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Pulsed Laser Heating of a Thermoelastic Medium with Two-temperature under Three-phase-lag Model

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In this paper, the problem of the generalized thermoelastic medium for three different theories under the effect of a laser pulse and two-temperature is investigated. The Lord–Shulman (L-S), Green-Naghdi of type III (G-N III) and three-phase-lag (3PHL) theories are discussed with two-temperature. The normal mode analysis is used to obtain the analytical expressions of the displacement components, force stress, thermodynamic temperature and conductive temperature. The numerical results are given and presented graphically and the thermal force was applied. Comparisons are made with the results predicted by (3PHL), (G-N III) and (L-S) in the presence and absence of two-temperature. The boundary plane surface is heated by a non-Gaussian laser beam.

Nomenclature

σ_{ij} Components of stress tensor

e_{ij} Components of strain tensor

$e = e_{kk}$ Cubic dilatation

u, v Displacement vectors

T Thermodynamic temperature

α_t Coefficient of linear thermal expansion

K Coefficient of thermal conductivity

τ_T Phase lag of temperature gradient

τ_v Phase lag of thermal displacement gradient

T_0 Reference temperature $\left|(T - T_0)/T_0\right| < 1$

ϕ Conductive temperature

λ, μ Lame' constants

δ_{ij} Kronecker's delta

c_e Specific heat at constant strain

ρ Density

K^* Material characteristic of the theory

τ_q Phase lag of heat flux

1 Introduction

The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity. The thermoelasticity with finite wave speeds was investigated by (Ignaczak and Ostoja-Starzewski, 2010). Five generalizations of the coupled theory of thermoelasticity were explained by (Hetnarski and Ignaczak, 1999). The first generalization formulates the generalized thermoelasticity theory involving one thermal relaxation time by (Lord and Shulman, 1967). The temperature rate-dependent thermoelasticity is developed where includes two thermal relaxation times and does not violate the second law of thermodynamics of heat conduction, when the body under consideration has a center of symmetry by (Green and Lindsay, 1972). The influence of magnetic field on generalized piezo-thermoelastic rotating medium with two relaxation times was studied by (Othman et al., 2017). Hetnarski and Ignaczak (1996) were reviewed and presentation of generalized theories of thermoelasticity. The wave propagation in anisotropic solids in generalized theories of thermoelasticity was investigated by many authors (Marin, et al. 2014; Sharma and Marin, 2013; Sharma and Singh, 1985; Othman, et al. 2018; Tzou, 1995, Sangwan, et al. 2018). The third generalization of the coupled theory of thermoelasticity is developed by Hetnarski and Ignaczak and is known as low-temperature thermoelasticity. The fourth generalization to the coupled theory of thermoelasticity introduced by Green and Naghdi and this theory is concerned with the thermoelasticity theory without energy dissipation, referred to as (G-N II) in which the classical Fourier law is replaced by a heat flux rate-temperature gradient relation and Green and Naghdi with energy dissipation referred to as (G-N III). The fifth generalization of the coupled theory of thermo-elasticity is referred to the dual-phase-lag thermoelasticity as in (Othman and Abd-Elaziz, 2015; Othman and Atwa, 2013). Recently the (3PHL), heat conduction equation in which the Fourier law of heat conduction is replaced by an approximation to a modification of the Fourier law with the introduction of three different phases-lags for the heat flux vector, the temperature gradient and the thermal displacement gradient by (Roy Choudhuri, 2007). Quintanilla and Racke (2008) discussed the stability of the (3PHL), the heat conduction equation. Subsequently, this theory has employed of thermoelasticity with (3PHL) to discuss a problem of generalized magneto-thermoelastic half-space with diffusion under initial stress by Othman and Eraki (2017).

The two-temperature theory of thermoelasticity was introduced by many works (Chen and Gurtin, 1968; Khamis, et al. 2017), in which the classical Clausius-Duhem inequality was replaced by another one depending on two-temperatures; the conductive temperature and the thermo-dynamic temperature, the first is due to the thermal processes, and the second is due to the mechanical processes inherent between the particles and the layers of elastic material, this theory was also investigated by Ieşan (1970). The two-temperature model was underrated and unnoticed for many years thereafter. Only in the last decade has the theory been noticed, developed in many works, and find its applications, mainly in the problems in which the discontinuities of stresses have no physical interpretations. Among Quintanilla (2004) who contributes to develop this theory, it has studied existence, structural stability, convergence and spatial behavior of this theory, it was introduced the generalized Fourier law to the field equations of the two-temperature theory of thermoelasticity and proved the uniqueness of the solution for homogeneous isotropic material by Youssef (2006), the propagation of harmonic plane waves studied by Puri and Jordan (2006). Recently, authors have studied the uniqueness and growth solutions by Magaña and Quintanilla (2009), for the model proposed by (Youssef, 2006).

