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Mathematical modeling and analysis of elastic waves in a thermo piezoelectric multilayered rotating composite rod with LEMV/CFRP interface

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Abstract: In this present paper, we form the mathematical model for wave propagation in a thermo piezoelectric multilayered rotating composite rod made of inner and outer piezoelectric layer bonded together by Linear Elastic Materials with voids (LEMV).To uncouple the equation of motion, electric and heat conduction equations, displacement potential functions are introduced. The frequency equations are obtained for longitudinal and flexural modes of vibration and are studied numerically for heat conducting PZT-5A material.The computed non-dimensional frequency is presented in the form of dispersion curves against various physical quantities. Adhesive layer Linear Elastic Materials with Voids (LEMV) is compared with Carbon Fibre Reinforced Polymer (CFRP).We found that the frequency wave characteristics are more stable and realistic in the presence of thermal, electrical and the rotation parameters

Keywords: Piezoelectric cylinder, Thermal cylinders, Rotating rod, Vibration, Stress analysis, LEMV, CFRP, multilayered structures, Composite cylinder.

1 Introduction

Solid state materials in engineering is hovering to provide significant inputs to the areas of constructive design of structural components as well as creating trends of its own. The cross disciplinary fields of mechanical materials and interfacial component are shows potential developments. Further interdisciplinary materials research will likely to continue to acquiesce materials with improved properties for application that is both common place and specialized. piezoelectric polymers allow their use in a massive amount of compositions and geometrical shapes for a huge variety of applications from transducers in acoustics, ultrasonics and hydrophone applications to resonators in band pass filters, power supplies, delay lines, medical scans and some industrial non-destructive testing instruments.The frequency responses of rotating cylindrical structures has numerous applications in a variety of fields of science and technology, specifically, submarine structures, pressure vessel, bore wells, ship building industries and have many other engineering applications.

Honarvar et al. (2007) developed a wave propagation model for a transversely isotropic cylinders and verified their physical characteristics. Thermo-piezoelectric materials are intelligent materials that display individual electro-mechanical coupling.In view of this, Paul and Raman (1991) studied wave propagation in a hollow pyroelectric circular cylinder of crystal class 6.Mindlin (1974)analyzed equations of high frequency vibrations of thermopiezoelectric crystal plates.Also Paul and Raman (1993) discussed wave propagation in a pyroelectric cylinder of arbitrary cross section with a circular cylindrical cavity.Storozhev (2013),investigated propagation of electro elastic waves in multilayer piezoelectric cylinders with a sector notch.Nelson and Karthikeyan (2008a) discussed axisymmetric vibration of pyrocomposite solid cylinder.Nelson and Karthikeyan (2008b)studied axisymmetric vibration of pyrocomposite hollow cylinder.Shulga (2002) observed Propagation of harmonic waves in anisotropic piezoelectric cylinders:Compound waveguides.Hua et al. (2013) observed guided wave propagation and focusing in multi-layer pipe with viscoelastic coating and infinite soil media.Tasdemirci et al. (2004) discussed Stress wave propagation effects in two-and three-layered composite materials.Singh and Saxena (1995) discussed axisymmetric vibration of a circular plate with double linear variable thickness.Presented clear statement for modal shapes and natural frequencies of materials into account the effects of length, shear deformation, and rotary inertia.Abd-Alla and Mahmoud (2010) observed magneto-thermoelastic problem in rotating non homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model.El-Naggar et al. (2002) discussed thermal stresses in a rotating non-homogeneous orthotropic hollow cylinder.Abd-Alla et al. (2000) examined thermal stresses in a non-homogeneous orthotropic elastic multilayered cylinder.Abd-All et al. (1999) observed transient thermal stresses in a rotating non-homogeneous cylindrically orthotropic composite tubes. Selvamani (2015) has discussed wave propagation in a rotating disc of polygonal cross-section immersed in an inviscid fluid.Several researches are performedto incorporate the interfacial material in analysis of composite multilayered mechanical structure.Among that, the study of Cowin and Nunziato (1983) with Linear Elastic Materials with Voids as interface bonding materialis noted. However, the rotational effect on the thermo-electro-mechanical vibration responses of multiylarerd cylinder with different bonding material(LEMV/CFRP) not been reported thus far. Therefore, the objective of the present work is to investigate the influence of the thermos piezo elasticity and rotation on the vibrating of

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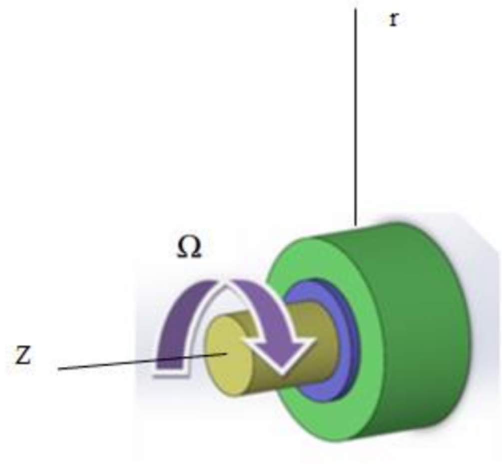

Fig. 1: Geometry which shows the problem

piezoelectric multilayered cylinder with different bonding material.
The present article intended to revise mathematical model for wave propagation in a thermo piezoelectric multilayered rotating composite rod made of inner and outer piezoelectric layer bonded together by linear Elastic materials with voids (LEMV).To uncouple the equation of motion, electric and heat conduction equations, displacement potential functions are introduced. The frequency equations are obtained for longitudinal and flexural modes of vibration and are studied numerically for heat conducting PZT-5A material.The computed non-dimensional frequency is presented in the form of dispersion curves against various physical quantities. Adhesive layer LEMV is compared with Carbon Fibre Reinforced Polymer (CFRP).

