TECHNISCHE MECHANIK 10 (1989) Heft 2
Manuskripteingang: 17. 01. 1989

Anisotropy of the eddy viscosities and their pressure dependence in the near wall region of three-dimensional turbulent boundary layers

M. Hoffmeister

0. Introduction

The anisotropy of the eddy viscosities in three-dimensional turbulent boundary layers (3-d b.l.) is known from many investigations. Large differences among the experimental findings appeared. For the near wall region of pressure driven 3-d b.l. over hydraulically smooth plane walls this is presumably due to the pressure influence.

An attempt was made to describe the anisotropy of the eddy viscosities in 3-d b.l. with the aid of an extended mixing length model [1]. Better approximations of certain experimental findings compared with the application of an isotropic eddy viscosity model could be obtained. But essential assumptions involved in the model refer to the proportionality between the mixing length and the wall distance and to the independence of the mixing length on the wall pressure gradient. Already from two-dimensional turbulent boundary layers (2-d b.l.) it is known that these prerequisites are not sufficiently fulfilled close to the wall, i. e. in the viscous sublayer and in the buffer layer. Consequently, the investigations in [1] are restricted to wall distances larger than about 20 wall units.

Another proposal to include the anisotropy of the eddy viscosities for the calculation of 3-d b.l. is described in [2]. The basic idea is the introduction of a constant eddy viscosity ratio in a local streamwise oriented coordinate system neglecting possible influences of the static pressure gradient on this ratio. But there is some experimental evidence that this can be only a first and very rough approximation of the real flow properties. So further investigations, especially with respect to the region close to the wall and including the wall pressure distribution seemed to be advisable.

1. Dimensionless mean velocity distributions in the near wall region of 3-d b.l.

Some years ago experimental findings from 3-d b.l. ahead and after a swept wall bounded bar and from the swirling flow in a short vaneless radial diffuser revealed some evidence for the existence of simple dimensionless mean velocity distributions

$$\text{with} \quad \varphi = \varphi(\eta), \qquad (0 \leqslant \eta < \eta^{(2)}) \qquad \text{(1a)}$$

$$\tilde{\varphi} = \tilde{\varphi}(\tilde{\eta}), \qquad (0 \leqslant \tilde{\eta} < \tilde{\eta}^{(2)}) \qquad \text{(1b)}$$

$$\varphi = \overline{c_s}/v_o^*; \qquad \tilde{\varphi} = \overline{c_n}/w_o^* \qquad \text{(2a)}$$

$$\eta = y \cdot v_o^*/\nu; \qquad \tilde{\eta} = y \cdot w_o^*/\nu \qquad \text{(2b)}$$

in a wall stream line oriented, s, n, y-co-ordinate system [3]. Here, s has the direction of the local mean wall shear stress vector $\vec{\tau}_o$, y is the wall distance, and n completes the local Cartesian system; $\overline{c_s}$ and $\overline{c_n}$ are the interesting mean velocity components, $v_o^* = |\vec{\tau}_o/\rho|^{1/2} > 0$ denotes the wall friction velocity and $w_o^* = \left(\frac{\nu}{\rho}\frac{\partial \overline{p}_o}{\partial n}\right)^{1/3} \neq 0$ with the fluid density ρ, with the mean static wall pressure $\overline{p}_o$, and with the kinematic viscosity ν is the scaling velocity for the n-direction.

The upper validity bounds $\eta^{(2)}$, $\tilde{\eta}^{(2)}$ in (1) depend – among others – on the pressure gradient number for the n-direction

$$\alpha_n^{1/3} = w_o^*/v_o^*. \qquad \text{(3a)}$$

An influence of the corresponding parameter for the s-direction

$$\alpha_s^{1/3} = \left(\frac{\nu}{\rho}\frac{\partial \overline{p}_o}{\partial s}\right)^{1/3} \Big/ v_o^* \qquad \text{(3b)}$$

could not be detected within the intervals

$$\alpha_n^{1/3} \approx 0.18 \ldots 0.38$$
$$\qquad \text{(4)}$$
$$\alpha_s^{1/3} \approx -0.28 \ldots +0.25.$$

To confirm and to extend these investigations with respect to smaller wall distances ($\eta \lesssim 20$) further measurements with the aid of a rotated single hot-wire probe were carried out [4]. The special design of this probe1) made the interference between the prongs and the hot-wire being parallel to the wall small and permitted a proper determination of the distance between the wall and the sensing element with the help of a microscope. The prongs penetrated a PVC plug which was flush mounted in the wall.