The so-called ultra-short lasers are those with pulse duration ranging from nano-seconds to femtoseconds. In the case of ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam have introduced situations where very large thermal gradients or an ultra-high heating rate may exist on the boundaries by Sun et al. (2008). Researchers have proposed several models to describe the mechanism of heat conduction during short-pulse laser heating. It has been found that usually the microscopic two-step models, that is, parabolic and hyperbolic are useful for modification material as thin films. When a metal film is heated by a laser pulse, a thermoelastic wave is generated due to thermal expansion near the surface. The effect of magnetic field on a rotating thermoelastic medium with voids under thermal loading due to laser pulse with energy dissipation has investigated by Othman et al. (2018).

In this paper, the generalized thermoelastic theory is applied to study the effect of two-temperature on thermoelastic medium due to laser pulse using three-phase-lag model. The (L-S), (G-N III) and (3PHL) theories are discussed with two-temperatures. The normal mode analysis is used to obtain the exact solution of the physical quantities. The effect of laser pulse as well as two-temperature are discussed numerically and illustrated graphically.

2 Basic Equations

The governing equations for an isotropic, homogeneous elastic solid with the generalized thermoelastic medium in the absence of body forces using (3PHL) model are:

The constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e - \gamma(T - T_0)], \tag{1}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{2}$$

The equation of motion

$$\rho \ddot{u}_i = 2\mu e_{ij,j} + [\lambda e_{,j} - \gamma T_{,j}]\delta_{ij}. \tag{3}$$

The equation of heat conduction

$$K^*\nabla^2\phi + \tau_v^*\nabla^2\dot{\phi} + K\tau_t\nabla^2\ddot{\phi} = (1 + \tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2})[\rho c_e\ddot{T} + \gamma T_0\ddot{e} - \rho\dot{Q}]. \tag{4}$$

Where, $\tau_v^* = (K + K^*\tau_v)$, $\gamma = (3\lambda + 2\mu)\alpha_t$.

The equation of two-temperatures

$$T = (1 - b\nabla^2)\phi. \tag{5}$$

Where, the list of symbols is given in the nomenclature.

The plate surface is illuminated by laser pulse given by the heat input

$$Q = I_0 \frac{t}{t_0^2} \exp(\frac{-t}{t_0}).\frac{1}{2\pi r^2} \exp(\frac{-x^2}{r^2})\, \gamma e^{-\gamma y}\,. \qquad (6)$$

where, I_0 is the energy absorbed, t_0 is the pulse rise time, r is the beam radius, y is a function of the depth of the heat deposition due to the laser pulse is assumed to decay exponentially within the solid.

3 Formulation of the Problem

We consider an isotropic, homogeneous elastic solid with the generalized thermo-elastic medium. All quantities are considered are functions of the time variable t and of the coordinates x and y. We consider the normal source acting on the plane surface of generalized thermoelastic half-space under the effect of two-temperatures, we assume $\boldsymbol{u} = (u, v, 0)$.

The equation of motion in the absence of body force

$$\rho\ddot{u} = \mu\nabla^2 u + (\lambda+\mu)e_{,x} - \gamma(1-b\nabla^2)\phi_{,x}\,, \qquad (7)$$

$$\rho\ddot{v} = \mu\nabla^2 v + (\lambda+\mu)e_{,y} - \gamma(1-b\nabla^2)\phi_{,y}\,, \qquad (8)$$

$$K^*\nabla^2\phi + \tau_v^*\nabla^2\frac{\partial\phi}{\partial t} + K\tau_t\nabla^2\frac{\partial^2\phi}{\partial t^2} = (1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2})[(\rho c_e(1-b\nabla^2)\ddot{\phi} + \gamma T_0\frac{\partial^2 e}{\partial t^2} - \rho\dot{Q}]. \qquad (9)$$

To facilitate the solution, the following dimensions quantities are introduced

$$(x', y') = c\eta(x, y), \qquad (u', v') = c\eta(u, v), \qquad \{t', \tau', \tau_T', \tau_q', \tau_v'\} = c^2\eta\{t, \tau, \tau_T, \tau_q, \tau_v\},$$

$$(T', \phi') = \frac{\gamma}{(\lambda+2\mu)}(T, \phi), \quad \sigma_{ij}' = \frac{\sigma_{ij}}{(\lambda+2\mu)}, \quad Q' = \frac{\gamma}{c_e\eta c_1^2(\lambda+2\mu)}Q, \quad \eta = \rho c_e / K,$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \text{ and } c^2 = (\lambda+2\mu)/\rho. \qquad (10)$$