2 Formulation of problem

In this section we consider a homogeneous transversely isotropic, thermally and electrically conducting piezoelectric rotating composite rod of infinite length with uniform temperature T_0 in the undistributed state initially. The composite rod is assumed to be rotating with consistent angular velocity $\bar{\Omega}$. In cylindrical coordinates (r, z), the equations of motion in axisymmetric direction and absence of body force and including the Coriolis Effect and centripetal forces are

$$\sigma^l_{rr,r} + \sigma^l_{rz,z} + r^{-1}\sigma^l_{,rr} + \rho(\bar{\Omega} \times (\bar{\Omega} \times \bar{u}) + 2(\bar{\Omega} \times \bar{u}_{,t})) = \rho u_{,tt} \tag{1a}$$

$$\sigma^l_{rz,r} + \sigma^l_{zz,z} + r^{-1}\sigma^l_{,rz} + \rho(\bar{\Omega} \times (\bar{\Omega} \times \bar{u}) + 2(\bar{\Omega} \times \bar{u}_{,t})) = \rho w_{,tt} \tag{1b}$$

The electric displacement equation is given as

$$\frac{1}{r}\frac{\partial}{\partial r}(rD^l_r) + \frac{\partial}{\partial z}(D^l_z) = 0 \tag{1c}$$

The heat conduction equation is defined as

$$K_{11}(T^l_{,rr} + r^{-1}T^l_{,r} + r^{-2}T^l_{,\theta\theta}) + K_{33}T^l_{,zz} - \rho^l c_v T_{,t} = T_0\frac{\partial}{\partial t}[\beta_1(e^l_{,rr} + e^l_{,\theta\theta}) + \beta_3 e^1_{,zz} - p_3\phi_{,z}] \tag{1d}$$

Relation between the stress and strain of mechanical and electrical field is

$$\sigma^l_{rr} = c_{11}e^l_{rr} + c_{12}e^l_{\theta\theta} + c_{13}e^l_{zz} - \beta_1 T^l - e_{31}E^l_z \tag{2a}$$

$$\sigma^l_{zz} = c_{13}e^l_{rr} + c_{13}e^l_{\theta\theta} + c_{33}e^l_{zz} - \beta_3 T^l - e_{33}E^l_z \tag{2b}$$

$$\sigma^l_{rz} = c_{44}e^l rz \tag{2c}$$

$$D_r = e_{15}e^l_{rz} + \varepsilon_{11}E^l_r \tag{2d}$$

$$D_z = e_{31}(e^l_{rr} + e^l_{\theta\theta}) + e_{33}e^l_{zz} + \varepsilon_{33}E^l_z + p_3 T \tag{2e}$$

where $\sigma^l_{rr}, \sigma^l_{r\theta}, \sigma^l_{rz}, \sigma^l_{\theta\theta}, \sigma^l_{zz}, \sigma^l_{z\theta}$ denotes mechanical stress component, $e^l_{\theta\theta}, e^l_{r\theta}, e^l_{z\theta}, e^l_{rz}$ are the strain components, T^l is the temperature change about the equilibrium temperature,$c_{11}, c_{12}, c_{13}, c_{33}, c_{44}, c_{66}$ are the five elastic constants,β_1, β_3 and K_1, K_3 respectively thermal expansion coefficients and thermal conductivities along and perpendicular to the symmetry, ρ is the mass density, c_v is the specific heat capacity,p_3 is the pyroelectric effect.

The strains e_{ij} are related to the displacements given by

$$e^l_{rr} = u^l_{,r} e^l_{\theta\theta} = r^{-1}(u^l + v^l_{,\theta}), e^l_{zz} = w^l_{,rz} \tag{3a}$$

$$e^l_{r\theta} = v^l_r - r^{-1}(v^l - u^l_{,\theta}), \tag{3b}$$

$$e^l_{z\theta} = v^l_{,z} + r^{-1} w^l_{,r\theta} e^l_{rz} = w^l_{,r} + u^l_{,z} \tag{3c}$$

Substitution of the (3) and (2) into (1) results in the following three dimensional equation of motion and heat conduction:

$$c_{11}(u^l_{,rr} + r^{-1}u^l_{,r} - r^{-2}u) + c_{44}u^l_{,zz} + (c_{44}+c_{13})w^l_{rz} + (e_{31}+e_{15})E^l_{rz} - \beta_1 T_{,r} + \rho(\Omega^2 u^l + 2\Omega u_{,t}) = \rho u^l_{,tt} \tag{4a}$$

$$c_{44}(w^l_{rr} + r^{-1}w^l_{,r}) + r^{-1}(c_{44}+c_{13})u^l_{,z} + (c_{44}+c_{13})u^l_{,rz} + c_{33}w^l_{,zz} + e_{33}E_{,zz}(E_{,rr} + r^{-1}E_{,r})$$
$$-\beta_3 T_{,z} + \rho(\Omega^2 w^l + 2\Omega u_{,t}) = \rho w^l_{,tt} \tag{4b}$$

$$e_{15}(w^l_{rr} + r^{-1}w^l_{,r}) + e_{31} + e_{15}(u^l_{,rz} + r^{-1}u^l_{,z}) + e_{33}w^l_{,zz} - \eta_{33}E^l_{,zz} - \eta_{11}(E^l_{,rr} + r^{-1}E^l_{,r}) + p_3 T^l_{,z} = 0 \tag{4c}$$

$$K_{11}(T^l_{,rr} + r^{-1}T^l_{,r} + r^{-2}T^l_{,\theta\theta}) + K_{33}T^l_{,zz} - \rho^l c_v T_{,t} = T_0 \frac{\partial}{\partial t}[\beta_1(u^l_{,r} + r^{-1}v^l_{,\theta} + r^{-1}u^l) + \beta_3 w^l_{,z}] - p_3 E^l_{,z} \tag{4d}$$

