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Ferromagnetic Convection in a Rotating Medium with Magnetic Field Dependent Viscosity. A Correction Applied

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The effect of magnetic field dependent (MFD) viscosity on the thermal convection in a ferrofluid layer, heated from below, has been investigated in the simultaneous presence of a uniform vertical magnetic field and a uniform vertical rotation. A correction is applied to Vaidyanathan et al. (Ind. J. Pure Appl. Phy., 2001, 40, 159-165), which is very important in order to predict the correct behavior of MFD viscosity. A linear stability analysis has been carried out for stationary modes and oscillatory modes separately. The critical wave number and critical Rayleigh number for the onset of instability, for the case of free boundaries, are determined numerically for sufficiently large values of the magnetic parameter M_1. Numerical results are obtained and are illustrated graphically. It is shown that MFD viscosity has a destabilizing effect on the system for the case of stationary mode and stabilizing effect for the case of oscillatory mode, whereas magnetization has a destabilizing effect. Further, it is also shown that rotation has a stabilizing effect on the system.

1 Introduction

Synthetic magnetic fluids, also known as Ferrofluids, are the colloidal suspensions of solid single- domain ferromagnetic nano-particles, with typical dimensions of 10 nm, dispersed in an organic carrier (e.g. kerosene or ester) or water. In the recent past the studies on ferrofluids attracted several researchers due to their manifold applications in various fields such as acoustics, lubrication, vacuum technology, metals recovery, instrumentation, vibration damping etc. These researches have led to many commercial uses of ferrofluids which includes chemical reactor, medicine, novel zero-leakage rotary shaft seals used in computer disk drives, high speed silent printers, contrast enhancement of magnetic resonance imaging (MRI), pressure seals of compressors and blowers, cooling of loud speakers (Rosensweig, 1985; Odenbach, 2002a).

Ferrohydrodynamics, the study of the magnetic properties of colloidal suspensions has drawn considerable interest since the 1930 (Elmore, 1938), but the investigations on ferroconvection intensified noticeably, starting from the fundamental paper of Finlayson (1970). An authoritative introduction to ferrohydrodynamics is provided in a beautiful monograph by Rosensweig (1985). This book and the references therein laid a serious scientific foundation for further investigations in this field of enquiry. Currently, a significant body of literature exists devoted to ferroconvection. For a broad view of the subject one may referred to Lalas and Carmi (1971), Shliomis (1972), Aniss et al. (2001), Odenbach (2002b), Sunil et al. (2005), Suslov (2008), Lee and Shivakumara (2011), Prakash (2013a, b), Rahman and Suslov (2015, 2016) and Labusch et al. (2016).

The most specific characteristic property of a ferrofluid is the possibility to exert a significant influence to their flow and physical properties by means of moderate magnetic fields (Odenbach, 2002a). The effect on the viscous behavior of fluid due to the presence of an external magnetic field seems to be most prominent and is one of the most challenging topics of magnetic fluid research. Several research papers have been published by eminent researchers in this direction. Rosensweig et al. (1969) reported the investigation of a viscosity increase observed in ferrofluids containing nanosized magnetic particles in magnetic fields. The effect of a homogeneous magnetic field on the viscosity of the fluid with solid particles possessing intrinsic magnetic moments has been investigated by Shliomis (1974). Vaidyanathan et al. (2001) studied the influence of MFD viscosity on ferroconvection in a rotating medium heated from below using linear stability analysis. Vaidyanathan et al. (2002) further investigated the same problem of ferroconvection in a rotating sparsely distributed porous medium for the case of stationary and oscillatory modes. Ramanathan and Suresh (2004) studied the effect of magnetic field dependent viscosity and anisotropy of porous medium on ferroconvection. Sunil et al. (2005) investigated the effect of magnetic field dependent viscosity on a rotating ferromagnetic fluid heated and soluted from below saturating a porous medium. Prakash and Gupta (2013) derived upper bounds for the complex growth rate of oscillatory motions in ferromagnetic convection with MFD viscosity in a rotating fluid layer.

It is worth mentioning here that in the above cited papers on MFD viscosity, the researchers performed their analysis by considering MFD viscosity in the form $\mu = \mu_1(1 + \vec{\delta}.\vec{B})$, where μ_1 is fluid viscosity in the absence of magnetic field $\vec{B}$ and $\vec{\delta}$ is the variation coefficient of viscosity. They resolved μ into components μ_x, μ_y and μ_z which is not technically correct. Since μ, being a scalar quantity, cannot be resolved in such a manner. Undoubtedly, they have investigated a very important problem of ferrohydrodynamics, but their results cannot be relied upon due to this wrong assumption. Recently, Prakash and Bala (2016) and Prakash et al. (2017, 2018a, b) have rectified the above problem for some ferromagnetic convection configurations with MFD viscosity. In the present communication the attention has, particularly, been given to the above cited paper by Vaidyanathan et al. (2001) on ferromagnetic convection in a rotating medium with MFD viscosity. Keeping in view the above fact the basic equations have been reformulated and then mathematical and numerical analysis has been performed to remedy the weaknesses in the existing results and to give correct interpretation of the problem. It is also important to point out here that the role of viscosity for stationary convection is observed to destabilize the system which is in confirmation with the result obtained by Chandrasekhar (1981) for the case of ordinary fluid.

2 Mathematical Formulation

Consider a ferromagnetic fluid layer of infinite horizontal extension and finite vertical thickness dheated from below which is kept under the simultaneous action of a uniform vertical magnetic field $\vec{H}$ and uniform vertical rotation $\vec{\Omega}$ (see Fig.1). The magnetic fluid is assumed to be incompressible having a variable viscosity, given by $\mu = \mu_1(1 + \vec{\delta}.\vec{B})$, where μ_1 is the viscosity of the magnetic fluid when there is no magnetic field applied, μ is the magnetic field dependent viscosity and $\vec{B}$ is the magnetic induction. The variation coefficient of viscosity $\vec{\delta}$ has been taken to be isotropic, i.e. $\delta_1 = \delta_2 = \delta_3 = \delta$. The effect of shear dependence on viscosity is not considered since it has negligible effect for a mono dispersive system of large rotation and high field. As a first approximation for small field variation, linear variation of magneto viscosity has been used (Vaidyanathan et al., 2002).