The displacement components $u(x, y, t)$ and $v(x, y, t)$ may be written in terms of potential functions $q(x, y, t)$ and $\Psi(x, y, t)$ as

$$u = q_{,x} - \Psi_{,y}\,, \qquad v = q_{,y} + \Psi_{,x}\,. \qquad (11)$$

Using Eqs. (10) and (11), in the Eqs. (7)-(9) become in the following form (after suppressing the primes)

$$\nabla^2 q - (1-b\nabla^2)\phi = \ddot{q}, \qquad (12)$$

$$\nabla^2\Psi = \beta^2\frac{\partial^2}{\partial t^2}\Psi, \qquad (13)$$

$$\varepsilon_1\nabla^2\phi + \varepsilon_2\nabla^2\dot{\phi} + \tau_t\nabla^2\ddot{\phi} = (1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2})[(1-b^*\nabla^2)\ddot{\phi} + \varepsilon_3\ddot{q} - \dot{Q}]. \qquad (14)$$

Where $\beta^2 = \frac{\lambda+2\mu}{\mu}$, $\varepsilon_1 = \frac{k^*}{\rho c_e c_0^2}$, $\varepsilon_2 = 1+\varepsilon_1\tau_v$, $\varepsilon_3 = \frac{\gamma^2 T_0}{\rho c_e(\lambda+2\mu)}$, $b^* = b c^2\eta^2$.

Also, by using Eqs. (1) and (10)-(11), we obtain the components of stress in the form

$$\sigma_{xx} = u_{,x} + (1-\frac{2}{\beta^2})v_{,y} - T\,, \qquad (15)$$

$$\sigma_{yy} = (1-\frac{2}{\beta^2})u_{,x} + v_{,y} - T\,, \qquad (16)$$

$$\sigma_{xy} = \frac{1}{\beta^2}(u_{,y} + v_{,x}). \qquad (17)$$

The solution of the considered physical variables can be decomposed in terms of normal modes in the form

$$[u, v, e, \phi, \Psi, q, T](x, y, t) = [u^*, v^*, e^*, \phi^*, \Psi^*, q^*, T^*](y)\exp i(\omega t + kx), \qquad (18)$$

Where ω is the complex time constant (frequency), i is the imaginary unit, k is the wave number in the $x-$direction and $[u^*, v^*, e^*, \phi^*, \Psi^*, q^*, T^*]$ are the amplitudes of the functions.
Using equation (18), equations (12)-(14) become respectively

$$(\mathrm{D}^2 - A_1)q^* + (b^*\mathrm{D}^2 - A_2)\phi^* = 0, \tag{19}$$

$$(\mathrm{D}^2 - A_3)\Psi^* = 0, \tag{20}$$

$$(\mathrm{A}_4\mathrm{D}^2 - A_5)\phi^* + (A_6\mathrm{D}^2 - A_7)q^* = -Q_0 f(x,t)e^{-\gamma y}. \tag{21}$$

Where $A_1 = k^2 - \omega^2, \quad \mathrm{A}_2 = 1 + b^* k^2, \quad A_3 = k^2 - \beta^2\omega^2,$

$$A_4 = \varepsilon_1 + i\varepsilon_2\omega - \tau_t\omega^2 - b^*\omega^2[1 + i\tau_q\omega - \frac{\tau_q^2}{2!}\omega^2],$$

$$A_5 = (\varepsilon_1 k^2 + i\varepsilon_2\omega k^2 - \tau_t\omega^2 k^2) + (\omega^2 + k^2\omega^2\,\mathrm{b}^*)[1 + i\tau_q\omega - \frac{\tau_q^2}{2!}\omega^2],$$

$$A_6 = \varepsilon_3\omega^2[1 + \mathrm{i}\tau_q\omega - \frac{\tau_q^2}{2!}\omega^2], \qquad A_7 = \varepsilon_3\omega^2 k^2[1 + i\tau_q\omega - \frac{\tau_q^2}{2!}\omega^2], \qquad Q_0 = \frac{I_0\gamma}{2\pi r^2 t_0^2},$$

$$f(x,t) = [(1 - \frac{t}{t_0}) + \tau_q(\frac{-2}{t_0} + \frac{t}{t_0^2}) + \frac{\tau_q^2}{2}(\frac{3}{t_0^2} - \frac{t}{t_0^3})]\exp(-\frac{x^2}{r^2} - \frac{t}{t_0} - i\omega t - ikx), \quad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}y}.$$