Solution of the field Equation

$$u^l(r,z,t) = (\phi^l_{,r}) \exp i(kz + \omega t) \tag{5a}$$

$$w^l(r,z,t) = \frac{i}{a} W^l \exp i(kz + \omega t) \tag{5b}$$

$$E^l(r,z,t) = \frac{ic_{44}}{ae_{33}} V^l \exp i(kz + \omega t) \tag{5c}$$

$$T^l(r,z,t) = \frac{c_{44}}{\beta_3 a^2} T^l \exp i(kz + \omega t) \tag{5d}$$

Here $i = \sqrt{-1}$, k is the wave number,ω is the frequency,$u^l, w^l E^l, T^l$ are all displacement potentials, electric conduction and thermal change. By introducing the dimensionless quantities such as $x = \frac{r}{a}$,$\eta = ka$, $\Omega^2 = \frac{\rho\omega^2 a^2}{c_{44}}$,$\bar{c}_{11} = \frac{c_{11}}{c_{44}}$,
$\bar{c}_{13} = \frac{c_{13}}{c_{44}}$,$\bar{c}_{33} = \frac{c_{33}}{c_{44}}$,$\bar{c}_{66} = \frac{c_{66}}{c_{44}}$,$\bar{\beta} = \frac{\beta_1}{\beta_3}$,$\bar{k}_i = \frac{(\rho c_{44})^{\frac{1}{2}}}{\beta_3^2 T_0 a\Omega}$,$\bar{d} = \frac{\rho c_v c_{44}}{\beta_3^2 T_0}$,$\bar{p}_3 = \frac{p_3 c_{44}}{\beta_3 e_{33}}$
Substituting (5) in (4) we obtain

$$(\bar{c}_{11}\nabla^2 + (\Omega^2 - \zeta^2))\phi^l - \zeta(1+\bar{c}_{13})W^l - \zeta(\bar{e}_{31}+\bar{e}_{15})E^l - \bar{\beta}T^l = 0 \tag{6a}$$

$$\zeta(1+\bar{c}_{13})\nabla^2\phi^l + (\nabla^2 + (\Omega^2 - \zeta^2\bar{c}_{33}))W^l + (\bar{e}_{33}\nabla^2 - \zeta^2)E^l - \zeta T^l = 0 \tag{6b}$$

$$\zeta(\bar{e}_{31}+\bar{e}_{15})\nabla^2\phi^l + (\bar{e}_{15}\nabla^2 - \zeta^2)W^l + (\zeta^2\bar{e}_{33} - \bar{\epsilon}\nabla^2)E^l + \zeta\bar{p}_3 T^l = 0 \tag{6c}$$

$$\bar{\beta}\nabla^2\phi^l - \zeta W^l + \zeta\bar{p}_3 E^l + (\bar{d} + i\bar{k}_1\nabla^2 + i\bar{k}_3\zeta^2)T^l = 0 \tag{6d}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + x^{-1}\frac{\partial}{\partial x}$

$$\begin{vmatrix} [\bar{c}_{11}\nabla^2 + (\Omega^2 - \zeta^2)] & -\zeta(1+\bar{c}_{13}) & \zeta(\bar{e}_{31}+\bar{e}_{15}) & -\bar{\beta} \\ \zeta(1+\bar{c}_{13})\nabla^2 & [\nabla^2 + (\Omega^2 - \bar{c}_{33}\zeta^2)] & (\bar{e}_{33}\nabla^2 - \zeta^2) & -\zeta \\ \zeta(\bar{e}_{31}+\bar{e}_{15})\nabla^2 & \bar{e}_{15}\nabla^2 - \zeta^2 & (\zeta^2\bar{e}_{33} - \bar{\epsilon}\nabla^2) & \zeta\bar{p}_3 \\ \bar{\beta}\nabla^2 & -\zeta & \zeta\bar{p}_3 & (\bar{d} + i\bar{k}_1\nabla^2 - i\bar{k}_3\zeta^2) \end{vmatrix} (\phi^l_n, W^l_n, E^l, T^l_n)^T = 0 \tag{7}$$

Evaluating the given in (7), we obtain a partial differential equation of the form

$$(p^l\nabla^8 + q^l\nabla^6 + r^l\nabla^4 + s^l\nabla^2 + t^l)(\phi^l, W^l, E^l, T^l)^T = 0 \tag{8}$$

Factorizing the relation given in (8) into biquadrate equation for $(\alpha_j^2 a)^2$, $j = 1,2,3,4$ the solutions for the symmetric modes are obtained as

$$\phi^l = \sum_{j=1}^{4}[A^l_j J_n(\alpha^l_j ax)]cosn\theta \tag{9a}$$

$$W^l = \sum_{j=1}^{4}[a^l_j A^l_j J_n(\alpha^l_j ax)]cosn\theta \tag{9b}$$

$$E^l = \sum_{j=1}^{4}[b^l_j A^l_j J_n(\alpha^l_j ax)]cosn\theta \tag{9c}$$

$$T^l = \sum_{j=1}^{4}[c^l_j A^l_j J_n(\alpha^l_j ax)]cosn\theta \tag{9d}$$

Here $(\alpha_j^l ax) > 0, for(j = 1, 2, 3, 4)$ are the roots of algebraic equation

$$(p^l(\alpha_j^l ax)^8 + q^l(\alpha_j^l ax)^6 + r^l(\alpha_j^l ax)^4 + s^l(\alpha_j^l ax)^2 + t^l)(\phi^l, W^l, E^l, T^l)^T = 0$$

Here a_j^l, b_j^l, c_j^l are the arbitrary constants and $J_n(\alpha_l^j ax)$ denotes the Bessel functions first kind of order n. The constants a_j^l, b_j^l, c_j^l are calculated using the following relations with $\nabla^2 = \alpha_j^l ax$

$$\begin{aligned}(\bar{c}_{11}\nabla^2 + (\Omega^2 - \zeta^2)) - \zeta(1 + \bar{c}_{13})a_j^l - \zeta(\bar{e}_{31} + \bar{e}_{15})b_j^l - \bar{\beta}c_j^l = 0\\ \zeta(1 + \bar{c}_{13})\nabla^2 + (\nabla^2 + (\Omega^2 - \zeta^2\bar{c}_{33}))a_j^l + (\bar{e}_{33}\nabla^2 - \zeta^2)b_j^l - \zeta c_j^l = 0\\ \zeta(\bar{e}_{31} + \bar{e}_{15})\nabla^2 + \bar{e}_{15}\nabla^2 - \zeta^2)a_j^l + \zeta\bar{p}_3 b_j^l + (\zeta^2\bar{\epsilon}_{33} - \bar{\epsilon}\nabla^2)c_j^l = 0\\ \bar{\beta}\nabla^2 - \zeta a_j^l + (\bar{d} + i\bar{k}_3\zeta^2)b_j^l + \zeta\bar{p}_3 c_j^l = 0\end{aligned}$$

