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Study of dynamic plastic buckling of cylindrical shell impacted by sudden constant load

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Based on Literature 3, in this paper, the characteristic of dynamic plastic buckling was studied of cylindrical shell impacted by sudden constant load. Furthermore, critical buckling load and mode of buckling load and mode of buckling were also analysed.

1. Introduction

Cylindrical shell is a typical part in engineering. It is necessary to study response and buckling characteristic of it impacted by shocking load. There was a few papers about the study of dynamic plastic buckling of cylindrical shell impacted by shocking load. In 1968, Florence [1] presented a approximating theory by which somethingh about buckling mode was discussed. In 1970, Henry [2] studied the buckling mode of short cylindrical shell forced by explosive load by means of experiment. In 1983, Wang [4] studied the critical shocking velocity of cylindrical shell impacted by bullet. In 1988, Wang Guotai and Tan Huimin [5] made some experiment research of cylindrical shell by means of Hopkinson column principle. They gave some experimental results about critical time and critical load. It is known that, research about critical time and critical load is limited in experimental area; In theoretical area only buckling mode is investigated. Therefore, Tan Huimin and Gao Shiqiao [3] studied the critical time of cylindrical shell impacted by constant velocity shocking load by both theoretical and experimental methods. In this paper, based on literiture [3], the critical load of cylindrical shell impacted by sudden constant load was analysed and discussed.
In this paper, assumptions are as follows.

1° Cylindrical shell is long but thin shell;
2° Effection of stress wave is neglected;
3° Effection of boundary is neglected;
4° The material of shell is strength material.

2. Govering equations

2.1. Equilibrium equations

In terms of bending theory of cylindrical shell, the dynamic equation of cylindrical shell can be written by

$$\frac{\partial^2 M_x}{\partial x^2} + N_x \frac{\partial^2 w}{\partial x^2} + \frac{N_y}{R} = \Gamma \cdot h \cdot \frac{\partial^2 w_x}{\partial t^2} \qquad (1)$$

When the load is sudden constant load, for $t \geq 0$, there is $N_x = -N$, so that equation (1) is changed to

$$\frac{\partial^2 M_x}{\partial x^2} - N \frac{\partial^2 w}{\partial x^2} + \frac{N_y}{R} = \Gamma \cdot h \cdot \frac{\partial^2 w_x}{\partial t^2} \qquad (2)$$

where w is normal displacement of middle surface of shell; R is radius of middle surface; h is thickness of shell; Γ is mass density; x is axial coordinate; y is circumferential coordinate; M_x and N_y are axial moment and circumferential internal force respectively; t is time (shown in Fig. 1)

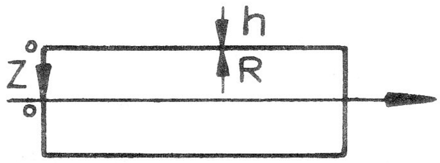

Figure 1
Structure and coordinates of cylindrical shell

The equation of moment M_x and internal force N_y can be written by

$$\begin{cases} M_x = -\int_{-h/2}^{h/2} \sigma_x \cdot z \cdot dz \\ N_y = \int_{-h/2}^{h/2} \sigma_y \cdot dz \end{cases} \qquad (3)$$

where σ_x and σ_y are axial stress and circumferential stress respectively, z is normal coordinate.

2.2. Deformation equations

When the effection of stress wave and the effection of boundary are neglected, relations between strain ratio and velocity can be written by

$$\begin{cases} \dot{\varepsilon}_x = \dot{\varepsilon}_x + z \cdot \dot{w}'' \\ \dot{\varepsilon}_y = -(1 - z/R) \cdot \dfrac{\dot{w}}{R} \end{cases} \qquad (4)$$

Because $z/R << 1$, equation (4) can be written by

$$\begin{cases} \dot{\varepsilon}_x = \dot{\varepsilon}_x + z \cdot \dot{w} \\ \dot{\varepsilon}_y = -\dfrac{\dot{w}}{R} \end{cases} \qquad (5)$$

where $\dot{\varepsilon}_x$ and $\dot{\varepsilon}_y$ are strain ratio along axial and circumferential directions respectively; $\dot{\varepsilon}_x$ is strain ratio along axial direction in middle surface; "." indicates derivative for time t; "," indicates derivate for coordinate x.

