TECHNISCHE MECHANIK 12 (1991) Heft 2
Manuskripteingang: 02. 12. 1990

Inverse Problem in Vibration of Lumped Non Conservative Systems Generating Symmetric Coefficient Matrices

Ladislav Starek

This paper presents a solution of the inverse problem for a linear damped assymmetric system with singular velocity and displacement coefficient matrices. Next the conditions for given spectral and modal properties are defined. When these conditions are fulfilled then the inverse formulas determine real symmetric coefficient matrices H_0 and H_1.

1. Introduction

Inverse problem in vibration of lumped nonconservative systems are concerned with the construction of the coefficient matrices of the mathematical models of the vibrating systems which have given spectral and modal properties. This problem has been solved by Danek (1982, Lancaster and Maroulas (1987) and Starek (1989).

Danek has solved this problem for the case of real nonsingular coefficient matrices and he has defined the inverse formulas which determine the coefficient matrices $\boldsymbol{A}_3$, $\boldsymbol{A}_2$ *and* $\boldsymbol{A}_1$ of the above mentioned systems with given spectral and modal properties.

Lancaster and Maroulas have solved the inverse problem by means of the spectral theory of matrix polynomials. They have defined Jordan pairs that determine real matrix polynomials and selfadjoint polynomials.

Starek has solved the inverse problem in the state space form and he has derived the inverse formulas which directly determine real coefficient matrices $\boldsymbol{H}_0 = \boldsymbol{A}_1^{-1}\boldsymbol{A}_3$ and $\boldsymbol{H}_1 = \boldsymbol{A}_1^{-1}\boldsymbol{A}_2$ in the case of singular coefficient matrices $\boldsymbol{A}_3$ and $\boldsymbol{A}_2$ too.

The goal of this paper is to derive the conditions for given spectral and modal properties. When these conditions are fulfilled then the inverse formulas will determine real symmetric coefficient matrices $\boldsymbol{H}_0$ and $\boldsymbol{H}_1$.

2. Mathematical Models

This section introduces the mathematical models of linear lumped nonconservative systems for which the inverse problem will be solved. There are introduced formulations of these models in n space, 2n space and the state space form. Next the relations are defined by means of which the left modal vectors for systems of simple as well as general Jordan structures are defined.

Here we consider linear lumped parameter systems which can be modeled by vector differential equation of the form

$$\boldsymbol{A}_1\ddot{\boldsymbol{q}}(t) + \boldsymbol{A}_2\dot{\boldsymbol{q}}(t) + \boldsymbol{A}_3\boldsymbol{q}(t) = \boldsymbol{f}(t) \tag{2.1}$$

where $\boldsymbol{q}(t)$ is an n vector of time varying elements representing the displacement of the masses in the lumped mass model. The vectors $\ddot{\boldsymbol{q}}(t)$ and $\dot{\boldsymbol{q}}(t)$ represent the accelerations and velocities, respectively. The overdot means that each element of $\boldsymbol{q}(t)$ is differentiated with respect to time. The coefficients $\boldsymbol{A}_1$, $\boldsymbol{A}_2$ and $\boldsymbol{A}_3$ are n square matrices of constant real elements representing the various physical parameters of the system. The n vector $\boldsymbol{f} = \boldsymbol{f}(t)$ represents applied external forces and also is time varying. The matrices $\boldsymbol{A}_1$, $\boldsymbol{A}_2$ and $\boldsymbol{A}_3$ are in general asymmetric, the matrix $\boldsymbol{A}_1$ is nonsingular and the matrices $\boldsymbol{A}_2$, and $\boldsymbol{A}_3$ can be singular.

Premultiplying the equation (2.1) by the inverse matrix $\boldsymbol{A}_1^{-1}$ yields

$$\ddot{\boldsymbol{v}}(t) + \boldsymbol{H}_1\dot{\boldsymbol{v}}(t) + \boldsymbol{H}_0\boldsymbol{v}(t) = \boldsymbol{p}(t) \tag{2.2}$$

where $\boldsymbol{H}_0 = \boldsymbol{A}_1^{-1}\boldsymbol{A}_3$, $\boldsymbol{H}_1 = \boldsymbol{A}_1^{-1}\boldsymbol{A}_2$, $\boldsymbol{p}(t) = \boldsymbol{A}_1^{-1}\boldsymbol{f}(t)$ and $\boldsymbol{v}(t) = \boldsymbol{q}(t)$.

