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Random Vibration of an Unconstrained Viscoelastically Damped Beam

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1. Introduction

Structural members of most of the modern machines are subjected to a severe vibratory environment spread over a wide range of frequencies. Such vibrations result in malfunctioning of machine parts and also give rise to annoying noise radiation. These vibrations are of random type and it is not generally possible to effectively control these vibration by the use of conventional design methods. This is especially so when existing structural members of a machine cannot be replaced. For control of such vibrations in existing structural members it is useful to add damping without removing the member from the machine. The damping is provided by covering the structural member with a sheet of viscoelastic material, so that it becomes a composits structure in the form of a two layer arrangement, in which the viscoelastic layer acts as an unconstrained damping treatment. The reduction of vibrations occurs because of dissipation of energy due to deformation in the form of direct strains produced in the viscoelastic layer.

Materials like long chain polymers, rubbers and plastics, which possess high energy dissipation capacity, are used for the viscoelastic layer. Examples of such materials are buna N rubber (butadiene acrylonitrile), PVC (polyvinylchloride), styrofoam etc. The use of viscoelastic damping in the form of layered structural members has been gaining widespread industrial applications, as for example in aircraft fuselage structures, missile frames, electronic equipment mounts, automobile door panels, rocket launch pads and building construction. Most of these structures are subjected to random loads in addition to conditions of free and forced vibrations and shock excitation.

The analysis of flexural vibration of unconstrained (two-layer) viscoelastically damped structures has been carried out by several investigators for sinusoidal [1 – 5] and shock [6 – 9] excitations. Also considerable work has been reported on the random excitations of elastic homogeneous structures [10 – 19]. However, the work reported on the analysis of unconstrained structures to random excitation is rather meagre and this paper is an effort in that direction.

Mead [18, 19] has estimated the effect of a viscoelastic damping compound on the vibration stresses and amplitudes of an aircraft fuselage structure subjected to jet efflux excitation. In his analysis, the damping and the stiffness of the viscoelastic material have been assumed to be represented by a complex modulus, but they fail to represent adequately the dynamic properties of the viscoelastic material. Borisov et al [4] have considered the excitation by bandlimited white noise of structures, on which vibration absorbing coatings are applied. They have also used the complex modulus representation. Tarnóczy [20] has conducted experimental study of loss factors of thin metal plates covered with unconstrained viscoelastic layers for reduction of radiated sound energy when subjected to white-noise type of excitation.

In the present paper analysis of an unconstrained viscoelastically damped beam subjected to random excitations is considered. A fourelement model is used to represent the dynamic proporties of the viscoelastic material. Two types of random excitation are considered – namely the white-noise excitation at a point and the turbulent boundary layer excitation over the span of the structure. The analysis carried out here ist for a beam and it can suitably be extended for application to a plate. Simply supported end conditions are considered.

2. Theoretical Analysis

2.1. Equation of motion

Figure 1 shows an unconstrained beam arrangement. The authors have derived the equation of motion for such a beam [21] based on the following assumptions:

1) Both layers bend according to Bernoulli-Euler theory.
2) Transverse displacement at a section is constant along the thickness.
3) Only extensional and bending effects occur, while shear deformations are negligible.
4) Longitudinal displacement at a transverse section varies linearly with layer thickness, but its rate is different in different layers.
5) Rotational and longitudinal inertia terms are negligible and only transverse inertia terms are included.
6) All displacements are small as in the linear theory of elasticity.
7) No slipping occurs at the interface of the layers when the beam bends.
8) The material of the viscoelastic layer is linear and its dynamic properties are represented by a four-element model.

The equation of motion is obtained by combining the relationships obtained by considering the equilibrium of forces and the continuity of displacements. The equation of motion thus obtained for a general two layer beam arrangement is

$$\Theta \frac{\partial^4 w}{\partial x^4} + \mu \frac{\partial^2 w}{\partial t^2} = f(x, t) \qquad (1)$$

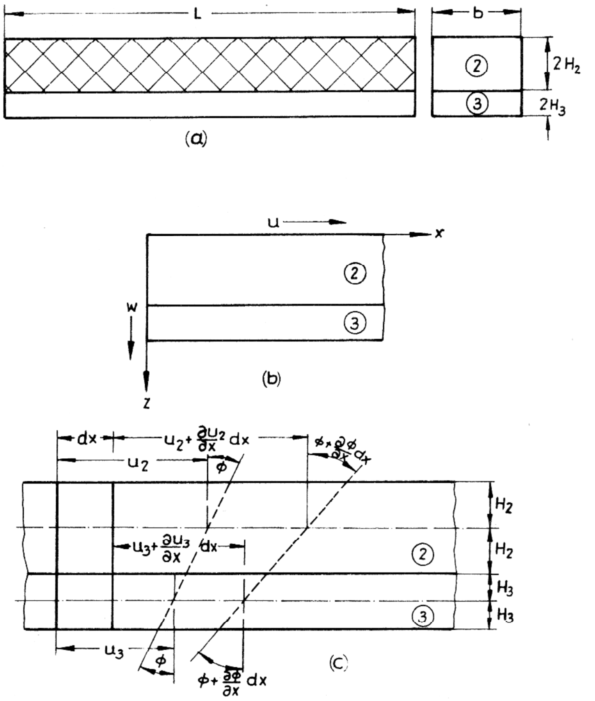

Figure 1
Two-layer sandwich beam — 2 is viscoelastic layer and 3 is elastic layer. (a) Layer arrangement, (b) co-ordinate system and (c) relative deformations.