Eliminating ϕ^* and q^* among Eqs. (19) and (21) respectively, we obtain the following differential equations

$$\{\mathrm{D}^4 - A\mathrm{D}^2 + B\}q^* = -Q_0 N_1 f(x,t)e^{-\gamma y}, \tag{22}$$

$$\{\mathrm{D}^4 - A\mathrm{D}^2 + B\}\phi^* = -Q_0 N_2 f(x,t)e^{-\gamma y}, \tag{23}$$

where, $A = \dfrac{A_2A_6 + A_7b^* - A_5 - A_1A_4}{-A_4 + A_6b^*}, \quad B = \dfrac{A_2A_7 - A_5A_1}{-A_4 + A_6b^*}, \quad N_1 = b^*\gamma^2 - A_2,$

$N_2 = \gamma^2 - A_1, \qquad \mathrm{i} = 1, 2.$

Equation (22) and (23) can be factored as

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)q^* = -N_1 Q_0 f(x,t)e^{-\gamma y}, \tag{24}$$

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)\phi^* = -N_2 Q_0 f(x,t)e^{-\gamma y}, \tag{25}$$

where $k_n^2\ (n = 1, 2)$ are the roots of the characteristic equation of Eqs. (24) and (25).
The general solutions of Eqs. (20), (22) and (23) are given by:

$$\Psi = M_3 e^{(-k_n y + i\omega t + ikx)}. \tag{26}$$

$$q = \sum_{n=1}^{2} M_n e^{(-k_n y + i\omega t + ikx)} - \ell_1 N_1 Q_0 f_1(x,t)e^{-\gamma y}, \tag{27}$$

$$\phi = \sum_{n=1}^{2} H_{1n} M_n e^{(-k_n y + i\omega t + ikx)} - \ell_1 N_2 Q_0 f_1(x,t)e^{-\gamma y}, \tag{28}$$

From Eq. (5) and (28) we obtain

$$T = \sum_{n=1}^{2} H_{2n} M_n e^{(-k_n y + i\omega t + ikx)} - \ell_1 N_2 Q_0 J(x) f_1(x,t)e^{-\gamma y}, \tag{29}$$

where, $M_n\ (n = 1, 2)$ are some constants,

$$f_1(x,t) = [(1 - \frac{t}{t_0}) + \tau_q(\frac{-2}{t_0} + \frac{t}{t_0^2}) + \frac{\tau_q^2}{2}(\frac{3}{t_0^2} - \frac{t}{t_0^3})]\exp(-\frac{x^2}{r^2} - \frac{t}{t_0}), \ \ell_1 = \frac{1}{\gamma^4 - A\gamma^2 + B},$$

$$H_{1n} = \frac{-(k_n^2 - A_1)}{b^* k_n^2 - A_2}, \qquad H_{2n} = H_{1n}[1 - b^*(k_n^2 - k^2)], \qquad J(x) = 1 - b^*(\frac{-2}{r^2}[1 - \frac{2x}{r^2}] + \gamma^2).$$

To obtain the components of the displacement vector, substituting from Eqs. (26) and (27) in Eq. (11), then

$$u = \sum_{n=1}^{2} ikM_n e^{(-k_n y + i\omega t + ikx)} + \frac{2x\,\ell_1}{r^2} N_1 Q_0 f_1(x,t) e^{-\gamma y} - mM_3 e^{(-my + i\omega t + ikx)}, \tag{30}$$

$$v = \sum_{n=1}^{2} -k_n M_n e^{(-k_n y + i\omega t + ikx)} + \ell_1 N_1 \gamma Q_0 f_1(x,t) e^{-\gamma y} + ikM_3 e^{(-my + i\omega t + ikx)}, \tag{31}$$

Substituting from Eqs. (29), (30) and (31) in Eqs. (15)-(17), we obtain the stress components as follows:

$$\sigma_{xx} = H_3 \ell_1 Q_0 f_1(x,t) e^{-\gamma y} - H_4 M_3 e^{(-my + i\omega t + ikx)} - \sum_{n=1}^{2} H_{5n} M_n e^{(-k_n y + i\omega t + ikx)}, \tag{32}$$

$$\sigma_{yy} = H_6 \ell_1 Q_0 f_1(x,t) e^{-\gamma y} - H_7 M_3 e^{(-my + i\omega t + ikx)} + \sum_{n=1}^{2} H_{8n} M_n e^{(-k_n y + i\omega t + ikx)}, \tag{33}$$

$$\sigma_{xy} = H_9 \ell_1 N_1 Q_0 f_1(x,t) e^{-\gamma y} + H_{10} M_3 e^{(-my + i\omega t + ikx)} - \sum_{n=1}^{2} H_{11n} M_n e^{(-k_n y + i\omega t + ikx)}. \tag{34}$$

