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An One-Dimensional Mathematical Modelling for Unsteady Tidal Flow and Salinity Intrusion in River Network

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1. Introduction

The nature of environmental behavior is complex and a superfical analysis of a problem may leave the solution with many uncertainties, so reducing project realiability. The mathematical modelling approach in flow and water quality modelling in river networks is neccessary for more precise answers to such problems. If the mathematical modelling has to be used as a predictive tool in water resources management, first of all it must accurately reproduce natural processes an an approximate forecast future behavior. Accuracy is based on available field data and on the manner to simulate the natural processes.

The salinity intrusion process including advection and dispersion has been dealt with in some models [1] – [3], but most of them are only descriptive.

The purpose of this study is to develop a descriptive and predictive model for unsteady tidal flow that simulates the hydraulic and substance transport response of a one-dimensional river network which made up for connected branches and loops.

2. Governing equations

The tidal hydrodynamic and salt balance equations are the tool in this study to predict the salinity intrusion in river networks. The continuity and longitudinal momentum equations constitute the tidal hydrodynamic model, while the conservation of salt represents the salinity intrusion model. The basic assumptions made in the derivation of the equations are:

a) The river is fairly straight and uniform so that the effects of bend can be neglected and the flow characteristics can be physically represented by a one-dimensional model.
b) Vertical acceleration is neglected and hydrostatic pressure prevails at each point in the river.
c) The wind effect as well as surface stress is negligible.
d) The influence of density gradient on the flow has been neglected.
e) The resistance coefficient is assumed to be the same as for steady flow in open channels and can be approximated by resistance law applicable to open or by field survey.
f) Well-mixed salinity condition exists at all points in the river.

The basic equations are:

$$B\frac{\partial H}{\partial t}+\frac{\partial Q}{\partial x}=q \tag{2.1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial H}{\partial x}+\frac{gn^2QIQI}{A^2R^{4/3}}=0 \tag{2.2}$$

$$\frac{\partial(A_tC)}{\partial t}+\frac{\partial(QC)}{\partial x}=\frac{\partial}{\partial x}\left(A_tE\frac{\partial C}{\partial x}\right)+\sigma(C) \tag{2.3}$$

where B-total width of the surface H- water level to bottom, Q-cross-sectional descharge, g- acceleration due to gravity, A- conveyance area, n- Manning's coefficient, R- hydraulic radius, E- dispersion coefficient, C- cross-sectional averaged salinity, A_t- total area of the cross-section, $\sigma(C)$-source and sink of salinity, x- longitudinal coordinate, t- time, q- the lateral inflow.

The tidal model and salt transport model are linked together, and an uncoupled numerical solution method is used to solve the above equations. By this method one solves eqs. (2.1) and (2.2) first and then in the same time step eqs. (2.3) using unsteady flow conditions determined from the solution of eqs. (2.1) – (2.2).

3. Numerical solution for the tidal model

3.1. River network schematization

A river network consists of junctions (interior nodes), boundaries and river branches each of which is subdivided into river reaches by cross-sections. The flow of every branch has a certain direction. A river network in which the flow in every branch has a determined direction can be considered as a graph with directional arcs. Each arc corresponds to a branch and the positive directions are numbered in an increasing order along the positive direction of the branch. There are two types of branch interior branch and boundary branch. Every interior branch connects two junctions I and J; I is called the beginning and J the end of the arc (I, J). Boundary branch connects a junction and a boundary. In order to conform to the algorithm described in what follows we always impose the boundary as the end of an arc.

3.2. Difference schemes for eqs. (2.1) – (2.2)

For the tidal model the following four-point implicit scheme is used in order to solve eqs. (2.1) – (2.2):

$$f=\left\{\frac{1}{2}\ \Theta(\Delta f_i+\Delta f_{i-1})+f_i^n+f_{i-1}^n\right\}$$

$$\frac{\partial f}{\partial x}=\frac{1}{x_i-x_{i-1}}\left\{\Theta(\Delta f_i-\Delta f_{i-1})+f_i^n-f_{i-1}^n\right\} \tag{3.1}$$

$$\frac{\partial f}{\partial t} = \frac{1}{2\Delta t}(\Delta f_i + \Delta f_{i-1})$$

where $f_i^n = f(x_i, t_n)$; $\Delta f_i = f_i^{n+1} - f_i^n$; Θ-stability weighting factor; Δt the time step; f a variable representing Q, H . . .