3 Solution for linear elastic materials with voids

The displacement equations of motion and equation of equilibrated inertia for an isotropic LEMV are

$$(\lambda + 2\mu)(u_{,rr} + r^{-1}u_{,r} - r^{-2}u) + \mu u_{,zz} + (\lambda + \mu)w_{,zz} + \beta\aleph_{,r} = \rho u_{,tt} \tag{10a}$$

$$(\lambda + \mu)(u_{,rz} + r^{-1}u_{,z}) + \mu(w_{,rr} + r^{-1}w_{,r}) + (\lambda + 2\mu)w_{,zz} + \beta\aleph_{,z} = \rho w_{,tt} \tag{10b}$$

$$-\beta(u_{,r} + r^{-1}u) - \beta w_{,z} + \alpha(\aleph_{,rr} + r^{-1}\aleph_{,r} + \aleph_{,zz}) - \delta k\aleph_{,tt} - \omega\aleph_{,t} - \xi\aleph = 0 \tag{10c}$$

u,v,w represents displacements components along r, θ, and z directions $\alpha, \beta, \xi, \omega$ and k are LEMV material constants characterizing the core in the equilibrated inertial state,ρ is the density and λ, μ are the lame constants and $\aleph$ is the new kinematical variable associated with another material without voids. The stress in the LEMV core materials are

$$\begin{aligned}\sigma_{,rr} = (\lambda + 2\mu)u_{,r} + \lambda r^{-1}u + \lambda w_{,z} + \beta\aleph\\ \sigma_{,rz} = \mu(u_{,r} + w_{,r})\end{aligned}$$

The solution of (10) is taken as

$$u = U_{,r}\exp i(kz + wt) \tag{11a}$$

$$w = (\frac{i}{h})W\exp i(kz + wt) \tag{11b}$$

$$\aleph = (\frac{1}{h^2})\aleph\exp i(kz + wt) \tag{11c}$$

Substitution of the (11) into (10) with non-dimensional variables x and ζ, can be reduced as

$$\begin{vmatrix}(\lambda + 2\mu)\nabla^2 + F_1 & -F_2 & F_3\\ F_2\nabla^2 & \bar{\mu}\nabla^2 + F_4 & F_5\\ -F_3\nabla^2 & F_5 & \alpha\nabla^2 + F_6\end{vmatrix}(U, W, \aleph)^T = 0 \tag{12}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x}$ $F_1 = \frac{\rho}{\rho^1}(ch)^2 - \bar{\mu}\zeta^2$ $F_2 = (\bar{\lambda} + \bar{\mu})\zeta$,$F_3 = \bar{\beta}$ $F_4 = \frac{\rho}{\rho^1}(ch)^2 - (\bar{\lambda} + \bar{\mu})\zeta^2$ $F_5 = \bar{\beta}\zeta$ $F_6 = \frac{\rho}{\rho^1}(ch)^2\bar{k} - \bar{\alpha}\zeta^2 - i\bar{\omega}(ch) - \bar{\xi}$ and $\bar{\lambda} = \frac{\lambda}{c_{44}^l}$,$\bar{\mu} = \frac{\mu}{c_{44}^l}$,$\bar{\alpha} = \frac{\alpha}{h^2 c_{44}^l}$,$\bar{\beta} = \frac{\beta}{c_{44}^l}$,$\bar{\xi} = \frac{\xi}{c_{44}^l}$,$\bar{\omega} = \frac{\omega}{h}(c_{44}^l\rho)^{\frac{1}{2}}$,$\bar{K} = \frac{k}{h^2}$. The above (12) can be specified as,

$$(\nabla^6 + P\nabla^4 + Q\nabla^2 + R)(U, W, \aleph) = 0 \tag{13}$$

Thus the solution of (13) can be given as follows,

$$U = \sum_{j=1}^{3}[A_j J_0(\alpha_j ax) + B_j Y_0(\alpha_j x)] \tag{14a}$$

$$W = \sum_{j=1}^{3} d_j[A_j J_0(\alpha_j x) + B_j Y_0(\alpha_j x)], \tag{14b}$$

$$\aleph = \sum_{j=1}^{3} e_j[A_j J_n(\alpha_j x) + B_j Y_n(\alpha_j x)] \tag{14c}$$

$(\alpha_j x)^2$ are the roots of the equation when replacing $\nabla^2 = -(\alpha_j x)^2$. The arbitrary constant d_j and e_j are obtained from

$$\begin{aligned}F_2\nabla^2 + (\bar{\mu}\nabla^2 + F_4)d_j + F_5 e_j = 0\\ -F_3\nabla^2 + F_5 d_j + (\alpha\nabla^2 + F_6)e_j = 0\end{aligned}$$

The governing equation for CFRP core material can be obtained from (10) by taking the void volume fraction $\aleph = 0$ and the lames constants as $\lambda = c_{12}, \mu = \frac{(c_{11} - c_{12})}{2}$.

4 Boundary condition and frequency equation

We consider a wave propagation problem of piezo thermo elastic multilayered rotating rod with adhesive as LEMV/CFRP under the effect of the axisymmetric rotation field. The frequency equations of the problem can be obtained using the following boundary conditions.
(i) At the inner and outer surface of solid rod with free traction
$\sigma^l_{rr} = \sigma^l_{rz} = E^l = T^l = 0$ With $l = 1, 3$.
(ii) At the solid and adhesive interface
$\sigma^l_{rr} = \sigma_{rr}; \sigma^l_{rz} = \sigma_{rz};$
(iii)Thermally insulated and Electrically shorted Boundary conditions
$T^l = 0$ and $E^l = 0$;

Using the values of ϕ^l, W^l, E^l, T^l and $U, W, \aleph$ from the (9) and (14), the above boundary conditions will give a system of homogeneous algebraic equations for the unknown constants $A^l_1, A^l_2, A^l_3, A^l_4, A_1, A_2$ and A_3.This set of equations can be written in the following 20×20 vanishing determinant form

$$|(Y_i j)| = 0, (i, j = 1, 2, 3, ..., 20) at x = x_1, where j = 1, 2, 3, 4 \tag{15}$$

