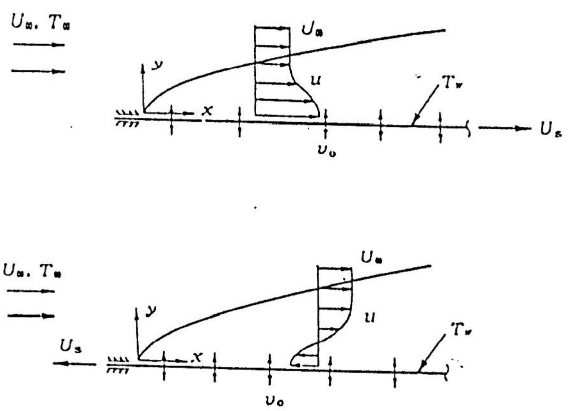

Figure 1. Physical model and coordinate system

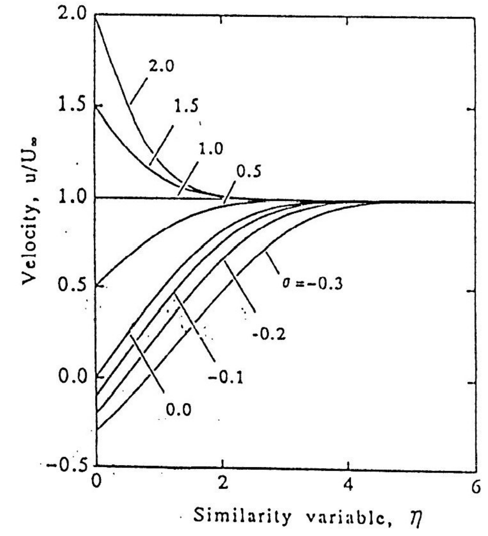

Figure 2. Velocity profiles for $s = 0.0$ and different values of σ

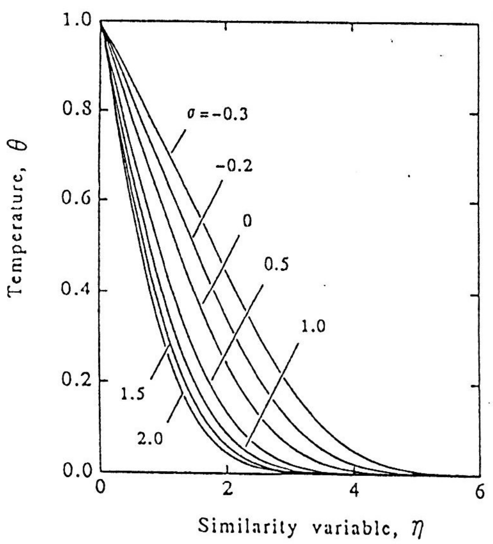

Figure 3. Temperature profiles for $s = 0.0$, $Pr = 0.7$ and different values of σ

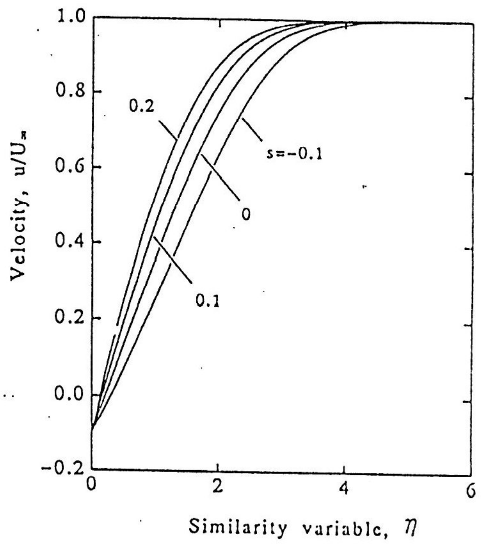

Figure 4. Velocity profiles for σ = −0.1 and different values of s

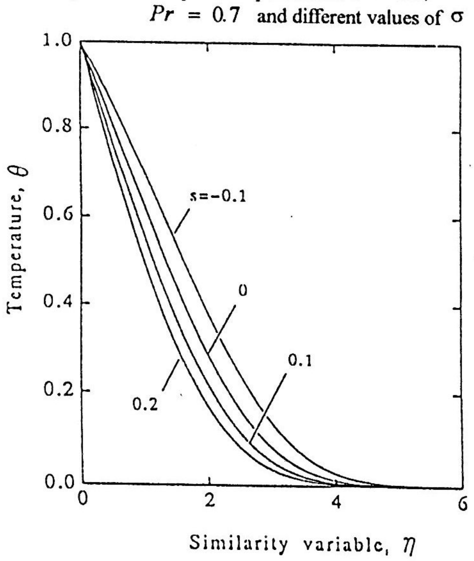

Figure 5. Temperature profiles for $Pr = 0.73$, σ = −0.1 and different values of s

