TECHNISCHE MECHANIK
an open access journal

journal homepage: www.ovgu.de/techmech

Tech. Mech., Vol. 41, Is. 1, (2021), 14–23

Received: 15.02.2021
Accepted: 25.06.2021
Available online: 28.06.2021

Viscothermoelastic waves in a gravitated piezoelectric multilayered LEMV /CFRP cylinder coated with thin film

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Abstract: The present paper is concerned the effects of gravitational force and rotation in a composite multilayered hollow cylinder which contain inner and outer piezo-thermoelasticity layers bonded by Linear Elastic Material with Voids (LEMV) within the frame of dual-phase-lag model.Also the composite multilayered hollow cylinder coated with thin film is considered.The equation of displacement components, temperature, and electric are obtained using linear theory of elasticity. The dispersion equations are acquired by means of traction free boundary conditions and are numerically analyzed for CdSe material.The enumerated frequency, thermal and electrical nature against wave number in the presence of gravatity and rotation is presented graphically. Adhesive layer LEMV is compared with Carbon Fiber Reinforced Polymer (CFRP) in presence of gravity and rotation.

Keywords: Thermoelascity, multilayered cylinders, LEMV, CFRP, Dual phase lagging model,thin film coated.

1 Introduction

Piezoelectric materials are ordinarily utilized for savvy structure applications because of their direct and converse piezoelectric impacts which enable them to be used as the both actuators and sensors. The structure and development of piezoelectric whirligigs and other pivoting sensors have significant applications in innovation. The investigation of the impacts of turn on the proliferation of waves in piezo-thermoelastic cylinder has been broadly examined in the previous two decades.

Lord and Shulman (1967) at first investigation the generalized dynamical hypothesis of thermoelasticity.Singh et al. (2017) studied proliferation of Rayleigh wave in two temperature dual phase lag thermoelasticity.Green and Lindsay (1972) explored different parts of thermoelasticity. Othman et al. (2017) examined impact of magnetic field on generalized piezo-thermoelastic rotating medium with two relaxation times. Assessment of the fundamental properties of thermomechanics, by Green and Naghdi (1991).Mindlin (1974) determined the conditions of high recurrence vibrations of thermo-piezo-electric plate.Green and Naghdi (1992) talked about undamped heat waves in a elastic solid. Abou-Dina et al. (2017) figure a model for nonlinear thermo-electroelasticity in broadened thermo-electroelasticity in expanded thermoelasticity. Green and Lindsay (1972) clarify thermoelasticity without vitality scattering.Abo-Dahab (2015) examined proliferation of Stoneley waves in magnetothermoelastic materials with voids and two unwinding times. Abd-Alla et al. (2013) considered propagation of rayleigh waves inmagneto-thermo-versatile half-space of a homogeneous orthotropic material under the impact of the rotating, starting pressure and gravity field.Impact of magnectic field on poroelastic bone model for inward rebuilding by Abo-Dahab and Abd-alla Abd-Alla and Abo-Dahab (2013).Othman and Lotfy Othman and Lotfy (2013) discussed about the impact of magnectic field and rotate of the 2-D issue of a fiber-fortified thermoelastic under three hypotheses with impact of gravity.Samal and Chattaraj (2011) detail another advancement for surface wave proliferation in fiber reinforced anisotropic elastic layer between fluid immersed permeable half space and uniform fluid layer.Paul and Raman (1993) found wave engendering in a pyroelectric cylinder of arbitrary cross segment with a round cylindrical cavity.Paul and Nelson (1996) plan ideas of axisymmetric vibration of piezocomposite hollow circular cylinder.Puri and Cowin (1985) found plane waves in direct elastic materials with voids. The old style pressure vessal issues for direct elastic material with voids talked about by Cowin and Puri (1983).Ponnusamy (2013) examined wave proliferation in a piezoelectric solid bar of circular cross-section immersed in fluid. Impact of rotation on generalized thermo-viscoelastic Rayleigh-Lamb waves by Sharma and Othman (2007).Assaf et al. (2010) explored vibration and acoustic reaction of damped sandwich plates drenched in a light or heavy fluid. Ebenezer and Ramesh (2003) analysis of axially polarized piezoelectric cylinders with arbitrary boundary conditions on the flat surfaces.Botta and Cerri (2007) described wave propagation in Reissner-Mindlin piezoelectric coupled cylinder with non-constant electric field through the thickness.Waves in rotating and conducting piezoelectric media is described by Wauer (1999).Roychoudhuri and Mukhopadhyay (2000) investigated effect of rotation and relaxation times on plane waves in generalized thermo visco elasticity.Dragomir et al. (2014) discussed about of energy dissipation and critical speed of granular flow in a rotating cylinder.Wang (2002) investigated axi-symmetric wave propagation in a cylinder coated with a piezoelectric layer.Barshinger (2001) studied about guided waves in pipes with viscoelastic coatings.Mahesh and Selvamani (2020) investigated bending analysis of generalized thermoelastic waves in a multilayered cylinder using theory of dual phase lagging.

The present paper, DPL theory is used to study the influence of gravity and rotation on piezo-thermoelastic cylinder with thin

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film coated.The outer surface area of the cylinder is coated by a perfectly conducting material.The equation of displacement components, temperature and electric are obtained using linear theory of elasticity. The computed non-dimensional frequency is presented in the form of dispersion curves against various physical variables. Adhesive layer LEMV is compared with Carbon Fiber Reinforced Polymer (CFRP) in presence of gravity and rotation.

