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Projects-Based Instruction of Intermediate Strength of Materials Course: Preparing Students for Future Workforce

Isaac Elishakoff [1]⋆, Abhishek Ratanpara [2]

[1] Distinguished Research Professor, Department of Ocean and Mechanical Engineering, Florida Atlantic University, Boca Raton, FL 33431-0991
[2] Doctoral Candidate, Department of Ocean and Mechanical Engineering, Florida Atlantic University, Boca Raton, FL 33431-0991

Abstract: This paper is devoted to the transformative instruction of Intermediate Strength of Materials or Aerospace Structures courses. It is argued that instead of placing heavy emphasis on tests and exams it is preferable to engage students with small size projects covering main topics of the course. Each student is assigned a serial number. The parameters of the loads and/or parameters describing geometric dimensions in offered project problems are made dependent on the serial number. This creates individualized project and takes care that students perform these individually even in case they cooperate. The latter is being welcomed since it promotes discussions between students, thus resulting in the better understanding of the material. Projects create natural interaction between the faculty, teaching assistant, and the students, who pose questions via Canvas system or any other accepted software in use at the given University.

Keywords:

Introduction

According to the an American lawyer and educator and former president of Harvard University, Derek Bok, “The college that takes students with modest entering abilities and improves their abilities substantially contributes more than the school that takes very bright students and helps them develop only modestly.” [1]. From this point of view teaching at a non-Ivy League school is extremely rewarding. We are getting ample opportunities to improve our incoming students.

As Hadim and Eshe (2002) stress, “In recent years, the engineering education community is showing increasing interest in project-based learning approaches” [2]. This trend is illustrated by the large and continuously expanding body of related educational literature as summarized below. The roots of project-based education were traced by Brown and Brown (1997) back to the early 1980s [3]. Felder et al (2001) and his co-workers (Rosati and Felder, 1995) developed an Index of Learning Styles that can be used to categorize the various dimensions of learning [4,5]. While the traditional lecture-based teaching approach is well known to address only certain learning styles, the use of design projects provides the student with a broad context to the material presented in the lectures. With PBL [project-based learning], students are encouraged to assume responsibility for their learning experience and to shift from passive to more active learning patterns. This is likely to improve the knowledge retention as well as the ability to integrate material from different courses. Woods et al. [SI demonstrated the benefits of project-based learning by comparing the problem-based and the lecture- based learning environments through analysis of data obtained from two questionnaires of the same students exposed to both environments.” [6].

In the class of Intermediate Strength of Materials, we implemented five different projects. In each problem of every project the serial number “s” was incorporated instead of other possible numbers. The serial number is the sequence number as the students’ names appear in the class roster. In the beginning of the semester each student was assigned with the unique serial number. The teaching assistant solves the problem analytically and numerically and thus is in possession of some critical part of answers. Students are required to seek solution for his/her specific serial number value, since it is also easier than to pursue solution for arbitrary value of s. This makes problem an individualized one. Cooperation is welcomed. The 4 assigned projects were in (1) using singularity functions for determining the deflections of a statically determinate beam of length of 10 meters, when at each cross section equal j varying from 1 util and including 9 there is either an external load or an external moment applied, or a distributed load starts and ends at some other location; (2) the second project is obtained by placing additional support(s) on the problems in the previous project resulting in statically indeterminate problems; (3) the third project offers 5 problems in column buckling; (4) the last, fourth project offers 5 problems in using various failure criteria and design. Hereinafter, we demonstrate two problems in the project 3, dealing with buckling of the columns.

1 Buckling Project: Stepped Simply Supported Column

For example, in one project, the problem was asked to determine the critical load in the simply supported compound column. As shown in figure 1.1, half portion of the column with the rigidity EI and the other half was assigned as (s+1)EI. Here “s” was different for each student based on their individual serial number. That project was based on determination of critical load in the column with different boundary condition and compound structures. To guide the student one problem was solved with s=49. Since the total students were 48 in the class, serial number 49 would not be used by any student. Here we have explained the

⋆ E-mail address: elishako@fau.edu
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procedure of two problems of different set of boundary conditions where column is either simply supported or clamped at both ends. These problems are as follows:
Determine the critical load of the column given in Figure 1.1. Keep the values of the modulus of elasticity E and moment of inertia I as variables. Try to verify that the answer makes sense.

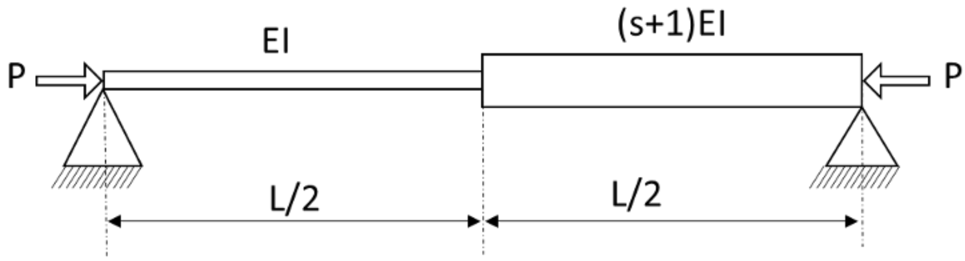

Fig. 1.1: Simply supported stepped column

Here, s is the individualized serial number. The question arises that why the coefficient s+1 was adopted in the assignment? This is in order to avoid the case of treating of uniform column that already was covered during the lectures. Indeed, sEI for serial number 1 would make the column uniform.
The compound column has two different stiffnesses since we have a single step in the middle of the column. Based on their stiffnesses, the deflection in each step is governed by a different differential equation. We have denoted as y_1 and y_2 deflections in the part with the stiffness of EI and (s+1)EI, respectively. During Fall semester of the 2021/22 academic year, we had 49 students. In order to avoid providing the general solution, we will consider the specific serial number s=50 which was not in use during that semester.