Where, $H_3 = \frac{2N_1}{r^2} - \frac{4x^2 N_1}{r^4} - N_1\gamma^2 + \frac{2N_1\gamma^2}{\beta^2} + N_2 J(x), \quad H_4 = m + ikm - \frac{2ikm}{\beta^2},$

$$H_{5n} = \frac{2k_n^2}{\beta^2} + H_{2n}, \qquad H_6 = -N_1\gamma^2 + \frac{2N_1}{r^2} - \frac{4x^2 N_1}{r^4} - \frac{2N_1}{\beta^2}(\frac{2}{r^2} - \frac{4x^2}{r^4}) + N_2 J(x),$$

$$H_7 = m + ikm + \frac{2m}{\beta^2}, \; H_{8n} = \frac{2k^2}{\beta^2} - H_{2n}, \; H_9 = \frac{-4x\gamma}{r^2\beta^2}, \; H_{10} = \frac{m^2 - k^2}{\beta^2}, H_{11n} = \frac{2ikk_n}{\beta^2}.$$

4 The Boundary Conditions

In order to determine the parameters $M_n \left(n = 1, 2\right)$, we need to consider the boundary conditions at $y = 0$ as follows:

$$\sigma_{xx}(x,0,t) = -P(x,t) = -P^* e^{i(\omega t + kx)}, \; \sigma_{xy}(x,0,t) = 0, \; T(x,0,t) = 0. \tag{35}$$

Using the expressions of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parameters:

$$\sum_{n=1}^{2} H_{5n} M_n + H_4 M_3 = P^*, \tag{36}$$

$$\sum_{n=1}^{2} -H_{11n} M_n + H_{10} M_3 = 0, \tag{37}$$

$$\sum_{n=1}^{2} H_{2n} M_n = 0, \tag{38}$$

Solving Eqs. (36)-(38), the constants $M_n \; (n = 1, 2)$ are defined as follows:

$$M_1 = \frac{\Delta_1}{\Delta}, \qquad M_2 = \frac{\Delta_2}{\Delta}, \qquad M_3 = \frac{\Delta_3}{\Delta}. \tag{39}$$

Where, $\Delta = H_{10}(H_{52}H_{21} - H_{51}H_{22}) + H_4(-H_{111}H_{22} - H_{112}H_{21}),$

$$\Delta_1 = -H_{10}H_{22}P^*, \qquad \Delta_2 = P^*(H_{10}H_{21}), \qquad \Delta_3 = P^*(H_{112}H_{21} - H_{111}H_{22}).$$

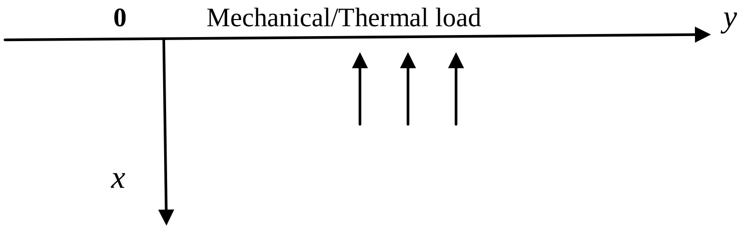

Figure 1. Geometry of the problem.

5 Numerical Results

To study the effect of time and two-temperatures, we now present some numerical results. For this purpose, copper is taken as the thermoelastic material for which we take the following values of the different physical constants as in Othman and Eraki (2017).

$\lambda = 7.7 \cdot 10^{10} N \cdot m^{-2}$, $\mu = 3.86 \cdot 10^{10} kg \cdot m^{-1} \cdot s^{-2}$, $K = 300\ w \cdot m^{-1} \cdot K^{-1}$, $\alpha_t = 1.78 \cdot 10^{-5} K^{-1}$,

$\rho = 8954 kg \cdot m^{-3}$, $c_e = 383.1\ J \cdot kg^{-1} K^{-1}$, $T_0 = 293K$, $k = 1.5$, $\omega = -0.7$, $x = 0.1$,

$K^* = 2.97 \cdot 10^{13}$, $p^* = -1$, $\tau_v = 0.3$, $\tau_t = 0.5$, $\tau_q = 1.5$.

The laser pulse parameters are $I_0 = 10^5$, $r = 10^{-5} m$, $\gamma = 2 \cdot 10^{-4} m^{-1}$, $t_0 = 0.1 \cdot 10^{-7}$ s.