2 PROBLEM FORMULATION

We deal with a homogeneous transversely isotropous thermally and electrically conducting composite multilayered hollow cylinder of limitless length with constant temperature T_0 in an unvaried state at the beginning. Cylinder rotating uniformly with an angular rate a couple of fastened axis in area with angular velocity Ω. In cylindrical coordinates (r, θ, z), the equations of motion within the absence of body force and as well as the outcome and centripetal forces are: Selvamani and Mahesh (2019)

$$c_{11}(u^l_{,rr} + r^{-1}u^l_{,r} + r^{-2}u^l) + c_{44}u^l_{,zz} + (c_{44} + c_{13})w^l_{rz} + (e_{31} + e_{15})E^l_{rz} + \rho g w_{,r} - \beta_1 T_{,r} + \rho(\Omega^2 u^l + 2\Omega u_{,t}) = \rho u^l_{,tt} \tag{1a}$$

$$c_{44}(w^l_{rr} + r^{-1}w^l_{,r}) + r^{-1}(c_{44} + c_{13})u^l_{,z} + (c_{44} + c_{13})u^l_{,rz} + c_{33}w^l_{,zz} + e_{33}E_{,zz}(E_{,rr} + r^{-1}E_{,r}) - \rho g u_{,r} - \beta_3 T_{,z} + \rho(\Omega^2 w^l + 2\Omega u_{,t}) = \rho w^l_{,tt} \tag{1b}$$

$$e_{15}(w^l_{rr} + r^{-1}w^l_{,r}) + e_{31} + e_{15}(u^l_{,rz} + r^{-1}u^l_{,z}) + e_{33}w^l_{,zz} - \eta_{33}E^l_{,zz} - \eta_{11}(E^l_{,rr} + r^{-1}E^l_{,r}) + p_3 T^l_{,z} = 0 \tag{1c}$$

$$k_1 T_{,rr} + k_3 T_{,zz} + \tau_\theta(k_1 T_{,rr} + k_3 T_{,zz})_{,t} = (1 + \tau_q \frac{\partial}{\partial t})[\rho C_T T + T_0(\beta_1 u_{,r} + \beta_3 w_{,z} - p_3 \phi_{,z})] \tag{1d}$$

The solutions of Equations (1) is considered in the form

$$u^l = U^l_{,r} \exp\{i(kz + pt)\} \tag{2a}$$

$$w^l = \left(\frac{i}{h}\right) W^l \exp\{i(kz + pt)\} \tag{2b}$$

$$\varphi^l = (ic_{44}/ae_{33})\, E^l e^{i(kz+pt)} \tag{2c}$$

$$T^l = \frac{c_{44}}{\beta_3}\left(\frac{T^l}{h^2}\right) \exp\{i(kz + pt)\} \tag{2d}$$

Where, u^l, w^l, φ^l, T^l are displacement potentials, k denotes wave number, p denotes angular frequency and $i = \sqrt{-1}$.We introduce the non dimensional quantities $x = \frac{r}{a}, \varepsilon = ka, c = \rho p$, 'a' denotes geometrical parameter of the composite hollow cylinder. $\bar{c}_{11} = c_{11}/c_{44}, \bar{c}_{13} = c_{13}/c_{44}, \bar{c}_{33} = c_{33}/c_{44}, \bar{c_{66}} = c_{66}/c_{44}, \bar{\beta} = \beta_1/\beta_3, \bar{k}_i = \dfrac{(\rho c_{44})^{\frac{1}{2}}}{\beta_3^2 T_0 a \Omega}$

Substituting the Equation (2) in Equation (1) we obtain

$$[c^l_{11}\nabla^2 + \varepsilon^2 - (ca)^2]U^l - \varepsilon(1 + \bar{c}^l_{13})W^l + \varepsilon(\bar{e}^l_{15} + \bar{e}^l_{31})E^l - \bar{\beta}T^l = 0 \tag{3a}$$

$$\varepsilon(1 + \bar{c}^l_{13})\nabla^2 U^l + [\bar{c}^l_{44}\nabla^2 + \varepsilon^2\bar{c}^l_{33} - (ca)]W^l + (\bar{e}_{15}\nabla^2 + A_6)\bar{c}_{44}E^l + \varepsilon T^l = 0 \tag{3b}$$

$$A_3\nabla^2 U^l + [\bar{e}_{15}\nabla^2 + \varepsilon^2]W^l + \left[-\frac{K_{33}}{K}\nabla^2 + \bar{K}^{2\ l}_{33}\varepsilon^2\right]E^l - p^l\varepsilon T^l = 0 \tag{3c}$$

$$\iota\nabla^2 U^l - MW^l + M_3 E^l + \left[\frac{K\nabla^2}{\beta^* a^2}(1 + \tau_t) - M_1 - M_2\right]T^l = 0 \tag{3d}$$

The above relation reformulate as follows

$$\begin{vmatrix} \bar{c}^l_{11}\nabla^2 + A_1 & -A_2 & A_3 & -A_4 \\ A_2\nabla^2 & \bar{c}^l_{44}\nabla^2 + A_5 & (\bar{e}_{15}\nabla^2 + A_6)\bar{c}_{44} & A_7 \\ A_3\nabla^2 & \bar{e}_{15}\nabla^2 + A_6 & -\dfrac{K_{33}}{K}\nabla^2 + A_8 & -A_9 \\ \iota\nabla^2 & -M & M_3 & \dfrac{K\nabla^2}{\beta^* a^2}(1 + \tau_t) \end{vmatrix} \tag{4}$$