By combining $\dot{\boldsymbol{v}}(t) - \dot{\boldsymbol{v}}(t) = \boldsymbol{o}$ with equation (2.2) this system can be written in 2n space

$$\boldsymbol{N}\dot{\boldsymbol{u}}(t) - \boldsymbol{P}\boldsymbol{u}(t) = \boldsymbol{g}(t) \tag{2.3}$$

where $\boldsymbol{u}(t) = \begin{bmatrix} \boldsymbol{v}(t) \\ \dot{\boldsymbol{v}}(t) \end{bmatrix}$, $\boldsymbol{g}(t) = \begin{bmatrix} \boldsymbol{p}(t) \\ \boldsymbol{o} \end{bmatrix}$

$$\boldsymbol{N} = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{P} = \begin{bmatrix} -\boldsymbol{H}_0 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}$$

Finally let the equation (2.3) be premultiplied by the inverse matrix $\boldsymbol{N}^{-1}$. This yields the standard state space formulation

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{h}(t) \tag{2.4}$$

where the state vector $\boldsymbol{x}(t) = \boldsymbol{u}(t)$, $\boldsymbol{h}(t) = \boldsymbol{N}^{-1}\boldsymbol{g}(t)$ and the state matrix

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{H}_0 & -\boldsymbol{H}_1 \end{bmatrix}.$$

Next consider solutions of (2.4) of the form $\boldsymbol{x}(t) = \boldsymbol{x}e^{st}$ for $\boldsymbol{h}(t) = \boldsymbol{o}$. Then (2.4) yields the eigenvalue problem

$$(\boldsymbol{A} - s\boldsymbol{I})\boldsymbol{x} = \boldsymbol{o} \tag{2.5}$$

The equation (2.5) fulfils the relation which expresses the solution of the associated eigenvalue problem

$$\boldsymbol{A}\boldsymbol{X} - \boldsymbol{X}\boldsymbol{D} = \boldsymbol{0} \tag{2.6}$$

The equation (2.5) of the eigenvalue problem can be transformed into the canonical form

$$\boldsymbol{X}^{-1}\boldsymbol{A}\boldsymbol{X} = \boldsymbol{D} \tag{2.7}$$

which is characterized by the spectral matrix $\boldsymbol{D} \in C^{2n,2n}$ (in general Jordan 2n square matrix). The transform matrix

$X \in C^{2n,2n}$ is the modal matrix of the eigenvalue problem (2.5).

Let's determine now the left modal vectors for the adjoint problem to the eigenproblem (2.5)

a) If A is a matrix of simple structure then D be the diagonal matrix and $X = [x_1, x_2, \ldots x_{2n}]$ in which x_j is an eigenvector of A corresponding to the eigenvalue s_j. Define the matrix $Y = (X^{-1})^T$ and write

   $Y = [y_1, y_2, \ldots, y_{2n}]$. Then $Y^T X = I$ and, comparing elements, we obtain

   $$y_j^T x_k = \delta_{jk} \tag{2.8}$$

   for j,k = 1,2, ..., 2n. Note that the systems $\{x_j\}_{j=1}^{2n}$ and $\{\bar{y}_j\}_{j=1}^{2n}$ are biorthogonal with respect to the standard inner product in C^{2n}. Let's take transposes in (2.7) and we have $X^T A^T (X^{-1})^T = D$, so that

   $$A^T Y = YD \tag{2.9}$$

   This implies

   $A^T y_j = s_j y_j$ for $j = 1, 2, \ldots, 2n$

   Thus, the columns of $Y = (X^{-1})^T$ are eigenvectors for A^T. *They are also known as left eigenvectors of A.*

b) If A is a matrix of general Jordan structure then $D \neq D^T$. Let's define now an (2n,2n) invertible matrix Q from the equality $Q = (NX)^{-1}$ with N defined by (2.3) and X from (2.7). Then [2]

   $$QNX = I \tag{2.10}$$

   Let's take the following multiplication

   $$B_1 = NAN^{-1} = \begin{bmatrix} H_1 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} 0 & I \\ -H_0 & -H_1 \end{bmatrix} \begin{bmatrix} 0 & I \\ I & -H_1 \end{bmatrix} = \begin{bmatrix} 0 & -H_0 \\ I & -H_1 \end{bmatrix} \tag{2.11}$$