At $x = x_1$ where $j = 1, 2, 3, 4$

$$Y_{1j} = 2\bar{c_{66}} n(n-1)J_n(\alpha^l_j a x_1) + (\alpha^l_j a x_1)J_{n+1}(\alpha^l_j a x_1)] - x^2[(\alpha_j a)^2 \bar{c_{11}} + \zeta \bar{c_{13}} a_j + \zeta b_j + \bar{\beta} c_j]J_n(\alpha^l_j a x_1)$$
$$Y_{1,j+4} = -[2\bar{\mu}(\alpha_j x_1)J_n(\alpha_j x_1) + (\bar{\lambda} + \bar{\mu})(\alpha_j)^2 + \bar{\beta} e_j - \bar{\lambda}\zeta \bar{d}_j]J_0(\alpha_j x_1)$$
$$Y_{2j} = 2n[(\alpha^1_j a x_1)J_{n+1}(\alpha^l_j a x_1) - (n-1)J_n(\alpha^l_j a x_1)]$$
$$Y_{2,j+4} = -\bar{\mu}(\epsilon + d_j)(\alpha_j)J_1(\alpha_j x_1)$$
$$Y_{3j} = (\zeta + a_j + \bar{e}_{15} b_j)[nJ_n(\alpha^1_j a x_1) - (\alpha_j a x)J_{n+1}(\alpha^1_j a x_1)]$$
$$Y_{3,j+4} = -(\alpha_j)J_1(\alpha_j x_1)$$
$$Y_{4,j} = (\bar{e_{15}}\zeta a_j - \bar{\epsilon_{11}} b_j)[nJ_n(\alpha^1_j a x_1) - (\alpha^1_j a x_1)J_{n+1}(\alpha^1_j a x_1)]$$
$$Y_{4,j+4} = -d_j J_0(\alpha_j x_1)$$
$$Y_{5j} = e^l_j J_0(\alpha^l_j x_1)$$
$$Y_{6j} = x_1 J_0(\alpha^l_j x_1) - (\alpha^l_j)J_1(\alpha^l_j x_1)$$
$$Y_{7,j+4} = e_j(\alpha_j)J_1(\alpha^l_j x_1)$$

and the other nonzero elements at the interface $x = x_1$ can be obtained on replacing J_0 by J_1 in the above elements. They are Y(i,j+7),(i=1,2,3,4,5,6,7).The components at the interface $x = x_2$ by varying j from 5 to 7 are replaced the subscript x_1 by x_2 and the other nonzero elements at the interface $x = x_2$ are calculated by replacing J_0 in to J_1 in the above elements. They are Y(i,j+3),Y(i,j+10)(i=8,9,10,11),Y(12,j+10),Y(13,+10) and Y(14,j+3). At the outer surface $x = x_3$ the non zero elements by varying j=11,12,13,14 are replaced subscript x_1 by x_3 in the above elements. The other nonzero element at the interface $x = x_3$ can be obtained on replacing J_0 by J_1 in the above elements. They are Y(i,j+4),(i=15, 16, 17, 18).

5 Numerical computation and discussion

The purpose of the present study is demonstrating the various applications of composite materials with different adhesive core materials. The material chosen for the numerical calculation is PZT-5A. The physical data of PZT-5A are taken from Sharma et al. (2004) and are used for the numerical calculation:
$C_{11} = 13.9 \times 10^{10} Nm^{-2}$;$C_{12} = 7.78 \times 10^{10} Nm^{-2}$;$C_{13} = 7.43 \times 10^{10} Nm^{-2}$;$C_{33} = 11.5 \times 10^{10} Nm^{-2}$;$C_{44} = 2.56 \times 10^{10} Nm^{-2}$;$C_{66} = 3.06 \times 10^{10} Nm^{-2}$;$\beta_1 = 1.52 \times 10^6 NK^{-1}m^{-2}$; $\beta_3 = 1.53 \times 10^6 NK^{-1}m^{-2}$;$T_0 = 298K$;$C_v = 420 jkg^{-1}K^{-1}$; $p_3 = -452 \times 10^{-6} CK^{-1}m^{-2}$;$K_1 = K_3 = 1.5 Wm^{-1}K^{-1}$;$e_{13} = -6.98 Cm^{-2}$;$e_{33} = 13.8 Cm^{-2}$;$e_{15} = 13.4 Cm^{-2}$; $\epsilon_{11} = 60^{10} CN^{-1}m^{-2}$;$\epsilon_{33} = 54.7 \times 10^1 0 CN^{-1}m^{-2}$;$\rho = 7750 Kgm^{-2}$;
The frequency (15) are essentially an implicit transcendental equation of the unknown frequency parameter and wave number for the given boundary conditions. The results of the present investigation are displayed in Figs. $2-7$. In Figs. 2 and 3 the variations of the non-dimensional frequency of a thermo piezoelectric multilayered solid rod with respect to the parameter ($\Omega = 0, 0.2, 0.4$) have been shown for different values of the wave number to the solid rod. From Fig. 2 it is observed that the non-dimensional frequency of the solid rod shows some dispersion from the linear behavior of frequency with respect to Ω for the increasing wave number parameter k.But in Fig. 3 almost linear variation with respect to Ω for the increasing wave number parameter k without thermal impact. In both cases at small values of the parameter k the values of frequency are almost steady for different values of the rotating parameter Ω,where the higher values of k the frequency starting dispersive only including thermal impact but maintain the same behavior for without thermal impact of the rod.