Substituting (3.1) into eqs. (2.1) – (2.2) for all reaches, after linearizing, one gets a set of linear equations:

$$\begin{aligned} &A1.H_{i-1} + B1.Q_{i-1} + C1.H_i + D1.Q_i = E1 \\ &A2.H_{i-1} + B2.Q_{i-1} + C2.H_i + D2.Q_i = E2 \end{aligned} \qquad (3.2)$$

where A1, B1, . . . , D2, E2 are known coefficients. It is noted that (3.1) is the Preissmann's scheme which has been presented in detail in [3]. Beside its advantage of being implicit and well-approximating the integral conservation laws, the above scheme allows the simultaneous computation of the unknown functions at the same grid-point. This feature is convenient for giving boundary conditions and getting field data. It was shown in [5], furthermore, that the Preissmann's scheme being quickly stable with respect to initial conditions.

3.3. *Initial condition, boundary condition and condition at junctions*

In order to proceed with the computation of (3.2) it is necessary to specify the stages and discharges at all grid points at the initial time step. Usually these are not known and owing to the avantage of a four-point scheme we should estimate by setting Q = 0 and H = constant at all computational sections.

In a river reach with two boundary sections it is necessary to specify stages, discharges or stage-discharge relations as the boundary conditions.

In a river network the junction condition is often treated as an interior boundary. Assuming a steady-state condition at the junction shown in Fig. 1, mass conservation is satisfied by the following equation:

$$Q_1 = Q_2 + Q_3 \qquad (3.3)$$

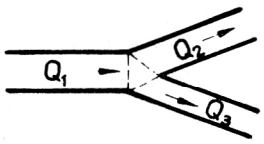

Fig. 1
Junction of three channels

and the conservation of energy is represented by equations

$$\begin{aligned} H_1 + \frac{u_1^2}{2g} &= H_2 + \alpha_1 \frac{u_2^2}{2g} + h_1, \\ H_2 + \frac{u_2^2}{2g} &= H_3 + \alpha_2 \frac{u_3^2}{2g} + h_2. \end{aligned} \qquad (3.4)$$

Here are α_i – the correction factors for energy loss; h_i – the energy main loss given by the product of the friction slope and the distance between sections; H_i – water level; u_i – the velocity; Q_i – the discharge. Eqs. (3.4) can be simplified by disregarding the velocity and main loss terms.

3.4. *Computational sequences*

For the computational purposes two types of relations are developed: the recurrent equation for every branch and the general junction equation.

a) Recurrent equation

To solve (3.2) for every branch of a river network the following recurrent equation is used:

$$\begin{aligned} H_i &= p_i Q_i + q_i H_i + r_i \\ Q_{i-1} &= t_i Q_i + v_i H_i + z_i H_i + S_i \end{aligned} \qquad (3.5)$$

where p_i, q_i, r_i, t_i, v_i, z_i, S_i – are called recurrent coefficients and are known functions of A1, B1, . . ., D2, E2.

In fact (3.5) is a double sweep algorithm. The first sweep computes recurrent coefficients and the reverse sweep computes H, Q at all cross-sections. It is shown in [6] that (3.5) is stable to the round-off errors. Equation (3.5) is called a forward sweep. In the following we use a procedure similar to (3.5) but the process starts from the end back to the beginning of the branch in order to compute the recurrent coefficients:

$$\begin{aligned} H_i &= \bar{p}_i Q_i + \bar{q}_i H_N + \bar{r}_i \\ Q_{i+1} &= \bar{t}_i Q_i + \bar{v}_i H_i + \bar{z}_i H_N + \bar{S}_i \end{aligned} \qquad (3.6)$$

where the subscript i refers to the beginning and the subscript N corresponds to the end of a branch. Equation (3.6) is called the inverse sweep.

b) Junction equation

To set the equation at junction, the relations (3.5) – (3.6) are used. Consider, for example, a branch (I, J), (I is an interior junction while J is either junction or boundary)

b1) The case where I, J are both junctions.