   We now have

   $$B_1 = NAN^{-1} = NXDX^{-1}N^{-1} = Q^{-1}DQ \tag{2.12}$$

   Now represent Q as a block matrix $Q = [Q_1, Q_2]$ where $Q_{1,2} \in C^{2n,n}$. Then

   $$Q_1 = Q \begin{bmatrix} I \\ 0 \end{bmatrix} = X^{-1}N^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} = X^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} = W^T \tag{2.13}$$

   From (2.12) we have $Q B_1 = D Q$.

   Substituting (2.11) and Q yields $Q_2 = DQ_1 = DW^T$ and so

   $$Q = [W^T, D W^T] \tag{2.14}$$

   The rows of $W^T \in C^{2n,n}$, partitioned consistently with the partition of the spectral matrix D into its Jordan blocks and taken in each block in the reverse order form left modal vectors for the matrix A.

   We shall refer to the equality (2.10) as the biorthogonality condition for X and Q.

   From this condition we have $XQ = N^{-1}$.

   Substituting for N, Q a $X = \begin{bmatrix} V \\ VD \end{bmatrix}$ into the above mentioned relation we yield the following useful equalities

   $$VW^T = 0 \tag{2.15}$$

   $$VDW^T = I$$

Note that the eigenvalues (and their multiplicities) of a lumped linear system described by real coefficient matrices are symmetric with respect to the real axis of the complex plane. This implies that there is a Jordan matrix for such a system with the block diagonal form

$$D = (J_c, J_R, \bar{J}_c) \tag{2.16}$$

where J_c is a matrix with all its eigenvalues in the open upper half of the complex plane, J_R is real matrix and the entries of $\bar{J}_c$ are the complex conjugate of those in J_c.

The modal matrices V and W are partitioned in a compatible way as (2.16)

$$V = (V_c, V_R, \bar{V}_c), \quad W = (W_c, W_R, \bar{W}_c) \tag{2.17}$$

3. Inverse formulas

Here we derive formulas for the coefficient matrices $H_o = A_1^{-1}A_3$ and $H_1 = A_1^{-1}A_2$ by inversion of the equation (2.7). From the equation (2.7) we obtain

$$A = XDX^{-1} \tag{3.1}$$

Assuming the partitioning of the matrices X, Q given by (2.14) and recalling that $X^{-1} = QN$ we obtain

$$XDX^{-1} = \begin{bmatrix} V \\ VD \end{bmatrix} D \left[W^T D W^T \right] \begin{bmatrix} H_1 & I \\ I & 0 \end{bmatrix}$$

after some manipulation and by using (2.4) we have the equality

$$\begin{bmatrix} O & I \\ -H_0 & -H_1 \end{bmatrix} = \begin{bmatrix} VDW^T H_1 + VD^2 W^T VDW^T \\ VD^2 W^T H_1 + VD^3 W^T VD^2 W^T \end{bmatrix}$$

From the equality of associated submatrices we obtain the inverse formulas

$$H_o = (VD^2 W^T)^2 - VD^3 W^T \tag{3.2}$$

$$H_1 = -VD^2 W^T \tag{3.3}$$

and the condition

$$VDW^T = I \tag{3.4}$$

The formulas (3.2) and (3.3) determine the two desired coefficient matrices of the system (2.1) if the third is chosen with spectral and modal properties which must satisfy the condition (3.4).

From the condition (3.4) the suitable matrices D, V and W can be defined for the inverse problem.

In that case:

- the matrix D must have rank(D) $\geq n$
- the modal matrices V, $W \in C^{n,2n}$ must be nonsingular.

If we choose such spectral and modal properties that (3.4) is valid then the system determined by the coefficient matrices H_o and H_1 is unique.