1) designed by Dipl.-Ing. H. Grützner

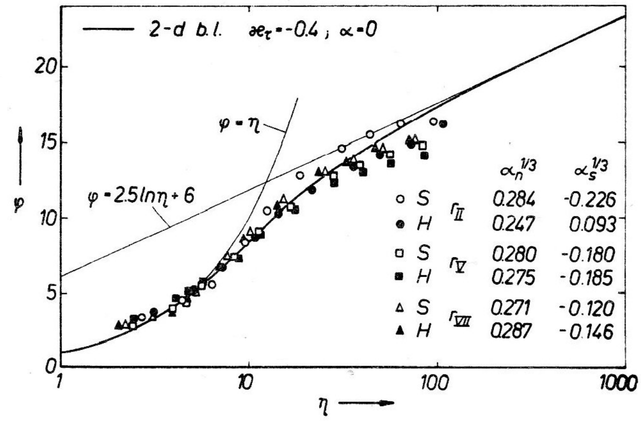

Fig. 1
Mean velocity components of the s-direction
($\alpha_s \to \alpha$ for 2-d b.l.)

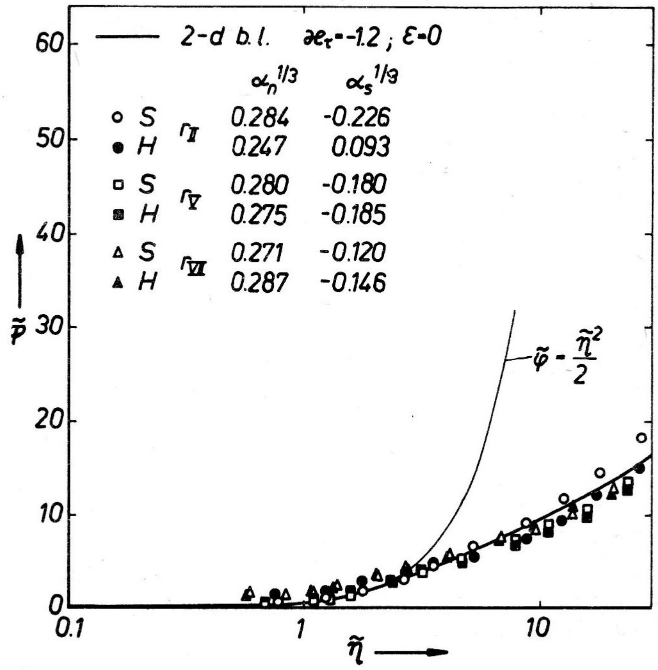

Fig. 2
Mean velocity components of the n-direction
($\epsilon = \text{sign}(\tau_o)\,|\alpha|^{-2/3}$ for 2-d b.l., see [6])

The remaining wall effects on the hot-wire readings for small distances were not corrected. The method was applied to the strongly swirling flow in a vaneless radial diffuser with parallel walls and an outer to inner radius ratio of $r_A/r_E \approx 1.7$. The width of the diffuser referred to the inlet radius amounts to ≈ 0.16, and the Reynolds number based on a characteristic velocity u_B and on the inner diameter equals $\approx 1.86 \cdot 10^5$ (for further details see [4]).