where,

$$\Theta = \frac{K_3 B_3 + E_2 \Delta + E_2^2 A_2 I_2}{K_3 + E_2 A_2}, \qquad (2)$$

$$\Delta = K_3 I_2 + A_2 (K_3 \delta_2^2 + B_3) \qquad (3)$$

and

$$\delta_2 = H_2 + H_3 \qquad (4)$$

2.1.1. *Four-element model*

We now consider layer 2 to be viscoelastic with its dynamic properties in extension represented by a four-element model (Figure 2). For such a model the dynamic elastic modulus of the viscoelastic material can be shown to be given by [22]

$$E_2 = \frac{\beta + \gamma D + \psi D^2}{1 + \alpha D} \qquad (5)$$

where α, β, γ and ψ, called the model constants, are related to the elements ζ_1, ζ_3, η_2 and η_3 of the model by the relations

$$\alpha = \eta_3 / \zeta_3, \quad \beta = \zeta_1, \quad \gamma = \eta_2 + \eta_3 + \frac{\zeta_1 \eta_3}{\zeta_3},$$

$$\psi = \eta_2 \eta_3 / \zeta_3 \qquad (6)$$

Substituting the value of E_2 from equation (5) into equation (2), we obtain after simplification

$$\Theta = \frac{\Theta_1 D^4 + \Theta_2 D^3 + \Theta_3 D^2 + \Theta_4 D + \Theta_5}{\Theta_6 D^3 + \Theta_7 D^2 + \Theta_8 D + \Theta_9} \qquad (7)$$

where

$$\left.\begin{aligned}
\Theta_1 &= \psi^2 A_2 I_2, \\
\Theta_2 &= \psi (\alpha \Delta + 2 \gamma A_2 I_2), \\
\Theta_3 &= K_3 B_3 \alpha^2 + (\psi + \alpha \gamma) \Delta + (\nu^2 + 2 \beta \psi) A_2 I_2, \\
\Theta_4 &= 2 K_3 B_3 \alpha + (\gamma + \beta \alpha) \Delta + 2 \gamma \beta A_2 I_2, \\
\Theta_5 &= K_3 B_3 + \beta \Delta + \beta^2 A_2 I_2, \\
\Theta_6 &= \psi A_2 \alpha, \\
\Theta_7 &= K_3 \alpha^2 + (\gamma \alpha + \psi) A_2, \\
\Theta_8 &= 2 K_3 \alpha + (\beta \alpha + \gamma) A_2, \\
\Theta_9 &= K_3 + \beta A_2
\end{aligned}\right\} \qquad (8)$$

Substituting the value of Θ from equation (7) into equation (1) and introducing the dimensionless coordinate $\bar{x} = x/L$, we get the following equation of motion for an unconstrained viscoelastically damped beam

$$\Theta_1 \frac{\partial^8 w}{\partial \bar{x}^4 \partial t^4} + \Theta_2 \frac{\partial^7 w}{\partial \bar{x}^4 \partial t^3} + \Theta_3 \frac{\partial^6 w}{\partial \bar{x}^4 \partial t^2} + \Theta_4 \frac{\partial^5 w}{\partial \bar{x}^4 \partial t} + \Theta_5 \frac{\partial^4 w}{\partial \bar{x}^4} + \mu L^4 \left[\Theta_6 \frac{\partial^5 w}{\partial t^5} + \Theta_7 \frac{\partial^4 w}{\partial t^4} + \Theta_8 \frac{\partial^3 w}{\partial t^3} + \Theta_9 \frac{\partial^2 w}{\partial t^2}\right] = L^4 \left[\Theta_6 \frac{\partial^3 w}{\partial t^3} + \Theta_7 \frac{\partial^2 w}{\partial t^2} + \Theta_8 \frac{\partial w}{\partial t} + \Theta_9\right] f(\bar{x}, t) \qquad (9)$$

2.2. *Response of a simply supported, unconstrained viscoelastically damped beam to random excitation*

Let us define the transfer function $H_w(n, \omega)$ of the beam as its response to unit harmonic force of the type $e^{i\omega t} \sin n\pi\bar{x}$, then on substituting $H_w(n, \omega)\, e^{i\omega t} \sin n\pi\bar{x}$ for w and $e^{i\omega t} \sin n\pi\bar{x}$ for $f(\bar{x}, t)$ in equation (9), we obtain the transfer function of the beam as

$$H_w(n, \omega) = \frac{L^4 (-i\omega^3 \Theta_6 - \omega^2 \Theta_7 + i\omega \Theta_8 + \Theta_9)}{[i\omega^5 \Theta_6 \mu L^4 + \omega^4 (\Theta_1 n^4 \pi^4 + \Theta_7 \mu L^4) - i\omega^3 (\Theta_2 n^4 \pi^4 + \Theta_8 \mu L^4) - \omega^2 (\Theta_3 n^4 \pi^4 + \Theta_9 \mu L^4) + i\omega \Theta_4 n^4 \pi^4 + \Theta_5 n^4 \pi^4]} \qquad (10)$$

In equation (9), $f(\bar{x}, t)$ represents the intensity of any general excitation force acting on the beam. For determining the response of the beam to such an excitation, we decompose the loading $f(\bar{x}, t)$ into the space modes of $\sin n\pi\bar{x}$, so that we may write

$$f(\bar{x}, t) = \sum_{n=1}^{\infty} f_n(t) \sin n\pi\bar{x} \qquad (11)$$

where,

$$f_n(t) = 2\int_0^1 f(\bar{x}, t)\sin n\pi\bar{x}\ d\bar{x} \tag{12}$$

The impulse response function $h_w(n, t)$ of the beam can be considered as the Fourier transform of its transfer function. So it can be expressed as

$$h_w(n, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_w(n, \omega)\, e^{i\omega t}\, d\omega \tag{13}$$

The displacement response $w(\bar{x}, t)$ of the beam corresponding to the excitation $f(\bar{x}, t)$ is obtained by using the superposition or convolution integral, and is given by

$$w(\bar{x}, t) = \int_{-\infty}^{\infty} f(\bar{x}, \tau)\, h_w(n, t-\tau)\, d\tau \tag{14}$$

Substituting equations (11) and (12) into equation (14) and obtaining from the resulting expression the statistical average over the sample space of the product $w(\bar{x}_1, t_1)\ w(\bar{x}_2, t_2)$, which reduces to mean square of w when $\bar{x}_1 = \bar{x}_2$ and $t_1 = t_2$, we get

$$E[w(\bar{x}_1,t_1)\,w(\bar{x}_2,t_2)] = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin n\pi\bar{x}_1 \sin m\pi\bar{x}_2$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_w(n, t_1-\tau_1)\, h_w(m, t_2-\tau_2)\, d\tau_1\, d\tau_2$$

$$4\int_0^1\int_0^1 E[f(\bar{\xi}_1,\tau_1)\, f(\bar{\xi}_2,\tau_2)] \tag{15}$$

$$\sin n\pi\bar{\xi}_1\ \sin m\pi\bar{\xi}_2\ d\bar{\xi}_1\ d\bar{\xi}_2$$

Putting

$$\left.\begin{array}{ll} t_1 - \tau_1 = T_1, & t_2 - \tau_2 = T_2, \\ t_1 - t_2 = \tau, & t_2 = t \end{array}\right\} \tag{16}$$

in equation (15), we get the statistical average as

$$E[w(\bar{x}_1, t+\tau)\ \ w(x_2, t)]$$

$$= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin n\pi\bar{x}_1\ \ \sin m\pi\bar{x}_2$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_w(n, T_1)\ h_w(m, T_2)\, dT_1\, dT_2$$

$$4\int_0^1\int_0^1 E[f(\bar{\xi}_1, t+\tau-T_1)\ f(\bar{\xi}_2, t-T_2)]$$

$$\sin n\pi\bar{\xi}_1\ \sin m\pi\bar{\xi}_2\ d\bar{\xi}_1\ d\bar{\xi}_2 \tag{17}$$