The numerical technique, outlined above, was used for the distribution of the real part of the temperature T, the displacement components u, v and the stress components σ_{xx}, σ_{yy}, σ_{xy} for the problem. All the variables are taken in non-dimensional form the result. Figs. 2-7 depict the variety of the displacement components u, v, the temperature T, the stress components σ_{xx}, σ_{yy} and σ_{xy} in the absence and the presence of two-temperature (i.e. $b = 0, 0.1$) in the presence of the laser pulse.

Fig. 2 shows that the distribution of the displacement u, in the context of (3PHL), (L-S) and (G-N III) theories, always begins from positive values for $b = 0, 0.1.$ It shows that, in the presence of two-temperature (i.e. $b = 0.1$), the values of u based on (3PHL), (L-S) and (G-N III) theories decrease in the range $0 \le y \le 5.$ However, in the absence of two-temperature (i.e. $b = 0$), the values of u based on (3PHL), (L-S) and (G-N III) theories decrease in the range $0 \le y \le 1.$ Fig. 3 is plotted the distribution of the displacement v with distance y. The behavior of v for both theories is almost similar for $b = 0, 0.1.$ It decreases in the range $0 \le y \le 0.9,$ and begin to increase in the range $1 \le y \le 9.$ Even approaching the final to zero. The change in the temperature distribution T with the distance y represents in Fig. 4. The temperature distribution is exhibiting the similar trend for both theories for $b = 0, 0.1.$ It is an increasing function in the domain $0 \le y \le 0.8$ and a decreasing function in the domain $0.8 \le y \le 4,$ at $b = 0$. It is an increasing function in the domain $0 \le y \le 0.8$ and a decreasing function in the domain $0.8 \le y \le 4,$ for $b = 0$. It is noticed that the temperature distribution is strongly affected by the presence of two-temperature because for $b = 0.1$, temperature distribution increases in the range $0 \le y \le 1,$ while decreases in the range $1 \le y \le 6.$ The variation of the stress component σ_{xx} with distance y has shown in Fig. 5. The behavior of σ_{xx} for both theories is alike. It satisfied the boundary conditions and decreasing in the range $0 \le y \le 6$ for $b = 0,\ 0.1$ and finally decays to zero. Fig. 6 shows the variation of the stress component σ_{yy} with distance y. The behavior of σ_{yy} for $b = 0.1$ begins to decrease, then smooth decreases and takes the form of wave and try to return to zero in three theories. While, for $b = 0$, the behavior of σ_{yy} begins to increase, then smooth decreases and takes the form of wave and try to return to zero in three theories. The stress component σ_{xy} with distance y indicated in Fig. 7. The behavior of σ_{xy} for both theories is alike. It satisfied the boundary conditions and increasing in the range $0 \le y \le 1$ for $b = 0,\ 0.1$ and decreasing in the range $1 \le y \le 8$ and finally decays to zero.

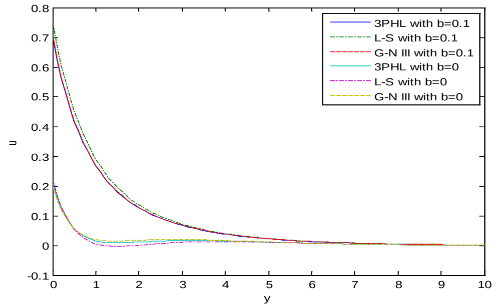

Figure 2. Horizontal displacement distribution u for $b = 0, 0.1$.

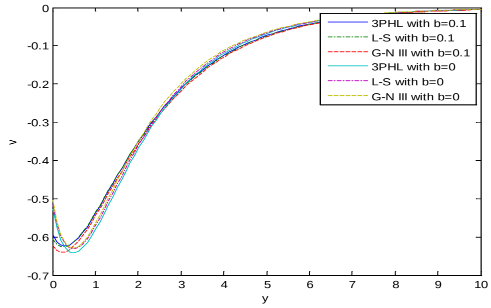

Figure 3. Vertical displacement distribution v for $b = 0, 0.1$.

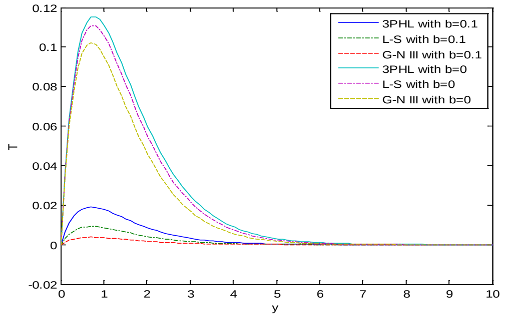

Figure 4. Temperature distribution T for $b = 0, 0.1$.

