TECHNISCHE MECHANIK
an open access journal

journal homepage: www.ovgu.de/techmech

Tech. Mech., Vol. 43, Is. 2, (2023), 249–258

Received: 07.06.2023
Accepted: 02.11.2023
Available online: 02.11.2023

Stability analysis of hygro-magneto-flexo electric functionally graded nanobeams embedded on visco-Pasternak foundation

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Abstract: In this paper, the stability analysis of hygro-magneto-flexo electricity (HMFE) on functionally graded (FG) viscoelastic nanobeams accommodate in viscoelastic foundation based on nonlocal elasticity theory is addressed. Higher order refined beam theory is used for the expositions of the displacement components and the viscoelastic foundation is included with Winkler-Pasternak layer. The governing equations of nonlocal gradient viscoelastic FG nanobeam are obtained by Hamilton's principle and solved by administrating an analytical solution for different boundary conditions. A power-law index model is adopted to describe continuous variation of temperature-dependent material properties of FG nanobeam. A parametric study is presented to inquire the effect of the nonlocal parameter on various physical variables.

Keywords: Damping vibration, Magneto-electro-viscoelastic, FG nanobeam, Visco-Pasternak foundation, Nonlocal strain gradient elasticity.

1 Introduction

The significance of functionally graded materials (FGMs) in contrast to conventional materials, as well as their effectiveness in overcoming technical challenges, especially in demanding hygro-thermal conditions, has greatly expanded the utilization of FGM structures. Therefore, hygro-thermal analysis of FGM structures is a benefit study in the research community. For this purpose, Akbarzadeh and Chen (2013) verified effects of hygro-thermal loading of functionally graded piezoelectric material. Lee and Kim (2013) investigated Post-buckling analyses of FG plates in hygro-thermal environment. Moreover, Mansouri and Shariyat (2015) verified hygro-thermal buckling behavior of FGMs plate with negative Poisson ratio. Ebrahimi and Salari (2015a,b) performed vibration study of orthotropic nanoplates incorporating nonlocal effects using a semi-analytical approach. Ansari and Gholami (2016) explored nonlocal vibrational response of buckled thermo-electro-elastic nanoplates considering different boundary conditions. Surface energy effect on nonlinear free vibration behavior of orthotropic piezoelectric cylindrical nano-shells and double shell structures are analyzed elaborately by Zhu et al. (2018, 2017). Merabet et al. (2017) adopted a novel nonlocal model to examine the buckling temperature of a single-walled boron nitride nanotube. So many researchers contributed towards bending of carbon nano tube(CNT) via nonlocal form and their modified versions (Merabet et al. (2017); Kheroubi et al. (2016); Ebrahimi and Barati (2016); Rakrak et al. (2016); Arda and Aydogdu (2020)). It is clear from the previous papers that, on nanoplates only the nonlocal elasticity theory is applied to capture small scale effects. However, nonlocal elasticity theory has some limitations in accurate prediction of mechanical behavior of nanostructures which is unable to examine the stiffness increment observed in experimental works and strain gradient elasticity. Description of size-dependency is still a challenging issue in modeling of nanostructures. To evaluate the mechanical behavior of nanoscale structural elements used in nano electro mechanical systems(NEMS), two types of nonlocal models are proposed, i.e. nonlocal hardening model and nonlocal softening model. Arani et al. (2016) examined nonlocal vibration of axially moving graphene sheet resting on orthotropic visco-Pasternak foundation under longitudinal magnetic field. Ke et al. (2015) examined free vibrational response of nonlocal piezoelectric nanoplates considering different boundary conditions. Also, Li et al. (2014) investigated stability and vibrational behavior of magneto-electro-elastic nanoplate according to the nonlocal theory. Based on the thin plate theory of Kelvin-Voigt model, they concluded the effect of some important parameters of the problem such as input voltage, damping coefficient, viscoelastic parameter and excitation frequency on the results. Theoretical studies are conducted on CNT incorporated with surface effect, linear varying load, low velocity

 doi: 10.24352/UB.OVGU-2023-061

and initial imperfections(Ebrahimi and Barati (2017a,b); Ebrahimi and Dabbagh (2018); Ehyaei and Daman (2017)). Vibration analysis of variable thickness rectangular viscoelastic nanoplates was studied by Mohammadsalehi et al. (2016). Nevertheless, the influence of viscoelastic properties is not considered in these studies. Ansari and Gholami (2016) explored nonlocal vibrational response of buckled magneto-electro-thermo-elastic nanoplates considering different boundary conditions. Lim et al. (2015) proposed the nonlocal strain gradient theory to introduce both of the length scales into a single theory. The nonlocal strain gradient theory captures the true influence of the two length scale parameters on the physical and mechanical behavior of small size structures. Ebrahimi and Barati (2016, 2017a,b) applied the nonlocal strain gradient theory in analysis of nanobeams. They mentioned that mechanical characteristics of nanostructures are significantly affected by stiffness-softening and stiffness-hardening mechanisms due to the nonlocal and strain gradient effects, respectively. Ebrahimi and Dabbagh (2018) extended the nonlocal strain gradient theory for analysis of nanoplates to obtain the wave frequencies for a range of two scale parameters. Hosseini and Jamalpoor (2015) derived an analytical solution for thermal vibration of double-viscoelastic FGM nanoplates. They reported that both viscoelastic medium and viscoelastic internal damping have a significant influence on the vibration characteristics of FG nanosystems. So, it is crucial to incorporate both nonlocal and strain gradient effects in analysis of graphene sheets for the first time. Lei et al. (2013) studied vibration of size-dependent Kelvin-Voigt viscoelastic damped Timoshenko nanobeams. Pouresmaeeli et al. (2013) analyzed vibration of viscoelastic orthotropic nanoscale plates resting on viscoelastic foundation. Hashemi et al. (2015) presented free vibration evaluation of double layered viscoelastic graphene sheets embedded in visco-Pasternak medium.