$(U^l, W^l, E^l, T^l)^T = 0$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x}, A_1 = \varepsilon^2 - (ca)^2, A_2 = \varepsilon(1 + \bar{c}^l_{13}), A_3 = \varepsilon(\bar{e}^l_{15} + \bar{e}^l_{31}), A_4 = \bar{\beta}$, $A_5 = \varepsilon^2\bar{c}^l_{33} - (ca), A_6 = \varepsilon^2, A_7 = \varepsilon, A_8 = \bar{K}^{2\ l}_{33}\varepsilon^2, M_3 = -\dfrac{ip_3\beta^* T_0 zkp}{e_{33}a^2}, M = \dfrac{\tau_q p^2 ia}{c_{44}}$, $M_1 = \frac{k^2}{\beta^*}(1 + ip\tau_t), M_2 = \frac{\rho^l c_v ip}{\beta^*}(1 - \tau_q p), A_9 = p^l\varepsilon, \iota = \frac{\beta^* T_0 z}{c_{44}}$.

Equation (4), reformulated as the following form

$$(A\nabla^8 + B\nabla^6 + C\nabla^4 + D\nabla^2 + E)(U^l, W^l, E^l, T^l)^T = 0 \tag{5}$$

The solution of Equation (5) is obtained as

$$U^l = \sum_{j=1}^{4}[A_j J_n(\alpha_j x) + B_j y_n(\alpha_j x)], \tag{6a}$$

$$W^l = \sum_{j=1}^{4} a_j^l [A_j J_n(\alpha_j x) + B_j y_n(\alpha_j x), \tag{6b}$$

$$E^l = \sum_{j=1}^{4} b_j^l [A_j J_n(\alpha_j x) + B_j y_n(\alpha_j x)], \tag{6c}$$

$$T^l = \sum_{j=1}^{4} c_j^l [A_j J_n(\alpha_j x) + B_j y_n(\alpha_j x)], \tag{6d}$$

The values a_j^l, b_j^l, c_j^l are the arbitrary constants and $J_n(\alpha_l^j ax)$ and $Y_n(\alpha_l^j ax)$ and denotes the Bessel functions first kind of order n. Here $(\alpha_i^l ax) > 0$, for $(i = 1, 2, 3, 4)$ are the zeros of algebraic equation

$$(A(\alpha_j^l a)^8 + B(\alpha_j^l a)^6 + C(\alpha_j^l a)^4 + D(\alpha_j^l a)^2 + E)(U^l, W^l, E^l, T^l)^T = 0$$

The constants a_j^l, e_j^l, and c_j^l can be evaluated using the following relations:

$$[c_{11}^l \nabla^2 + \varepsilon^2 - (ca)^2] - \varepsilon(1 + \bar{c}_{13}^l)a_j^l + \varepsilon(\bar{e}_{15}^l + \bar{e}_{31}^l)e_j^l - \bar{\beta}c_j^l = 0 \tag{7a}$$

$$\varepsilon(1 + \bar{c}_{13}^l)\nabla^2 + [\bar{c}_{44}^l \nabla^2 + \varepsilon^2 \bar{c}_{33}^l - (ca)]a_j^l + (\bar{e}_{15}\nabla^2 + A_6)\bar{c}_{44}b_j^l + \varepsilon c_j^l = 0 \tag{7b}$$

$$A_3\nabla^2 + [\bar{e}_{15}\nabla^2 + \varepsilon^2]a_j^l + \left[-\frac{K_{33}}{K}\nabla^2 + \bar{K}_{33}^{2}{}^l\varepsilon^2\right]b_j^l - p^l\varepsilon c_j^l = 0 \tag{7c}$$

$$\iota\nabla^2 - Ma_j^l + M_3 b_j^l + \left[\frac{K\nabla^2}{\beta^* a^2}(1 + \tau_t) - M_1 - M_2\right]c_j^l = 0 \tag{7d}$$

3 EQUATION OF MOTION FOR LINEAR ELASTIC MATERIALS WITH VOIDS LEMV

The equations of motion for isotropic LEMV materials are given as Cowin and Puri (1983)

$$(\lambda + 2\mu)(u_{,rr} + r^{-1}u_{,r} - r^{-2}u) + \mu u_{,zz} + (\lambda + \mu)w_{,zz} + \beta E_{,r} = \rho u_{,tt} \tag{8a}$$

$$(\lambda + \mu)(u_{,rz} + r^{-1}u_{,z}) + \mu(w_{,rr} + r^{-1}w_{,r}) + (\lambda + 2\mu)w_{,zz} + \beta E_{,z} = \rho w_{,tt} \tag{8b}$$

$$-\beta(u_{,r} + r^{-1}u) - \beta w_{,z} + \alpha(E_{,rr} + r^{-1}E_{,r} + \phi_{,zz}) - \delta k E_{,tt} - \omega E_{,t} - \xi E = 0 \tag{8c}$$

u,v,w represents displacements components along r, θ, and z directions $\alpha, \beta, \xi, \omega$ and k are LEMV material constants characterizing the core in the equilibrated inertial state,ρ is the density and λ, μ are the lame constants and $\aleph$ is the new kinematical variable associated with another material without voids. The stress in the LEMV core materials are

$$\sigma_{,rr} = (\lambda + 2\mu)u_{,r} + \lambda r^{-1}u + \lambda w_{,z} + \beta\phi$$