$$EIy_1'' + Py_1 = 0 \tag{1.1}$$
$$50EIy_2'' + Py_2 = 0 \tag{1.2}$$

Equation 1.1 and 1.2 can be obtained in the lecture. Now let us introduce the following notation:

$$k^2 = \frac{P}{50EI} \tag{1.3}$$

We can rewrite the equation 1.1 and 1.2 as equation 1.4 and 1.5 respectively.

$$y_1'' + 50k^2y_1 = 0 \tag{1.4}$$
$$y_2'' + k^2y_2 = 0 \tag{1.5}$$

The solution of equations 1.4 and 1.5 is given by equations 1.6 and 1.7 respectively,

$$y_2 = c_3 sin(kx) + c_4 cos(kx) \tag{1.6}$$
$$y_1 = c_1 sin\left(\sqrt{50}kx\right) + c_2 cos\left(\sqrt{50}kx\right) \tag{1.7}$$

The deflection y1 will be zero at the simple support at x=0, and y2 will be zero at other simple support at x=L. By applying these boundary condition, we get the constants value as follows.

$$x = 0;\ y_1 = 0: \qquad c_2 = 0 \tag{1.8}$$
$$x = L;\ y_2 = 0: \qquad c_3\, sin(kL) + c_4\, cos(kL) = 0 \tag{1.9}$$

At the cross section $x = \frac{L}{2}$, we apply the continuity conditions, which means the deflection and slope will be continues at the cross section.

$$c_1 sin\left(\frac{\sqrt{50}kL}{2}\right) - c_3 sin\left(\frac{kL}{2}\right) - c_4 cos\left(\frac{kL}{2}\right) = 0 \tag{1.10}$$

With $y_1' = y_2'\ at\ x = \frac{L}{2}$; we get

$$\sqrt{50}kc_1 cos\left(\frac{\sqrt{50}kL}{2}\right) - kc_3 cos\left(\frac{kL}{2}\right) + kc_4 sin\left(\frac{kL}{2}\right) = 0 \tag{1.11}$$

From equation 1.11, 1.12 and 1.13, we create the system of equations as follows,

$$\begin{vmatrix} sin\left(\frac{\sqrt{50}kL}{2}\right) & -sin\left(\frac{kL}{2}\right) & -cos\left(\frac{kL}{2}\right) \\ \sqrt{50}kcos\left(\frac{\sqrt{50}kL}{2}\right) & -kcos\left(\frac{kL}{2}\right) & ksin\left(\frac{kL}{2}\right) \\ 0 & sin(kL) & cos(kL) \end{vmatrix} = 0 \tag{1.12}$$

Denoting, $\frac{kL}{2} = \alpha$, we get

$$\begin{vmatrix} sin\left(\alpha\sqrt{50}\right) & -sin(\alpha) & -cos(\alpha) \\ \sqrt{50}cos\left(\alpha\sqrt{50}\right) & -cos(\alpha) & sin(\alpha) \\ 0 & sin(2\alpha) & cos(2\alpha) \end{vmatrix} = 0 \tag{1.13}$$

The given determinantal equation can be solved with the help of software like Maple or MATLAB. We used Maple software to solve the determinantal equation, which gave a transcendental equation, as follows, (figure 1.2 shows the variation of the determinant vs α). The first non-trivial solution of this equation is,

$$\alpha = 0.2856 \qquad \text{or} \qquad \frac{kL}{2} = 0.2856$$

Considering equation 1.3, we obtain the buckling load which coincides with Feodosiev's results for this part of problem [7].

$$P = \frac{16.3134EI}{L^2} \tag{1.14}$$

Based on the serial number every student will get different value of critical load and verify by the Euler's equation for critical load. Students were asked to try to verify their answer. One way of doing this is to compare obtained result with buckling loads of uniform column of stiffness EI and (S+1)EI, respectively.

$$\frac{\pi^2 EI}{L^2} < \frac{16.3134EI}{L^2} < \frac{50\pi^2 EI}{L^2} \tag{1.15}$$

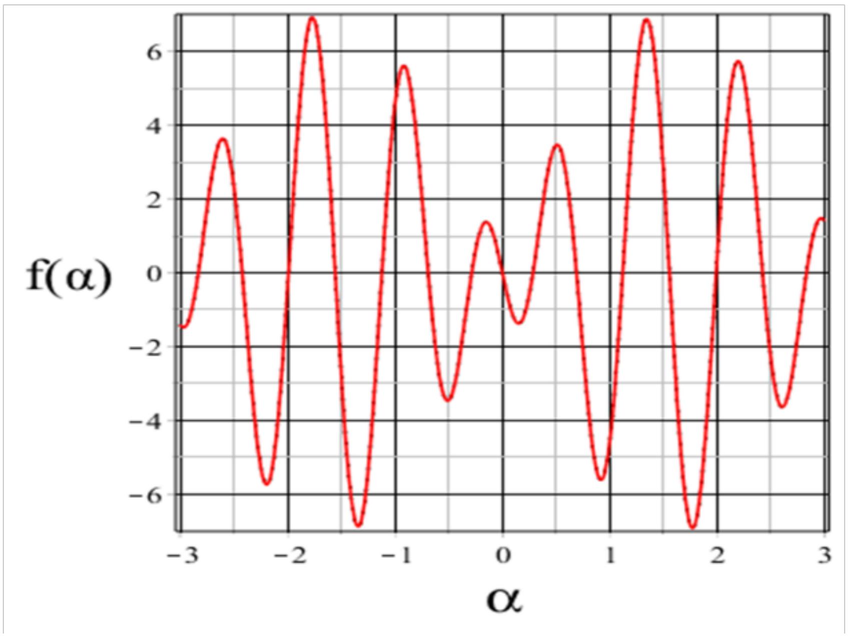

Fig. 1.2: Graph showing the solution of the transcendental equation (S=49)

In the Figure 1.2, we have shown a graph, where transcendental equation obtained from the determinant as a function of α, is plotted against α. Each student was instructed to obtain the graph based on their respective serial number (see Table 1.1).