The relation (3.4) gives $H_i = H_{(I)}$, $H_N = H_{(J)}$, where $H_{(I)}$ is the water level at the junction I.

The forward sweep from I to J provides

$$\frac{1}{p_N} H_{(J)} - \frac{q_N}{p_N} H_{(I)} = Q_N + \frac{r_N}{p_N} \qquad (3.7)$$

The inverse sweep from J to I provides

$$\frac{\bar{q}_1}{\bar{p}_1} H_{(J)} - \frac{1}{\bar{p}_1} H_{(I)} = -Q_1 - \frac{\bar{r}_1}{\bar{p}_1} \qquad (3.8)$$

b2) The case when I is a junction and J is a boundary

i) $Q_N = BQ$ is known (boundary condition Q in the end point). Both sweep procedures are used and the results are:

$$\frac{\bar{q}_1 \cdot q_N - 1}{\bar{p}_1} H_{(1)} = -Q_1 - \frac{\bar{r}_1 + \bar{a}_1 (r_N + p_N \cdot BQ)}{\bar{p}_1} \quad (3.9)$$

ii) $H_N = BH$ is known (boundary condition H at the end point). The inverse sweep provides

$$-\frac{1}{\bar{p}_1} H_{(1)} = -Q - \frac{\bar{r}_1 + \bar{a}_1 \cdot BH}{\bar{p}_1}$$

The procedure is similar if the boundary condition is given in the form $\alpha H_N + \beta Q_N = \gamma$. So for an interior branch we have two relations (3.7) – (3.8), for a boundary branch we have either (3.9) or (3.10). Since the sum of the discharges at junction has to be zero we can link (3.7) – (3.10) together to get a system that only consists of water level unknowns at all junctions. Solving this system and using (3.5), the H, Q at all cross-sections of the present branch of the network can be computed.

4. Numerical solution for the salinity intrusion model

The diffusion equation (2.3) desribes two physical phenomena : transport (or advection) and diffusion, both are longitudinal. For salinity intrusion, diffusion is a pseudophenomenon. The equation (2.3.) was set up with the assumption of neglecting molecular diffusion velocity and the diffusion term was an averaged result over the cross-section. This term represents jointly two phenomena : the turbulent diffusion and the complement due to irregularity of velocity at the cross-section. The parabolic equation (2.3) requires one boundary condition at each end and one initial condition. The two boundary conditions of a branch can usually be specified by the following procedures:

a) The salinity at the boundaries is a known function of time $C(0,t) = C_u(t)$ for upstream, and $C(L,t) = C_d(t)$ for downstream.

b) Assuming the salinity does not change at the downstream boundary.

c) Assuming the second partial derivative of the salinity is equal to zero, which means the salinity has a linear relationship with x at the downstream.

To make the model more predictive some authors [1], [2] divide the salinity process into two parts: during the flood tide salinity at downstream must be given and during the ebb tide salinity variation is assumed linear with x. The detail discussions of boundary conditions for the salinity model can be seen in [5], [6].

As considered in detail in physics, pure diffusion is a phenomenon occurring so slowly that for (2.3), during outflow towards boundary (at ebb tide) or inflow towards junction this phenomenon has neglegible influence on the variation of salinity there within a time step. This hypothesis together with the fractioned-step method enable us to formulate easily the salinity equation at the junction. Following [4], solving (2.3) within t to $t + \Delta t$ the problem may be separated into two processes:

$$\frac{\partial C}{\partial t} + \frac{Q}{A} \frac{\partial C}{\partial x} = 0 \quad (4.1)$$

$$\frac{\partial C}{\partial t} = \frac{1}{A} \frac{\partial}{\partial x} \left(EA \frac{\partial C}{\partial x}\right) + S(C) \quad (4.2)$$

The solutions of (4.1) will be used as initial condition of (4.2) (for details, see [4]). The decomposition of (2.3) into (4.2) is purely mathematical, while physically salinity C at a point x and at the time t is the result of two simultaneous processes: transport and dispersion.