$$\sigma_{,rz} = \mu(u_{,t} + w_{,r})$$

The solution of for (8) is taken as

$$u = U_{,r} \exp i(kz + pt) \tag{9a}$$

$$w = (\frac{i}{h})W \exp i(kz + pt) \tag{9b}$$

$$E = (\frac{1}{h^2})E \exp i(kz + pt) \tag{9c}$$

The above solution in (9) and nondimensionl variables x and ε, equation can be reduced as

$$\begin{vmatrix} (\lambda + 2\mu)\nabla^2 + M_1 & -M_2 & M_3 \\ M_2\nabla^2 & \bar{\mu}\nabla^2 + M_4 & M_5 \\ -M_3\nabla^2 & M_5 & \alpha\nabla^2 + M_6 \end{vmatrix}(U, W, E)^T = 0 \tag{10}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x}$, $M_1 = \frac{\rho}{\rho^1}(ch)^2 - \bar{\mu}\varepsilon^2$,$M_2 = (\bar{\lambda} + \bar{\mu})\varepsilon$,$M_3 = \bar{\beta}$, $M_4 = \frac{\rho}{\rho^1}(ch)^2 - (\bar{\lambda} + \bar{\mu})\varepsilon^2$, $M_5 = \bar{\beta}\varepsilon$, $M_6 = \frac{\rho}{\rho^1}(ch)^2\bar{k} - \bar{\alpha}\varepsilon^2 - i\bar{\omega}(ch) - \bar{\xi}$

The Equation (10) can be specified as,

$$(\nabla^6 + P\nabla^4 + Q\nabla^2 + R)(U, W, E) = 0 \tag{11}$$

Thus the solution of Equation (11) is as follows,

$$U = \sum_{j=1}^{3}[A_j J_0(\alpha_j x) + B_j y_0(\alpha_j x)] \tag{12a}$$

$$W = \sum_{j=1}^{3} a_j[A_j J_0(\alpha_j x) + B_j y_0(\alpha_j x)], \tag{12b}$$

$$E = \sum_{j=1}^{3} b_j[A_j J_0(\alpha_j x) + B_j y_0(\alpha_j x)] \tag{12c}$$

$(\alpha_j x)^2$ are zeros of the equation when replacing $\nabla^2 = -(\alpha_j x)^2$. The arbitrary constant a_j and b_j are obtained from

$$M_2\nabla^2 + (\bar{\mu}\nabla^2 + M_4)a_j + M_5 b_j = 0$$
$$-M_3\nabla^2 + M_5 a_j + (\alpha\nabla^2 + M_6)b_j = 0$$

For the governing equation of CFRP core material, we assume void volume fraction $E = 0$, and the lame's constants as $\lambda = c_{12}, \mu = \dfrac{c_{11} - c_{12}}{2}$ in the Equation(8) .

4 BOUNDARY CONDITIONS AND FREQUENCY EQUATIONS

In this problem, the free axisymmetric vibration of transversely isotropic piezoelectric viscothermoelastic multilayered composite LEMV /CFRP cylinder coated with thin film is considered. For the coated surface, the mechanical boundary conditions can be written as

$$\sigma_{,rj} = -\delta'_{jb}2\mu' h'[(\frac{3\lambda' + 2\mu'}{\lambda' + 2\mu'})(u_r)_{b,ab}] + 2h'\rho'(u_{rr})_j \tag{13}$$

and the shorted electrical boundary condition is $E = 0$ where λ,μ,ρ and h are Lames constants, density, thickness of the coated material, respectively, δ_{jb} is the Kronecker delta function in which a,b takes the value of θ and z,and j takes r,θ and z.In order to get the axisymmetric waves a,b can takes only z.Then the transformed boundary conditions along axisymmetric direction is taken as follows
The frequency equations can be obtained for the following boundary condition

(i) On the traction free inner surface

$\sigma^l_{rr} = \sigma^l_{rz} = E^l = T^l = 0$ With $l = 1$.

(ii) On the traction free outer surface
$\sigma^l_{rr} = 2h'\rho' u_{r,tt}$
$\sigma^l_{rz} = -2h'\mu' G^2 w_{,zz} + 2h'\rho' w_{tt}, E^l = T^l = 0$ With $l = 3$. where $G = \frac{1+c'_{12}}{c'_{11}}$

(iii) At the interface
$\sigma^l_{rr} = \sigma_{rr}; \sigma^l_{rz} = \sigma_{rz}; E^l = T^l = D^l = 0$

Substituting the above boundary condition we obtained as a 22×22 determinant equation

$$|(Y_i j)| = 0, (i, j = 1, 2, 3, ..., 22) \tag{14}$$

$At\ x = x_0\ where\ j = 1, 2, 3, 4$

$$Y_{1j} = 2\bar{c_{66}}\left(\frac{\alpha^1_j}{x_0}\right) J_1(\alpha^l_j x_0) - [(\alpha^l_j a)^2 \bar{c_{11}} + \zeta\bar{c_{13}} a^l_j + \bar{e}_{31}\zeta b^l_j + \bar{\beta} c^l_j] J_0(\alpha^l_j a x_0)$$

$$Y_{2j} = (\zeta + a^1_j + \bar{e}_{15} b^l_j)(\alpha^l_j) J_1(\alpha^l_j x_0)$$

$$Y_{3j} = (\zeta + a_j + \bar{e}_{15} b_j)[nJ_n(\alpha^1_j a x_1) - (\alpha_j a x) J_{n+1}(\alpha^1_j a x_1)]$$

$$Y_{4,j} = (\bar{e_{15}}\zeta a_j - \bar{\epsilon_{11}} b_j)[nJ_n(\alpha^1_j a x_1) - (\alpha^1_j a x_1) J_{n+1}(\alpha^1_j a x_1)]$$

