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The effects of suction or injection in boundary layer flow and heat transfer on a continuous moving surface

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An analysis has been made to determine the effects of uniform suction or injection in the steady laminar boundary layers over an isothermal continuous moving surface. The non-similar partial differential equations are transformed into ordinary differential equations by means of the difference-differential method. The solutions of these equations are expressed in integral form, which are then calculated by iterative numerical quadratures. The results of the velocity and temperature fields as well as for the skin friction and heat transfer coefficients at the surface are presented and discussed for different values of the suction or injection parameter and for several values of the Prandtl number and exponent of surface velocity.

1 Introduction

Boundary layer flows on moving solid surfaces are frequently encountered in transport processes occuring both in nature and industry. To cite a few practical examples, industrial processes such as the extursion of metals and plastics, cooling and drying of paper and textiles, and material handling involve boundary layers on moving surfaces in an ambient fluid. Due to entrainment of the ambient fluid, this boundary layer is physically different from that of the classical Blasius flow over a stationary flat plate and that the two problems cannot be mathematically transformed from one to the other. After a pioneering work by Sakiadis [1], the flow field past a continuous moving surface has drawn considerable attention and a good amount of literature has been generated on this problem. Tsou et al. [2]. Rhodes and Kaminer [3], Crane [4], Chida and Katto [5], Vleggaar [6], Banks [7], Banks and Zaturska [8], Abdelhaftez [9], Jeng et al. [10], Ingham and Pop [11], and Takhar et al. [12]. However, in all these papers the plate was assumed to be impermeable and hence it was not possible to apply suction or injection.

The significance of suction or injection for the boundary layer control has been well recognised, see Hartnett [13]. It is often necessary to prevent (or postpone) separation of the boundary layer to reduce drag and attain high lift values. It is also well known that suction or injection of fluid through the surface, as in mass transfer cooling can significantly modify the flow field and affect the rate of heat transfer in forced, free and mixed convection. Hence, Murty and Sarma [14] have studied the effect of suction of injection on boundary layer flow and heat transfer over a continuously moving flat plate and similarity solutions were presented. They found that this was possible when the transpiration velocity varied as $x^{-1/2}$, where x denotes the distance along the plate. Though giving a good insight into the nature of the problem, this situation has the disadvantage that the boundary condition that is necessarily imposed on the transpiration velocity is unrealistic.

The present paper is concerned with the effect of uniform suction or injection on the flow and heat transfer characteristics of laminar boundary layers over a flat plate moving continuously in a quiescent ambient fluid. The analogous problem of boundary layers flow and heat transfer over a wedge with constant suction or injection has been treated recently by Watanabe [15] and methods similar to those given in [15] to [17] are used to solve the present problem. Results were given for the velocity and temperature distributions, the coefficient of skin friction and Nusselt number for various values of the power law variation of the plate velocity, suction or injection parameter and different Prandtl numbers.

2 Analysis

The physical system under consideration is illustrated in Fig. 1. An *o-x-y* Cartesian coordinate system is fixed in space, and that at $x = y = 0$ (die slot), a thin solid permeable flat plate is extruded and moves in the positive diretion of the x axis with a surface velocity $U_s(x) = U_o x^m$, where U_o and m are constants. The motion of the fluid in the region $y > 0$ is assumed to be generated solely by the action of viscosity at the moving surface at $y = 0$. The plate is maintained at a constant temperature T_w, while the ambient fluid is at a uniform temperature T_∞ (with $T_w > T_\infty$). Assuming steady state flow of an incompressible viscous fluid at a large Reynolds number, the governing boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(\frac{\partial^2 u}{\partial y^2} \right) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{Pr} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