Here the expression $E[f(\bar{\xi}_1, t+\tau-T_1)\ f(\bar{\xi}_2, t-T_2)]$ represents the correlation of any random pressure field.

2.2.1. White noise random excitation

2.2.1.1. Field excitation

Let us consider an excitation process, such that its correlation is of the form

$$E[f(\bar{\xi}_1, t+\tau-T_1)\ f(\bar{\xi}_2, t-T_2)]$$

$$= \frac{1}{2}\,\delta(\bar{\xi}_1 - \bar{\xi}_2)\ \ R_f(\tau + T_2 - T_1) \tag{18}$$

where $\delta(\ \ )$ is Dirac's delta funktion. Equation (18) implies that spacewise the loads are completely correlated only for $\bar{\xi}_1 = \bar{\xi}_2$ and completely uncorrelated for $\bar{\xi}_1 \neq \bar{\xi}_2$, and that there exists some correlation in the time domain.

Substituting equation (18) into equation (17) and using the random vibration analysis and applying the Wiener-Khintchine relationships [23], we obtain after some simplification

$$E[w(\bar{x}_1, t+\tau)\ \ w(\bar{x}_2, t)$$

$$= \sum_{n=1}^{\infty} \sin n\pi\bar{x}_1\ \sin n\pi\bar{x}_2 \int_{-\infty}^{\infty} S_f(\omega)\ |H_w(n,\omega)|^2 e^{i\omega\tau} d\omega \tag{19}$$

The mean square displacement at a location $\bar{x}$, obtained by putting $\bar{x}_1 = \bar{x}_2 = \bar{x}$ and $\tau = 0$ in equation (19), is

$$E[w^2(\bar{x})] = \sum_{n=1}^{\infty} \sin^2 n\pi\bar{x} \int_{-\infty}^{\infty} S_f(\omega)\ |H_w(n,\omega)|^2\, d\omega \tag{20}$$

For white noise excitation the spectral density $S_f(\omega)$ is constant over the entire frequency range. Denoting this value by S_o, we write equation (20) as

$$E[w^2(\bar{x})] = S_o \sum_{n=1}^{\infty} \sin^2 n\pi\bar{x} \int_{-\infty}^{\infty} |H_w(n,\omega)|^2\, d\omega \tag{21}$$

Equation (21) represents the mean square displacement response of the beam at any point $\bar{x}$ to white noise random excitation of distributed type. If equation (21) is averaged over the span of the beam, then

$$E[w^2] = \frac{S_o}{2}\sum_{n=1}^{\infty}\int_{-\infty}^{\infty} |H_w(n, \omega)|^2\, d\omega \tag{22}$$

2.2.1.2. Point excitation

Let the beam be acted upon by a random force $F(t)$ at a single point $\bar{a}$, then $f(\bar{x}, t)$ can be expressed as

$$F(\bar{x}, t) = \frac{F(t)}{L}\,\delta(\bar{x} - \bar{a}) \tag{23}$$

Substituting equation (23) into equation (17) and simplifying with the help of Wiener-Khintchine relationships, we get mean square displacement at any point $\bar{x}$ on the beam as

$$E[w^2(\bar{x})] = 4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin n\pi\bar{x}\, \sin m\pi\bar{x}\, \sin n\eta\bar{a}\, \sin m\eta\bar{a}$$

$$\int_{-\infty}^{\infty} H_w(n, \omega)\ H_w(m, -\omega)\ S_f(\omega)\, d\omega \tag{24}$$

On averaging over the span of the beam, the double summation will reduce to single summation, because of the orthogonality of the functions $\sin n\pi\bar{x}$ and $\sin m\pi\bar{x}$ we have for the white noise excitation

$$E[w^2] = 2 S_o \sum_{n=1}^{\infty} \sin^2 n\pi\bar{a} \int_{-\infty}^{\infty} |H_w(n,\omega)|^2 d\omega \quad (25)$$

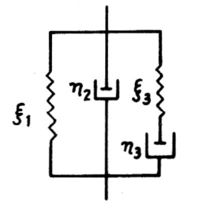

Figure 2
Four-element viscoelastic model.

2.2.2. Turbulent boundary layer excitation

For this case the statistical average of the excitation $E[f(\bar{\xi}_1, t+\tau-T_1)\ f(\bar{\xi}_2, t-T_2)]$ of equation (17) would represent the correlation of the wall pressure fluctuations caused by the turbulent flow of a fluid over a flexible surface. In the present study the fluid considered is air.

Wall pressure correlation measurements by several authors indicate that

(i) the fluctuating pressure field is a random process, which may be considered stationary in time and homogeneous in space, so that the correlation is a function of spatial separation and time difference only

(ii) the spatial correlation function along each of the longitudinal and lateral directions can be represented by an exponentially decaying cosine wave.

In view of (i) above, we can write

$$E[f(\bar{\xi}_1, t+\tau-T_1)\ f(\bar{\xi}_2, t-T_2)] = R_f(\bar{\zeta}, \tau-T_1+T_2) \quad (26)$$

where $\bar{\zeta} = \bar{\xi}_1 - \bar{\xi}_2$ is the spatial separation. Substituting equation (26) into equation (17) and simplifying it with the help of Wiener-Khintchine relationships, we get

$$E[w(\bar{x}_1, t+\tau)\ w(\bar{x}_2, t)] = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin n\pi\bar{x}_1 \sin m\pi\bar{x}_2$$

$$\int_{-\infty}^{\infty} H_w(n,\omega)\ H_w(m,-\omega)\ e^{i\omega\tau}$$

$$4\int_0^1\int_0^1 \underline{S}_f(\bar{\zeta},\omega) \sin n\pi\bar{\xi}_1 \sin m\pi\bar{\xi}_2\, d\bar{\xi}_1\, d\bar{\xi}_2\, d\omega \quad (27)$$

where $\underline{S}_f(\bar{\zeta},\omega)$ is the spectral density of the excitation process due to pressure fluctuations.