Other relevant projects can be drawn from papers by Venkataraman and Haftka (2008) [8], Storch et al (2018) [9], Sinha (2020) [10], Elishakoff et al (2021)[11], Gavioli, and Bisagni (2021) [12]. Educational issues are elucidated in papers [13-15].

Serial Number	α	Critical load	Serial Number	α	Critical load
1	1.265671905	12.8154	26	0.387167356	16.18904
2	1.079774654	13.99096	27	0.380306147	16.19887
3	0.955316618	14.60208	28	0.373797102	16.20802
4	0.865253147	14.97326	29	0.367611101	16.21655
5	0.796393494	15.22182	30	0.361722287	16.22453
6	0.741611577	15.39966	31	0.356107611	16.23202
7	0.696713106	15.53309	32	0.350746448	16.23905
8	0.659058036	15.63687	33	0.345620283	16.24566
9	0.626894451	15.71987	34	0.340712439	16.2519
10	0.599009497	15.78774	35	0.336007852	16.25778
11	0.57453396	15.84428	36	0.331492871	16.26335
12	0.552826748	15.89211	37	0.327155096	16.26863
13	0.533403436	15.93308	38	0.322983233	16.27363
14	0.515890336	15.96857	39	0.31896697	16.27839
15	0.499993988	15.99962	40	0.315096872	16.28291
16	0.485480323	16.027	41	0.311364284	16.28722
17	0.472160071	16.05133	42	0.307761253	16.29132
18	0.459878318	16.07309	43	0.304280454	16.29524
19	0.448506892	16.09267	44	0.300915131	16.29898
20	0.437938708	16.11039	45	0.297659037	16.30257
21	0.428083514	16.12648	46	0.294506389	16.30599
22	0.418864634	16.14118	47	0.291451825	16.30928
23	0.410216457	16.15464	48	0.288490362	16.31243
24	0.402082461	16.16703	49	0.285617366	16.31546
25	0.394413659	16.17846			

Tab. 1.1: Buckling loads for each serial number is obtained via MAPLE code (critical load values should be multiplied by EI/L^2)

2 Buckling Project: Buckling of stepped column clamped at both ends

Students were assigned the following problem:

Determine the critical load of the column with fixed support given in figure 2.1. Keep the values of the modulus of elasticity E and moment of inertia I as variables. Try to verify that the answer makes sense.

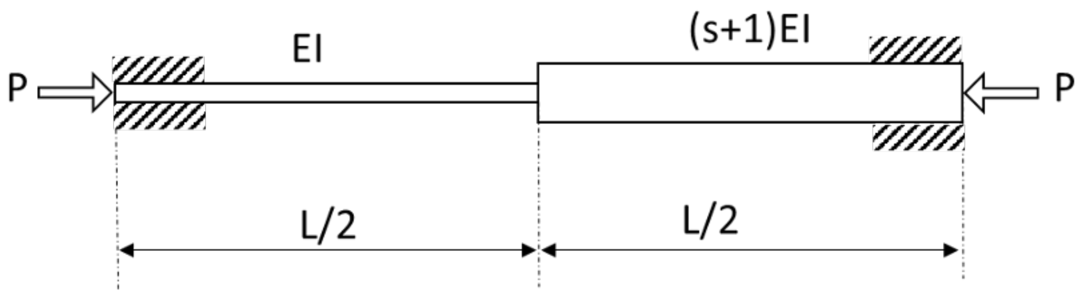

Fig. 2.1: Fixed compound column

The associated differential equations are as follows,

$$EIy_1'''' + Py_1'' = 0 \tag{2.1}$$

$$EIy_2'''' + Py_2'' = 0 \tag{2.2}$$

Equation 2.1and 2.2 can be obtained based on the deflection Euler's formula using notations,

$$50k^2 = \frac{P_{\text{cr}}}{EI} \tag{2.3}$$

We rewrite solution of equation 2.1and 2.2 as following:

$$y_1 = A_1 \sin\left(\sqrt{50}kx\right) + A_2 \cos\left(\sqrt{50}kx\right) + A_3 kx + A_4 \tag{2.4}$$

$$y_2 = B_1 \sin(kx) + B_2 \cos(kx) + B_3 kx + B_4 \tag{2.5}$$

The deflection y_1 and the slope y_1' are zero at the fixed support at x=0, and similarly y_2 and the slope y_2' are zero at other fixed support at x=L. By applying these boundary condition, we obtain following equation.
Applying boundary conditions, at $x = 0, y_1 = 0, y_1' = 0$ yields;

$$A_2 + A_4 = 0 \tag{2.6}$$

$$A_1\sqrt{50}k + A_3 k = 0 \tag{2.7}$$

At $x = L, y_2 = 0, y_2' = 0$, we get

$$B_1 sin(kL) + B_2 cos(kL) + B_3 kL + B_4 = 0 \tag{2.8}$$

$$B_1 kcos(kL) - B_2 ksin(kL) + B_3 k = 0 \tag{2.9}$$

Continuity conditions at $x = \frac{L}{2}$, demand the continuity of deflection, slope, bending moment and effective shear force.

$$y_1\left(\frac{L}{2}\right) = y_2\left(\frac{L}{2}\right) \tag{2.10}$$

$$y_1'\left(\frac{L}{2}\right) = y_2'\left(\frac{L}{2}\right) \tag{2.11}$$

$$EIy_1''\left(\frac{L}{2}\right) = 50EIy_2''\left(\frac{L}{2}\right) \tag{2.12}$$

$$EIy_1'''\left(\frac{L}{2}\right) + Py_1'\left(\frac{L}{2}\right) = 50EIy_2'''\left(\frac{L}{2}\right) + Py_2'\left(\frac{L}{2}\right) \tag{2.13}$$

In view of equation 2.11, equation 2.13 becomes;

$$EIy_1'''\left(\frac{L}{2}\right) = 50EIy_2'''\left(\frac{L}{2}\right) \tag{2.14}$$