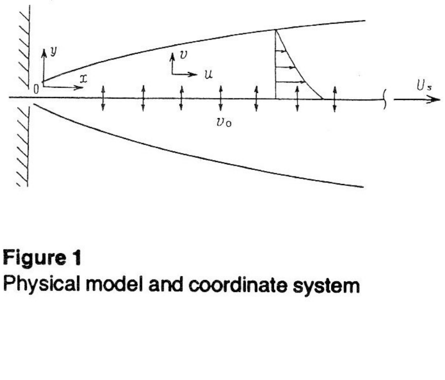

Figure 1
Physical model and coordinate system

with boundary conditions

$$y = 0 \;:\; u = U_s = U_o x^m, v = v_o \text{ (const)}, T = T_w \tag{4a}$$

$$y \to \infty \;:\; u = 0, T = T_\infty. \tag{4b}$$

Here u, v are the velocity components along x and y axes, T is the temperature, ν is the kinematic viscosity, $P\gamma$ is the Prandtl number and v_o is the velocity of suction or injection, when either $v_o < 0$ or $v_o > 0$, respectively.

To solve Eqs. (1) to (4) numerically, we follow Watanabe [15] and introduce the new variables

$$\psi = \left(\frac{2}{m+1}\, \nu x U_s\right)^{\frac{1}{2}} f(x, \eta), \; \theta(x, \eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5a}$$

and

$$\eta = y \left(\frac{m+1}{2} \frac{U_e}{\nu x}\right)^{\frac{1}{2}} \tag{5b}$$

Here ψ is the stream function defined by

$$u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x} \tag{6}$$

so that, using (5), the velocity components become

$$u = U_s \frac{\partial f}{\partial \eta} \tag{7a}$$

$$v = -\left(\frac{2}{m+1} \frac{\nu U_s}{x}\right)^{\frac{1}{2}} \left(\frac{1}{2} f + \frac{1}{2} \frac{x}{U_s} \frac{dU_s}{dx} f + x \frac{\partial f}{\partial \eta} \frac{\partial \eta}{\partial x} + x \frac{\partial f}{\partial x}\right). \tag{7b}$$

Substituting (7) into Eqs. (2) and (3), the dimensionless stream function f and dimensionless temperature θ satisfy the following transformed momentum and energy equations

$$\frac{\partial^3 f}{\partial \eta^3} + f \frac{\partial^2 f}{\partial \eta^2} - \frac{2m}{m+1} \left(\frac{\partial f}{\partial \eta}\right)^2 = \frac{2}{m+1} x \left(\frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial x \partial \eta} - \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial \eta^2}\right) \tag{8}$$

$$\frac{1}{P\gamma} \frac{\partial^2 \theta}{\partial \eta^2} + f \frac{\partial \theta}{\partial \eta} = \frac{2}{m+1} x \left(\frac{\partial \theta}{\partial x} \frac{\partial f}{\partial \eta} - \frac{\partial f}{\partial x} \frac{\partial \theta}{\partial \eta}\right). \tag{9}$$

The boundary conditions (4) are also transformed to

$$\eta = 0 \;:\; \frac{\partial f}{\partial \eta} = 1, \; \frac{1}{2}(m+1) f + x \frac{\partial f}{\partial x} = s, \; \theta = 1 \tag{10a}$$

$$\eta \to \infty \;:\; \frac{\partial f}{\partial \eta} = 0, \; \theta = 0 \tag{10b}$$

where s is the parameter of suction or injection, which is defined by

$$s = k x^{(1-m)/2} = -v_o \left(\frac{m+1}{2} \frac{x}{\nu U_s}\right)^{\frac{1}{2}}. \tag{11}$$