If $S_f(\omega)$ be the power spectral density of the pressure fluctuations, which is assumed independent of position, then we may write

$$\underline{S}_f(\bar{\zeta},\omega) = S_f(\omega)\, Q_f(\bar{\zeta},\omega) \quad (28)$$

where $Q_f(\bar{\zeta},\omega)$ is the spatial correlation coefficient of the pressure.

Bull et al [24] and Crocker [25] have, on the basis of experiments conducted by them, suggested the spatial correlation coefficient of the following form

$$Q_f(\bar{\zeta},\omega) = \exp(-\bar{\beta}_1\bar{\zeta})\ \cos(\bar{\gamma}\bar{\zeta}),\ \bar{\zeta} \geqslant 0 \quad (29)$$

where

$$\bar{\beta}_1 = 0.1\frac{L\omega}{U_c} \text{ and } \bar{\gamma} = 1.0\frac{L\omega}{U_c} \quad (30)$$

After substituting for $\underline{S}_f(\bar{\zeta},\omega)$ from equation (28) into equation (27), the mean square displacement at any point $\bar{x}$ is obtained as

$$E[w^2(\bar{x})] = 4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin n\pi\bar{x} \quad \sin m\pi\bar{x}$$

$$\int_{-\infty}^{\infty} S_f(\omega)\ H_w(m,-\omega)$$

$$\int_0^1\int_0^1 Q_f(\bar{\zeta},\omega) \sin n\pi\bar{\xi}_1 \sin m\pi\bar{\xi}_2\, d\bar{\xi}_1\, d\bar{\xi}_2\, d\omega \quad (31)$$

which on averaging over the span of the beam reduces to

$$E[w^2] = 2\sum_{n=1}^{\infty}\int_{-\infty}^{\infty} S_f(\omega)\ |H_w(n,\omega)|^2$$

$$\int_0^1\int_0^1 Q_f(\bar{\zeta},\omega) \sin n\pi\bar{\xi}_1 \sin n\pi\bar{\xi}_2\, d\bar{\xi}_1\, d\bar{\xi}_2\, d\omega \quad (32)$$

Substituting for $Q_f(\bar{\zeta},\omega)$ from equation (29) and introducing

$$\bar{\xi}_1 + \bar{\xi}_2 = \bar{\eta},\ \bar{\xi}_1 - \bar{\xi}_2 = \bar{\zeta} \text{ and}$$

$$\sin n\pi\bar{\xi}_1 \sin n\pi\bar{\xi}_2 = \frac{1}{2}(\cos n\pi\bar{\zeta} - \cos n\pi\bar{\eta})$$

into equation (32), and then applying a coordinate transformation similar to that suggested by Crocker et al [26], we get after simplification

$$E[w^2] = \sum_{n=1}^{\infty}\int_{-\infty}^{\infty} S_f(\omega)\ |H_w(n,\omega)|^2 (J_1 + J_2)\, d\omega \quad (33)$$

where

$$J_1 = \frac{e^{-\bar{\beta}_1}[-\bar{\beta}_1\sin(n\pi+\bar{\gamma}) - (n\pi+\bar{\gamma})\cos(n\pi+\bar{\gamma})] + n\pi(1+\bar{\beta}_1) + \bar{\gamma}}{n\pi[(n\pi+\bar{\gamma})^2 + \bar{\beta}_1^2]^2}$$
$$+ \frac{e^{-\bar{\beta}_1}[\{\bar{\beta}_1^2 - (n\pi+\bar{\gamma})^2\}\cos(n\pi+\bar{\gamma}) - 2\bar{\beta}_1(n\pi+\bar{\gamma})\sin(n\pi+\bar{\gamma})] - \bar{\beta}_1^2 + (n\pi+\bar{\gamma})^2}{[(n\pi+\bar{\gamma})^2 + \bar{\beta}_1^2]^2} \quad (34)$$

J_2 is obtained from equation (34) simply by replacing $\bar{\gamma}$ by $-\bar{\gamma}$ throughout.

For $S_f(\omega)$ the Bull's model is used, because it gives the best emperical fit to the experimental data obtained by him [27] and others [24] and also because his model and models similar to his have been used by several other authors.

3. Theoretical Results and Discussion

To obtain the influence of various parameters on the response, a composite two layer beam with the elastic layer of aluminium and the viscoelastic layer of PVC is considered. The effect of various geometrical and physical parameters on the beam response is evaluated on the basis of equation (25) and the same is discussed below. The beam is considered to be excited at its mid-point, i. e. $\bar{a} = 0{,}5$.

Figures 3 and 4 show the effect of geometrical parameters on the response quantity $E[w^2]/S_o$, while figures 5, 8 to 10 show the effect of variation of the physical parameters on the response. The response quantity considered here represents the ratio of the mean square value of the displacement $E[w^2]$ to that of the constant value of the spectral density S_o of the white noise excitation.

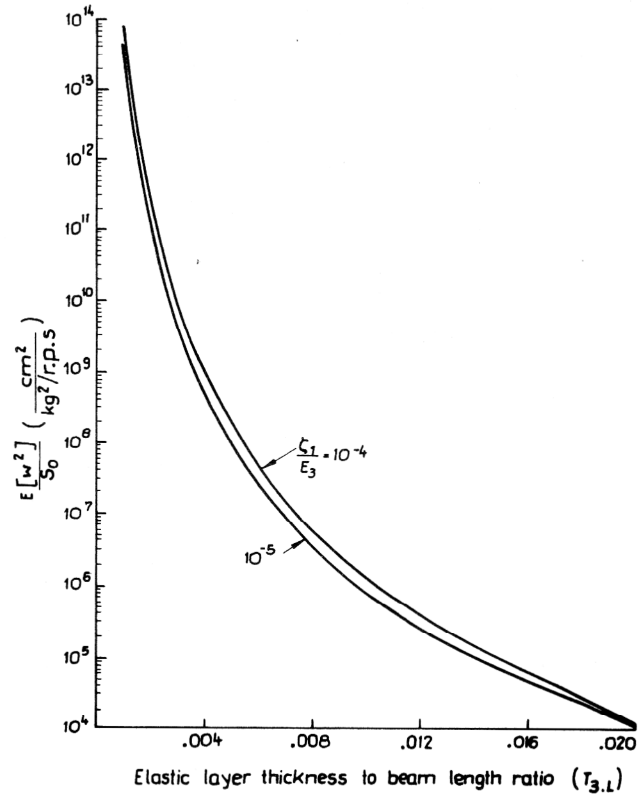

Figure 4
Variation of the ratio $E[w^2]/S_o$ with $T_{3.L}$.