In Figs. 4 and 5 the variations of the non-dimensional frequency of a thermo piezoelectric multilayered solid rod with respect to the parameter ($\Omega = 0, 0.2, 0.4$) have been shown for different values of the thickness to the solid rod. From Fig. 4 it is observed that the non-dimensional frequency of the solid rod shows almost linear behavior of frequency with respect to Ω for the increasing

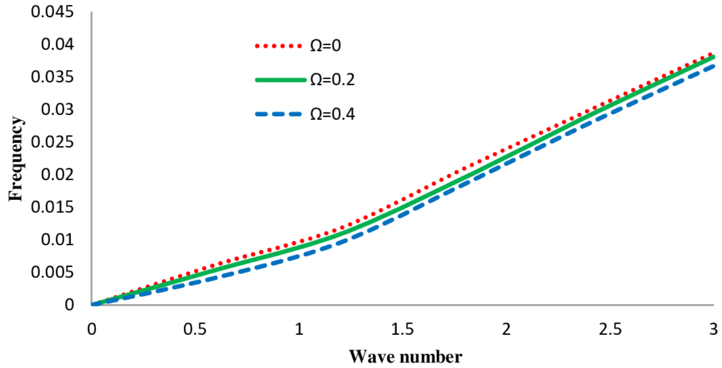

Fig. 2: Variation of dimensionless frequency against wave number at$\beta = 0$ with various rotating speed.

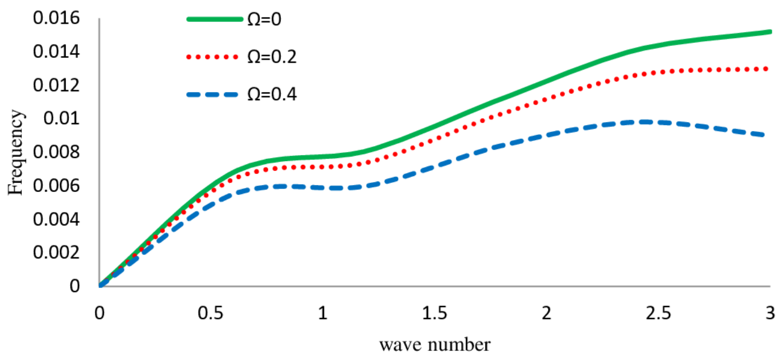

Fig. 3: Variation of dimensionless frequency against wave number at $\beta = 0.5$ with various rotating speed.

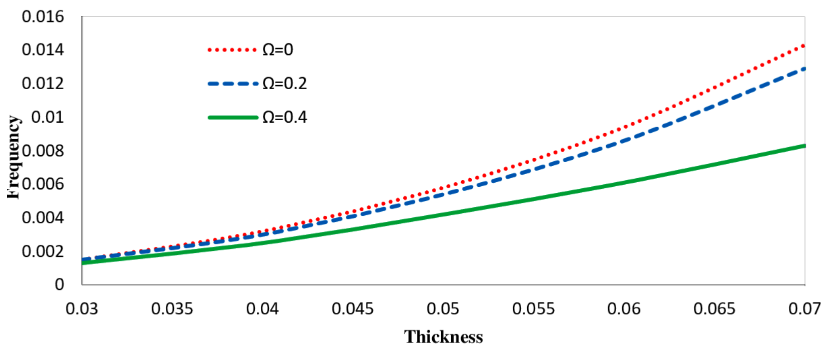

Fig. 4: Variation of dimensionless frequency against wave number at $\beta = 0$ with various rotating speed.

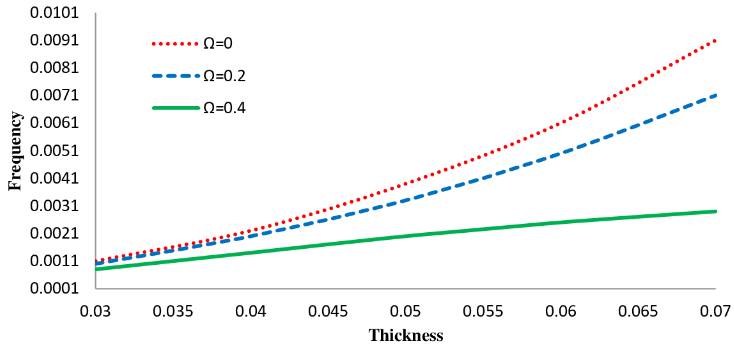

Fig. 5: Variation of dimensionless frequency against thickness at$\beta = 0.5$ with various rotating speed.

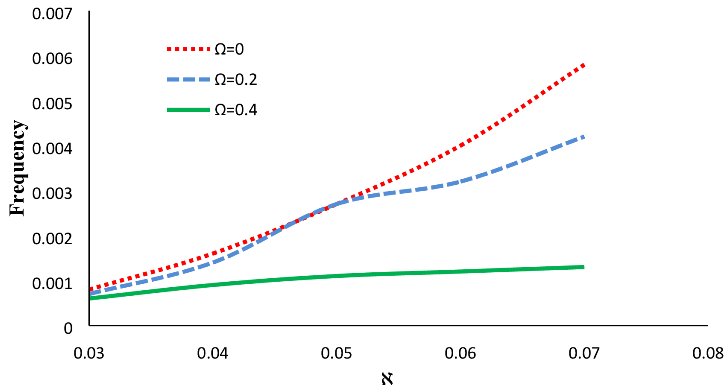

Fig. 6: Variation of dimensionless frequency against volume fraction $\aleph$ at $E = 0$ with various rotating speed.

thickness parameter.Also in Fig. 5 almost linear variation is observed with respect to Ω for the increasing thickness parameter.In both cases different values of the parameter the values of frequency are almost steady for different values of the rotating parameter Ω, further these two results shows the thermal impact of the solid rod is not significant in order increasing of parameters thickness h and rotating speed Ω of the solid rod.

The variation of the non-dimensional frequency with respect to the volume fraction $\aleph$ and different rotating speed for thermo piezoelectric multilayered rotating rod is shown in Figs. 6 and 7 respectively.From Fig 6 almost linear variation observed with respect to Ω for the increasing parameter $\aleph$ without electric impact.But in Fig.7 it is clear the frequency of the rod increases monotonically to attain maximum value in $\aleph = 2$ and slashes down to remaining range of $\aleph$ for different parameter ($\Omega = 0, 0.2, 0.4$) with electric impact. The 3D Figs 8-9 clarifies the variation of non dimensional frequency against wave number for the two different layers LEMV and CFRP of thermos piezo electric composite rod. These figures explain the dependence of the core materials in the frequency. The 3D Figs 10-11 clarifies the variation of non dimensional frequency against thickness for the two different layers LEMV and CFRP of thermopiezoelectric composite rod. These figures explain the dependence of the core materials in the frequency.