And the other nonzero elements $Y_{1,j+4}, Y_{2,j+4}, Y_{3,j+4}$ and $Y_{4,j+4}$ are obtained by replacing J_0 by J_1 and Y_0 by Y_1.
At $x = x_1$

$$Y_{5j} = 2\bar{c}_{66}\frac{\alpha_j^l}{x_1}J_1(\alpha_j^1 x_1) - \left[(\alpha_j^l a)^2\bar{c}_{11} + \zeta\bar{c}_{13}a_j^l + \bar{e}_{31}\zeta b_j^l + \bar{\beta}c_j^l\right]J_0(\alpha_j^l a x_1)$$

$$Y_{5,j+8} = -[2\bar{\mu}(\frac{\alpha_j}{x_1})J_1(\alpha x_1) + \{-(\bar{\lambda}+\bar{\mu})(\alpha_j)^2 + \bar{\beta}b_j - \bar{\lambda}\zeta a_j\}]J_0(\alpha_j x_1)$$

$$Y_{6j} = (\zeta + a_j^1 + \bar{e}_{15}b_j^l)(\alpha_j^l)J_1(\alpha_j^l a x_1)$$

$$Y_{6,j+8} = -\bar{\mu}(\zeta + a_j)(\alpha_j)J_1(\alpha_j x_1)$$

$$Y_{7j} = (\alpha_j^l)J_1(\alpha_j^l x_1)$$

$$Y_{7,j+8} = -(\alpha_j)J_1(\alpha_j^l x_1)$$

$$Y_{8j} = a_j^l J_0(\alpha_j^l x_1)$$

$$Y_{8,j+8} = -a_j^l J_0(\alpha_j^l x_1)$$

$$Y_{9j} = b_j^l J_0(\alpha_j^l x + 0)$$

$$Y_{10j} = e_j(\alpha_j)J_1(\alpha_j^l x_1)$$

$$Y_{11j} = \frac{c_j^l}{x_1}J_0(\alpha_j^l x_1) - (\alpha_j^l)J_1(\alpha_j^l x_1)$$

and the remaning nonzero element at the interfaces $x = x_1$ can be obtained on replacing J_0 by J_1 and Y_0 by Y_1 in the above elements. They are $Y_{i,j+4}, Y_{i,j+8}, Y_{i,j+11}, Y_{i,j+14}, (i = 5,6,7,8)$ *and* $Y_{9,j+4}, Y_{10,j+4}, Y_{11,j+4}$, .At the interface $x = x_2$, nonzero elements along the following rows Y_{ij} ,$(i = 12, 13, ..., 18)$ and $(j = 8, 9, ...20)$ are obtained on replacing x_1 by x_2 and superscript 1 by 2 in order. Similarly, at the outer surface $x = x_3$, the nonzero elements $Y_{ij}, (i = 19, 20, 21, 22)$ and $(j = 14, 15, ...22)$.

5 NUMERICAL DISCUSSION

The frequency equation is numerically carried out for the material CdSe and their material properties are given below: Mahesh and Selvamani (2020)

$C_{11} = 7.41 \times 10^{10}Nm^{-2}$,$C_{12} = 4.52 \times 10^{10}Nm^{-2}$, $C_{13} = 3.93 \times 10^{10}Nm^{-2}$, $C_{33} = 8.36 \times 10^{10}Nm^{-2}$, $C_{44} = 1.32 \times 10^{10}Nm^{-2}$, $T_0 = 298K$,$\rho = 5504kgm^{-3}$, $C_T = 260JKg^{-1}K^{-1}$, $e_{13} = -0.160Cm^{-2}$, $e_{33} = 0.347Cm^{-2}$, $e_{15} = -0.138Cm^{-2}$, $\beta_1 = \beta_3 = 0.621\times10^6 Nk^{-1}m^{-2}$, $P_3 = -2.94\times10^6 Ck^{-1}m^{-2}$, $K_1 = K_3 = 9Wm^{-1}K^{-1}$ $\epsilon_{11} = 8.26\times10^{-11}C^2N^{-1}m^{-2}$ $\tau_q = 0.9342\times10^{-12}s$,$\tau_q = 0.9342 \times 10^{-12}s$.
For the gold material $\rho' = 19.283g.cm^{-3}$, $\lambda' = 1.63 \times 10^{10}N.m^-2$, $\mu' = 0.42 \times 10^{10}N.m^-2$.

Eventually, in order to improved appraise the outcomes, effect of thermoelasticity theories on the non dimensional frequencies in the hollow cylinder are obtained for the traction free surfaces with continuity condition at the interfaces Nelson and Karthikeyan (2008) . The results relating to the thin film coated composite hollow circular cylinder together with LEMV/CFRP using LS theory in this paper are good agreement with those relating to the hollow cylinder of the previous references it shown in Table 1 . Also, the frequencies nature for variation of wave number in thermoelasticity theories maintaining same nature in both studies.