4.1. Numerical scheme for (4.1)

The numerical solutions of (4.1) must be carefully investigated. Most finite difference schemes have numerical diffusions that are sometimes stronger than those of (4.2) which spoil the physical significance of computation results.

After Lagrange's point of view, we can consider the problem (4.1) as follows: a liquid particle, starting from a point M at the time t_n along a trajectory (the dotted line in fig. 2) with a velocity U reaches the point x_{i+1} at the time t_{n+1}. In pure transport process the liquid particle does not change, so that

$$C(x_{i+1}, t_{n+1}) = C(M, t_n). \quad (4.3)$$

By this the problem (4.1) becomes that of going inversely along the trajectory back to the time. If M coincides with x_{i-1} (or x_i) where C has been known one can know $C(x_{i+1}, t_{n+1})$ precisely. If M lies between x_{i-1} and x_i one must use an interpolation that gives (4.1) the same accuracy. The above process is the content of the method of characteristics for solving (4.1). Following [3], [5], linear interpolation often leads to numerical diffusion. To overcome this phenomenon a cubic interpolation [3], cubic spline or Lagrange's interpolation can be used. The method of characteristics for (4.1) and procedure for interpolation can be seen in detail in [5], [6].

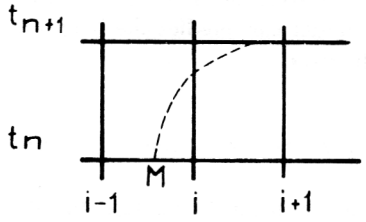

Fig. 2
Path of fluid particle

4.2. Numerical scheme for (4.2)

For solving (4.2) with a non-uniform grid we apply a weighted implicit finite difference scheme associated with the fractioned-step method:

$f = f_i^{n+1}$

$$\left|\frac{\partial f}{\partial x}\right|_{i+1/2} = \frac{1}{x_{i+1}-x_i}\left\{\Theta(\Delta f_{i+1}-\Delta f_i)+f_{i+1}^n-f_i^n\right\}$$

$$\frac{\partial f}{\partial t} = \frac{1}{\Delta t}\left\{\frac{x_{i+1}-x_1}{3(x_{i+1}-x_{i-1})}\Delta f_{i-1}\right. \tag{4.3}$$

$$\left.+\frac{2}{3}\Delta f_i+\frac{x_i-x_{i-1}}{3(x_{i+1}-x_{i-1})}\right\}$$

in which the symbols are similar to those in (3.1). Substituting (4.3) into (4.2) and working out some calculations, the result is the following tridiagonal system of equations

$$L_i C_{i-1}^{n+1} + M_i C_i^{n+1} + N_i C_{i+1}^{n+1} = P_i \tag{4.4}$$

where L_i, M_i, N_i, P_i are known functions.

(4.4) is solved by the double sweep method that has been well-considered in many papers.

4.3. Salinity boundary condition for a single river

i) During flood tide ($Q \leqslant 0$)

Using the boundary condition at downstream in order to solve (4.1) we get the values C_f for the entire river branch. These values are used as initial values for (4.2). The value C_f at downstream will be the boundary condition at the same end for (4.2).

ii) During ebb tide ($Q > 0$)

At upstream the salinity must be given and by solving (4.1) we get the values C_e which can used as initial condition for (4.2). The computed value C_e at downstream will be the boundary condition for (4.2) at this boundary.

It should be noted that for river network, the upstream boundary conditions are replaced by those values of salinity at the junction which are only available after treating the salinity at the junctions. So, it is only necessary to specify C at downstream during flood tide, the ebb tide C can be computed. If the computation process starts at the beginning of ebb tide C_{min} at downstream can be computed, one needs to know only C_{max}, then using an interpolation between C_{min} and C_{max} we can get the boundary C at downstream during flood tide. Such an organization needs only one information C_{max} and the model can predict the length of salinity intrusion into the river.