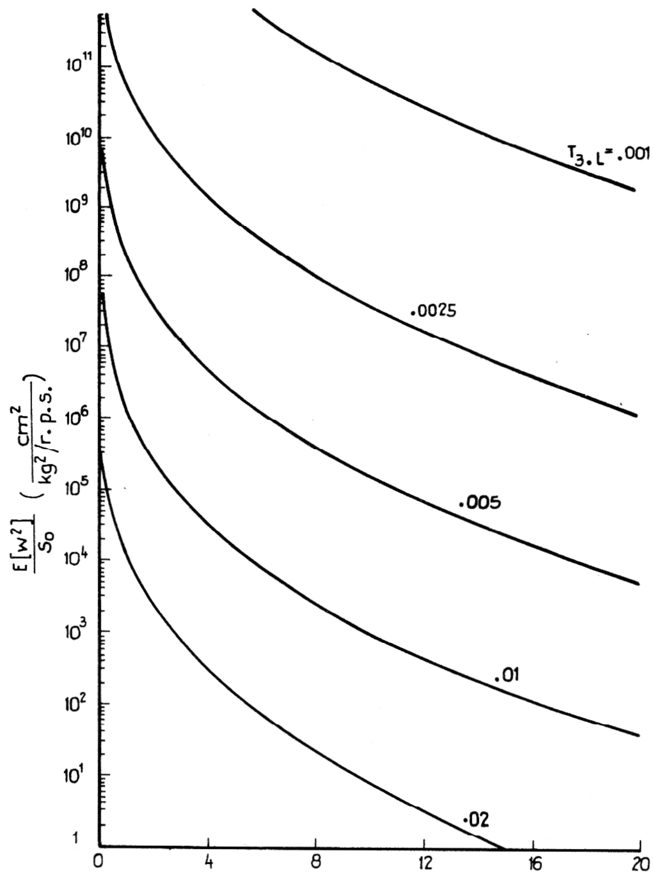

Figure 3
Variation of the ratio $E[w^2]/S_o$ with $T_{2.3}$.

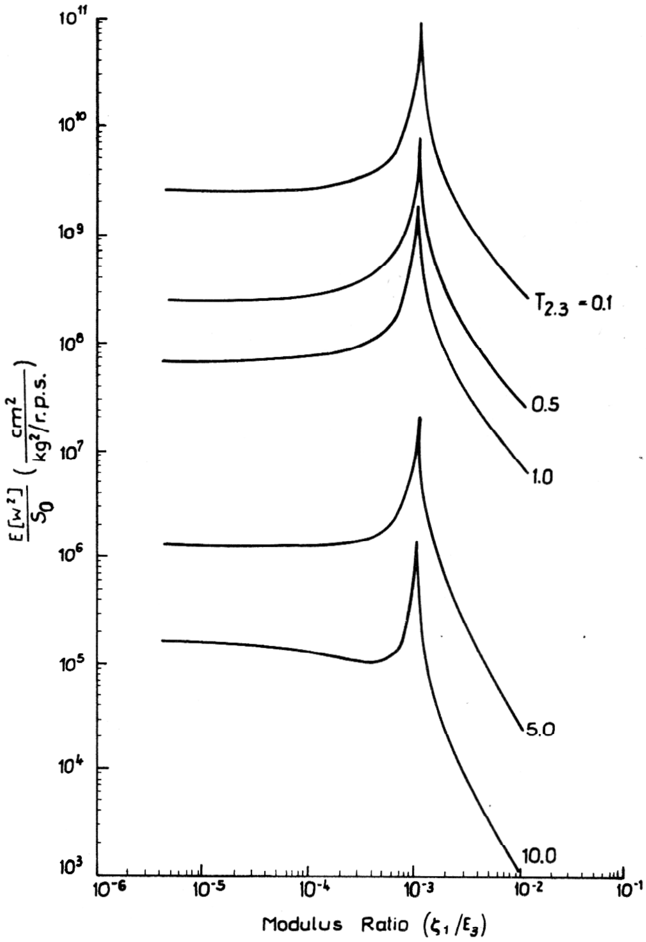

Figure 5
Variation of the ratio $E[w^2]/S_o$ with ζ_1/E_3 ($\zeta_3 = 330.06$ kg/cm^2, $\eta_2 = 0.207$ kg sec/cm^2, $\eta_3 = 1.038$ kg sec/cm^2).