These conditions result in following set of equations.

$$A_1 sin\left(\frac{\sqrt{50}kL}{2}\right) + A_2 cos\left(\frac{\sqrt{50}kL}{2}\right) + A_3\frac{kL}{2} + A_4 - B_1 sin\left(\frac{kL}{2}\right) - B_2 cos\left(\frac{kL}{2}\right) - B_3\frac{kL}{2} - B_4 = 0 \tag{2.15}$$

$$A_1\sqrt{50}k\, cos\left(\frac{\sqrt{50}kL}{2}\right) - A_2\sqrt{50}k sin\left(\frac{\sqrt{50}kL}{2}\right) + A_3 k - B_1 k\, cos\left(\frac{kL}{2}\right) + B_2 k\, sin\left(\frac{kL}{2}\right) - B_3 k = 0 \tag{2.16}$$

$$-A_1 50k^2 sin\left(\frac{\sqrt{50}kL}{2}\right) - A_2 50k^2 cos\left(\frac{\sqrt{50}kL}{2}\right) + B_1 50k^2 sin\left(\frac{kL}{2}\right) + B_2 50k^2 cos\left(\frac{kL}{2}\right) = 0 \tag{2.17}$$

$$-A_1 50\sqrt{50}k^3 cos\left(\frac{\sqrt{50}kL}{2}\right) + A_2 50\sqrt{50}k^3 sin\left(\frac{\sqrt{50}kL}{2}\right) + 50B_1 K^3 cos\left(\frac{KL}{2}\right) - 50B_2 K^3 sin\left(\frac{KL}{2}\right) = 0 \tag{2.18}$$

From equations 2.4-2.7 and 2.15-2.18, we obtain following determinantal equation.

$$\begin{vmatrix}
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
\sqrt{50}k & 0 & k & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & sin(kL) & cos(kL) & kL & 1 \\
0 & 0 & 0 & 0 & kcos(kL) & -ksin(kL) & k & 0 \\
sin\left(\frac{\sqrt{50}kL}{2}\right) & cos\left(\frac{\sqrt{50}kL}{2}\right) & \frac{kL}{2} & 1 & -sin\left(\frac{kL}{2}\right) & -cos\left(\frac{kL}{2}\right) & -\frac{kL}{2} & -1 \\
\sqrt{50}kcos\left(\frac{\sqrt{50}kL}{2}\right) & -\sqrt{50}ksin\left(\frac{\sqrt{50}kL}{2}\right) & k & 0 & -kcos\left(\frac{kL}{2}\right) & ksin\left(\frac{kL}{2}\right) & -k & 0 \\
-50k^2 sin\left(\frac{\sqrt{50}kL}{2}\right) & -50k^2 cos\left(\frac{\sqrt{50}kL}{2}\right) & 0 & 0 & 50k^2 sin\left(\frac{kL}{2}\right) & 50k^2 cos\left(\frac{kL}{2}\right) & 0 & 0 \\
-50\sqrt{50}k^3 cos\left(\frac{\sqrt{50}kL}{2}\right) & 50\sqrt{50}k^3 sin\left(\frac{\sqrt{50}kL}{2}\right) & 0 & 0 & 50K^3 cos\left(\frac{KL}{2}\right) & -50K^3 sin\left(\frac{KL}{2}\right) & 0 & 0
\end{vmatrix} = 0 \tag{2.19}$$

Multiplying 2nd, 3rd and 6th row by L, 7th row by L^2 and 8th row by L^3 results as follow;

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \sqrt{50}kL & 0 & kL & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & sin(kL) & cos(kL) & kL & 1 \\ 0 & 0 & 0 & 0 & kLcos(kL) & -kLsin(kL) & kL & 0 \\ sin\left(\frac{\sqrt{50}kL}{2}\right) & cos\left(\frac{\sqrt{50}kL}{2}\right) & \frac{kL}{2} & 1 & -sin\left(\frac{kL}{2}\right) & -cos\left(\frac{kL}{2}\right) & -\frac{kL}{2} & -1 \\ \sqrt{50}kLcos\left(\frac{\sqrt{50}kL}{2}\right) & -\sqrt{50}kLsin\left(\frac{\sqrt{50}kL}{2}\right) & kL & 0 & -kLcos\left(\frac{kL}{2}\right) & kLsin\left(\frac{kL}{2}\right) & -kL & 0 \\ -50k^2L^2sin\left(\frac{\sqrt{50}kL}{2}\right) & -50k^2L^2cos\left(\frac{\sqrt{50}kL}{2}\right) & 0 & 0 & 50k^2L^2sin\left(\frac{kL}{2}\right) & 50k^2L^2cos\left(\frac{kL}{2}\right) & 0 & 0 \\ -50\sqrt{50}k^3L^3cos\left(\frac{\sqrt{50}kL}{2}\right) & 50\sqrt{50}k^3L^3sin\left(\frac{\sqrt{50}kL}{2}\right) & 0 & 0 & 50K^3L^3cos\left(\frac{KL}{2}\right) & -50K^3L^3sin\left(\frac{KL}{2}\right) & 0 & 0 \end{vmatrix} = 0 \tag{2.20}$$

We denote $kL = \beta$, leading to determinantal equations into non-dimensional form,