2.3. Physical equations

In terms of Reference [6], the dynamic physical relation of cylindrical shell forced by load in one direction can be written by

$$\begin{cases} \dot{\sigma}_x = \dfrac{E}{(5-4\mu)\,\Theta-(1-2\mu)^2}\,[(\Theta+3)\dot{\varepsilon}_x+2(\Theta-1+2\mu)\dot{\varepsilon}_y] \\ \dot{\sigma}_y = \dfrac{E}{(5-4\mu)\,\Theta-(1-2\mu)^2}\,[4\Theta\dot{\varepsilon}_y+2(\Theta-1+2\mu)\dot{\varepsilon}_x] \end{cases} \quad (6)$$

where E is Young's modulus; μ is poisson ratio; $\Theta = E/E_h$ and E_h is tangient modulus.

Substituting equation (5) into equation (6) and integrating them in direction of thickness, leads to

$$\begin{cases} \dot{N}_x = \dfrac{E}{(5-4\mu)\Theta-(1-2\mu)^2}\,[(\Theta+3)\cdot h\cdot \dot{e}_x - 2(\Theta-1+2\mu)\cdot h\cdot \dfrac{\dot{w}}{R}] \\ \dot{N}_y = \dfrac{E}{(5-4\mu)\,\Theta-(1-2\mu)^2}\,[-4\cdot\Theta\cdot h\,\dfrac{\dot{w}}{R} + 2(\Theta-1+2\mu)\cdot h\cdot \dot{e}_x] \end{cases} \quad (7)$$

For sudden constant load in axial direction, when $t>0$, $\dot{N}_x = 0$, leads to

$$\begin{cases} \dot{e}_x = \dfrac{2(\Theta-1+2\mu)}{\Theta+3}\cdot\dfrac{\dot{w}}{R} \\ \dot{N}_y = -\dfrac{4\cdot E\cdot h}{\Theta+3}\cdot\dfrac{\dot{w}}{R} \end{cases} \quad (8)$$

2.4. Vibrations equations

Substituting equation (5) into the first equation of equation (6) and integrating them for time t (initial conditions are considered), then substituting it into equation (3), the M_x can be obtained. Integrating the second equation of equation (8) for time t (considering the initial conditions), the N_y can be obtained. Substituting M_x and N_y into equation (2), leads to

$$A w^{(4)} + N w^{(2)} + B w + C\ddot{w} = 0 \quad (9)$$

where

$$A = \frac{(\Theta+3)\cdot E\cdot h^3}{12[(5-4\mu)\Theta-(1-2\Theta)^2]};$$

$$B = \frac{4\cdot E\cdot h}{(\Theta+3)\cdot R^2};$$

$$C = \Gamma\cdot h$$

3. Critical load of buckling

Equation (9) is a linear uniform partial equation. To solve this equation, we assume $w = X(x)T(t)$. Substituting it into equation (9), leads to

$$(A\cdot X^{(4)} + N\cdot X^{(2)} + B\cdot X)\cdot T + C\cdot X\cdot \ddot{T} =$$

that is

$$\frac{A\cdot X^{(4)} + N\cdot X^{(2)} + B\cdot X}{X} = -\frac{C\cdot\ddot{T}}{T} = \delta \quad (10)$$

where δ is a constant which is independent of x and t. Equation (10) can be written by

$$A\cdot X^{(4)} + N\cdot X^{(2)} + (B-\delta)\cdot X = 0 \quad (11)$$

$$C\cdot\ddot{T} + \delta\cdot T = 0 \quad (12)$$

From equation (12), it is known that, when $\delta > 0$, the vibration of cylindrical shell system is stable; when $\delta \le 0$, the vibration is unstable. It is considered that, when the vibration is divergent, the cylindrical shell bigin to plastiv buckling. According to this principle, $\delta = 0$ is a critical state.

From equation (11), the eigen-equation and eigenvalue can be written by

$$\begin{cases} A\cdot\alpha^4 + N\cdot\alpha^2 + (B-\delta) = 0 \\ \alpha = \pm\sqrt{-N/2A \pm \sqrt{N^2/4A^2-(B-\delta)/A}} \end{cases} \quad (13)$$

From further discussion, it is known that, the condition which make the cylindrical shell have non-zero solution in uniform boundary condition is as follows

$$B-\delta < \frac{N^2}{4\cdot A} \quad (14)$$

and

$$\alpha = \frac{n\cdot\pi}{L} \quad (15)$$

where L is length of cylindrical shell, n is number of harmonic wave.