Figs. 1 und 2 show the measured mean velocity profiles corresponding to (1a) and (1b) at the radii $r_{II} \approx 1.14\, r_E$; $r_V \approx 1.46\, r_E$; $r_{VII} \approx 1.67\, r_E$ on the hub (H) and on the shroud side (S). The validity bounds for these cases might be estimated to be $\eta^{(2)} \approx 15$ and $\tilde{\eta}^{(2)} \approx 10$. The friction velocities and the directions of the wall shear stress vectors $\hat{\alpha}_o$ employed for the evaluation are indicated by the following table:

Fig. 3 exhibits the variation of the directions of the mean velocity vectors $\hat{\alpha}$ with growing wall distance (the angle $\hat{\alpha}$ is defined against the circumferential direction of the diffuser in planes parallel to the diffuser wall; it is positive for an outward flow). Primarily the strong change of the flow direction for $\eta \lesssim 10$ is remarkable.

	r_{II}		r_V		r_{VII}	
location	S	H	S	H	S	H
v_o^*/u_B	0.124	0.141	0.110	0.113	0.101	0.093
$\hat{\alpha}_o$ [°]	–26.5	4.5	–13	–14	–4.5	–7

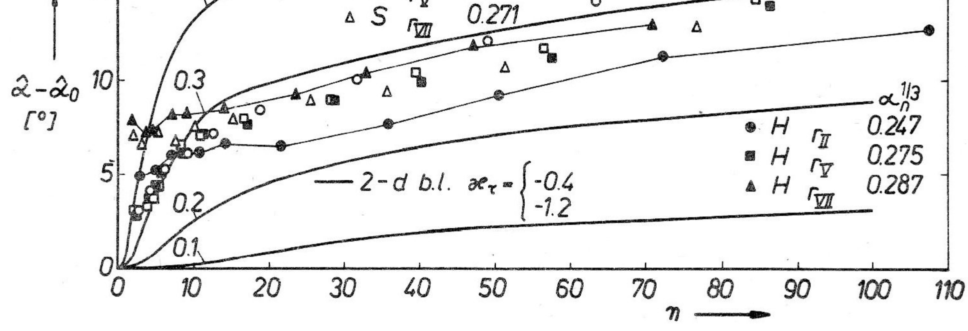

Fig. 3
Mean flow directions

2. A semi-empirical description of the near wall mean velocity profiles

Using the velocity scales of the s- and n-direction the mean total shear stress components $\overline{\tau}_s$ and $\overline{\tau}_n$ in the wall vicinity can be approximated by

$$g = 1, \qquad (0 \leqslant \eta \lesssim \eta^{(2)}) \tag{5a}$$

$$\tilde{g} = \tilde{\eta}, \qquad (0 \leqslant \tilde{\eta} \lesssim \tilde{\eta}^{(2)}) \tag{5b}$$

with the abbreviations

$$g = \frac{\overline{\tau}_s}{\rho} / v_o^{*\,2}; \qquad \tilde{g} = \frac{\overline{\tau}_n}{\rho} / w_o^{*\,2}. \tag{6}$$

Here, the second term on the right hand side of (5a) $\alpha_s \eta$ and all possible additional terms due to acceleration effects of the flow have been neglected, since in accordance with (1a) (1b) no further parameter influences should be expected.

Introducing eddy viscosities $\nu_{\tau ss}$ for the s-direction and $\nu_{\tau nn}$ for the n-direction the mean total shear stress components can be described by

$$g = (1 + Re_{\tau s})\, d\varphi / d\eta, \tag{7a}$$

$$\tilde{g} = (1 + Re_{\tau n})\, d\tilde{\varphi} / d\tilde{\eta} \tag{7b}$$

with

$$Re_{\tau s} = \nu_{\tau ss} / \nu, \qquad Re_{\tau n} = \nu_{\tau nn} / \nu. \tag{8}$$

An essential assumption involved in (7a), (7b) is that the eddy viscosity tensor $\vec{\vec{\nu}}_\tau$ has the proper directions s and n, i. e. it takes the form

$$\vec{\vec{\nu}}_\tau = \begin{pmatrix} \nu_{\tau ss} & 0 \\ 0 & \nu_{\tau nn} \end{pmatrix}. \tag{9}$$

This differs from earlier descriptions of the eddy viscosity anisotropy in 3-d b.l., which use for the proper directions a co-ordinate system locally aligned with the mean velocity vector, e. g. [5]. Up to date the justification of (9) is only given by the possibility of introducing a simple semi-empirical relation for the eddy viscosities which leads to (1a), (1b) and fulfills other plausible conditions. Further work is necessary to get a deeper explanation for the choice of the proper directions of $\vec{\vec{\nu}}_\tau$.