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \sqrt{50}\beta & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & sin(\beta) & cos(\beta) & \beta & 1 \\ 0 & 0 & 0 & 0 & \beta\cos(\beta) & -\beta\sin(\beta) & \beta & 0 \\ sin\left(\frac{\sqrt{50}\beta}{2}\right) & cos\left(\frac{\sqrt{50}\beta}{2}\right) & \frac{\beta}{2} & 1 & -sin\left(\frac{\beta}{2}\right) & -cos\left(\frac{\beta}{2}\right) & -\frac{\beta}{2} & -1 \\ \sqrt{50}\beta cos\left(\frac{\sqrt{50}\beta}{2}\right) & -\sqrt{50}\beta sin\left(\frac{\sqrt{50}\beta}{2}\right) & \beta & 0 & -\beta cos\left(\frac{\beta}{2}\right) & \beta sin\left(\frac{\beta}{2}\right) & -\beta & 0 \\ -50\beta^2sin\left(\frac{\sqrt{50}\beta}{2}\right) & -50\beta^2cos\left(\frac{\sqrt{50}\beta}{2}\right) & 0 & 0 & 50\beta^2sin\left(\frac{\beta}{2}\right) & 50\beta^2cos\left(\frac{\beta}{2}\right) & 0 & 0 \\ -50\sqrt{50}\beta^3cos\left(\frac{\sqrt{50}\beta}{2}\right) & 50\sqrt{50}\beta^3sin\left(\frac{\sqrt{50}\beta}{2}\right) & 0 & 0 & 50\beta^3cos\left(\frac{\beta}{2}\right) & -50\beta^3sin\left(\frac{\beta}{2}\right) & 0 & 0 \end{vmatrix} = 0 \tag{2.21}$$

The resultant transcendental equation is solved by MATLAB or MAPLE software. The first non-trivial solution is $\beta = 1.7148$ or in terms of P, we get $P = \frac{147.02EI}{L^2}$. This value is bracketed by the buckling loads of uniform column of stiffness EI and 50 EI, respectively.

$$\frac{4\pi^2EI}{L^2} < \frac{147.02EI}{L^2} < \frac{200\pi^2EI}{L^2} \tag{2.22}$$

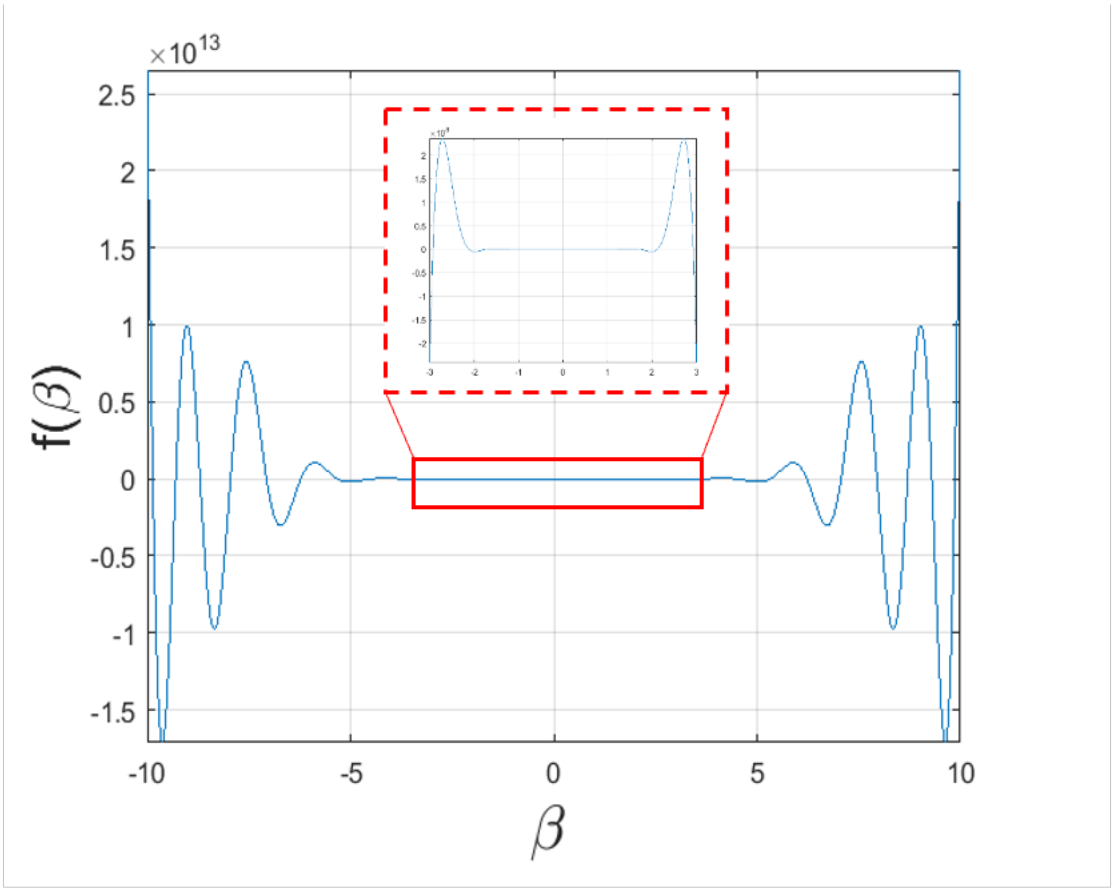

Fig. 2.2: Graph showing the solution of the transcendental equation for example 2(s=49).

Result of calculation of buckling load for student's serial number varying for s=1 to s=49 is listed in table 2.1.

Serial Number	β	Critical load	Serial Number	β	Critical load
1	5.0803	51.6189	26	2.1726	127.4452
2	4.3918	57.86372	27	2.1472	129.0931
3	3.9561	62.60291	28	2.1224	130.6329
4	3.6538	66.75127	29	2.0982	132.0733
5	3.4302	70.59763	30	2.0746	133.4229
6	3.2569	74.25178	31	2.0516	134.69
7	3.1179	77.7704	32	2.029	135.8558
8	3.0034	81.1837	33	2.007	136.9537
9	2.9068	84.49486	34	1.9854	137.9635
10	2.824	87.72474	35	1.9644	138.9192
11	2.7519	90.87544	36	1.9438	139.7993
12	2.6882	93.94345	37	1.9236	140.609
13	2.6312	96.92499	38	1.9039	141.3686
14	2.5798	99.83052	39	1.8847	142.0838
15	2.533	102.6574	40	1.8658	142.7296
16	2.4899	105.3932	41	1.8475	143.3568
17	2.4499	108.0362	42	1.8295	143.924
18	2.4126	110.5921	43	1.8119	144.4512
19	2.3775	113.0501	44	1.7948	144.9588
20	2.3444	115.4204	45	1.778	145.4191
21	2.3128	117.679	46	1.7617	145.8686
22	2.2827	119.8465	47	1.7457	146.2785
23	2.2537	121.8999	48	1.7301	146.6691
24	2.2258	123.8546	49	1.7148	147.027
25	2.1988	125.7028			

Tab. 2.1: Buckling loads for each serial number is obtained via MAPLE code (critical load values should be multiplied by EI/L^2)

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A Genesis of the Buckling Projects

The above buckling projects were inspired by the problem discussed in the book by Feodosiev (1968) [7]. This problem and its solution are exposed during the lecture. It is recommended that the lecturer present the derivations in detail in order to be able to make some didactic statements. The problem consists in determining the buckling load of the column showed in figure 3.