4.4. Formulation of the salinity equation at junction

Assuming that during inflow towards junction the dispersion process at that time has a negligible influence on the variation of salinity, the balance equation at junction can be established as follows:

Consider, for example, a junction I where four river branches intersect and at a certain time in branches 1, 2 flow goes into junction and in branches 3, 4 flow goes out from junction (see fig. 3).

Consider branches 1, 2, following the algorithm described above we go along the characteristics starting at the cross-section next to junction I (at t) back to the intersection of these characteristics with the straight line of the time level t. Using a certain interpolation the values at junction CN_1, CN_2 of branches 1, 2 can be computed. In flows going out of the junction the values CN_3, CN_4 must take the value at junction CN which results from mixing CN_1, CN_2 in branches 1, 2. So we have the following balance equation:

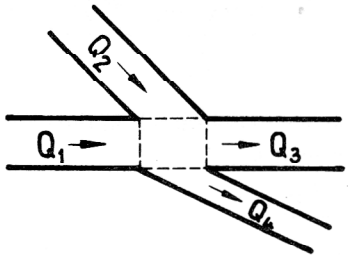

Fig. 3
Junction of four channels

$$Q_1.CN_1 + Q_2.CN_2 = Q_3.CN_3 + Q_4.CN_4$$
$$= CN.(Q_3 + Q_4)$$

or

$$CN = (Q_1.CN_1 + Q_2.CN_2)/(Q_3 + Q_4)$$

In general we have

$$CN = \sum_i Q_i^I.CN_i / \sum_j Q_j^o \tag{4.5}$$

where Q_i^I is the discharge of branch i going into junction and Q_j^o is the discharge of branch j going out of junction. (4.5) allows an easy estimation of salinity at junction.

5. Application of the model

The algorithm described above has been tested on some river networks in Vietnam. The Fig. 4 and 5 show the Camau peninsula and its schematization. This system consists of 30 junctions, 253 cross-sections, 55 branches, 7 boundaries. The peninsula is subjected to tides of two types: semidiurnal tide from the East sea and mixed but predominantly diurnal from the Gulf of Thailand. These two tidal flows when going into the very dense canal network create a complicated flow regime. The results of the hydrodynamic component of the first few tests are presented in Fig. 6a – d and 7a, b for discharge and for water level at some selected stations compared with field data. It can be seen that in general the model produces a good agreement with the observed data. The results obtained from the first few test runs of the model are presented in Fig. 8a – c. It can be observed that the results can meet the practical purpose.

6. Conclusion

A numerical model has been developed for computing water level, discharge and salinity in a tidal river network. The algorithm has been tested on some river systems with complexity of the topographic and hydrologic conditions. It can be seen that the model may be used for practical purposes. With some modifications the model can also used for water quality problems.