The 3D Figs 12-13 clarifies the variation of non dimensional frequency against β for the two different layers LEMV and CFRP of thermopiezoelectric composite rod. These figures explain the dependence of the core materials in the frequency.

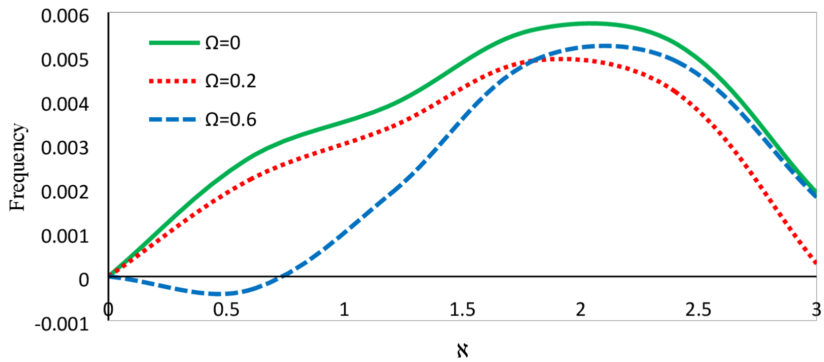

Fig. 7: Variation of dimensionless frequency against volume fraction $\aleph$ at $E = 0.5$ with various rotating speed.

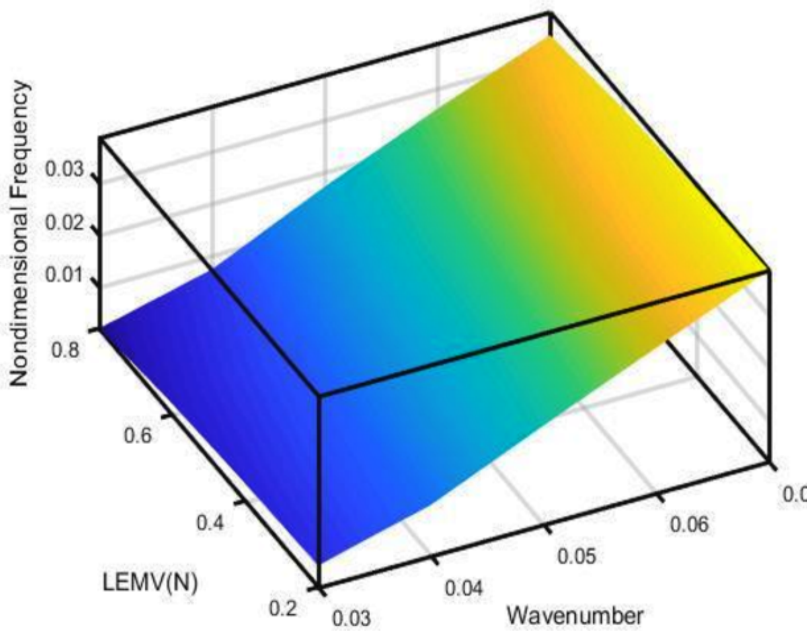

Fig. 8: Variation of dimensionless frequency against wave number and LEMV core in thermo piezoelectric multilayered rod with $\Omega = 0.5$

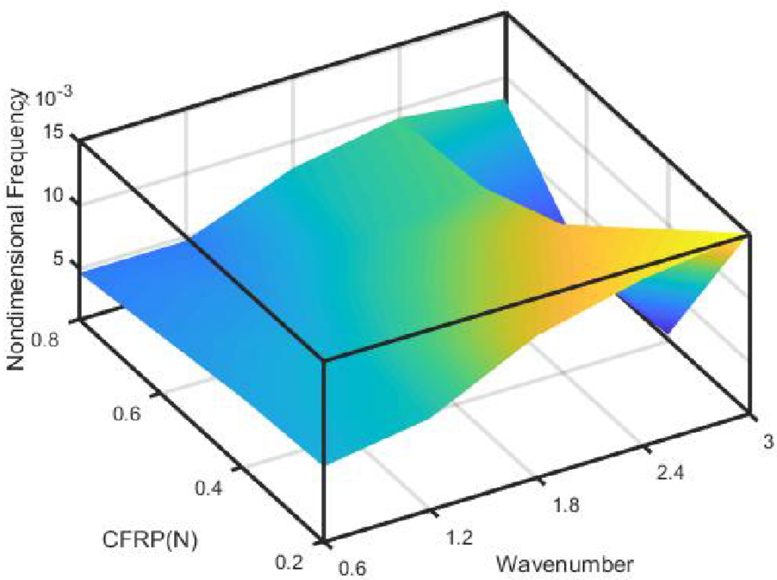

Fig. 9: Variation of dimensionless frequency against wave number and CFRP core in thermo piezoelectric multilayered rod with $\Omega = 0.5$

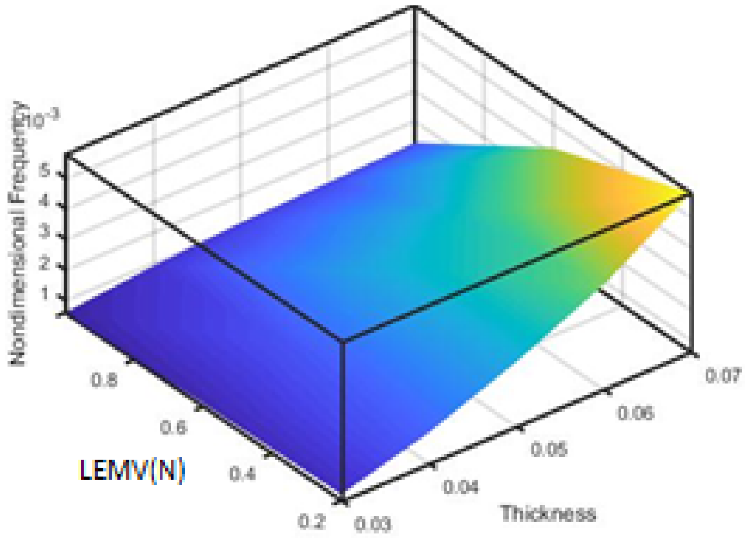

Fig. 10: Variation of dimensionless frequency against thickness and LEMV core in thermo piezoelectric multilayered rod with $\Omega = 0.5$