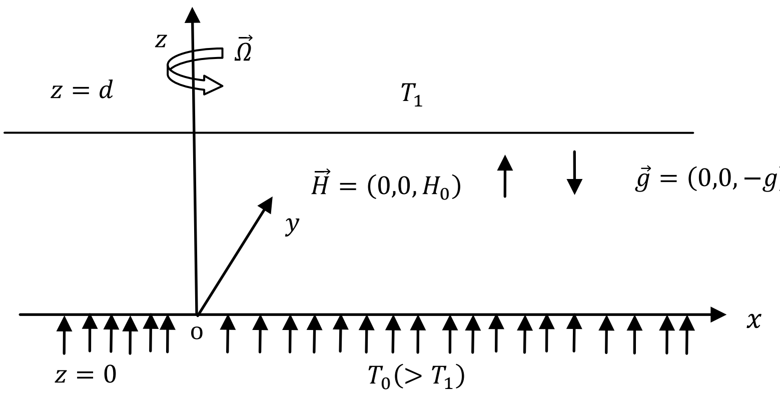

Fig.1 Geometrical configuration

The basic governing equations for the present problem are given by (Vaidyanathan et al., 2001):

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + \vec{q}.\nabla\vec{q}\right] = -\nabla\bar{P} + \rho\vec{g} + \mu\nabla^2\vec{q} + \nabla.(\vec{H}\vec{B}) + 2\rho_0(\vec{q}\times\vec{\Omega}) + \frac{\rho_0}{2}\nabla\left(\left|\vec{\Omega}\times\vec{r}\right|^2\right), \tag{2}$$

$$\left[\rho_0 C_{V,H} - \mu_0\vec{H}.\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt} + \mu_0 T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}.\frac{D\vec{H}}{Dt} = K_1\nabla^2 T + \phi, \tag{3}$$

$$\rho = \rho_0[1 + \alpha(T_0 - T)], \tag{4}$$

where $\vec{q} = (u, v, w)$ is the fluid velocity, $P = \bar{P} - \frac{\rho_0}{2}\nabla\left(\left|\vec{\Omega}\times\vec{r}\right|^2\right)$ is the pressure, $\vec{H}$ is the magnetic field, $\mu = \mu_1(1 + \vec{\delta}.\vec{B})$ is the variable viscosity, $\vec{g} = (0, 0, -g)$ is the acceleration due to gravity, $\vec{\Omega} = (0, 0, \Omega)$ is the angular velocity, $C_{V,H}$ is the heat capacity at constant volume and magnetic field, μ_0 is the magnetic permeability, T is the temperature, $\vec{M}$ is the magnetization, K_1 is the thermal conductivity, ϕ is the viscous dissipation containing second order terms in velocity, α is the coefficient of volume expansion and ρ_0 is the density at some reference temperature T_0.

For a non-conducting fluid with no displacement current, the Maxwell's equations are given by

$$\nabla.\vec{B} = 0, \nabla \times \vec{H} = 0,\ \vec{B} = \mu_0(\vec{H} + \vec{M}). \tag{5a,b}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field as well as the temperature as

$$\vec{M} = \frac{\vec{H}}{H} M(H, T). \tag{6}$$

The linearized magnetic equation of state is

$$M = M_0 + \chi (H - H_0) - K_2(T - T_0), \tag{7}$$

where M_0 is the magnetization when magnetic field is H_0 and temperature T_0, $\chi = \left(\frac{\partial \vec{M}}{\partial \vec{H}}\right)_{H_0,T_0}$ is magnetic susceptibility and $K_2 = -\left(\frac{\partial \vec{M}}{\partial T}\right)_{H_0,T_0}$ is the pyromagnetic coefficient.

The basic state is assumed to be quiescent state and is given by

$$\vec{q} = \vec{q}_b = 0, \rho = \rho_b(z), P = P_b(z),\ T = T_b(z) = -\beta\, z + T_0,\ \beta = \frac{T_0 - T_1}{d}, \vec{H}_b = \left(H_0 - \frac{K_2 \beta z}{1+\chi}\right)\hat{k},$$

$$\vec{M}_b = \left(M_0 + \frac{K_2\, \beta z}{1+\chi}\right)\hat{k}, \vec{H}_b + \vec{M}_b = H_0 + M_0. \tag{8}$$

The Perturbed State Solutions are given by

$$\vec{q} = \vec{q}_b + \overrightarrow{q'}, \rho = \rho_b(z) + \rho', P = P_b(z) + P', T = T_b(z) + \theta', \vec{H} = \vec{H}_b(z) + \vec{H}',$$

$$\vec{M} = \vec{M}_b(z) + \vec{M}', \tag{9}$$

where $\overrightarrow{q'} = (u', v', w')$, ρ', $P', \theta', \vec{H}'$ and $\vec{M}'$ are perturbations in velocity, density, pressure, temperature, magnetic field intensity and magnetization respectively and are assumed to be small.

Substituting equation (9) into equations (1) -(7) and using equation (8), we get the following linearized perturbation equations

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0, \tag{10}$$

$$\rho_0 \frac{\partial u'}{\partial t} = -\frac{\partial P'}{\partial x} + \mu_0(H_0 + M_0)\frac{\partial H_x'}{\partial z} + 2\rho_0 \Omega v' + \mu_1[1 + \delta\mu_0(H_0 + M_0)]\nabla^2 u', \tag{11}$$

$$\rho_0 \frac{\partial v'}{\partial t} = -\frac{\partial P'}{\partial y} + \mu_0(H_0 + M_0)\frac{\partial H_y'}{\partial z} - 2\rho_0 \Omega u' + \mu_1[1 + \delta\mu_0(H_0 + M_0)]\nabla^2 v', \tag{12}$$

$$\rho_0 \frac{\partial w'}{\partial t} = -\frac{\partial P'}{\partial z} + \mu_0(H_0 + M_0)\frac{\partial H_z'}{\partial z} - \mu_0 K_2 \beta H_z' + \frac{\mu_0 {K_2}^2 \beta \theta'}{(1+\chi)} + \rho_0 g \alpha \theta' + \mu_1[1 + \delta\mu_0(H_0 + M_0)]\nabla^2 w', \tag{13}$$

$$\rho_c \frac{\partial \theta'}{\partial t} - \mu_0 T_0\, K_2 \frac{\partial}{\partial t}\left(\frac{\partial \Phi'}{\partial z}\right) = \kappa_1 \nabla^2 \theta' + \left(\rho_c \beta - \frac{\mu_0 T_0\, {K_2}^2 \beta}{1+\chi}\right) w', \tag{14}$$

where $\rho_c = \rho_0\, C_{V,H} + \mu_0 K_2 H_0$, $H' = \nabla \Phi'$, Φ' is the perturbed magnetic potential

$$\text{and } H_z' + M_z' = (1 + \chi)H_z' - K_2 \theta', \tag{15}$$

$$H_i' + M_i' = \left(1 + \frac{M_0}{H_0}\right) H_i' (i = 1, 2), \tag{16}$$

where we have assumed $K_2 \beta d \ll (1 + \chi)H_0$, as the analysis is restricted to physical situations, in which the magnetization induced by temperature variations is small compared to that induced by the external magneticfield.

Using equations (5b), (15) and (16), we get

$$\left(1+\frac{M_0}{H_0}\right)\nabla_1{}^2\Phi' + (1+\chi)\frac{\partial^2\Phi'}{\partial z^2} - K_2\frac{\partial\theta'}{\partial z} = 0, \tag{17}$$

where $\nabla_1{}^2 = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)$.