Example 3.1

Consider the three degree of freedom system with semi-definite damping and stiffness given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ddot{\boldsymbol{v}}(t) + \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \dot{\boldsymbol{v}}(t) + \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \boldsymbol{v}(t) = \boldsymbol{o}$$

The spectral solution of the system yields that the matrices $\boldsymbol{D}$ and $\boldsymbol{X}$ are

$$\boldsymbol{D} = \begin{bmatrix} -1.41+1.51i & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.59+1.03i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.41-1.51i & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.59-1.03i \end{bmatrix}$$

$$\boldsymbol{X} = \begin{bmatrix} 0.27-0.09i & 0.17+0.62i & 1 & 0 & 0.27-0.09i & 0.17-0.62i \\ -0.33+0.35i & 0.25+0.11i & 1 & 0 & -0.33+0.35i & 0.25-0.11i \\ 0.06-0.26i & -0.42-0.73i & 1 & 0 & 0.06+0.26i & -0.42+0.73i \\ -0.52+0.27i & -0.74-0.19i & 0 & 1 & -0.52-0.27i & -0.74+0.19i \\ 1 & -0.26+0.19i & 0 & 1 & 1 & -0.26-0.19i \\ -0.48-0.27i & 1 & 0 & 1 & -0.48+0.27i & 1 \end{bmatrix}$$

Let the new spectral matrix be

$$\boldsymbol{D1} = \begin{bmatrix} -0.5+i & 0 & 0 & 0 & 0 & 0 \\ 0 & -1+2i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5-i & 0 \\ 0 & 0 & 0 & 0 & 0 & -1-2i \end{bmatrix}$$

and the right modal matrix be the original that is $\boldsymbol{V1} = \boldsymbol{V} = [\boldsymbol{I}\ \boldsymbol{0}]\boldsymbol{X}$. For the case of unique solution of the inverse problem for the matrixes $\boldsymbol{D1}$, $\boldsymbol{V1}$ and $\boldsymbol{W1}$ must hold $\boldsymbol{V1D1W1}^T = \boldsymbol{I}$. Then the left modal matrix is given by

$$\boldsymbol{W1} = \boldsymbol{X1}^{-1} \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{I} \end{bmatrix} \quad \text{where} \quad \boldsymbol{X1} = \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{VD1} \end{bmatrix}$$

By using the inverse formulas (3.2) and (3.3) $\boldsymbol{H}_o$ and $\boldsymbol{H}_1$ become

$$\boldsymbol{H}_o = \begin{bmatrix} 2.65 & -0.4 & -2.25 \\ 0.33 & 0.74 & -1.07 \\ -2.98 & -0.34 & 3.32 \end{bmatrix} \quad \boldsymbol{H}_1 = \begin{bmatrix} 1.6 & -0.86 & -0.74 \\ -0.18 & 0.42 & -0.24 \\ -1.42 & 0.44 & 0.98 \end{bmatrix}$$

To see that the method works note that (2.1) with $\boldsymbol{A}1 = \boldsymbol{I}$ and the determined matrices $\boldsymbol{H}_o$ and $\boldsymbol{H}_1$ yields the eigenvalue given by the matrix $\boldsymbol{D1}$.

4. Inverse formulas for symmetric coefficient matrices

The goal here is to derive the conditions for given spectral and modal properties. When these conditions are fulfilled then the inverse formulas will determine real symmetric coefficient matrices $\boldsymbol{H}_0$ and $\boldsymbol{H}_1$. In that case let the matrix $\boldsymbol{A}_1$ is positive definite. Then by substituting $\boldsymbol{q}(t) = \boldsymbol{A}_1^{-1/2}\boldsymbol{v}(t)$ into equation (2.1) and premultiplying the result by $\boldsymbol{A}_1^{-1/2}$ the corresponding matrices becomes $\boldsymbol{H}_0 = \boldsymbol{A}_1^{-1/2}\boldsymbol{A}_3\boldsymbol{A}_1^{-1/2}$ and $\boldsymbol{H}_1 = \boldsymbol{A}_1^{-1/2}\boldsymbol{A}_2\boldsymbol{A}_1^{-1/2}$. If the inverse formulas are to generate real symmetric coefficient matrices then the spectral and modal matrices must fulfil some further requirements. From the theory of matrix polynomial it is known that the spectral matrix $\boldsymbol{D}$ and modal matrices $\boldsymbol{V}$ and $\boldsymbol{W}$ generate hermitian coefficient matrices if a left modal matrix is of the form

$$\boldsymbol{W}^T = \boldsymbol{P}_D\boldsymbol{W}^* \tag{4.1}$$

where $\boldsymbol{P}_D$ is given by the formula (Gohberg, . . .)

$$\boldsymbol{P}_D = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{P}_c \\ \boldsymbol{0} & \boldsymbol{P}_R & \boldsymbol{0} \\ \boldsymbol{P}_c & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{4.2}$$

Note that $\boldsymbol{P}_D^* = \boldsymbol{P}_D$, $\boldsymbol{P}_D^2 = \boldsymbol{I}$ and

$$\boldsymbol{D}^*\boldsymbol{P}_D = \boldsymbol{P}_D\boldsymbol{D} \tag{4.3}$$

Now we will derive the conditions that must be fulfilled by the spectral and modal matrices to generate real symmetric coefficient matrices.