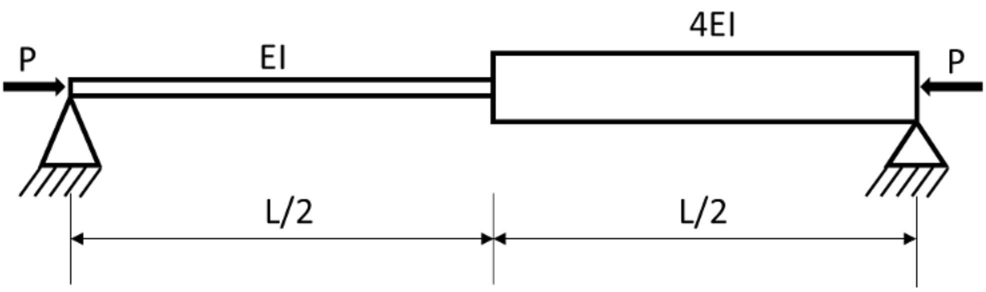

Fig. A.1: Simply supported stepped column

$$EIy_1'' + Py_1 = 0 \tag{A.1}$$
$$4EIy_2'' + Py_2 = 0 \tag{A.2}$$

Let us introduce the following notation:

$$k^2 = \frac{P}{4EI} \tag{A.3}$$

We rewrite the equation A.1 and A.2 as equation A.3 and A.4 respectively.

$$y_1'' + 4k^2 y_1 = 0 \tag{A.4}$$
$$y_2'' + k^2 y_2 = 0 \tag{A.5}$$

Solutions read, respectively,

$$y_1 = C_1 sin(2kx) + C_2 cos(2kx) \tag{A.6}$$
$$y_2 = C_3 sin(kx) + C_4 cos(kx) \tag{A.7}$$

The deflection y_1 must vanish at the simple support at x=0, and y_2 equals zero at other simple support at x=L. By applying these boundary conditions, we get $C_2 = 0$. The other boundary condition at x=L yields;

$$C_3 sin(kL) + C_4 cos(kL) = 0 \tag{A.8}$$

At $x = \frac{L}{2}$, we apply the continuity conditions, which means the respective deflections and slopes should be same at the cross section. Deflection continuity condition at $x = \frac{L}{2}$, $y_1 = y_2$ becomes,

$$C_1 sin\left(\frac{2kL}{2}\right) - C_3 sin\left(\frac{kL}{2}\right) - C_4 cos\left(\frac{kL}{2}\right) = 0 \tag{A.9}$$

The slope continuity conditions at $x = \frac{L}{2}$,namely, $y_1' = y_2'$ takes the form;

$$2kC_1cos\left(\frac{2kL}{2}\right) - kC_3cos\left(\frac{kL}{2}\right) + kC_4sin\left(\frac{kL}{2}\right) = 0 \tag{A.10}$$

Non-triviality for constant yields the determinantal equation A.10,

$$\begin{vmatrix} sin\left(\frac{2kL}{2}\right) & -sin\left(\frac{kL}{2}\right) & -cos\left(\frac{kL}{2}\right) \\ 2kcos\left(\frac{\sqrt{50}kL}{2}\right) & kcos\left(\frac{kL}{2}\right) & ksin\left(\frac{kL}{2}\right) \\ 0 & sin(kL) & cos(kL) \end{vmatrix} = 0 \tag{A.11}$$

Notation $kL = \alpha$, changes the equation A.10 into;

$$\begin{vmatrix} sin(\alpha) & -sin\left(\frac{\alpha}{2}\right) & -cos\left(\frac{\alpha}{2}\right) \\ 2cos(\alpha) & cos\left(\frac{\alpha}{2}\right) & sin\left(\frac{\alpha}{2}\right) \\ 0 & sin(\alpha) & cos(\alpha) \end{vmatrix} = 0 \tag{A.12}$$

Evaluation of the determinant equation results in

$$3\sin\alpha\cos\alpha\cos\frac{\alpha}{2} - 2\cos\alpha\cos\alpha\sin\frac{\alpha}{2} + \sin\alpha\sin\frac{\alpha}{2}\sin\alpha = 0 \tag{A.13}$$

or

$$6\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\cos\alpha\cos\frac{\alpha}{2} - 2\cos\alpha\cos\alpha\sin\frac{\alpha}{2} + \sin\alpha\sin\frac{\alpha}{2}\sin x = 0 \tag{A.14}$$

Finally, the transcendental equation is reduced to:

$$\sin\frac{\alpha}{2}\left[6\cos\frac{\alpha}{2}\cos x\cos\frac{\alpha}{2} - 2\cos\alpha\cos\alpha + \sin\alpha\sin\alpha\right] = 0 \tag{A.15}$$