Now we eliminate u' and v' between equations (11) and (12) by operating equation (11) by $\frac{\partial}{\partial x}$ and equation (12) by $\frac{\partial}{\partial y}$, adding the resulting equations and using equation (10). We obtain

$$\rho_0\frac{\partial}{\partial t}\left(\frac{\partial w'}{\partial z}\right) = \left(\frac{\partial^2 P'}{\partial x^2}+\frac{\partial^2 P'}{\partial y^2}\right) - 2\rho_0\Omega\zeta' + \mu_1[1+\delta\mu_0(H_0+M_0)]\nabla^2\left(\frac{\partial w'}{\partial z}\right) - \mu_0(H_0+M_0)\frac{\partial}{\partial z}\left(\frac{\partial H_x'}{\partial x}+\frac{\partial H_y'}{\partial y}\right), \tag{18}$$

where $\zeta' = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}$ is the z component of vorticity.

Now eliminating P' between equations (13) and (18), we get

$$\rho_0\frac{\partial}{\partial t}\nabla^2 w' = -2\rho_0\Omega\frac{\partial\zeta'}{\partial z} + \mu_1[1+\delta\mu_0(H_0+M_0)]\frac{\partial^2}{\partial z^2}(\nabla^2 w') + \rho_0 g\alpha\nabla_1{}^2\theta' + \frac{\mu_0 K_2{}^2\beta\ \nabla_1{}^2\theta'}{1+\chi} + \mu_1\nabla_1{}^2(\nabla^2 w')$$

$$+\mu_0\mu_1\delta(H_0+M_0)\nabla_1{}^2(\nabla^2 w') - \mu_0 K_2\beta\frac{\partial}{\partial z}\nabla_1^2\Phi'. \tag{19}$$

Further, operating equation (11) by $\frac{\partial}{\partial y}$ and equation (12) by $\frac{\partial}{\partial x}$, subtracting the resulting equations and using equation (10), we get an equation describing vorticity as

$$\rho_0\frac{\partial\zeta'}{\partial t} = 2\rho_0\Omega\frac{\partial w'}{\partial z} + \mu_1[1+\delta\mu_0(H_0+M_0)]\nabla^2\zeta'. \tag{20}$$

Now we analyze the perturbations w', θ', ζ' and Φ' into two dimensional periodic waves and consider disturbances characterized by a particular wave number k. Thus we assume to all quantities describing the perturbation a dependence on x, y and t of the form

$$(w', \theta', \zeta', \Phi') = [w''(z), \theta''(z), \zeta''(z), \Phi''(z)]exp\left[i\left(k_x x + k_y y\right) + nt\right], \tag{21}$$

where k_x and k_y are the horizontal wave numbers and $k = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number.

On using equation (21) in equations (19), (14), (17) and (20) and non-dimensionalizing the variables by setting

$$z_* = \frac{z}{d},\quad w_* = \frac{dw''}{\nu},\quad a = kd,\quad \zeta_* = \frac{d^2}{\nu}\zeta'',\quad D = d\frac{d}{dz},\ \theta_* = \frac{K_1 aR^{1/2}}{\rho_c\beta\nu\, d}\theta'',\ \Phi_* = \frac{(1+\chi)K_1 aR^{1/2}}{K_2\rho_c\beta\nu\, d^2}\Phi'',\quad \nu = \frac{\mu}{\rho_0},\ \sigma = \frac{\nu\rho_c}{K_1},$$

$$\delta_* = \mu_0\delta H_0(1+\chi),\quad R = \frac{g\alpha\beta d^4\rho_c}{K_1\nu},\quad M_1 = \frac{\mu_0 K_2{}^2\beta}{(1+\chi)\alpha\rho_0 g},\ M_2 = \frac{\mu_0 T_0 K_2{}^2}{(1+\chi)\rho_c},\quad M_3 = \frac{1+\frac{M_0}{H_0}}{(1+\chi)},\ T_a = \frac{4\Omega^2 d^4}{\nu^2},\ p = \frac{nd^2}{\nu} \tag{22}$$

we obtain the following non dimensional equations (dropping the asterisks for simplicity)

$$(D^2-a^2)\{(1+\delta M_3)(D^2-a^2)-p\}w = aR^{\frac{1}{2}}\{(1+M_1)\theta - M_1 D\Phi\} + T_a{}^{\frac{1}{2}}D\zeta, \tag{23}$$

$$(D^2-a^2-p\sigma)\theta + pM_2\sigma D\Phi = -(1-M_2)aR^{\frac{1}{2}}w, \tag{24}$$

$$\{(1+\delta M_3)(D^2-a^2)-p\}\zeta = -T_a{}^{\frac{1}{2}}Dw, \tag{25}$$

$$(D^2-a^2M_3)\Phi = D\theta. \tag{26}$$

Since, M_2 is of very small order (Finlayson, 1970), it is neglected in the subsequent analysis and thus equation

(24) takes the form

$$(D^2 - a^2 - p\sigma)\theta = -aR^{\frac{1}{2}}w. \tag{27}$$

The constant temperature boundaries are considered to be free. Thus the boundary conditions are given by

$$w = 0 = \theta = D^2 w = D\zeta = D\Phi \text{ at } z = 0 \text{ and } z = 1, \tag{28}$$

where z is the real independent variable such that $0 \leq z \leq 1$, represent the two boundaries. $D = \frac{d}{dz}$ is the differentiation along the vertical coordinate, a^2 is square of the wave number, $\sigma > 0$ is the Prandtl number, $R > 0$ is the Rayleigh number, $T_a > 0$ is the Taylor number, $M_1 > 0$ is the magnetic number which defines ratio of magnetic forces due to temperature fluctuation to buoyant forces, $M_3 > 0$ is the measure of the nonlinearity of magnetization, $M_2 > 0$ is a non-dimensional parameter which defines the ratio of thermal flux due to magnetization to magnetic flux, $p = p_r + ip_i$ is a complex constant in general such that p_r and p_i are real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$, $\Phi(z) = \Phi_r(z) + i\Phi_i(z)$ and $\zeta(z) = \zeta_r(z) + i\zeta_i(z)$ are complex valued functions of the real variable z where $w_r(z)$, $w_i(z)$, $\theta_r(z)$, $\theta_i(z)$, $\Phi_r(z)$, $\Phi_i(z)$, $\zeta_r(z)$ and $\zeta_i(z)$ are real valued functions of the real variable z.

Further, it may be noted that the equation (23) and equations (25) -(28) describe an eigenvalue problem for p and govern ferromagnetic convection, with MFD viscosity, in the presence of uniform rotation.