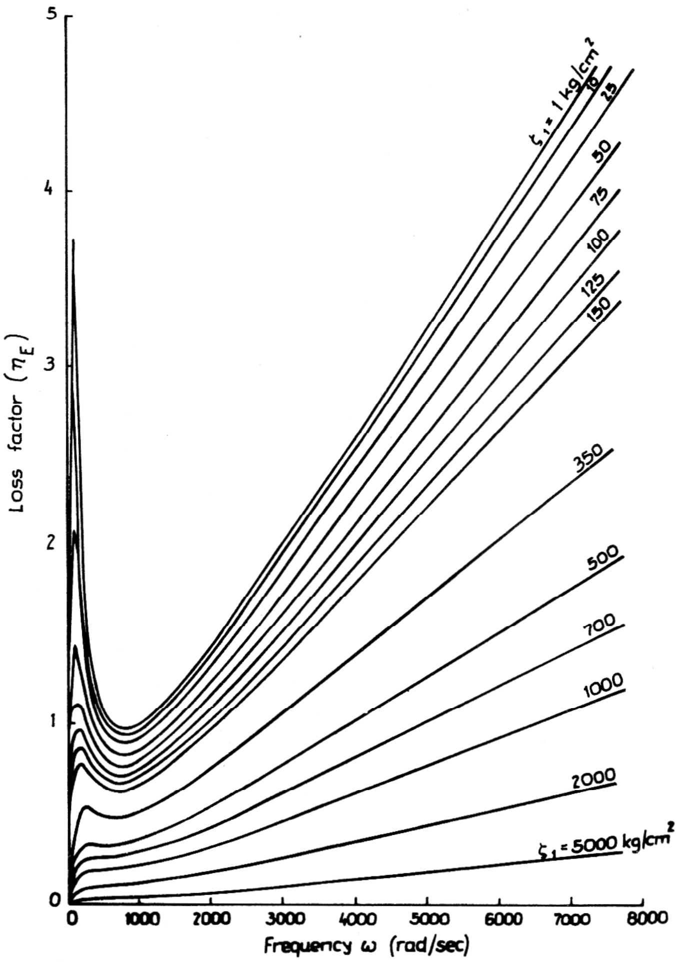

Figure 6
Variation of η_E with ω and ζ_1 (ζ_3 = 330.06 kg/cm^2, η_2 = 0.207 kg sec/cm^2, η_3 = 1.038 kg sec/cm).

In figure 3 the response ration E $[w^2]/S_o$ is found to decrease with the increase in the layer thickness ratio $T_{2.3}$ for all values of $T_{3.L}$ considered, but the rate of decrease is rapid at low values of $T_{2.3}$ and slow at high values. The initial rapid rate of decrease is due to the increase of the beam loss factor with the increase of $T_{2.3}$. As the increase of $T_{2.3}$ is continued, the beam loss factor reaches a maximum and then decreases at a fairly uniform rate [28]. This decrease of loss factor would tend to increase the response, but it is offset by the increase in stiffness associated with the increase in $T_{2.3}$, thereby resulting in a slowly decreasing nature of the response for the continued increase in the thickness ratio $T_{2.3}$.

The variation of the response ratio with the ratio $T_{3.L}$ of the elastic layer thickness to beam length is shown in figure 4. From this figure it is seen that, as expected, the response of the beam increases with increase in beam span (lower values of $T_{3.L}$). The effect of change of ζ_1/E_3 on the response is not significant.

The effect of the geometrical parameters on the response of the two layer viscoelastic beam discussed above is similar to that for sinusoidal and impact excitations [3, 8, 9].

The physical parameters considered are the elements of the viscoelastic model, because a change in any one of the elements results in a change in the physical characteristics of the viscoelastic material. In figures 5, 8 to 10 is illustrated the effect of change of each of the four elements separately on the beam response. Only one of the elements is varied at a time, while the other three are kept at fixed values indicated in each of the figures. Thus in figure 5 the modulus ration ζ_1/E_3 varied for several values of the thickness ration $T_{2.3}$. The response ratio E $[w^2]/S_o$ is seen to behave in a typical manner unlike that for sinusoidal and impact excitations. The response increases gradually at low values of ζ_1/E_3 and it attains a peak value at $\frac{\zeta_1}{E_3} = 10^{-3}$. Further increase of this ratio causes a rapid decrease of the response from the peak value. The variation of response is similar for all values of $T_{2.3}$ considered herein. The typical variation of response can be explained with the help of figures 6 and 7. From figure 6 it is seen, that at all frequencies the loss factor is high for small values of ζ_1, which causes an initial low response. The loss factor subsequently decreases with the increase in the value of ζ_1, but at the same time, as figure 7 indicates, higher values of ζ_1, give rise to large values of storage modulus, i. e. more rigidity to the

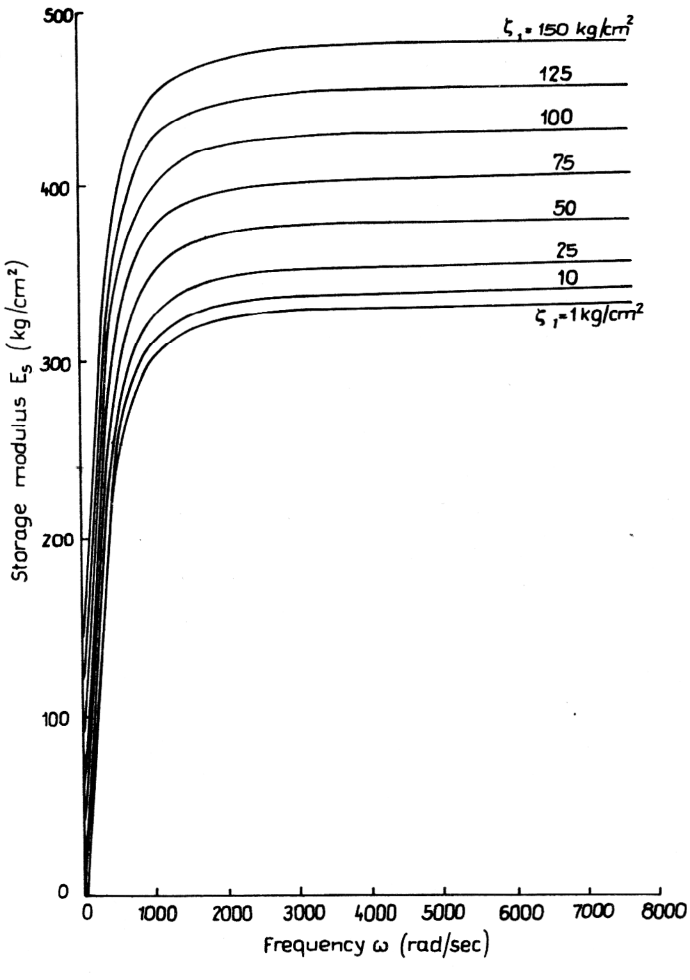

Figure 7
Variation of E_s with ω and ζ_1 (ζ_3 = 330.06 kg/cm^2, η_2 = 0.207 kg sec/cm^2, η_3 = 1.038 kg sec/cm^2).