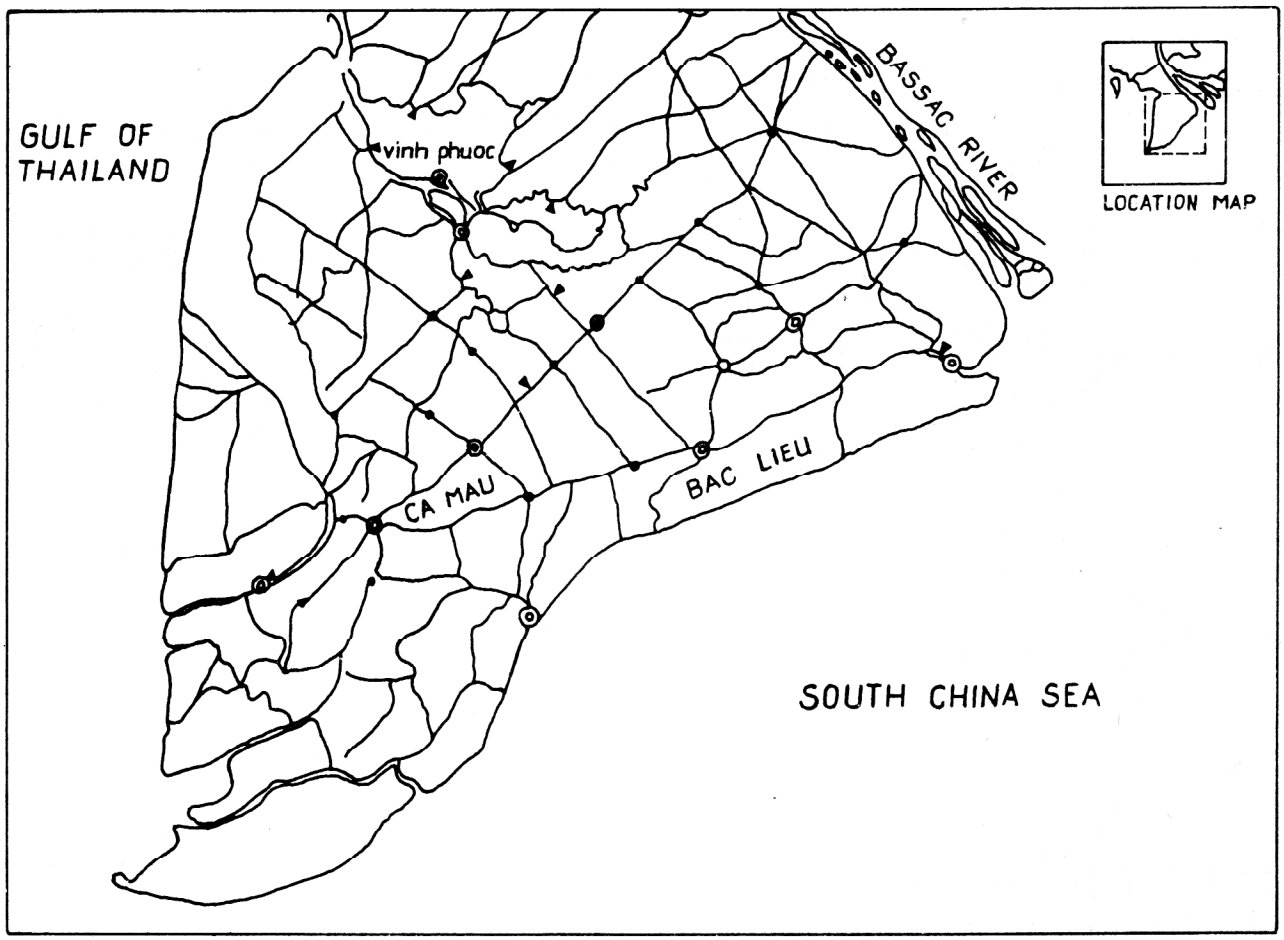

Fig. 4
Channel system in the Camau peninsula

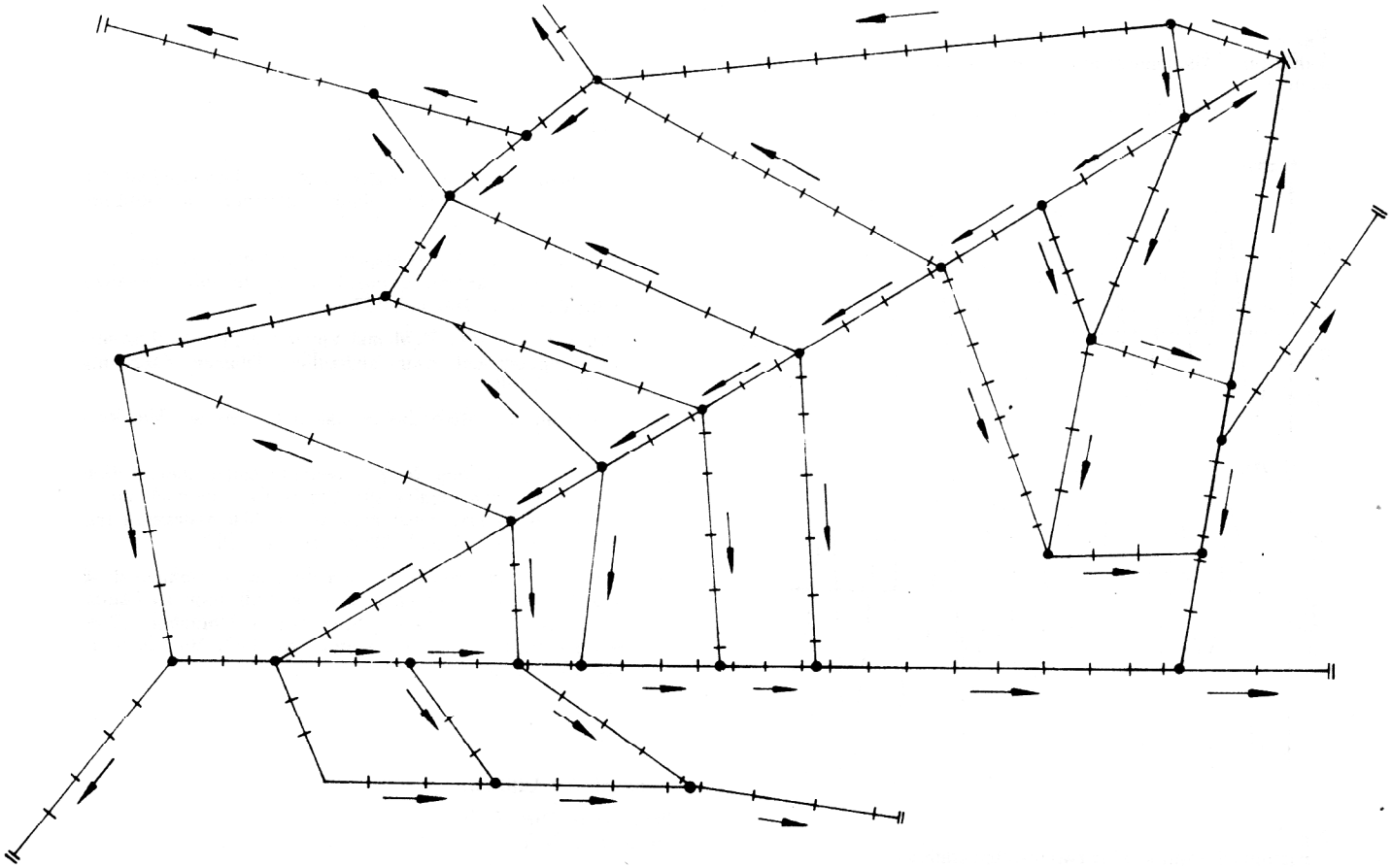

Fig. 5
Schematization of Camau channel system