From (2.13) we have

$$\boldsymbol{X}\boldsymbol{W}^T = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \tag{4.4}$$

Substituting for $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{VD} \end{bmatrix}$ and $\boldsymbol{W}^T$ from (4.1) we obtain

$$\begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{VD} \end{bmatrix} \boldsymbol{P}_D\boldsymbol{V}^* = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \tag{4.5}$$

After substituting for $\boldsymbol{V}$ and $\boldsymbol{D}$ from (2.16) and (2.17) we obtain the following conditions

$$\boldsymbol{V}_c\boldsymbol{P}_c\boldsymbol{V}_c^T + \boldsymbol{V}_R\boldsymbol{P}_R\boldsymbol{V}_R^T + \bar{\boldsymbol{V}}_c\bar{\boldsymbol{P}}_c\bar{\boldsymbol{V}}_c^T = \boldsymbol{0} \tag{4.6}$$

$$\boldsymbol{V}_c\boldsymbol{J}_c\boldsymbol{P}_c\boldsymbol{V}_c^T + \boldsymbol{V}_R\boldsymbol{J}_R\boldsymbol{P}_R\boldsymbol{V}_R^T + \bar{\boldsymbol{V}}_c\bar{\boldsymbol{J}}_c\bar{\boldsymbol{P}}_c\bar{\boldsymbol{V}}_c^T = \boldsymbol{I} \tag{4.7}$$

If the spectral matrix $\boldsymbol{D}$ and modal matrix $\boldsymbol{V}$ are of the form (2.16) and (2.17) and they fulfil the conditions (4.6) and (4.7) then the inverse formulas (3.2) and (3.3) generate the real symmetric coefficient matrices. For determining such spectral and modal matrices that will determine the real symmetric coefficient matrices we arrange the conditions (4.6) and (4.7) in the following way:

Let the matrices V_c and J_c are of the form $V_c = V_r + iV_i$ *and* $J_c = J_r + iJ_i$, where $V_r, V_i, J_r, J_i \in R$. After some manipulations we obtain

$$2(V_r P_c V_r^T - V_i P_c V_i^T) = -V_R P_R V_R^T \tag{4.8}$$

$$2(V_r J_r P_c V_r^T - V_i J_i P_c V_r^T - V_r J_i P_c V_i^T - V_i J_r P_c V_i^T) = \\ = I - V_R J_R P_R V_R^T \tag{4.9}$$

The conditions (4.8) and (4.9) in depending on the matrix A (the matrix A can be a matrix of simple or general Jordan structure, it has no real eigenvalues or real eigenvalues too) determine such modal vectors that generate the real symmetric coefficient matrices for the given spectral matrix.

Let's arrange the relations (4.8) and (4.9) in the following way. Let $V_i = V_r C$, where $C \in R^{n,n}$ and is nonsingular. Substituting for V_i *into (4.8) and (4.9) yields*

$$2V_r E V_r^T = V_R P_r V_R^T \tag{4.10}$$

$$2V_r F V_r^T = I - V_R J_R P_R V_R^T \tag{4.11}$$

where

$$E = C P_c C^T - P_c \tag{4.10a}$$

$$F = J_r P_c - C J_i P_c - J_i P_c C^T - C J_r P_c C^T \tag{4.11a}$$

Example 4.1

The coefficient matrices of the system (2.2) H_0 and H_1 must be determined in such a way that these matrices will be symmetric and the system will have the spectral properties given by the matrix D, where

$$D = \begin{bmatrix} -1-i & 0 & 0 & 0 & 0 & 0 \\ 0 & -1-2i & 0 & 0 & 0 & 0 \\ 0 & 0 & -2-3i & 0 & 0 & 0 \\ 0 & 0 & 0 & -1+i & 0 & 0 \\ 0 & 0 & 0 & 0 & -1+2i & 0 \\ 0 & 0 & 0 & 0 & 0 & -2+3i \end{bmatrix}$$