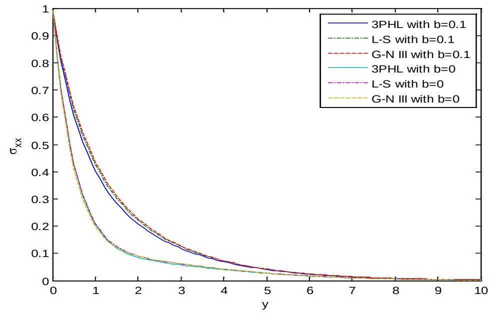

Figure 5. Distribution of stress component σ_{xx} for $b = 0, 0.1$.

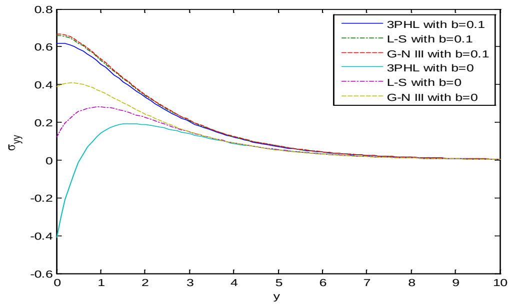

Figure 6. Distribution of stress component σ_{yy} for $b = 0, 0.1$.

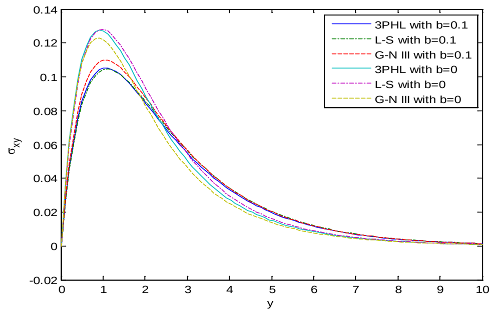

Figure 7. Distribution of stress component σ_{xy} for $b = 0, 0.1$.

Figs. 8-13 depict the variety of the displacement components u, v, the temperature T, the stress components σ_{xx}, σ_{yy} and σ_{xy} for different values of time ($t = 0.05 \cdot 10^{-13}$, $t = 0.09$) in the presence of laser pulse and two-temperatures.
Figs. 8 and 9 show the distributions of the displacement components u and v in the context of (3PHL), (L-S) and (G-N III) theories for $t = 0.05 \cdot 10^{-13}$ and $t = 0.09$. It is noticed that the distribution of u decreases for ($t = 0.05 \cdot 10^{-13}$, $t = 0.09$) while the distribution of v decreases for ($t = 0.05 \cdot 10^{-13}, t = 0.09$) in the range $0 \le y \le 0.2$ and increases in the range $0.2 \le y \le 6$ in three theories. Fig. 10 demonstrates that the distribution of the temperature T always begins from zero and satisfies the boundary conditions. In the context of the (3PHL), (L-S) and (G-N III) theories, the values

of T increase in the beginning to a maximum value in the range $0 \le y \le 1$, then decrease in the range $1 \le y \le 5$ and also move in wave propagation for $t = 0.05 \cdot 10^{-13}$ and $t = 0.09$. It is also noticed that the values of T for both (L-S) and (G-N III) theories are less in comparison to (3PHL) model. Fig. 11 depicts the distribution of the stress component σ_{xx} in the context of (3PHL), (L-S) and (G-N III) theories, for $t = 0.05 \cdot 10^{-13}$ and $t = 0.09$. It is observed that the distribution of σ_{xx} in the context of (3PHL), (L-S) and (G-N III) theories is decreasing for $t = 0.05 \cdot 10^{-13}$ and $t = 0.09$, until it decay to zero. Fig. 12 depicts the distribution of the stress component σ_{yy} in the context of (3PHL), (L-S) and (G-N III) theories, for $t = 0.05 \cdot 10^{-13}$ and $t = 0.09$. It is observed that the distribution of σ_{yy} in the context of (3PHL), (L-S) and (G-N III) theories are decreasing for $t = 0.05 \cdot 10^{-13}$ and $t = 0.09$, until it decay to zero. The distribution of the stress components σ_{xy} always begins from zero and satisfies the boundary conditions as demonstrated in Fig. 13. In the context of (3PHL), (L-S) and (G-N III) theories, the values of σ_{xy} increase in the beginning to a maximum value in the range $0 \le y \le 1$, then decrease in the range $1 \le y \le 6$ for $t = 0.05 \cdot 10^{-13}$ and $t = 0.09$, until it decay to zero.