We shall further transform Eqs. (8) and (9) by introducing the following variable

$$x^* = k x^{(1-m)/2}. \tag{12}$$

Equations (8) and (9) then become

$$\frac{\partial^3 f}{\partial \eta^3} + \left(f + \frac{1-m}{1+m} x^* \frac{\partial f}{\partial x^*}\right) \frac{\partial^2 f}{\partial \eta^2} - \left(\frac{1-m}{1+m} x^* \frac{\partial^2 f}{\partial x^* \partial \eta}\right) \frac{\partial f}{\partial \eta} - \frac{2m}{1+m} \left(\frac{\partial f}{\partial \eta}\right)^2 = 0 \tag{13}$$

$$\frac{1}{P\gamma} \frac{\partial^2 \theta}{\partial \eta^2} + \left(f + \frac{1-m}{1+m} x^* \frac{\partial f}{\partial x^*}\right) \frac{\partial \theta}{\partial \eta} - \frac{1-m}{1+m} x^* \frac{\partial \theta}{\partial x^*} \frac{\partial f}{\partial \eta} = 0 \tag{14}$$

and the boundary conditions (10) read

$$\eta = 0 \;:\; \frac{\partial f}{\partial \eta} = 1,$$

$$\frac{1}{2}(m+1) f + \frac{1}{2}(1-m) = x^* \frac{\partial f}{\partial x^*} = x^*, \; \theta = 1. \tag{15a}$$

$$\eta \to \infty \;:\; \frac{\partial f}{\partial \eta} = 0, \; \theta = 0 \tag{15b}$$

Equations (13) and (14) subject to the boundary conditions (15) are solved by employing the diefference-differential method. We will approximate these equations by replacing the partial derivatives with respect to x^* by finite-differences, example, by using a four-point formula of Gregory-Newton backward difference with a uniform step size h. The solution of resulting differential equations at the i-th station of $x^*_i = i h$ can be expressed in the form of integral equations as

$$\frac{df_i}{d\eta} = 1 + \int_0^\eta E(\eta) \int_0^\eta \frac{R(\eta)}{E(\eta)} d\eta\, d\eta - \left\{1 + \int_0^\infty E(\eta) \int_0^\eta \frac{R(\eta)}{E(\eta)} d\eta\, d\eta\right\} \frac{G(\eta)}{G(\infty)} \tag{16}$$

$$f_i = \pm i h + \int_0^\eta \frac{df_i}{d\eta} d\eta \tag{17}$$

$$\theta_i = 1 + \int_0^\eta P(\eta) \int_0^\eta \frac{S(\eta)}{P(\eta)} d\eta\, d\eta - \left\{1 + \int_0^\infty P(\eta) \int_0^\eta \frac{S(\eta)}{P(\eta)} d\eta\, d\eta\right\} \frac{Q(\eta)}{Q(\infty)} \tag{18}$$

where plus and minus signs in (17) denote the suction or injection, respectively, and

$$E(\eta) = \exp\left[\int_0^\eta \left\{-f_i - \frac{1-m}{1+m} \frac{i}{6} (11 f_i - 18 f_{i-1} + 9 f_{i-2} - f_{i-3})\right\} d\eta\right] \tag{19}$$

$$R(\eta) = \frac{1-m}{1+m} \frac{i}{6} \left(11 \frac{df_i}{d\eta} - 18 \frac{df_{i-1}}{d\eta} - 9 \frac{df_{i-2}}{d\eta} - 2 \frac{df_{i-3}}{d\eta}\right) \frac{df_i}{d\eta} + \frac{2m}{1+m} \left(\frac{df_i}{d\eta}\right)^2 \tag{20}$$

$$G(\eta) = \int_0^{\eta} E(\eta)\,d\eta \quad (21)$$

$$P(\eta) = \exp\left[\int_0^{\eta}\left\{-P_\gamma f_l - \frac{1-m}{1+m}\frac{iP_\gamma}{6}(11f_l - 18f_{l-1} + 9f_{l-2} - 2f_{l-3})\right\}d\eta\right] \quad (22)$$

$$Q(\eta) = \int_0^{\eta} P(\eta)\,d\eta \quad (23)$$

$$S(\eta) = \frac{1-m}{1+m}\frac{iP_\gamma}{6}(11\theta_l - 18\theta_{l-1} + 9\theta_{l-2} - 2\theta_{l-3})\frac{df_l}{d\eta}. \quad (24)$$