The investigation of pressure effects in the near wall mean velocity distributions of two-dimensional turbulent boundary layers (2-d b.l.) with the aid of a local similarity analysis yielded the following semi-empirical relation for the eddy viscosity [6]:

$$\frac{\nu_\tau}{\nu} \left[\left(\frac{a_\tau}{\nu_\tau / \nu} \right)^{1/n_\tau} \frac{n_\tau}{-1 + n_\tau} + 1 \right] = -\frac{\kappa_\tau}{\nu} \int_0^y \left| \frac{\tau}{\rho} \right|^{1/2} dy. \tag{10}$$

With $a_\tau = 3/4$, $n_\tau = 4/3$ and

$$\nu_\tau \longrightarrow \nu_{\tau ss}; \quad \overline{\tau} \longrightarrow \overline{\tau}_s = \overline{\tau}_o; \quad -\kappa_\tau \longrightarrow -\kappa_{\tau s} = 0.4 \tag{11}$$

this equation can be exploited for the integration of (7a). The result is indicated in fig. 1. Note the special case

$$Re_{\tau s} \approx -\kappa_{\tau s} \cdot \eta \qquad (Re_{\tau s} \gg 1) \tag{12}$$

which leads generally to

$$\varphi \approx -\frac{1}{\kappa_{\tau s}} \ln \eta + C_s \quad (\eta^{(1)} \lesssim \eta \lesssim \eta^{(2)}) \tag{13}$$

where $\eta^{(1)}$ represents a lower validity bound and $a_\tau = 3/4$ yields $C_s = 6.0$. Very close to the wall one obtains from (10) with (11)

$$Re_{\tau s} \approx \left[-\kappa_{\tau s} \cdot a_\tau^{-1/n_\tau} \left(\frac{n_\tau - 1}{n_\tau} \right) \eta \right]^{\frac{n_\tau}{n_\tau - 1}}, \tag{14}$$

$$(Re_{\tau s} \ll 1)$$

$$\text{i. e. } Re_{\tau s} \sim \eta^4 \text{ for } \eta \longrightarrow 0. \tag{14a}$$

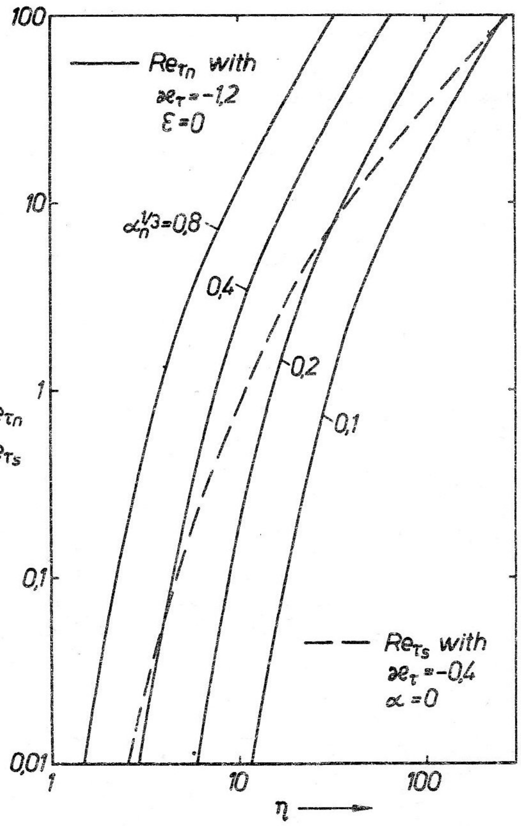

Fig. 4
Eddy viscosities

With unchanged constants $a_\tau = 3/4$, $n_\tau = 4/3$ and

$$\nu_\tau \longrightarrow \nu_{\tau nn};\ \bar{\tau} \longrightarrow \bar{\tau}_n = (\partial \bar{p}_o / \partial n) y;\ -\kappa_\tau \longrightarrow -\kappa_{\tau n} = 1.2 \qquad (15)$$

the integration of (7b) with (10) yields the curve plotted in fig. 2. Large lateral eddy viscosities give

$$Re_{\tau n} \approx -\frac{2}{3}\,\kappa_{\tau n}\,\tilde{\eta}^{3/2} \qquad (Re_{\tau n} \gg 1) \qquad (16)$$