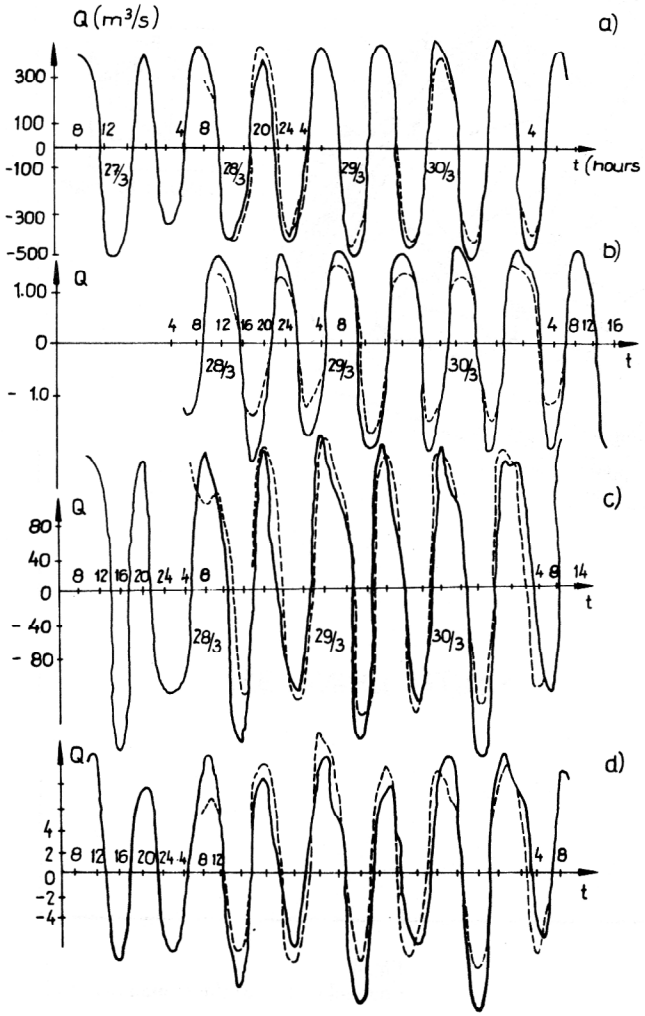

Fig. 6
Variation of discharge at four selected stations in time.
——— observed, ------- computed