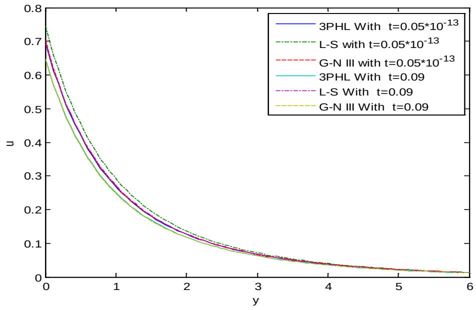

Figure 8. Horizontal displacement distribution u for $t = 0.05 \cdot 10^{-13}$, $t = 0.09$.

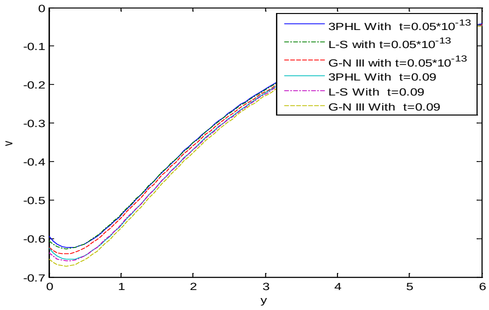

Figure 9. Horizontal displacement distribution v for $t = 0.05 \cdot 10^{-13}$, $t = 0.09$.

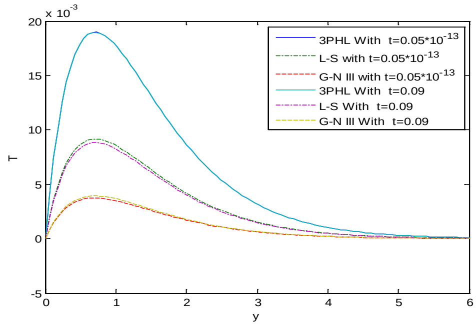

Figure 10. Temperature distribution T for $t = 0.05 \cdot 10^{-13}, t = 0.09$.

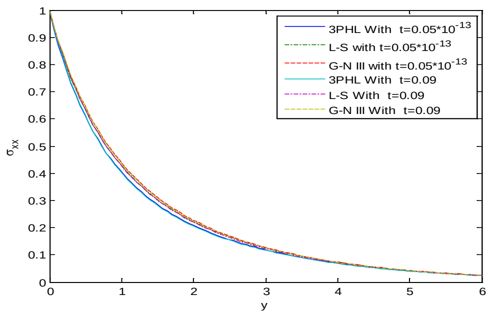

Figure 11. Distribution of stress component σ_{xx} for $t = 0.05 \cdot 10^{-13}, t = 0.09$.

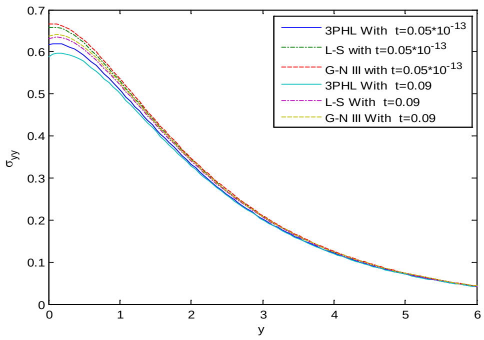

Figure 12. Distribution of stress component σ_{yy} for $t = 0.05 \cdot 10^{-13}, t = 0.09$.

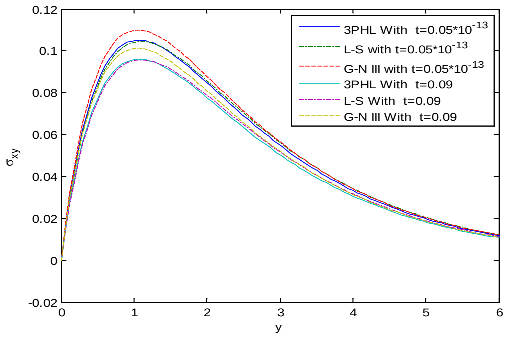

Figure 13. Distribution of stress component σ_{xy} for $t = 0.05 \cdot 10^{-13}, t = 0.09$.

6 Conclusions

By comparing the figures obtained under the three theories, important phenomena are observed:

(a) Analytical solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed.
(b) The method that is used in the present article is applicable to a wide range of the problems in the hydrodynamics and thermoelasticity.
(c) There are significant differences in the field quantities under (L-S), (GN-III), (3PHL) theories.
(d) The presence of the laser pulse and two-temperature play a significant role on all the physical quantities.
(e) The comparison of the three theories of thermoelasticity, (L-S), (G-N III) and (3PHL) theories are carried out.
(f) The value of all the physical quantities converges to zero.

Analysis of the temperature, stress generated and displacement components in a body due to the application of the effect of a laser pulse and two-temperature are an interesting problem of thermoelasticity. The problem assumes great significance when we consider the real behavior of the material characteristics with appropriate geometry of the model.

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