We have two possible cases to consider. The first possibility is that the first factor vanishes, namely

$$\sin\frac{\alpha}{2} = 0 \tag{A.16}$$

Then $\frac{\alpha}{2} = n\pi$, where $n = 0, 1, 2, \ldots.$

$$\frac{kL}{2} = n\pi \tag{A.17}$$

If n=0, then k=0 which means in first part of the column will have no deflection at all, this cannot be true in reality, since we discuss the case when buckling occurs. So, n=1,2,3, ...
Let us consider the case n=1,

$$k = \frac{2\pi}{L} \tag{A.18}$$

From our previous notation,

$$k = \sqrt{\frac{P_{cr}}{4EI}} \tag{A.19}$$

Then we get,

$$P_{cr} = 4EI\left(\frac{2\Pi}{L}\right)^2 = \frac{16\pi^2 EI}{L^2} \tag{A.20}$$

The candidate with the second factor equals zero, yields;

$$6\cos\frac{\alpha}{2}\cos\alpha\cos\frac{\alpha}{2} - 2\cos\alpha\cos\alpha + \sin\alpha\sin\alpha = 0 \tag{A.21}$$

or

$$6\left(\cos\frac{\alpha}{2}\right)^2\cos\alpha - 2\cos^2\alpha + \sin^2\alpha = 0 \tag{A.22}$$

Further simplification leads to;

$$6\left(\cos\frac{\alpha}{2}\right)^2\cos\alpha + 3\sin^2\alpha - 2 = 0 \tag{A.23}$$

Alternatively,

$$6\left(\cos\frac{\alpha}{2}\right)^2\cos\alpha + 12\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2} - 2 = \qquad 0 \tag{A.24}$$

Using here the formula for double argument we get,

$$\begin{aligned} 6\cos^2\frac{\alpha}{2}\left(\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}\right) + 12\sin^2\frac{\alpha}{2}\cdot\cos^2\frac{\alpha}{2} - 2 &= 0 \\ 6\cos^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2} - 6\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2} + 12\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2} - 2 &= 0 \\ 6\cos^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2} + 6\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2} - 2 &= 0 \\ 6\cos^2\frac{\alpha}{2}\left[\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\right] - 2 &= 0 \\ 6\cos^2\frac{\alpha}{2} - 2 &= 0 \\ \frac{6}{1+\tan^2\frac{\alpha}{2}} - 2 &= 0 \\ 6 - 2\left(1 + \tan^2\frac{\alpha}{2}\right) &= 0 \\ 2 - \tan^2\frac{\alpha}{2} &= 0 \end{aligned} \tag{A.25}$$

Finally, we obtain

$$\begin{aligned} \tan^2\frac{\alpha}{2} - 2 &= 0 \\ \tan\frac{\alpha}{2} &= \sqrt{2} \\ \frac{\alpha}{2} &= \tan^{-1}\left(\sqrt{2}\right) \\ \frac{\alpha}{2} &= 0.955 \\ \frac{kL}{2} &= 0.955 \\ \frac{P_{cr}}{4EI} &= \left(\frac{2\times 0.955}{L}\right)^2 \\ P_{cr} &= \frac{14.6EI}{L^2} \end{aligned} \tag{A.26}$$

The value is in between the critical load of uniform beam with rigidity EI and 4EI

$$\frac{\pi^2 EI}{L^2} < \frac{14.6EI}{L^2} < \frac{\pi^2 4EI}{L^2} \tag{A.27}$$

B Buckling of Column with Absolutely Rigid Element.

As described above the stiffness of one element equals EI whereas the stiffness of another element equals (s+1)EI. Where s is a student's serial number. In a large class, the number s=1 will therefore be large. The question arises if for large s one can resort to absolutely rigid column approximation.
For the simply supported beam;

$$EIy_1'' + P_{cr}y_1 = 0\,(s+1)\,EIy_2'' + P_{cr}y_2 \qquad = 0 \tag{B.1}$$

Now, if s is large, we can rephrase eq. (B.2) by;

$$y_1'' + \frac{P_{cr}}{(s+1)\,EI}y_1 = 0 \tag{B.2}$$

For unbounded s, we can neglect second term and get;

$$y_2'' = 0 \tag{B.3}$$

The solutions of eq B.1 and B.4 are;

$$y_1 = A_1\cos(kx) + A_2\sin(kx) \tag{B.4}$$

$$y_2 = B_1x + B_2 \tag{B.5}$$

Boundary condition $y_1 = 0$ at x=0 results in;

$$A_1 = 0 \tag{B.6}$$

Likewise, condition $y_2 = 0$at x=L results in;

$$B_1 L + B_2 = 0 \tag{B.7}$$

Continuity condition $y_1 = y_2$ at $x = \frac{L}{2}$ gives;

$$A_2 \sin\left(\frac{kL}{2}\right) - B_1 \frac{L}{2} - B_2 = \qquad 0 \tag{B.8}$$

Likewise, condition $y_1' = y_2'$ at $x = \frac{L}{2}$ reads;

$$A_2 k cos\left(\frac{kL}{2}\right) - B_1 = 0 \tag{B.9}$$

The three equations B.7, B.8 and B.9 leads to determinantal equation:

$$\begin{vmatrix} 0 & L & 1 \\ sin\left(\frac{kL}{2}\right) & -\frac{L}{2} & -1 \\ kcos\left(\frac{kL}{2}\right) & -1 & 0 \end{vmatrix} = 0 \tag{B.10}$$

$$-kLcos\left(\frac{kL}{2}\right) - sin\left(\frac{kL}{2}\right) + \frac{kL}{2} cos\left(\frac{kL}{2}\right) = 0 \tag{B.11}$$

$$-sin\left(\frac{kL}{2}\right) - \frac{kL}{2} cos\left(\frac{kL}{2}\right) = 0 \tag{B.12}$$