Table 1 Comparison of dimensionless frequency distribution of hollow cylinder and hollow cylinder with thin film coated against the increasing value of wave number

	Wave number	Nelson and Karthikeyan (2008)	wave number	Present study
Non Dimensional Frequency	0.2	0.01999	0.2	0.02199
	0.6	0.6000	0.6	0.7666
	1.2	1.2000	1.2	1.3866
	1.8	1.800	1.8	1.9587
	2.4	2.400	2.4	2.5381
	3.0	3.000	3.0	3.1424

Figure 1 depicts the variety of non-dimensional frequency against wave number in the casings of the L-S hypothesis and the DPL model with presence and absences of gravity .Initially, whenever the wave number are in a lower level automatically the frequencies increases. When the wave number increases then the frequencies reduced. The impact of gravity is makes significances impact in both L-S and DPL Theory.Figure 2 portrays the appropriation of the non-dimensional frequency against wave number in the existences and absents of rotation. It shows that this frequency part pitifully relies upon revolution. The supreme estimation of this Non dimensional Frequency part for L-S is expanding, and Non-dimensional Frequency esteems are expanding in lower estimations of wave number and diminishing the rest of the scope of wave number in DPL. From this observation the impact of

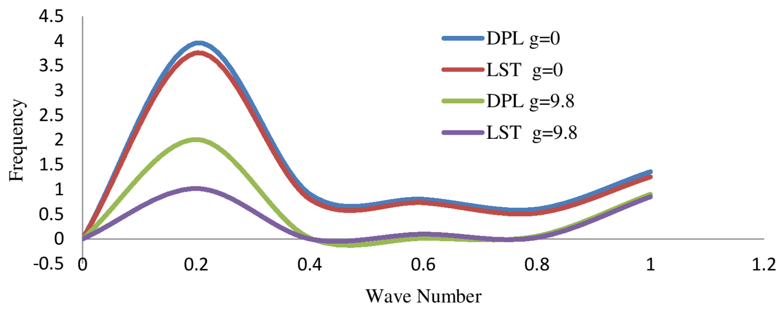

Fig. 1: Distribution of non dimensional frequency against the wave number with and without of gravity.

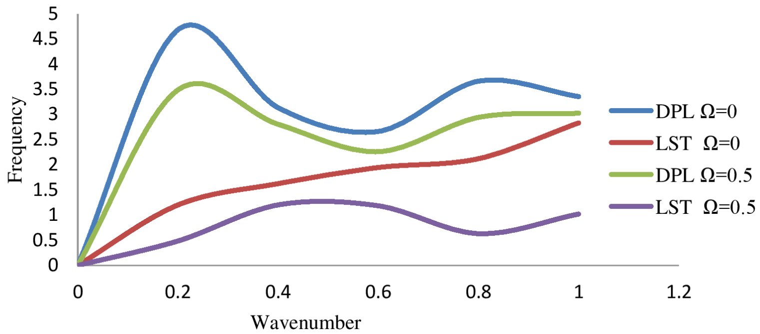

Fig. 2: Distribution of non-dimensional frequency against the wave number with and without of rotation.

rotation is makes significance impact in both L-S and DPL Theory. Figure 3 shows that within the sight of gravity the estimations of the temperature T in the two models decay bit by bit and quickly with the expansion of wave number. However, without gravity the estimations of T decay till achieving a specific neighborhood with least worth in the point which inclined to accomplish a nearby most extreme incentive before diminishing to bring down the qualities. Its shows that the role of gravity in temperature change against wavenumber in both L-S theory and DPL Theory, especially in DPL theory gravity creates more impact. Figure 4 represents the conduct of temperature T against the wave number. While considering the impact of rotation, it remains in negligible position. The nature of temperature in L-S is monotonic and diminishing to least esteem and it is littler than that of an acquired from DPL method in the underlying stage and afterward remains to be enormous qualities to the wave number. Its observe that the role of rotation in temperature change against wavenumber in both L-S theory and DPL Theory, Especially in DPL theory gravity creates more impact.
Figure 5 depicts the nature of the electric potential Component in the presence of both L-S hypothesis and DPL model. In the existences and nonexistence of gravity the component gets increasing symmetrically to lowest values of wave number and then follows to a constant nature for highest range of wave number. The impact of gravity here is negligible.Figures 6 exhibits the distributions with wave number of the electrical potential in the presences and absences of rotation . In both DPL and L-S theory the electric potential remains in a standard position without any changes in the presences and absences of rotation. The effect of rotation is not significant in this case, and the electric potential components gradually increasing for larger values of wave number.Figures 7 and 8 exhibits the 3D plots of the thermal damping in LEMV and CFRP layers for various values of N using DPL theory in the presences of gravity and Rotation.

Figures 9-11 represent the propagation of electric displacement with respect to the thickness of the coated layer with different

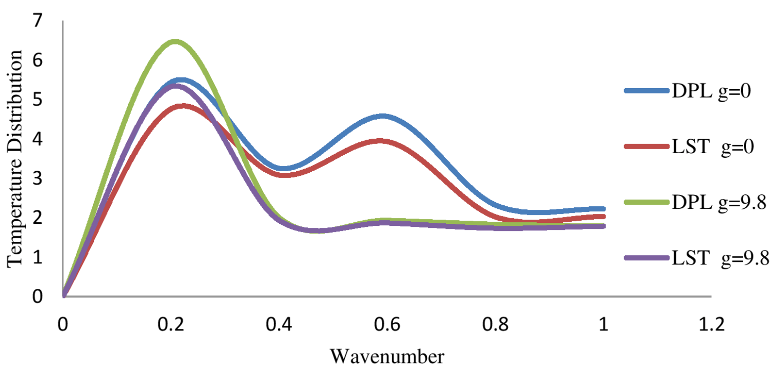

Fig. 3: Distribution of temperature against the wave number in the with and without of gravity.