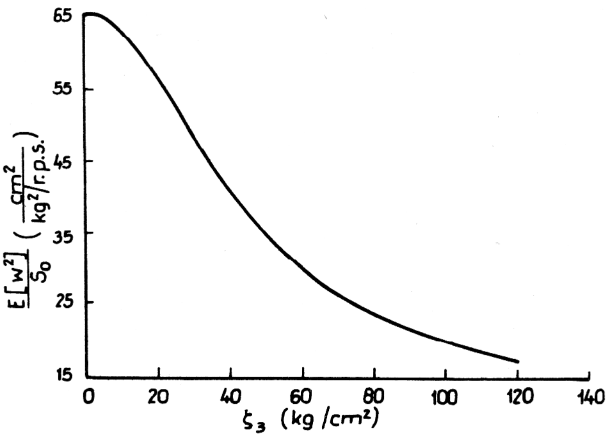

Figure 8
Variation of the ratio E $[w^2]/S_0$ withe the model element ζ_3 (ζ_1 = 275.94 kg/cm^2, η_2 = 0.207 kg sec/cm^2, η_3 = 1.038 kg sec/cm^2).

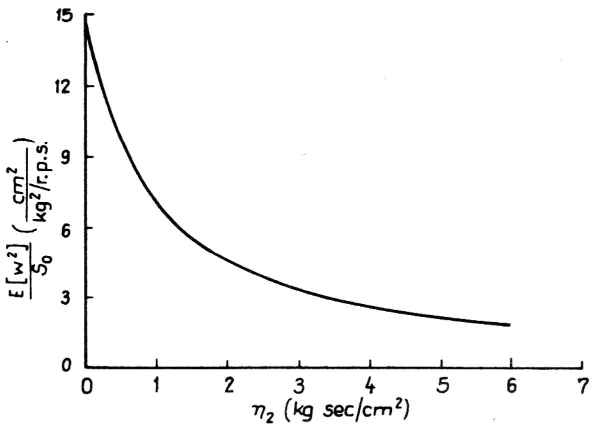

Figure 9
Variation of the ratio E $[w^2]/S_0$ with the model element η_2 (ζ_1 = 275.94 kg/cm^2, ζ_3 = 330.06 kg/cm^2, η_3 = 1.038 kg sec/cm^2).

beam, which leads to the reduction in response. Thus at some intermediate value of ζ_1, which in this case is 700 kg/cm^2, the response attains a peak value. Thereafter the response decreases because of the dominance of the increase in rigidity over the effect due to decrease in loss factor.

Figure 8 shows the variation of the response ratio E $[w^2]/S_0$ with the variation of the model element ζ_3. The response ratio is seen to decrease continuously with increasing values of ζ_3.

Figures 9 and 10 give the variation of the response ratio with the model elements η_2 and η_3 respectively. In both cases the reponse in seen to decrease continonosly with increasing values of each of the elements.

For turbulent boundary layer excitation the response quantity E $[w^2]$ is considered and it is found that the variation of response with the various geometrical and physical parameters is similar to that given in the above figures 3 to 5 and 8 to 10 for the white noise excitation case.

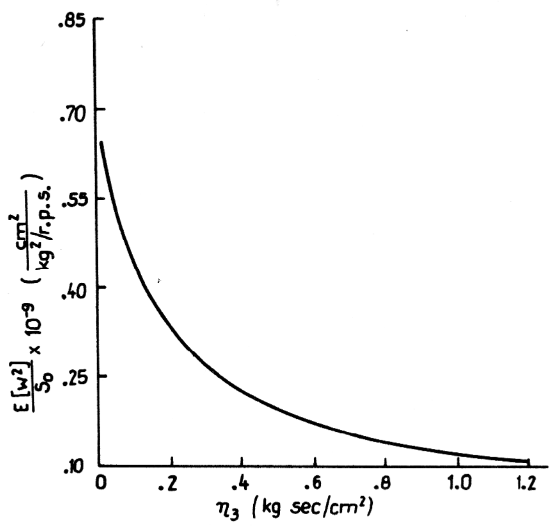

Figure 10
Variation of the ratio E $[w^2]/S_0$ with the model element η_3 (ζ_1 = 275.94 kg/cm^2, ζ_3 = 330.06 kg/cm^2, η_2 = 0.207 kg sec/cm^2).

4. Experimental Verification

A few experiments are conducted on test samples of the two layer elastic-viscoelastic beams for verifying some of the theoretical results. Both white noise and turbulent boundary layer excitations are used. The details of the

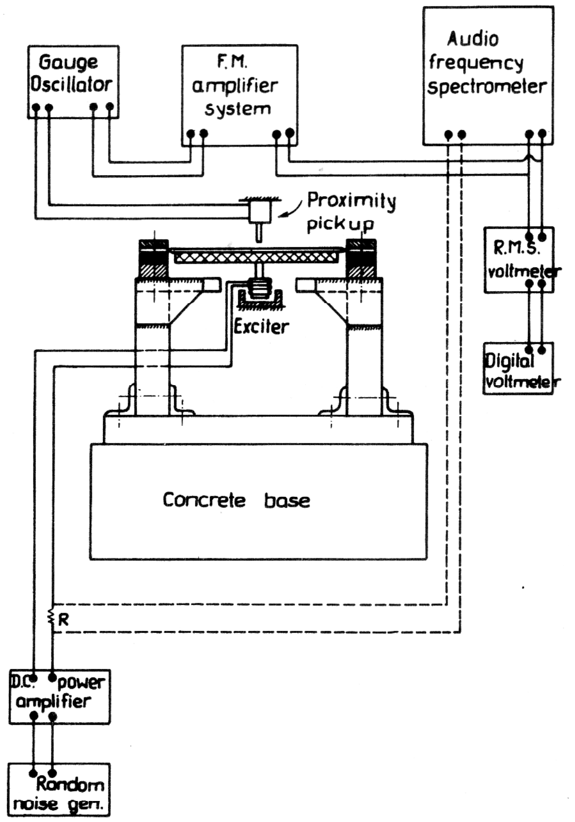

Figure 11
Block diagramm of testing equipment for point load random excitation.