and generally

$$\tilde{\varphi} \approx -\frac{3}{\kappa_{\tau n}}\,\tilde{\eta}^{1/2} + C_n,\ (\tilde{\eta}^{(1)} \lesssim \tilde{\eta} \lesssim \tilde{\eta}^{(2)}) \qquad (17)$$

with $C_n \approx 1.3$. For small eddy viscosities the relation (10) leads to

$$Re_{\tau n} \approx \left[-\frac{2}{3}\,\kappa_{\tau n}\,a_\tau^{-1/n_\tau} \left(\frac{n_\tau - 1}{n_\tau}\right) \tilde{\eta}^{3/2}\right]^{\frac{n_\tau}{n_\tau - 1}}, \qquad (18)$$

i. e. to

$$Re_{\tau n} \sim \tilde{\eta}^6 \quad \text{for} \quad \tilde{\eta} \longrightarrow 0. \qquad (18a)$$

Fig. 4 shows the anisotropy of the eddy viscosities ensuing from the model described above. It becomes clear that this anisotropy depends on the wall distance as well as on the pressure gradient number α_n, and that the ratio $\nu_{\tau nn}/\nu_{\tau ss}$ can be larger or smaller than unity.

3. Conclusions

The mean velocity distributions in the near wall region of pressure driven 3-d b.l. over hydraulically smooth and plane walls can be described with the aid of simple dimensionless relations in wall stream line oriented local co-ordinate systems for a certain range of the pressure gradient numbers. Two velocity scales must be introduced, the friction velocity for the direction of the mean wall shear stress vector and a second one for the direction orthogonal to the mean wall shear stress vector and parallel to the wall. This second scale is related to the gradient of the mean wall pressure in this direction. Experimental findings of the mean velocities ir 3-d b.l. taken with the aid of a rotated single hot-wire probe can be appoximated by a semi-empirical eddy viscosity model and simple assumptions on the behaviour of the mean total shear stresses close to the wall. A strong anisotropy of the eddy viscosities appears, which is a function of the wall distance and of the wall pressure gradient number perpendicular to the mean wall shear stress vector.

Comparatively large changes of the directions of the mean velocity vectors appear for wall distances smaller than about 10 wall units.

REFERENCES

[1] I. L. Ryhming, T. K. Fannelop: A Three-Dimensional Law-of-the-Wall Including Skewness and Roughness Effects. Three-Dimensional Turbulent Boundary Layers. Editors H. H. Fernholz and E. Krause. Springer-Verlag Berlin, Heidelberg, New York 1982, pp. 325/342.

[2] G. R. Schneider: Die Berechnung der turbulenten Grenzschicht am schiebenden Flügel unendlicher Spannweite mittels eines dreidimensionalen Mischungswegmodells. Dissertation der Georg-August-Universität zu Göttingen 1978.

[3] M. Hoffmeister: Mean velocity distributions in the near wall region of three-dimensional turbulent boundary layers. 1st Conference on Mechanics, Praha 1987 Proceedings Volume 8, pp. 179/183.

[4] M. Hoffmeister, H. Grützner: Beitrag zur Untersuchung wandnaher dreidimensionaler turbulenter Strömungen. 3. Tagung Turbulenzmodelle und ihre Anwendungen Reinhardsbrunn, 1986. Akademie der Wissenschaften der DDR, Institut für Mechanik, Report No. 7, Karl-Marx-Stadt 1987, S. 199/210.

[5] J. P. Johnston: EXPERIMENTAL STUDIES IN DIMENSIONAL TURBULENT BOUNDARY LAYERS. Thermosciences Division, Department of Mechanical Engineering Stanford University, Stanford California, July 1976 Report MD-34.

[6] M. Hoffmeister: Zur lokalen Ähnlichkeit in wandnahen Gebieten ebener turbulenter Grenzschichten. Wissenschaftliche Zeitschrift der Technischen Universität Dresden 33(1984)4, S. 287/293.

Address:

Prof. Dr. M. Hoffmeister
Institute for Mechanics
Academy of Sciences of the GDR
DDR – 9010 Karl-Marx-Stadt, PSF 408