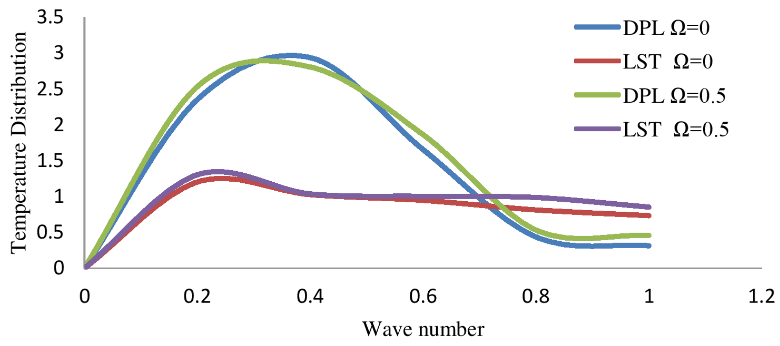

Fig. 4: Distribution of temperature against the wave number with and without of rotation.

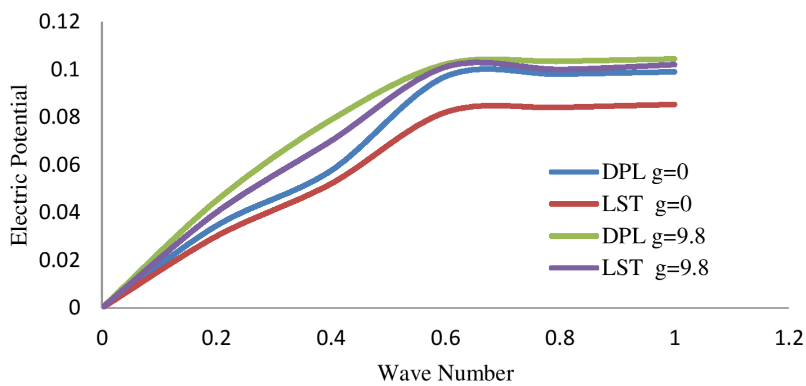

Fig. 5: Distribution of electric potential against the wave number with and without of gravity.

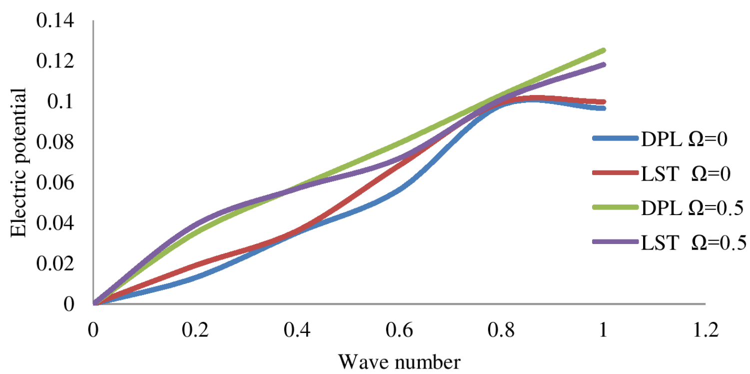

Fig. 6: Distribution of electric potential against the wave number in the absence and presence of rotation.

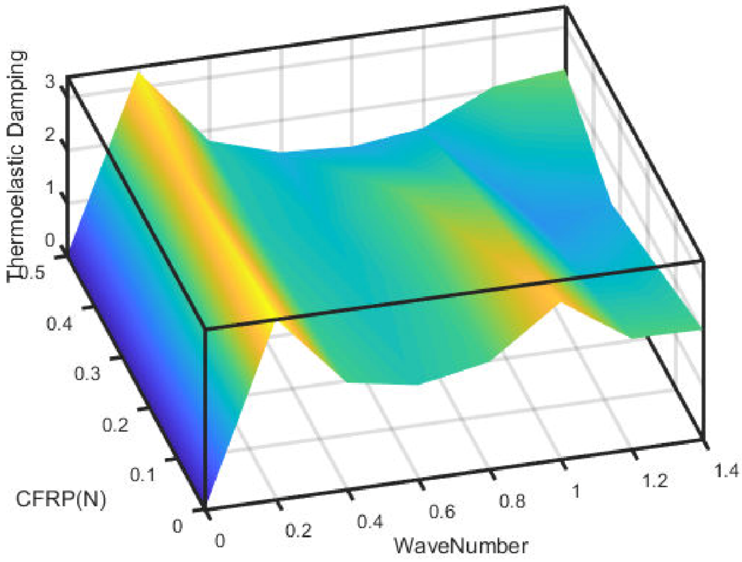

Fig. 7: Distribution of thermal damping against the wave number in the various values of LEMV (N) [g=9.8,Ω = 0.5].