3 Mathematical Analysis

Following the analysis of Finlayson (1970), the exact solutions satisfying the boundary conditions (28) are given by

$$w = A\, sin\pi z,\ \theta = B\, sin\pi z,\ \Phi = -\frac{C}{\pi}\, cos\pi z,\ \zeta = -\frac{D}{\pi}\, cos\pi z, D\Phi = C\, sin\pi z,\ D\zeta = D\, sin\pi z,$$

where A, B, C and D are constants. Substitution of above solutions in equations (23) and (25) -(27) yields a system of four linear homogeneous algebraic equations in the unknowns *A*, *B*, *C* and *D*. For the existence of non-trivial solutions of this system, the determinant of the coefficients of *A*, *B*, *C* and *D* must vanish. This determinant on simplification yields

$$Up^3 + Vp^2 + Wp + X = 0, \tag{29}$$

where

$$U = \sigma(\pi^2 + a^2)(\pi^2 + a^2 M_3), \tag{30}$$

$$V = (\pi^2 + a^2)^2(\pi^2 + a^2 M_3)[2\sigma(1 + \delta M_3) + 1], \tag{31}$$

$$W = (\pi^2 + a^2 M_3)[(\pi^2 + a^2)^3(1 + \delta M_3)\{(1 + \delta M_3)\sigma + 2\} + T_a\pi^2\sigma] - Ra^2[\pi^2 + a^2 M_3(1 + M_1)], \tag{32}$$

$$X = (\pi^2 + a^2)(\pi^2 + a^2 M_3)[(\pi^2 + a^2)^3(1 + \delta M_3)^2 + T_a\pi^2] - Ra^2(1 + \delta M_3)(\pi^2 + a^2)[\pi^2 + a^2 M_3(1 + M_1)]. \tag{33}$$

Substitution of $p = ip_i$ in equation (29) yields marginal state of convection. For $p_i = 0$, we have a case of stationary convection, while $p_i \neq 0$ defines the oscillatory convection.

From equation (29), the Rayleigh number for stationary convection can easily be derived as

$$R = \frac{(\pi^2 + a^2 M_3)\left[(\pi^2 + a^2)^3(1 + \delta M_3)^2 + T_a\pi^2\right]}{a^2(1 + \delta M_3)[\pi^2 + a^2 M_3(1 + M_1)]}. \tag{34}$$

In the expression (34), if we put $\delta = 0, T_a = 0$, we obtain the Rayleigh number for classical ferroconvection (Finlayson, 1970). If we put $\delta = 0 = M_3, T_a \neq 0$, we obtain Rayleigh number for classical rotatory hydrodynamic convection (Chandrasekhar, 1981) and if we put $\delta = 0 = M_3, T_a = 0$, we obtain Rayleigh number for convection in ordinary fluid heated from below (Chandrasekhar, 1981). If we put $T_a = 0$, $M_3 \neq 0$, we obtain Rayleigh number for ferroconvection with MFD viscosity (Prakash et al., 2017). If we put $\delta = 0, T_a \neq 0, M_3 \neq$

0, we obtain Rayleigh number for ferroconvection in a rotating ferrofluid layer (Venkatasubramanian and Kaloni, 1994).

When M_1 is very large, the magnetic Rayleigh number $N\ (= RM_1)$ for stationary mode can be expressed as

$$N = \frac{(\pi^2+a^2M_3)\left[(\pi^2+a^2)^3(1+\delta M_3)^2+T_a\pi^2\right]}{a^4(1+\delta M_3)M_3}. \quad (35)$$

To find the minimum value N_c of N with respect to wave number a, equation (35) is differentiated with respect to a^2 and equated to zero and the following polynomial is obtained

$$a^4(1+\delta M_3)(\pi^2+a^2)M_3[(\pi^2+a^2M_3)\{(\pi^2+a^2)^3(1+\delta M_3)^2+T_a\pi^2\}+(\pi^2+a^2)M_3\{(\pi^2+a^2)^3(1+\delta M_3)^2+T_a\pi^2\}+(\pi^2+a^2)(\pi^2+a^2M_3)3(\pi^2+a^2)^2(1+\delta M_3)^2]-(\pi^2+a^2)(\pi^2+a^2M_3)\{(\pi^2+a^2)^3(1+\delta M_3)^2+T_a\pi^2\}\{2a^2(1+\delta M_3)(\pi^2+a^2)M_3+a^4M_3(1+\delta M_3)\}=0. \quad (36)$$

The above equation is solved numerically by using the software Scientific Work Place for various values of M_3, δ and T_a, and the minimum value of a is obtained each time, hence N_c is obtained.

Table 1: Marginal stability of MFD viscosity of a ferrofluid in a rotating medium heated from below for stationary mode having $M_1 = 1000, T_a = 10^4$ and 10^5.

Taylor no. T_a	Coefficient of viscosity δ	Magnetization M_3	Critical wave no. a_c	$N_c = (RM_1)_c$
10^4	0.01	1	6.0655	6905.6
		3	5.7997	5895.6
		5	5.7012	5674.5
		7	5.6351	5571.7
	0.03	1	6.027	6909.2
		3	5.6872	5877.2
		5	5.5207	5637.1
		7	5.3926	5518.8
	0.05	1	5.9896	6913.4
		3	5.5828	5863.3
		5	5.3603	5611.2
		7	5.1854	5485.9
	0.07	1	5.9531	6918.0
		3	5.4856	5853.3
		5	5.2165	5594.6
		7	5.0057	5468.4
	0.09	1	5.9175	6923.2
		3	5.3947	5847.3
		5	5.0867	5585.8
		7	4.8478	5463.2
10^5	0.01	1	8.8651	24009
		3	8.6385	22100
		5	8.5422	21631
		7	8.4687	21376
	0.03	1	8.8075	23931
		3	8.4718	21837
		5	8.2762	21203
		7	8.1124	20800
	0.05	1	8.7514	23856
		3	8.3168	21598
		5	8.0393	20831
		7	7.8069	20321
	0.07	1	8.6967	23784
		3	8.1723	21378
		5	7.8262	20505

		7	7.5408	19918
	0.09	1	8.6434	23715
		3	8.037	21175
		5	7.6332	20216
		7	7.3059	19573