Since the matrix D is the diagonal matrix and it has no real eigenvalues, then P_c is the identity matrix and $V_R = 0$, $J_R = 0$. The conditions (4.10) and (4.11) will be reduced to the form

$$V_r(I - C C^T)V_r^T = 0 \tag{4.12}$$

$$2V_r F V_r^T = I \tag{4.13}$$

where

$$F = J_r - C J_i - J_i C^T - C J_r C^T$$

The first condition is fulfilled for an arbitrary $V_r \in R^{n,n}$ if C^T is orthogonal. If we choose C^T as an orthogonal matrix and V_r arbitrary nonsingular matrix, then from the second condition we obtain

$$2V_r F V_r^T = Z \tag{4.14}$$

where $Z = Z^T$. From the theory of matrices it is known that for every (n,n) nonsingular symmetric matrix Z there exists an (n,n) matrix T such that $Z = T^T T$. Substituting for Z into (4.14) we have

$$2V_r F V_r^T = T^T T$$

and it follows that

$$V_r = (T^T)^{-1} V_r. \tag{4.15}$$

In our case the required matrices are

$$J_r = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad J_i = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

Let the matrices

$$V_r = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix} \quad C^T = 1/\sqrt{2} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

be chosen so that Z will be positive definite.

After some calculation we obtain

$$V_c = \begin{bmatrix} 0.339 + 0.120i & -0.169 - 0.361i & -0.169 - 0.169i \\ 0.535 + 0.606i & 0.323 - 0.149i & 0.087 + 0.087i \\ 0.053 - 0.149i & -0.263 - 0.223i & 0.217 + 0.217i \end{bmatrix}$$

Since the modal matrix is $V = [V_c, \bar{V}_c]$ and after substituting for V, D and X or W^T into (3.2) and (3.3) for the coefficient matrices we obtain

$$H_1 = \begin{bmatrix} 2.087 & -1.007 & -1.098 \\ -1.007 & 2.671 & -0.258 \\ -1.089 & -0.258 & 3.242 \end{bmatrix}$$

$$H_0 = \begin{bmatrix} 7.516 & -3.423 & -2.915 \\ -3.423 & 3.494 & 1.091 \\ -2.915 & 1.091 & 10.276 \end{bmatrix}$$

To see that the method works note, that the system (2.2)with the determined matrices H_0 and H_1 yields the eigenvalues given by the matrix D.

5. Conclusion

In the paper the inverse formulas for linear lumped nonconservative systems of simple as well as general Jordan structures were derived in the case of singular velocity and displacement coefficient matrices. Next the conditions (4.6) and (4.7) for given spectral and modal properties are defined. When these conditions are fulfilled then the inverse formulas (3.2) and (3.3) determine real symmetric coefficient matrices for linear lumped nonconservative systems of simple as well as general Jordan structures. At the end the relations are defined by means of which the left modal vectors for system of simple as well as general Jordan structures are defined. The defined conditions (4.6) and (4.7) and the generated symmetric coefficient matrices can better approximate the results of the numerical simulation to the reality.

The derived conditions (2.15) for the spectral and modal properties can be used for checking numerical computations.

REFERENCES

[1] Danek, O., 1982, „Nonconservative Dynamic Systems", Strojnicky casopis 33, c. 6, pp. 667 – 680. (in Czech.)

[2] Gohberg, I., Lancaster, P., Rodman, L., 1982, „Matrix Polynomial", Academic Press, Inc., New York.

[3] Lancaster, P., Maroulas, J., 1987, „Inverse Eigenvalue Problem for Damped Vibrating Systems", Journal of Mathematical Analysis and Applications, Vol. 123, No. 1, pp. 238 – 261.

[4] Starek, L., 1989, „Inverse Problem in Vibration of Lumped Non Conservative Systems", Strojnicky casopis, 40, c. 6, pp. 677 – 685. (in Slovak.)

Address of the author:
Ladislav Starek
Faculty of Mechanical Engineering
Slovak Technical University at Bratislava
81231 Bratislava
Czechoslavakia