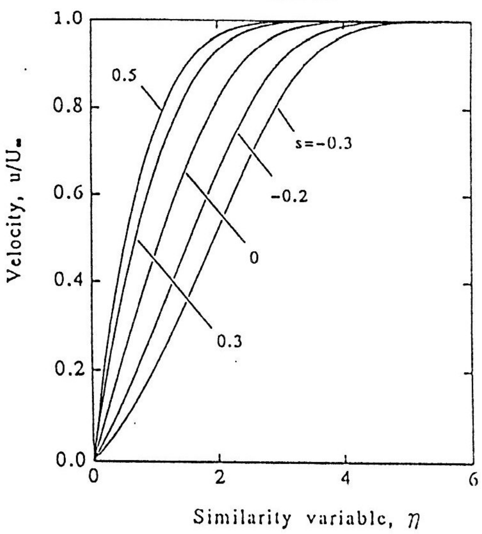

Figure 6. Velocity profiles for σ = 0.0 and different values of s

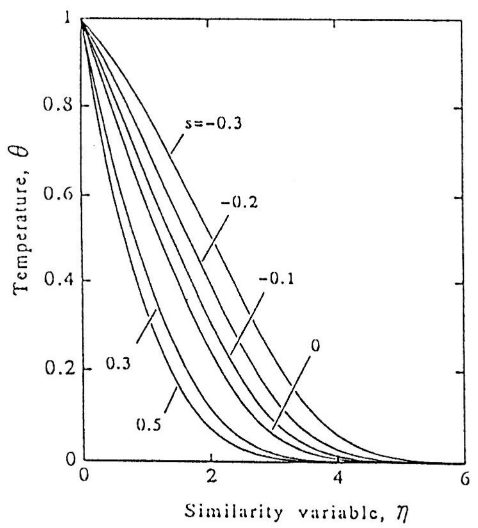

Figure 7. Temperature profiles for $Pr = 0.7$, $\sigma = 0.0$ and different values of s

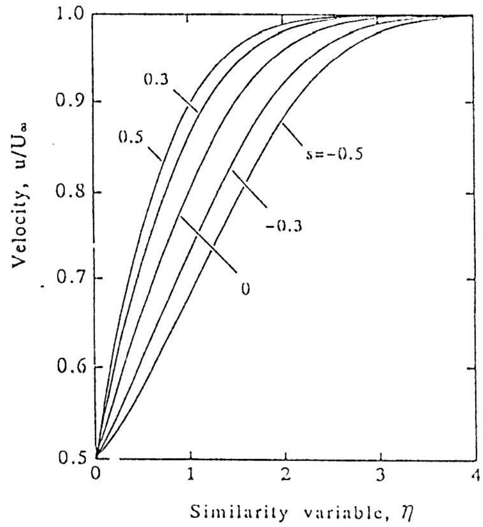

Figure 8. Velocity profiles for $\sigma = 0.5$ and different values of s

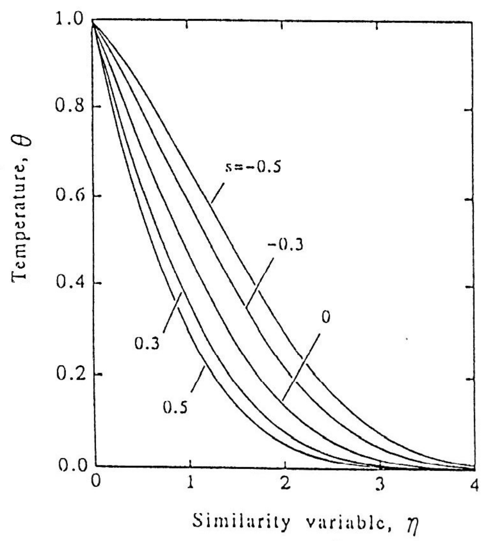

Figure 9. Temperature profiles for $Pr = 0.73$, $\sigma = 0.5$ and different values of s