By means of equation (15) and equation (13) and using the critical state principle mentioned above, for critical state, there is

$$\left(\frac{\pi^2 n^2}{L^2} - \frac{N}{2A}\right)^2 = \frac{N^2}{4A^2} - \frac{B}{A} \quad (16)$$

Solving equation (16), leads to

$$N = \frac{B\cdot L^2}{n^2\pi^2} + \frac{A\cdot n^2\pi^2}{L^2} \quad (17)$$

From extrem value theory, we obtain that, when

$$n = \frac{L}{\pi}\sqrt[4]{\frac{B}{A}} \quad (18)$$

N has minimum value which is

$$N_{min} = 2\sqrt{A\cdot B} \quad (19)$$

Using equation (9), leads to

$$N_{min} = \frac{2Eh^2}{R}\,\frac{1}{\sqrt{3[(5-4\mu)\Theta-(1-2\mu)^2]}} \quad (20)$$

where N_{min} is minimum buckling internal force, n is first buckling mode correspondingly. Furthermore, the minimum critical buckling load can be obtained as

$$P_{min} = 2\pi R\cdot N_{min} = \frac{4\pi E h^2}{\sqrt{3[(5-4\mu)\Theta-(1-2\mu)^2]}} \quad (21)$$

4. Discussion about critical buckling load

From equation (20), critical buckling stress can be written by

$$\sigma_{min} = N_{min}/h = \frac{2E}{\sqrt{3[(5-4\mu)\Theta-(1-2\mu)^2]}}\,\frac{h}{R} \quad (22)$$

Assuming that yield stress of cylindrical shell material is σ_s and that strength limit is σ_b, when $\sigma_{min} < \sigma_s$, the first buckling is elastic buckling; when $\sigma_s < \sigma_{min} < \sigma_b$, the first buckling is plastic buckling; when $\sigma_{min} > \sigma_b$, there is no buckling before damage due to strength limit. From equation (22), it is known that, it depends on the relative thickness h/R which damage will emerge. When $h/R < 0{,}5 \sqrt{3[(5-4\mu)\Theta-(1-2\mu)^2]} \cdot \sigma_s/E$, the structure will firstly be subjected to damage due to elastic buckling. When $\sqrt{3[(5-4\mu)\Theta-(1-2\mu)^2]} \cdot \sigma_s/2E < h/R < \sqrt{3[(5-4\mu)\Theta-(1-2\mu)^2]} \cdot \sigma_b/2E$, the cylindrical shell will firstly be subjected to plastic yield and then be subjected to damage due to plastic buckling. When $h/R > \sqrt{3[(5-4\mu)\Theta-(1-2\mu)^2]} \cdot \sigma_b/2E$, there is no buckling before damage due to strength limit.

5. Example and analysis of results

By means of the method mentioned above, in this paper, the critical buckling load are calculated and analysed of cylindrical shell impacted by axial sudden constant load (in Fig. 2). In Fig. 2 $R = 2.98$ mm; $L = 7.87$ mm; $h = 0.252$ mm. The material of cylindrical shell is copper whose Young's modulus $E = 20$GPA, Poissson ratio $\mu = 0.5$, $\Theta = 15$, yield stress $\sigma_s = 140MPA$, *strength limit* $\sigma_b = 320$MPA.
Stubstiuting the parameters above into equation (21), we obtain that, the critical load $P_{min} = 737.3$ Kg, the number of buckling mode correspondingly n = 4.65 ≈ 5. The change curves of critical load P_{min}, yield load P_s and limit load P_b with thickness h for $R = 2.98$ mm are given in Fig. 3. It is shown from Fig. 3 that, if $h < 0.12$ mm, the structure will be subjected to damage due to elastic buckling before plastic deformation; if 0.12 mm < h < 0.28 mm, the structure will firstly be subjected to plastic deformation and then will be subjected to damage due to plastic buckling; if $h > 0.28$ mm, it is impossible that buckling emerge before damage.

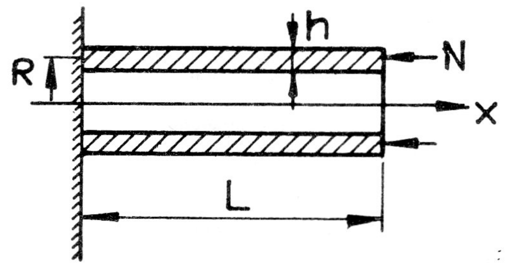

Figure 2
Structure of cylindrical shell

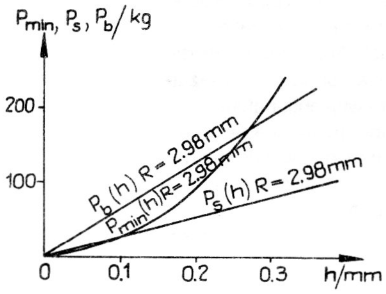

Figure 3
Curves of load

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