Our attention is now focused on the skin friction τ_w and heat flux q_w at the plate given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (25)$$

where μ and λ are the dynamic viscosity and thermal conductivity, respectively. With the definition of the local kin friction coefficient C_f and the local Nusselt number as

$$C_f = \frac{T_w}{pU_s^2}, \quad Nu = \frac{x q_w}{\lambda(T_w - T_\infty)} \quad (26)$$

we obtain

$$C_f Re_x^{\frac{1}{2}} = \left(\frac{m+1}{2}\right)^{\frac{1}{2}}\left(\frac{\partial^2 f}{\partial \eta^2}\right)_{\eta=0} \quad (27a)$$

$$Nu\, Re_x = -\left(\frac{m+1}{2}\right)^{\frac{1}{2}}\left(\frac{\partial \Theta}{\partial \eta}\right)_{\eta=0} \quad (27b)$$

where $Re_x = x\,U_s/v$ is the local Reynolds number.

3 Method of solution

The numerical integration of Eqs. (16) to (18) is performed by iterative numerical quadratures using a Simpson's rule. Numerical calculations are carried out in positive or negative direction of x^* according to $s > 0$ or $s < 0$, respectively. The numerical integration starts at $x^* = 0$, where Eqs. (13) and (14) reduce to

$$f''' + ff'' - \frac{2m}{1+m}f'^2 = 0 \quad (28)$$

$$\theta'' + P\gamma f\theta' = 0 \quad (29)$$

with the boundary conditions

$$f'.(0) = 1, f(0) = 0, \theta(0) = 1 \quad (30a)$$

$$f'(\infty) = 0, \theta(\infty) = 0 \quad (30b)$$

where primes denote differentiation with respect to η. The step size in Simpson's rule used in the present numerical quadrature at each station of x^* is adjusted so that solutions may be obtained at intervals of $h = 0.01$ in η and an upper limit of infinity in integrals is replaced by a finite value $\eta = 15$. By using the present numerical integrations, a four-point formula of backward differences is used except for the first three stations following $x^* = 0$, where two-point and three-point formulas, respectively, should be used. In the iteration, the criterion of convergence is put equal to 5×10^{-7} for $(\partial^2 f/\partial\eta^2)_{\eta=0}$ and $(\partial\theta/\partial\eta)_{\eta=0}$ in absolute magnitudes.

4 Results and discussion

Equations (16) to (18) were evaluated to determine the velocity and temperature fields as well as the skin friction and surface flows into the fluid. Further, Tables 3 and 4 show that for an impermeable surface the heat transfer coefficient increases monotonically with the increase of the Prandtl number and is more intense for larger values of $P\gamma$. In addition, the values of $-\theta'(0)$ are greater for $m = -0.35484$ than those for $m = 1{,}22222$.

Representative velocity profiles and temperature distributions versus the similarity variable η are illustrated in Figs. 2 to 9, exhibiting the effects of the parameters *m, s* and $P\gamma$. Specifically, Figs. 2 to 5 show that the suction or injection has a profound effect on the boundary layer thicknesses. In general, injection prompts *s*-shaped velocity profiles and