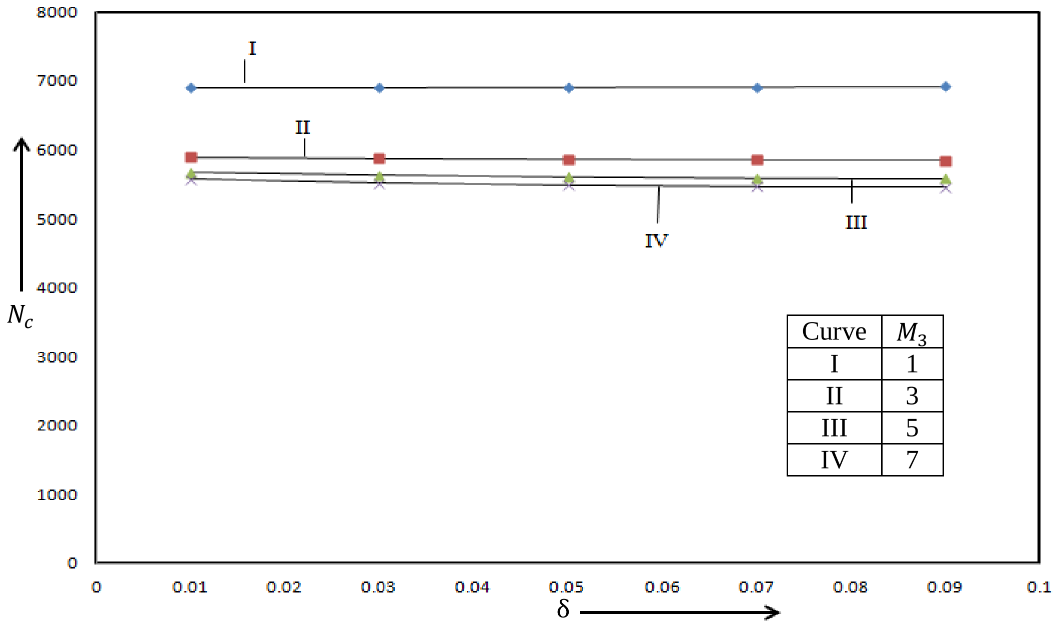

Fig.2 Effect of magnetic field on the variation of magnetic Rayleigh number (N_c) versus coefficient of field dependent viscosity (δ) for stationary mode for Taylor number $T_a = 10^4$.

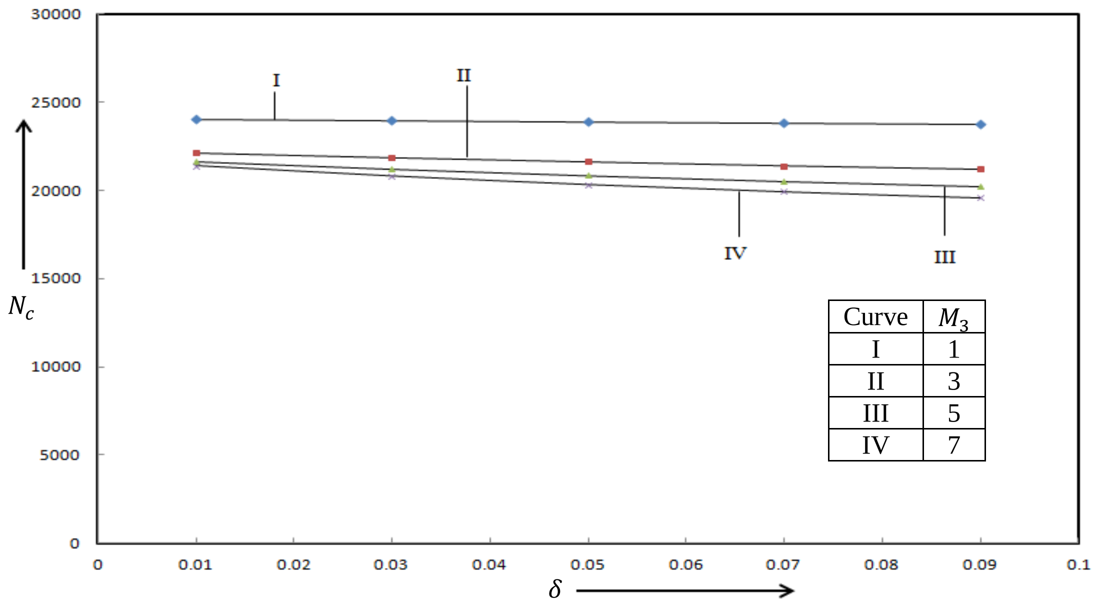

Fig.3 Effect of magnetic field on the variation of magnetic Rayleigh number (N_c) versus coefficient of field dependent viscosity (δ) for stationary mode for Taylor number $T_a = 10^5$.

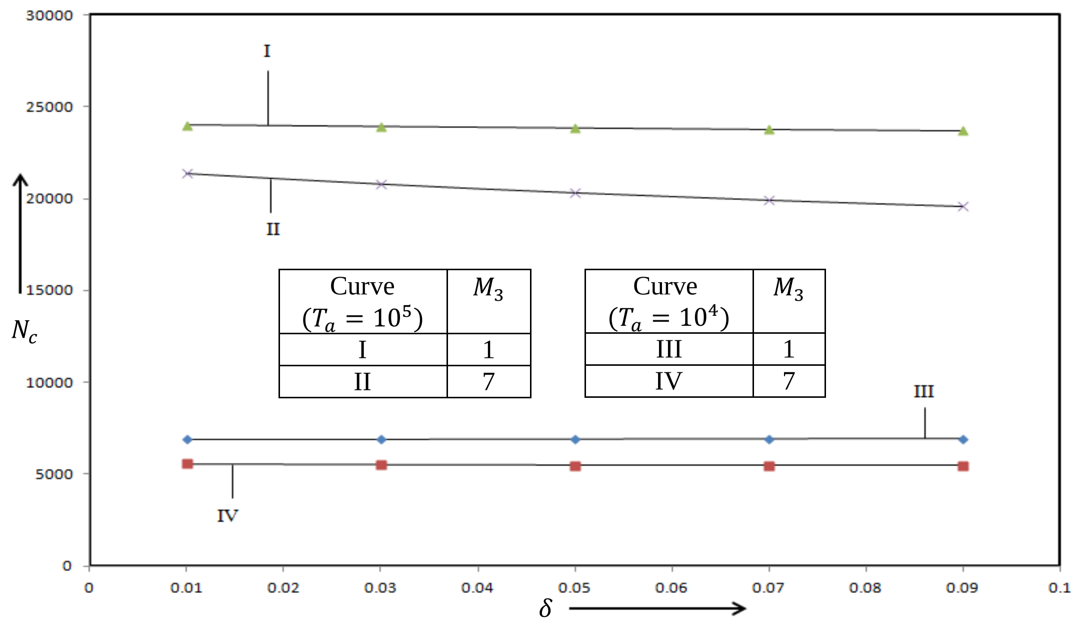

Fig.4 Effect of magnetic field on the variation of magnetic Rayleigh number (N_c) versus coefficient of field dependent viscosity (δ) for stationary mode for Taylor number $T_a = 10^4$ and 10^5.