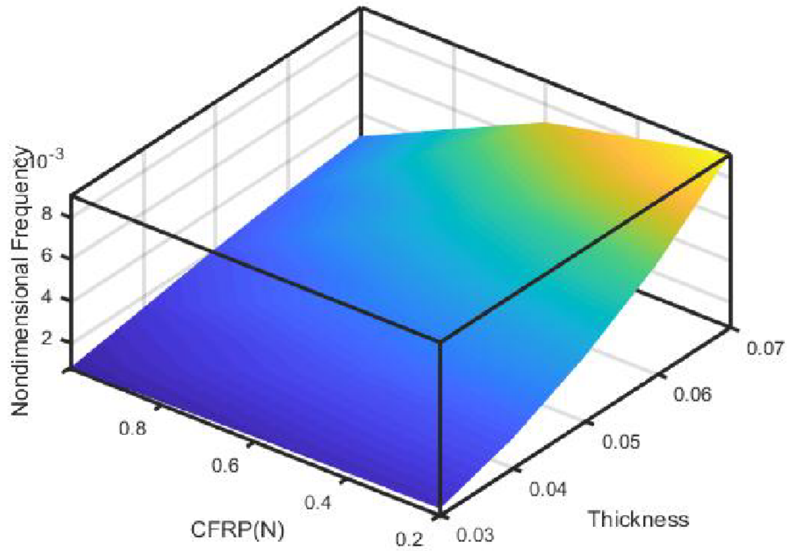

Fig. 11: Variation of dimensionless frequency against thickness and CFRP core in thermo piezoelectric multilayered rod with $\Omega = 0.5$

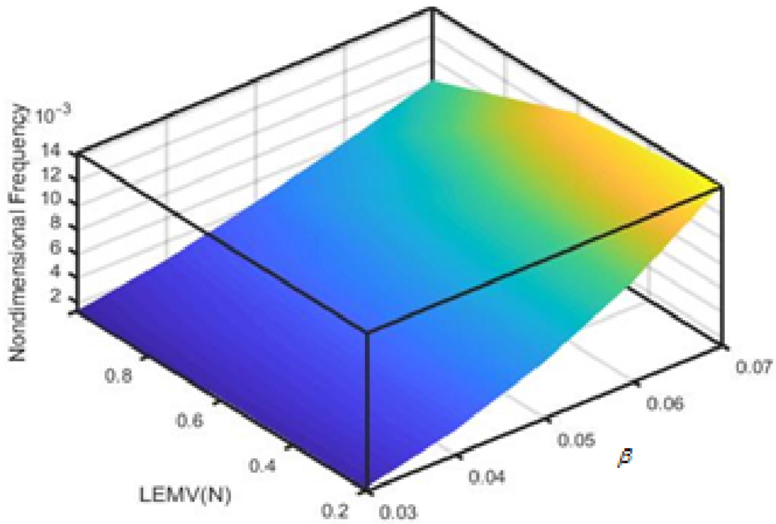

Fig. 12: Variation of dimensionless frequency against β and LEMV core in thermo piezoelectric multilayered rod with $\Omega = 0.5$

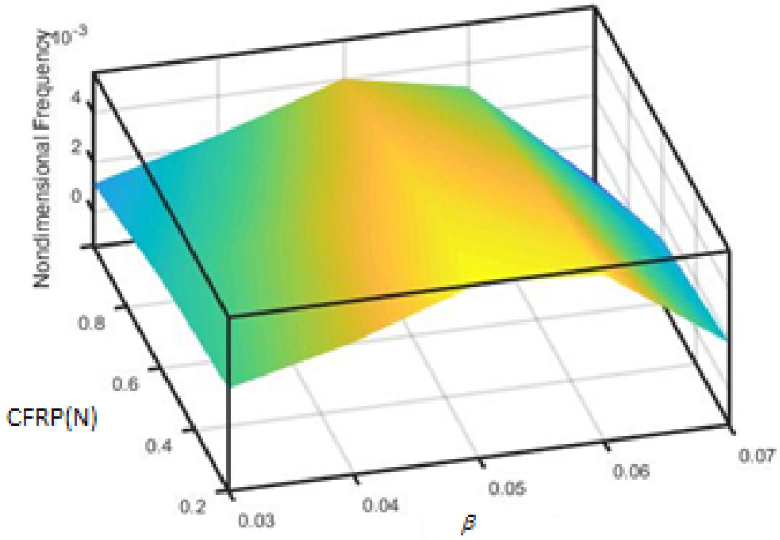

Fig. 13: Variation of dimensionless frequency against β and CFRP in thermo piezoelectric multilayered rod with $\Omega = 0.5$

6 Conclusion

The free wave propagation in a multilayered piezothermoelastic rotating rod with LEMV/CFRP interface is discussed using three-dimensional linear theory of elasticity. Three displacement potential functions are introduced to uncouple the equations of motion, electric and heat conduction. The frequency equation of the system consisting of rotating piezothermoelastic rod is developed under the assumption of thermally insulated and electrically shorted free boundary conditions at the surface of the rod. The analytical equations are numerically studied through the MATLAB programming for axisymmetric modes of vibration for PZT-5A material. The computed numerical results are presented as dispersion curves for the variation of frequency against the various physical quantities with thermal and electrical parameters together with rotational speeds. The rotation, thermal and electrical fields have a great role in frequency distribution, since the amplitudes of these frequency modes are varying with the increase of the field values. We concluded that the deformation of a body depends on the nature of the applied forces and rotational effect as well as the type of boundary conditions. The result provides a motivation to investigate the free wave propagation in a multilayered piezothermoelastic rotating rod with LEMV/CFRP core as a new class of applications elastic solids. The methods used in the present article are applicable to a wide range of problems in elasticity

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