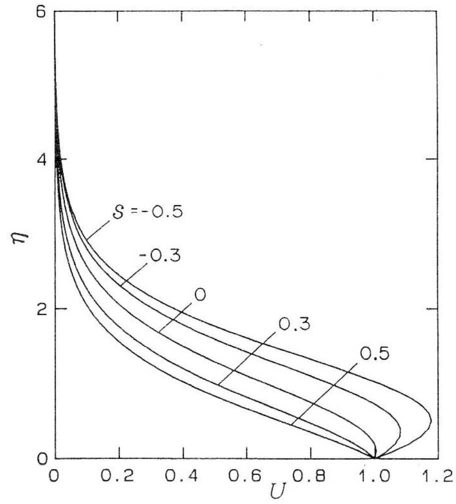

Figure 2
Velocity profiles for $m = -0.35484$ and $s = -0.5, -0.3, 0, 0.3, 0.5$

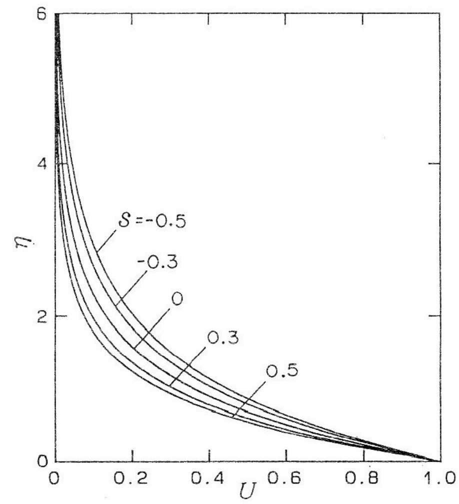

Figure 3
Velocity profiles for $m = 1.22222$ and $s = -0.5, -0.3, 0, 0.3, 0.5$

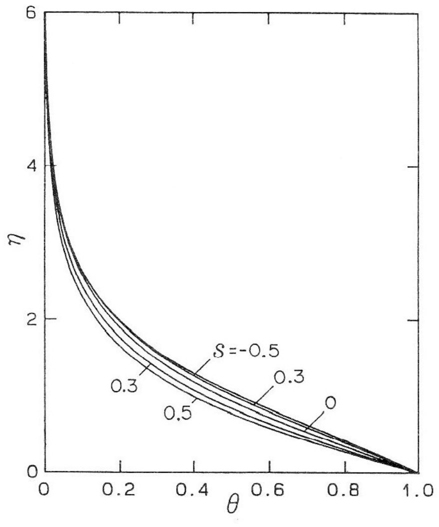

Figure 4
Velocity profiles for m=-0.35484, P_γ= 0.73 and s= -0.5,-0.3,0,0.3,0.5

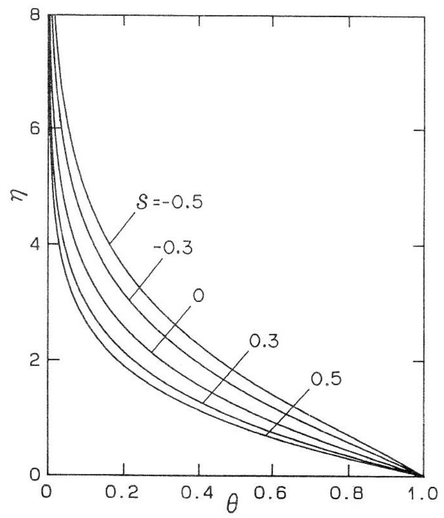

Figure 5
Temperature profiles for m = 1.22222, $P\gamma$ = 0.73 and s = − 0.5, − 0.3, 0, 0.3, 0.5

Table 1

Values of *f''(s, 0) and θ'(σ, 0)*
($P\gamma$ = 0.73, m = − 0.35484)

s	*f''(s, 0)*	*− θ'(s, 0)*
0.5	− 0.45884	0.79869
0.4	− 0.34621	0.75134
0.3	− 0.23377	0.70692
0.2	− 0.12169	0.66545
0.1	− 0.01017	0.62692
0	− 0.10053	0.59124
− 0.1	0.21019	0.55828
− 0.2	0.31851	0.52788
− 0.3	0.42523	0.49985
− 0.4	− 0.53006	0.47398
− 0.5	− 0.63272	0.45009