From equation (29), the Rayleigh number for oscillatory mode can easily be obtained as

$$R^o = \frac{[2\sigma(1+\delta M_3)+1]\left[(\pi^2+a^2)^3(1+\delta M_3)\{(1+\delta M_3)\sigma+2\}+T_a\pi^2\sigma\right](\pi^2+a^2M_3) - \sigma(\pi^2+a^2M_3)\left[(\pi^2+a^2)^3(1+\delta M_3)^2+T_a\pi^2\right]}{a^2[\pi^2+a^2M_3(1+M_1)][\sigma(1+\delta M_3)+1]}. \quad (37)$$

When M_1 is very large, the magnetic Rayleigh number $N^o(=RM_1)^o$ for oscillatory mode can be obtained using

$$N^o = \frac{[2\sigma(1+\delta M_3)+1]\left[(\pi^2+a^2)^3(1+\delta M_3)\{(1+\delta M_3)\sigma+2\}+T_a\pi^2\sigma\right](\pi^2+a^2M_3) - \sigma(\pi^2+a^2M_3)\left[(\pi^2+a^2)^3(1+\delta M_3)^2+T_a\pi^2\right]}{a^4M_3[\sigma(1+\delta M_3)+1]}. \quad (38)$$

To find the minimum value N_c^o of N^o with respect to wave number a, equation (38) is differentiated with respect to a^2 and equated to zero and the following polynomial is obtained

$a^4M_3[\sigma(1+\delta M_3)+1][2\sigma(1+\delta M_3)+1]M_3(\pi^2+a^2)^3(1+\delta M_3)[\sigma(1+\delta M_3)+2]+T_a\pi^2\sigma M_3 a^4M_3[\sigma(1+\delta M_3)+1][2\sigma(1+\delta M_3)+1]+a^4M_3[\sigma(1+\delta M_3)+1][2\sigma(1+\delta M_3)+1](\pi^2+a^2M_3)3(\pi^2+a^2)^2(1+\delta M_3)[\sigma(1+\delta M_3)+2]-a^4M_3[\sigma(1+\delta M_3)+1]\sigma(1+\delta M_3)^2(\pi^2+a^2M_3)3(\pi^2+a^2)^2-a^4M_3[\sigma(1+\delta M_3)+1]\sigma(1+\delta M_3)^2(\pi^2+a^2)^3M_3-a^4M_3[\sigma(1+\delta M_3)+1]T_a\pi^2\sigma M_3-[2\sigma(1+\delta M_3)+1](\pi^2+a^2)^3(1+\delta M_3)[\sigma(1+\delta M_3)+2]2a^2M_3[\sigma(1+\delta M_3)+1](\pi^2+a^2M_3)-[2\sigma(1+\delta M_3)+1]T_a\pi^2\sigma(\pi^2+a^2M_3)2a^2M_3[\sigma(1+\delta M_3)+1]+\sigma(\pi^2+a^2M_3)[(\pi^2+a^2)^3(1+\delta M_3)^2+T_a\pi^2]2a^2M_3[\sigma(1+\delta M_3)+1]=0.$

(39) The above equation is solved numerically by using the software Scientific Work Place for various values of M_3, δ and T_a, and the minimum value of a is obtained each time, hence N_c^o is obtained.

Table 2: Marginal stability of MFD viscosity of a ferrofluid in a rotating medium heated from below for oscillatory mode having $M_1 = 1000$, $T_a = 10^4$ and 10^5.

Taylor no. T_a	Coefficient of viscosity δ	Magnetization M_3	Critical wave no. a_c	$N_c^o = (RM_1)_c^o$
	0.01	1	4. 7997	13765
		3	4. 5176	11132
		5	4. 4251	10718
		7	4. 3727	10642
		1	4. 7861	14069
		3	4. 4763	11823

10^4	0.03	5	4. 3568	11793
		7	4. 2785	12101
	0.05	1	4. 7727	14375
		3	4. 4367	12521
		5	4. 2928	12882
		7	4. 1922	13588
	0.07	1	4. 7595	14682
		3	4. 3986	13226
		5	4. 2326	13988
		7	4. 1127	15103
	0.09	1	4. 7465	14682
		3	4. 3619	13228
		5	4. 1759	13995
		7	4. 0393	15117
10^5	0.01	1	6. 9344	40017
		3	6. 708	36018
		5	6. 6353	35660
		7	6. 5903	35840
	0.03	1	6. 913	40787
		3	6. 644	37979
		5	6. 5303	38799
		7	6. 4462	40148
	0.05	1	6. 8919	41557
		3	6. 5823	39935
		5	6. 4314	41919
		7	6. 3133	44417
	0.07	1	6. 8711	42327
		3	6. 5228	41885
		5	6. 3379	45022
		7	6. 1901	48656
	0.09	1	6. 8505	43097
		3	6. 4653	43830
		5	6. 2494	48112
		7	6. 0754	52872

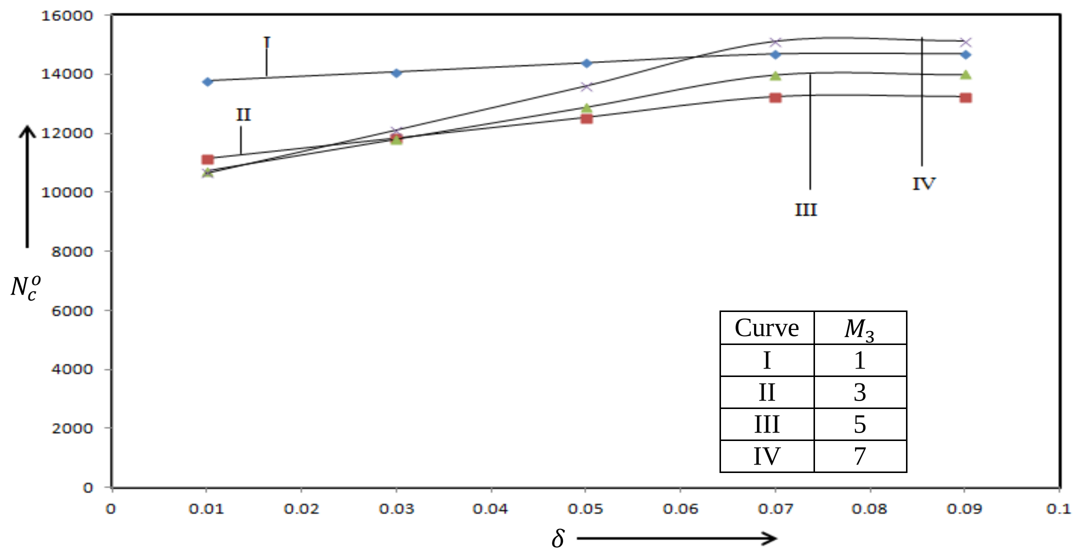

Fig.5 Effect of magnetic field on the variation of magnetic Rayleigh number (N_c^o) versus coefficient of field dependent viscosity (δ) for oscillatory mode for Taylor number $T_a = 10^4$ and $\sigma = 0.9$.