$$tan\left(\frac{kL}{2}\right) = -\frac{kL}{2} \tag{B.13}$$

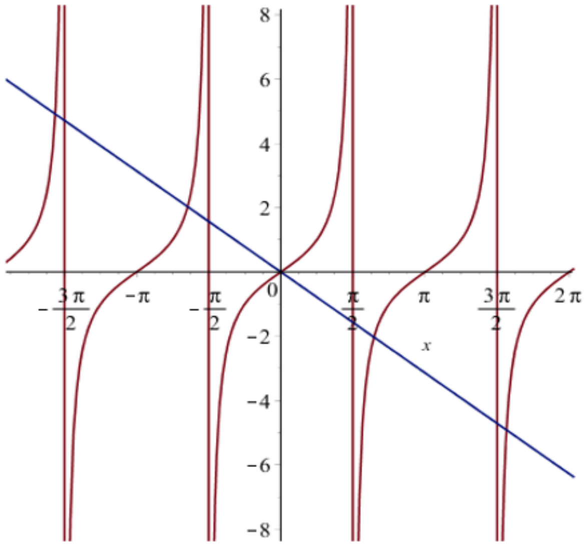

Fig. B.1: Graph showing solution for determinantal equation B.10

$$\frac{kL}{2} = 2.028757838 \tag{B.14}$$

$$k = \frac{4.057515676}{L} \tag{B.15}$$

$$k^2 = \frac{16.4634}{L^2} \tag{B.16}$$

$$k^2 = \frac{P_{cr}}{EI} \tag{B.17}$$

$$P_{cr} = \frac{16.4634 \times E\, I}{L^2} \tag{B.18}$$

In case of Fixed ended stepped column, where the rigidity of the second part of column is infinite, we get

$$EIy_1'''' + P_{cr}y_1'' = 0 \tag{B.19}$$
$$y_2'''' = 0 \tag{B.20}$$

Solution of the equation B.11 and B.12 are B.13 and B.14 respectively;

$$y_1 = A_1 \sin(kx) + A_2 \cos(kx) + A_3 x + A_4 \tag{B.21}$$
$$y_2 = \frac{B_1 x^3}{6} + \frac{B_2 x^2}{2} + B_3 x + B_4 \tag{B.22}$$

Boundary condition $y_1 = 0$ and$y_1' = 0$ at x=0 results in;

$$A_2 + A_4 = 0 \tag{B.23}$$
$$A_1 k + A_3 = 0 \tag{B.24}$$

Likewise, condition $y_2 = 0$ and $y_2' = 0$ at x=L results in;

$$\frac{B_1 L^3}{6} + \frac{B_2 L^2}{2} + B_3 L + B_4 = 0 \tag{B.25}$$
$$\frac{B_1 L^2}{2} + B_2 L + B_3 = 0 \tag{B.26}$$

At the cross section $x = \frac{L}{2}$, we apply the continuity conditions, which means the deflection, slope, bending moment and effective shear force will be continues at the cross section. Hence;

$$y_1\left(\frac{L}{2}\right) = y_2\left(\frac{L}{2}\right)$$
$$A_1 \sin\left(\frac{kL}{2}\right) + A_2 \cos\left(\frac{kL}{2}\right) + A_3 \frac{kL}{2} + A_4 - \frac{B_1 L^3}{48} - \frac{B_2 L^2}{8} - \frac{B_3 L}{2} - B_4 = 0 \tag{B.27}$$

$$y_1'\left(\frac{L}{2}\right) = y_2'\left(\frac{L}{2}\right)$$
$$A_1 k cos\left(\frac{kL}{2}\right) - A_2 k sin\left(\frac{kL}{2}\right) + A_3 K - \frac{B_1 L^2}{g} - \frac{B_2 L}{2} - B_3 K = 0 \tag{B.28}$$

$$EIy_1''\left(\frac{L}{2}\right) = \infty \cdot EIy_2''\left(\frac{L}{2}\right)$$
$$y_2''\left(\frac{L}{2}\right) = 0$$
$$B_1\left(\frac{L}{2}\right) + B_2 = 0 \tag{B.29}$$

$$EIy_1'''\left(\frac{L}{2}\right) = -\infty EIy_2'''\left(\frac{L}{2}\right)$$
$$y_2'''\left(\frac{L}{2}\right) = 0$$
$$B_1 = 0 \tag{B.30}$$

From the above equations B.15 -B.22, we get the determinantal equation

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \sin\left(\frac{kL}{2}\right) & \cos\left(\frac{kL}{2}\right) & \frac{L}{2} & 1 & -\frac{L}{2} & -1 \\ \cos\left(\frac{kL}{2}\right) & -\sin\left(\frac{kL}{2}\right) & 1 & 0 & -1 & 0 \end{vmatrix} = 0 \tag{B.31}$$

After solving the matrix with the help of software, one can get the answer as following:

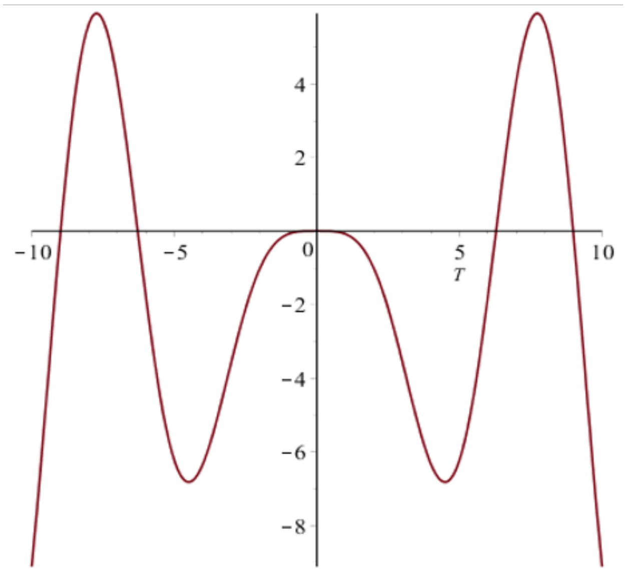

Fig. B.2: Graph showing the solution of determinantal equation B.23

$$\frac{kL}{2} = 6.283185307$$
$$k = \frac{12.566370614}{L}$$
$$k^2 = \frac{157.913670408}{L^2}$$
$$k^2 = \frac{P_{cr}}{EI}$$
$$P_c r = (157.913670408 \times EI)\,/L^2 \tag{B.32}$$