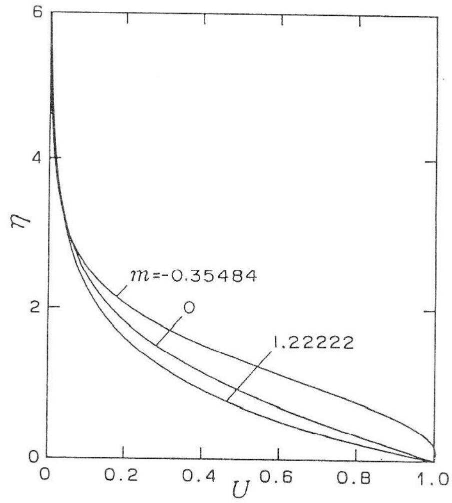

Figure 6
Velocity profiles for s = 0 and m = − 0.35484, 0, 1.22222

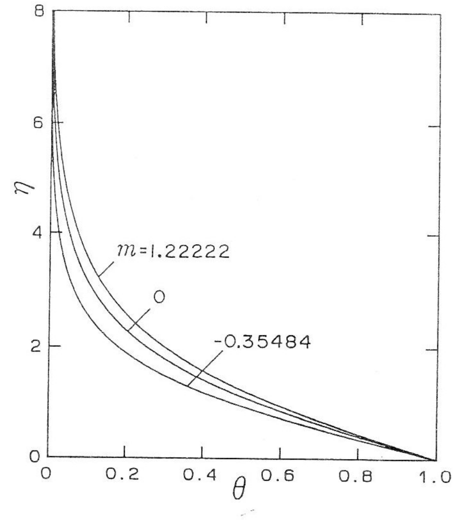

Figure 7
Temperature profiles for s = 0, $P\gamma$ = 0.73 and m = − 0.35484, 0, 1.22222

Table 2

Values of *f''(s, 0) and θ'(s, 0)*
($P\gamma$ = 0.73, m = − 1.22222)

s	*f''(s, 0)*	*− θ'(s, 0)*
0.5	− 1.30795	0.71373
0.4	− 1.24788	0.66061
0.3	− 1.19012	0.60901
0.2	− 1.13473	0.55905
0.1	− 1.08175	0.51082
0	− 1.03119	0.46446
− 0.1	− 0.98307	0.42007
− 0.2	− 0.93736	0.37775
− 0.3	− 0.89404	0.33762
− 0.4	− 0.85306	0.29977
− 0.5	− 0.81435	0.26430

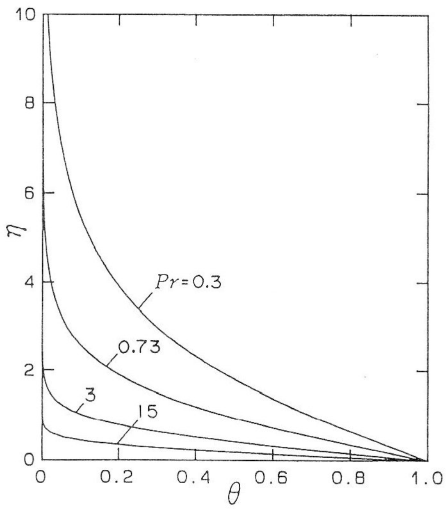

Figure 8
Temperature profiles for $s = 0$
$m = -0.35484$ and $P\gamma = 0.3, 0.73, 3, 15$

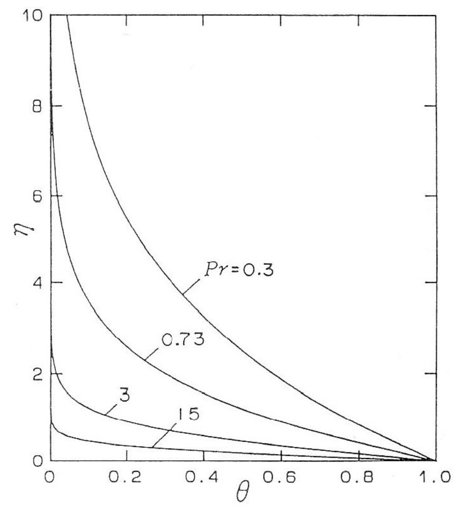

Figure 9
Temperature profiles for $s = 0$,
$m = 1.22222$ and $P\gamma = 0.3, 0.73, 3, 15$