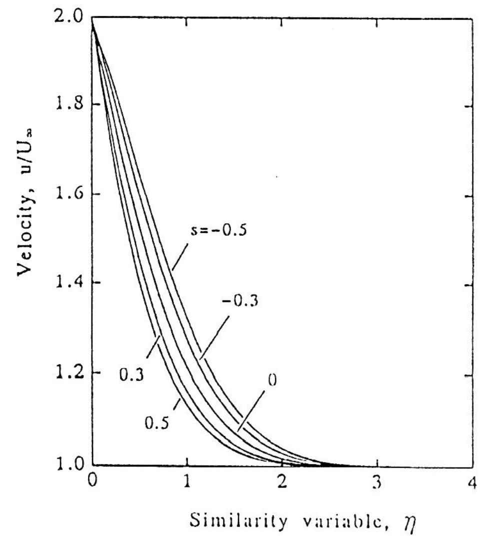

Figure 10. Velocity profiles for $\sigma = 2.0$ and different values of s

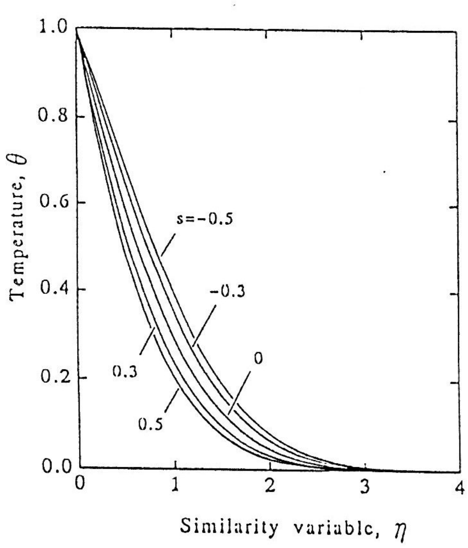

Figure 11. Temperature profiles for $Pr = 0.7$, $\sigma = 2.0$ and different values of s

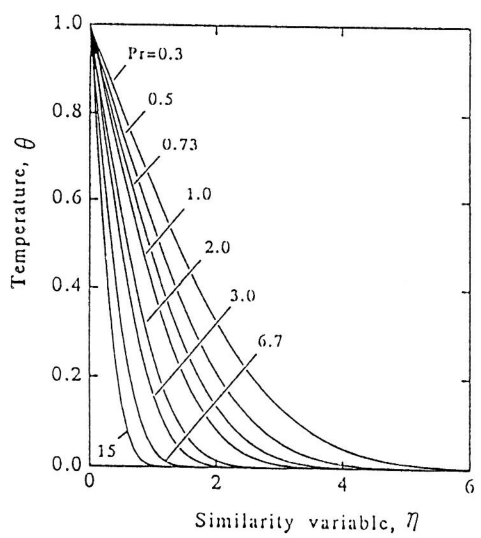

Figure 12. Temperature profiles for $\sigma = 0.5$, $s = 0.0$ and different values of Pr

Figures 2 to 12 show the variation of the dimensionless velocity profile f' and temperature θ for different values of the parameters Pr, σ and s. It is seen that u/U_∞ increases gradually to 1 for $\sigma \leq 1$ (Figures 2, 4, 6, 8) and decreases gradually to 1 for $\sigma > 1$ (Figures 2, 10). However, θ decreases monotonically with the increase of σ and Pr (Figures 3, 12). Further, we notice, as expected, that the effect of the suction or injection parameter s is to increase the velocity profiles for $\sigma \leq 1$ and decrease these profiles for $\sigma > 1$. But, the temperature profiles decrease with s for all values of σ.
Finally, it is worth mentioning that the results of the present paper are basically in good agreement with our previous results (Pop et al., 1992 b). However, the present analysis is more general than any previous investigation.

	Values of σ				
s	0.0	0.5	1.0	1.5	2.0
-0.5	0.02785	0.08889	0.0	-0.18030	-0.43168
-0.4	0.5761	0.11092	0.0	-0.20299	-0.48146
-0.3	0.10888	0.13722	0.0	-0.23056	-0.53521
-0.2	0.17485	0.16471	0.0	-0.25937	-0.59305
-0.1	0.24951	0.19698	0.0	-0.29045	0.65490
0.0	0.33206	0.23246	0.0	-0.32370	-0.72058
0.1	0.42241	0.27139	0.0	-0.35937	-0.79062
0.2	0.51972	0.31381	0.0	-0.39780	-0.86566
0.4	0.73218	0.40865	0.0	-0.48259	-1.03000
0.5	0.84582	0.46057	0.0	-0.52883	-1.11927

Table 1. Values of skin friction coefficient $C_f \cdot Re_x^{1/2}$ for different values of σ and s

	Values of σ					
Pr	-0.3	0.0	0.5	1.0	1.5	2.0
0.3	-0.15480	-0.21479	-0.26923	-0.30902	-0.34209	-0.37107
0.5	-0.17326	-0.25929	-0.33953	-0.39849	-0.44862	-0.49231
0.73	-0.18552	-0.29709	-0.40306	-0.48204	-0.54823	-0.60648
1.0	-0.19399	-0.33206	-0.46491	-0.56419	-0.64740	-0.72058
2.0	-0.20309	-0.42231	-0.63758	-0.79788	-0.93154	-1.04857
3.0	-0.19894	-0.48505	-0.76812	-0.97721	-1.15052	-1.30168
6.7	-0.15882	-0.63647	-1.11588	-1.46037	-1.74179	-1.98516
10.0	-0.12099	-0.72814	-1.34726	-1.78412	-2.13828	-2.44337
15.0	-0.07558	-0.83411	-1.63292	-2.18510	-2.62943	-3.01084

Table 2. Values of local Nusselt number $Nu \cdot Re_x^{1/2}$ for $s = 0.0$ and different values of Pr and σ

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