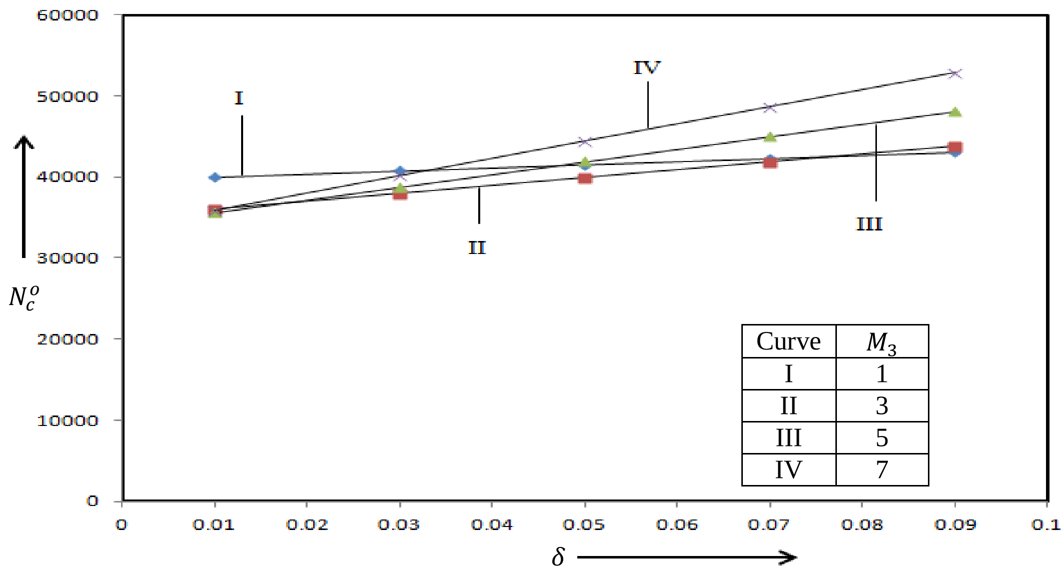

Fig.6 Effect of magnetic field on the variation of magnetic Rayleigh number (N_c^o) versus coefficient of field dependent viscosity (δ) for oscillatory mode for Taylor number $T_a = 10^5$ and $\sigma = 0.9$.

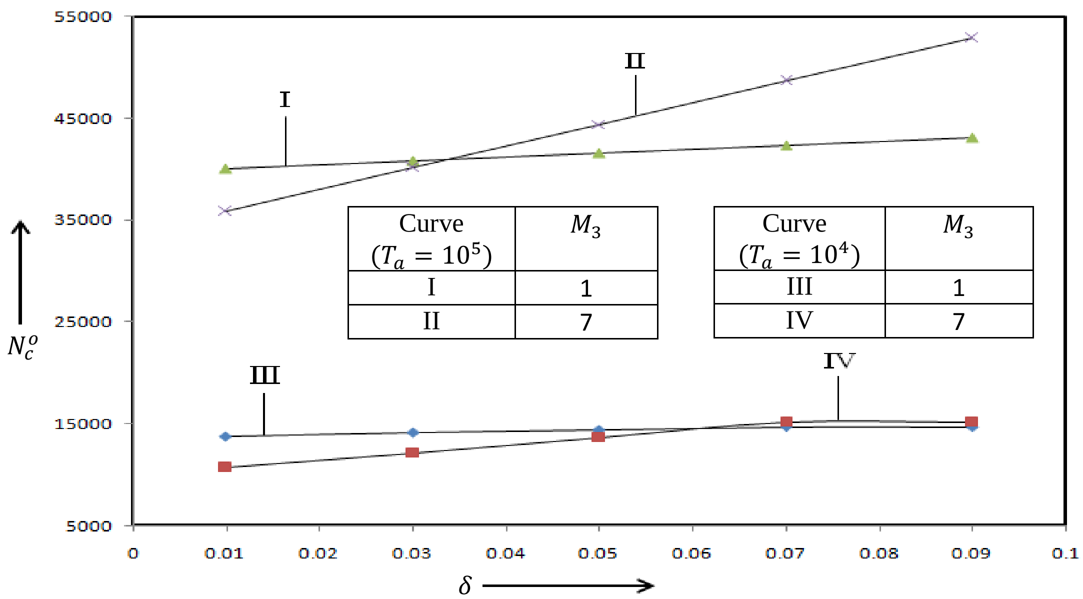

Fig.7 Effect of magnetic field on the variation of magnetic Rayleigh number (N_c^o) versus coefficient of field dependent viscosity (δ) for oscillatory mode for Taylor number $T_a = 10^4$ and $T_a = 10^5$ when $\sigma = 0.9$.

4 Discussion and Conclusion

In the present communication, the influence of magnetic field dependent viscosity on the thermal convection in a rotating ferrofluid layer heated from below in the presence of uniform vertical magnetic field has been investigated. The magnetization parameter M_1 is considered to be 1000 (Vaidyanathan et al., 1997). The value of M_2 being negligible (Finlayson, 1970), has been taken as zero. The values of the parameter M_3 are varied from 1 to 7. The values of the coefficient of magnetic field dependent viscosity δ, has been varied from 0.01 to 0.09.

Emphasize has been given to a paper published by Vaidyanathan et al. (2001). These researchers have carried out their analysis by considering MFD viscosity as $\mu = \mu_1(1 + \vec{\delta}.\vec{B})$. But they further resolved μ into components μ_x, μ_y and μ_z along the coordinate axes which is technically wrong. Since μ, being a scalar quantity, cannot be resolved into components. Thus a correction to their analysis is very much sought after in order to give a correct interpretation of the problem. Keeping these facts in mind, the basic equations have been reformulated to the correct perspective and then mathematical and numerical analysis has been performed. The results so obtained have significant variations from the existing results which were otherwise obtained by using wrong assumption.

From table 1 and from figures 2-4, it is evident that the critical value of magnetic Rayleigh number, $N_c = (RM_1)_c$ decreases with the increase in the magnetization parameter M_3. Hence the magnetization has destabilizing effect on the system. The physical interpretation of this may be given as follows: As the value of M_3 increases the departure of linearity in the magnetic equation of state increases resulting into the increase in the velocity of the ferrofluid in the vertical direction favoring the manifestation of instability. This increase in magnetization releases extra energy, which adds up to thermal energy to destabilize the flow more quickly. Thus the magnetization parameter destabilizes the system. The similar result also obtained by Vaidyanathan et al. (2001), but the difference in the values of N_c is quite significant and increases with the increase in the value of δ. It is also evident from figures 2-4 that for stationary convection, the value of magnetic Rayleigh number decreases as the MFD viscosity parameter δ increases, predicting the destabilizing behavior of viscosity parameter δ. This unexpected result that 'the role of viscosity is inverted in the presence of rotation', has also been predicted by Chandrasekhar (1981) for the case of ordinary fluid.