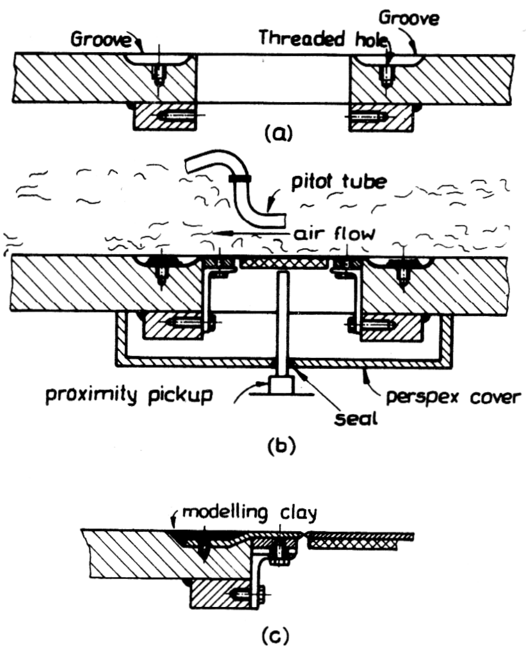

Figure 12
Wind tunnel test section for turbulent boundary layer excitation. (a) Details of the mild steel wall-plate, (b) specimen in position with testing arrangement and (c) details of fixing of beam end.

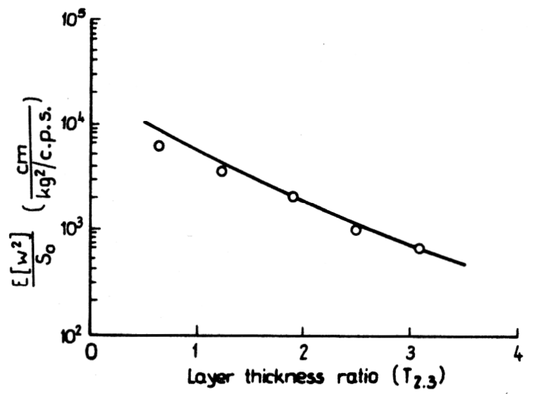

Figure 13
Variation of the ratio $E[w^2]/S_o$ with T_{23} for point load excitation at $\bar{a}$ = 0,5 (o experimental, —— theoretical).

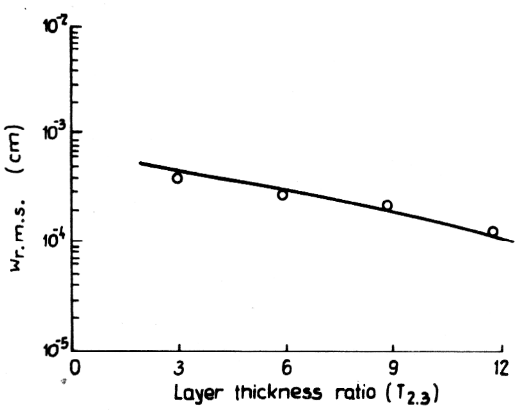

Figure 14
Variation of r. m. s. displacement with T_{23} for turbulent boundary layer excitation (o experimental, —— theoretical).

specimens, tested for both types of excitations, are given in Appendix I. For white noise excitation a white noise generator is employed and the detailed arrangement of the instrumentation is shown in figure 11. For the turbulent boundary layer excitation the test specimen is mounted on the wall of a wind tunnel as shown in figure 12, and the necessary instrumentation is used in the same manner as shown in figure 11.

The experimental results thus obtained are plotted in figures 13 and 14 and are shown by small circles, while the theoretical results are shown by continuous lines.

5. Conclusions

The agreement between the experimental and theoretical values is reasonably good. It is seen from figures 3 to 5 and 8 to 10, that the response to random excitations can be suitably controlled by applying an unconstrained viscoelastic layer over an elastic main layer. The values of the elements of the model can be suitably selected to obtain a certain desired level of the response. Similarly suitable values of the layer thickness ratio can be selected for a given viscoelastic material, so that the response is kept within a desired level.

APPENDIX I: DETAILS OF BEAM SPECIMENS TESTED

S.	No.	L (cm)	b (cm)	Layer No. (i)	H_i (cm)	Material
a)	White noise random excitation					
	1.	30	2.5	2	0.050	PVC
				3	0.081	Al
	2.	30	2.5	2	0.100	PVC
				3	0.081	Al
	3.	30	2.5	2	0.150	PVC
				3	0.081	Al
	4.	30	2.5	2	0.200	PVC
				3	0.081	Al
	5.	30	2.5	2	0.250	PVC
				3	0.081	Al
b)	Turbulent boundary layer excitation					
	1.	13	2.5	2	0.050	PVC
				3	0.017	Al
	2.	13	2.5	2	0.100	PVC
				3	0.017	Al
	3.	13	2.5	2	0.150	PVC
				3	0.017	Al
	4.	13	2.5	2	0.200	PVC
				3	0.017	Al

APPENDIX II: LIST OF SYMBOLS

a	coordinate of point load excitation
A_i	area of cross-section of layer i
b	width of beam
B_i	flexural rigidity of layer i (= $E_i I_i$)
D	operator $\partial/\partial t$
E_i	Young's modulus of layer i
E[ ]	mathematical expectation of the random quantity inside the brackets

$f(x, t)$	intensity of external loading
$F(t)$	external point load
$h_w(n, t)$	impulse response function
$H_w(n, \omega)$	transfer function
H_i	semi thickness of layer i
I_i	section area moment of layer i
K_i	longitudinal stiffness of layer i ($= E_i A_i$)
L	length of beam
n, m	modal number
$Q_f(\bar{\zeta}, \omega)$	spatial correlation coefficient
$R_f(\bar{\zeta}, \tau - T_1 + T_2)$	correlation of 'f' for turbulent boundary layer excitation
$\underline{S}_f(\bar{\zeta}, \omega)$	spectral density of pressure fluctuations
S_o	spectral density of white noise random excitation
$S_f(\omega)$	spectral density of 'f'
$S_w(\omega)$	spectral density of 'w'
t, t_1, t_2, T_1, T_2	time variable
$T_{2.3}$	ratio of thickness of layer 2 to that of layer 3
$T_{3.L}$	ratio of thickness of layer 3 to the span of beam
x, x_1, x_2	spatial coordinate
U_c	convection velocity of air
w	transverse displacement of beam
$\alpha, \beta, \gamma, \psi$	constants of the viscoelastic model
$\zeta_1, \zeta_3, \eta_2, \eta_3$	elements of the viscoelastic model
$\xi_1, \xi_2, \zeta, \eta$	spatial variable
ρ_i	density of layer i
Φ	slope of beam
μ	mass per unit length of beam $= 2b(\rho_2 H_2 + \rho_3 H_3)$
ω	circular frequency
τ, τ_1, τ_2	time variable

Viscoelastic layer is designated as layer 2 while elastic layer as layer 3. A bar over a spatial variable or coordinate represents its ratio to beam span L.

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