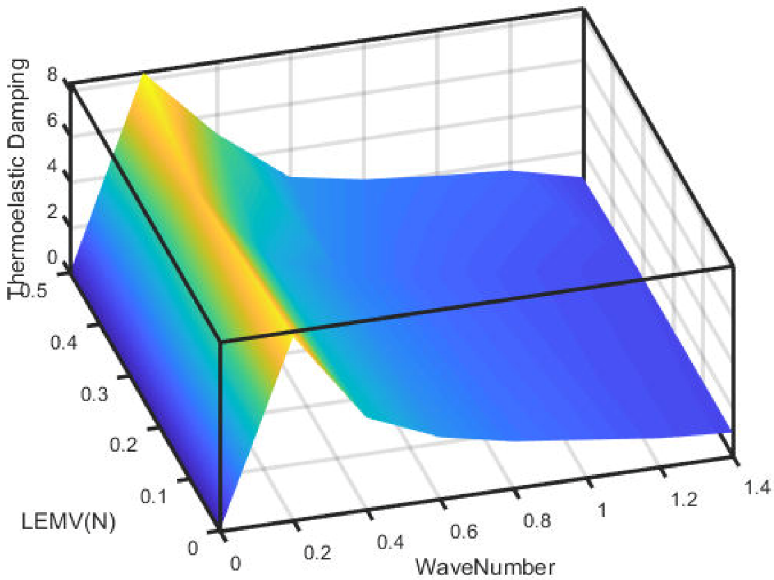

Fig. 8: Distribution of thermal damping against the wave number in the various values of CFRP (N) [g=9.8,Ω=0.5.

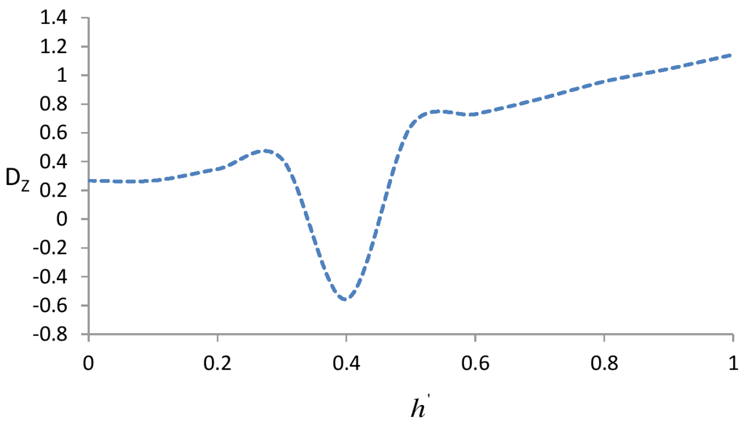

Fig. 9: Variation of electric displacement versus thickness of the coating material $h^{'}$ for [g=9.8,Ω=0]

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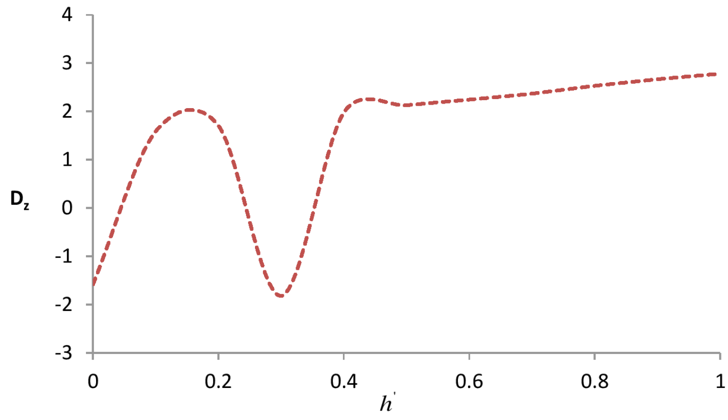

Fig. 10: Variation of electric displacement versus thickness of the coating material for [g=9.8,Ω=0.2].

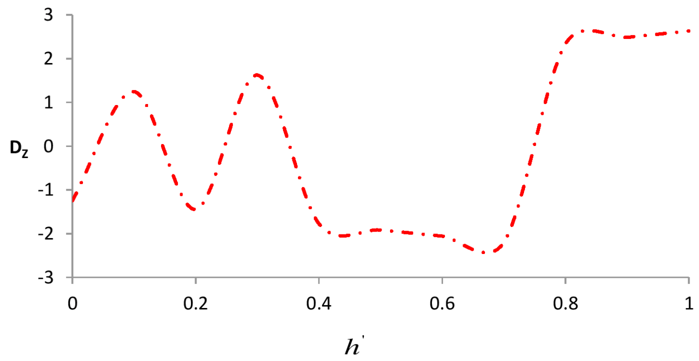

Fig. 11: Variation of electric displacement versus thickness of the coating material h' for [g=9.8,Ω=0.5].

rotational speeds. Whenever the thickness of the cylinder increases the electric displacement is decreasing and again increasing and travels in the wave propagation. Also, it is noticed in all the figures that the trend of the curve is oscillating when the rotational speed increases. These trends of the curves admit the elastic properties of the solid due to rotational effect and coating of the material.

6 Conclusion

The fundamental motivation behind the current work is to explore the impact of gravitation and turning power on a piezo-thermoelastic cylinder within DPL model and how they make a fundamental job in expanding or diminishing the adequacy of the diverse physical amounts. The outcomes acquired by applying both of the LâĂŞS hypothesis what's more, DPL model are extremely near one another aside from in deciding one of the segments of the electric dislodging where the outcomes contrast and when all is said in done the impact of the nearness of gravity is to debilitate the supreme qualities of the physical amounts with the exception of on account of the equivalent part of the electric relocation. Similarly the results of thin film coated multi layer LEMV/CFRP cylinders are also studied. In this manner also discussed thermal damping in LEMV/CFRP layers for DPL model. This result may be useful of various fields of engineering. Especially in Light weight and Heavy strength material manufacturing industries and Production engineering fields.

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