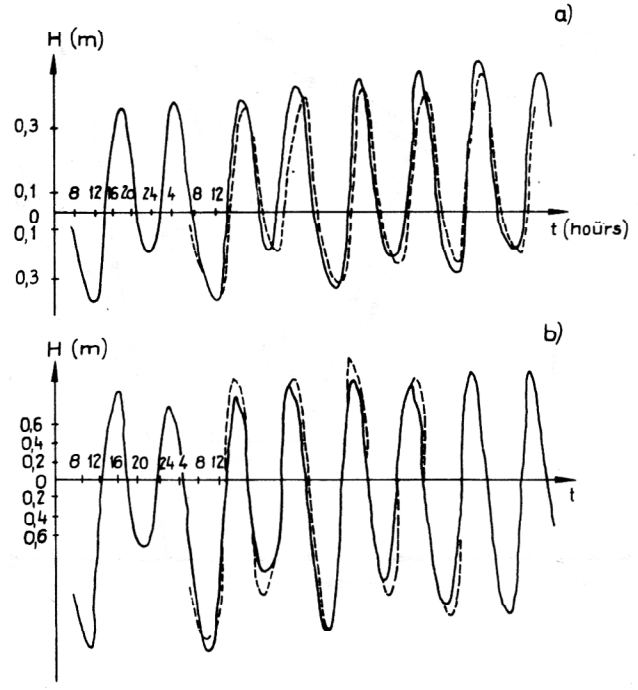

Fig. 7
Variation of water level at two selected stations in time:
——— observed, -------- computed

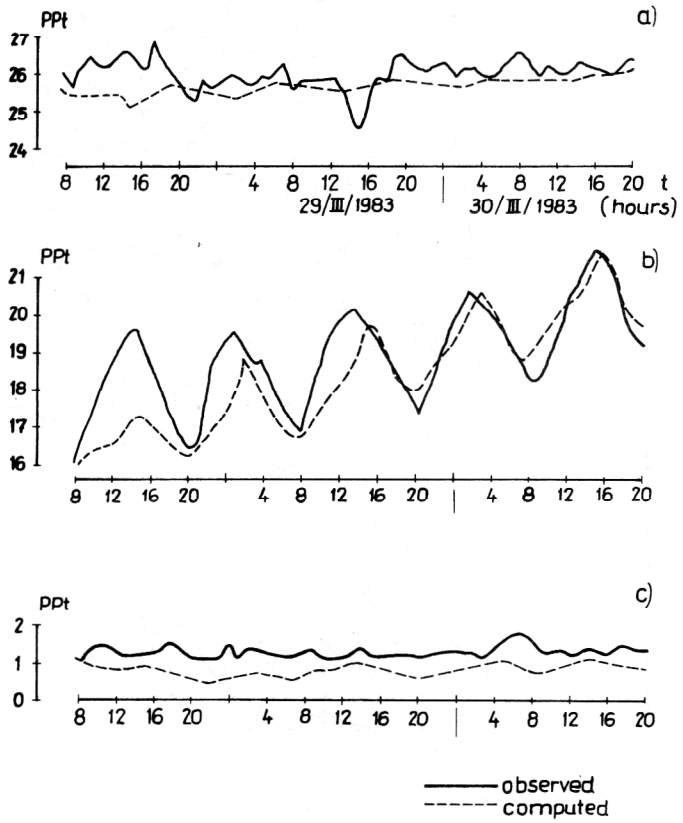

Fig. 8
Variation of salinity at three selected stations in time (salinity hydrograph):
——— observed, -------- computed

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