It is also found from table 1 and figure 4, that the magnetic Rayleigh number increases with increase in the values of Taylor number T_a. Thus the rotation has stabilizing effect on the system. Again the difference in the existing values (Vaidyanathan et al., 2001) and the values obtained herein is significant.

It is interesting to note from figures 5 and 6 that for the case of oscillatory motions the value of magnetic Rayleigh number increases as the MFD viscosity parameter δ increases, thus resulting into the postponement of instability. Thus, MFD viscosity has a stabilizing effect on the system for the case of oscillatory convection, which is a result also obtained by Vaidyanathan et al. (2001).

Further, we may note from figures 5 and 6 that for the case of oscillatory convection also, M_3 prepone the onset of convection. Thus magnetization M_3 has destabilizing effect on the system for the case of oscillatory convection also. Finally, figure 7 predicts the stabilizing behavior of rotation on the system for the case of oscillatory convection.

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References

Aniss, S., Souhar, M. and Belhaq, M.: Effect of magnetic modulation on the stability of a magnetic liquid layer heated from below. *ASME J. Heat Transfer*, **123(3)**, (2001), 428- 433.

Chandrasekhar, S.: *Hydrodynamic and Hydromagnetic* Stability. Dover publications, INC., New York, (1981).

Elmore, W.C.: The magnetization of ferromagnetic colloids. *Phys. Rev.*, **54**, (1938), 1092-1095.

Finlayson, B.A.: Convective instability of ferromagnetic fluids. *J. Fluid Mech.*, **40**, (1970), 753-767.

Labusch, M., Keip, M.A., Shvartsman, V.V., Lupascu, D.C. and Schroder, J.: On the influence of ferroelectric polarization states on the magneto-electric coupling in two phase composites. *Tech. Mech.*, **36(1-2)**, (2016), 73-87.

Lalas, D.P. and Carmi, S.: Thermoconvective stability of ferrofluids. *Phys. Fluids*, **14(2)**, (1971), 436-437.

Lee, J. and Shivakumara, I.S.: Onset of penetrative convection in a ferrofluid-saturated porous layer. *Spec. topics & Rev. Porous Media: An Int. Journal*, **2(3)**, (2011), 217-225.

Odenbach, S.: *Ferrofluids: Magnetically Controllable Fluids and their Applications*. Springer-Verlag, Berlin, Heidelberg, (2002b).

Odenbach, S.: *Magnetoviscous Effects in Ferrofluids*. Springer-Verlag, Berlin, Heidelberg, (2002a).

Prakash, J. and Bala, R.: On estimating the complex growth rates in ferromagnetic convection with magnetic field dependent viscosity in a rotating sparsely distributed porous medium. *J. Appl. Mech. Tech. Phys.,* **57(4)**, (2016), 623-636.

Prakash, J. and Gupta, S.: On arresting the complex growth rates in ferromagnetic convection with magnetic field dependent viscosity in a rotating ferrofluid layer. *J. Mag. Mag. Mater.,* **345**, (2013), 201-207.

Prakash, J., Kumar, P., Kumari, K. and Manan, S.: Ferromagnetic convection in a densely packed porous medium with magnetic field dependent viscosity- revisited. *Z. Naturforsch,* **73(3)**, (2018a), 181-189.

Prakash, J., Kumar, R. and Kumari, K.: Thermal convection in a ferromagnetic fluid layer with magnetic field dependent viscosity: A correction applied. *Studia Geotech. et Mech.,* **39(3)**, (2017), 39-46.

Prakash, J., Manan, S. and Kumar, P.: Ferromagnetic convection in a sparsely distributed porous medium with magnetic field dependent viscosity revisited. *J. Porous Media*, **21(8)**, (2018b), 749-762.

Prakash, J.: On arresting the complex growth rates in ferromagnetic convection in a ferrofluid saturated porous layer. *J. Porous Media*, **16(3)**, (2013b), 217-226.

Prakash, J.: On exchange of stabilities in ferromagnetic convection in a rotating porous medium. *Int. J. Fluid Mech. Res*., **40(5)**, (2013a), 391-404.

Rahman, H. and Suslov, S.A.: Magneto-gravitational convection in a vertical layer of ferrofluid in a uniform oblique magnetic Field. *J. Fluid Mech.,* **795**, (2016), 847-875.

Rahman, H. and Suslov, S.A.: Thermomagnetic convection in a layer of ferrofluids placed in a uniform oblique external magnetic field. *J. Fluid Mech.,* **764**, (2015), 316-348.

Ramanathan, A. and Suresh, G.: Effect of magnetic field dependent viscosity and anisotropy of porous medium on ferroconvection. *Int. J. Engng. Sc.*, **42(3-4)**, (2004), 411-425.

Rosensweig, R. E., Kaiser, R. and Miskolczy, G.: Viscosity of magnetic fluid in a magnetic field. *J. Colloid Inter. Sc.,* **29(4)**, (1969), 680-686.

Rosensweig, R.E.: *Ferrohydrodynamics*. Cambridge University Press, Cambridge, (1985).

Shliomis, M.I.: Effective viscosity of magnetic suspensions. *Soviet Physics JETP,* **34(6)**, (1972), 1291-1294.

Shliomis, M.I.: Magnetic fluids. *Soviet Phys. Uspekhi (Engl. Trans.),* **17(2)**, (1974), 153-169.

Sunil, Divya and Sharma, R.C.: Effect of magnetic field dependent viscosity on a rotating ferromagnetic fluid heated and soluted from below, saturating a porous medium. *J. Porous Media*, **8(6)**, (2005), 569-588.

Sunil, Sharma, A., Kumar, P. and Gupta, U.: The effect of magnetic field dependent viscosity and rotation on ferrothermohaline convection saturating a porous medium in the presence of dust particles. *J. Geophys. Eng.,* **2**, (2005), 238-251.

Suslov, S.A.: Thermo-magnetic convection in a vertical layer of ferromagnetic fluid. *Phys. Fluids,* **20(8)**, (2008), 084101.

Vaidyanathan, G., Sekar, R. and Ramanathan, A.: Ferrothermohaline convection. *J. Mag. Mag. Mater.,* **176**, (1997), 321-330.

Vaidyanathan, G., Sekar, R. and Ramanathan, A.: The effect of magnetic field dependent viscosity on ferroconvection in rotating medium. *Indian J. Pure Appl. Phys.,* **40**, (2001), 159-165.

Vaidyanathan, G., Sekar, R., Vasanthakumari, R. and Ramanathan, A.: The effect of magnetic field dependent viscosity on ferroconvection in a rotating sparsely distributed porous medium. *J. Mag. Mag. Mater.,* **250**, (2002), 65-76.

Venkatasubramanian, S. and Kaloni, P.N.: Effects of rotation on the thermoconvective instability of a horizontal layer of ferrofluids. *Int. J. Engng. Sc.,* **32(2)**, (1994), 237-256.

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