Table 3
Values of $\theta'(0)$
$(m = -0.35484)$

$P\gamma$	$-\theta'(0)$
0.3	0.31852
0.5	0.45987
0.73	0.59124
1	0.71960
2	1.07543
3	1.34167
5	1.75711
7	2.09126
10	2.50995
15	3.08325

Table 4
Values of $\theta'(0)$
$(m = 1.22222)$

$P\gamma$	$-\theta'(0)$
0.3	0.23829
0.5	0.35194
0.73	0.46446
1	0.57829
2	0.90688
3	1.16038
5	1.56280
7	1.88994
10	2.30235
15	2.86971

may exhibit overshoots in the velocity near the thin slot $(x = 0)$ (cf. Fig. 2). However, the net effect of suction is to reduce the overshooting tendency and slow down the flow. Complementary to the previous four figures, Figs. 6 to 9 depict the variation of the velocity and temperature profiles for an impermeable surface $(s = 0)$ with parameters m and $P\gamma$. Figures 6 and 7 show clearly that the magnitude of the velocity profiles decreases as m increases (confirming the results of Banks [7] while the temperature distributions increase as m is increased. The thermal boundary layer thickness is strongly affected by the heat transfer at the surface. Results were computed for values of the surface velocity exponent $m = -0.35484$, 0 and 1.22222, the suction or injection parameter s ranging from -0.5 to 0.5 and the Prandtl number $P\gamma$ was varied from 0.3 to 15. Tables 1 to 4 contain selected results for $f''(s, 0)$ and $-\theta'(s, 0)$, which are representative of the skin friction coefficient and the Nusselt number, respectively. The accuracy of the predictive results of the present method has been established for the special case of an impermeable surface $(s = 0)$ by comparisons with known data from literature. Thus, Banks [7] have obtained $f''(0) = 0.10053$ for $m = -0.35484$ and $f''(0) = -1.03119$ for $m = 1.22222$, which are in excellent agreement with our results shown in Table 1 and 2 (cf. $s = 0$).

It is seen from Table 1 that for $m = -0.35484$ the suction $(s > 0)$ leads to negative values of the skin friction coefficient compared to those for an impermeable surface. Injection $(s < 0)$ *of fluid has the opposite effect. However, this coefficient is negative for both porous and non-porous surface when* $m = 1.22222$ (cf. Table 2). On the other hand, Tables 1 and 2 indicate that the heat transfer parameter $-\theta'(s, 0)$ remains positive for all values of s considered; hence the heat flux at the Prandtl number also remains positive (cf. Figs. 8 and 9). As expected, this thickness decreases with increasing $P\gamma$ and as a result, heat transfer is enhanced. It is also important to note that there are lower temperature at $m = -0.35484$ than those for $m = 1.22222$.

5 Conclusion

The aim of the present paper was to study the steady boundary layers over a permeable and isothermal surface that moves continuously in its plane. Solutions of the governing partial differential equations are sought by employing the difference-differential method in combination with successive numerical quadratures. Fluid mechanics and heat transfer coefficients are evaluated for a wide range of the suction or injection parameter, Prandtl number and the surface velocity exponent. It was shown that the effect of suction or injection on both velocity profile and temperature distribution is significant as depicted in terms of $f''(s, 0)$ and $-\theta'(s, 0)$. Also the thermal boundary layer thickness is substantially affected by changes in $P\gamma$.

Finally, it should be emphasized that the principal aim of the paper was to investigate the effects of a wall mass transfer parameter on boundary layers occuring on a continuous moving surface by using a wellknown calculation technique. The agreement of the present results with those of an inpermeable surface (similarity solutions) reported by Banks [7] is very good. The lack of experimental data for comparisons does therefore not weaken the conclusions arrived at.

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