Hygro-thermo-mechanical analysis of nanostructures is carried out by a limited number of researchers. Kurtinaitiene et al. (2016) investigated the effect of additives on the hydrothermal synthesis of manganese ferrite nanoparticles. Alzahrani et al. (2013) investigated size effects on static behavior of nanoplates resting on elastic foundation subjected to hygro-thermal loadings. They extended nonlocal constitutive relations of Eringen to contain the hygro-thermal effects. Also, Sobhy (2015) studied frequency response of simply-supported shear deformable orthotropic graphene sheets exposed to hygro-thermal loading. Free vibration of magneto-electro-elastic (MEE) nanoplates based on the Eringen's nonlocal theory and Kirchhoff plate theory were studied by Ke et al. (2015). However, the MEVHT buckling of orthotropic higher order refined beam theory viscoelastic FG nanobeams resting on four-parameter viscoelastic foundation and magneto-electro properties based on nonlocal theory has not been reported so far.

Based on above studies, we verified the transient damping vibration of magneto-electro-viscoelastic-hygro-thermal (MEVHT) on functionally graded (FG) viscoelastic nanobeams accommodate in viscoelastic foundation based on nonlocal strain gradient elasticity equation. Higher order refined beam theory is used for the expositions of the displacement components. The governing equations of nonlocal strain gradient viscoelastic FG nanobeam are obtained by Hamilton's principle and solved by administrating an analytical solution for different boundary conditions. The importance of natural frequency to the mechanical loading, electric loading and magnetic loading, power-law exponent and slenderness ratio of viscoelastic FG nanobeams are verified.

2 Theory and formulation of the problem

2.1 Effective properties of FGM nanobeam based on neutral axis position

It is regarded that the neutral axis of a FG nanobeam liability to the variation in volume fractions of material phases may not coincide with its mid-axis which redounds to bending-extension coupling. Considering the exact position of neutral axis, this coupling can be eliminated. To capture exact position of neutral axis, the z_{ms}, z_{ns} are measured from the middle and neutral surfaces, respectively as depicted in Fig. 1.

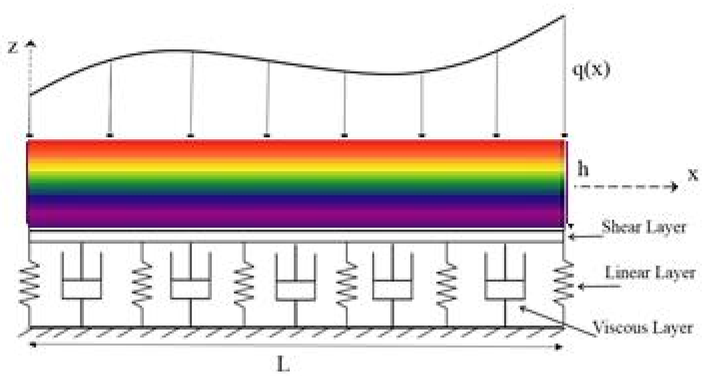

Fig. 1: Geometry of FG nanobeam resting on viscoelastic foundation

Satisfying the first moment with respect to Young's modulus being zero, the exact position of neutral axis is determined as follows:

$$\int_{-h/2}^{h/2} E(z_{ms})(z_{ms} - h_0)dz_{ms} = 0 \tag{1}$$

In which the neutral axis ($z = h_0$) is defined by:

$$h_0 = \frac{\int_{-h/2}^{h/2} E(z_{ms}) z_{ms} dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms}) dz_{ms}} \tag{2}$$

Assumption of a viscoelastic FG nanobeam bedded in viscoelastic medium having length L, width b and thickness h which its coordinates are depicted in Fig. 1. The hygro-thermo-viscoelastic material properties of nonlocal FGM beam including Young's modulus (E), mass density (ρ), thermal expansion (α) and moisture expansion coefficient (β) can be represented by:

$$E(z_{ns}) = (E_c - E_m)\left(\frac{z_{ns} + h_0}{h} + \frac{1}{2}\right)^p + E_m, \tag{3a}$$

$$\rho(z_{ns}) = (\rho_c - \rho_m)\left(\frac{z_{ns} + h_0}{h} + \frac{1}{2}\right)^p + \rho_m, \tag{3b}$$

$$\alpha(z_{ns}) = (\alpha_c - \alpha_m)\left(\frac{z_{ns} + h_0}{h} + \frac{1}{2}\right)^p + \alpha_m, \tag{3c}$$

$$\beta(z_{ns}) = (\beta_c - \beta_m)\left(\frac{z_{ns} + h_0}{h} + \frac{1}{2}\right)^p + \beta_m, \tag{3d}$$

Here p indicates the power-law exponent which evaluates the distribution of material properties across the thickness. It is reported in some studies that considering temperature-dependent material properties provide more accurate results for analysis of FGM structures.Therefore, the following relation can be adopted to represent the temperature-dependent material coefficients:

$$P = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{4}$$

in which P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients of material phases.

2.2 Kinematic relations

The displacement field of refined shear deformable FGM beam can be expressed by:

$$u_x(x, z_{ns}) = u(x) - z_{ns}\frac{\partial w_b}{\partial x} - f(z_{ns})\frac{\partial w_s}{\partial x} \tag{5a}$$

$$u_z(x, z_{ns}) = w_b(x) + w_s(x) \tag{5b}$$

where u is axial mid-plane displacement and w_b, w_s denote the bending and shear components of transverse displacement, respectively. Also, $f(z_{ns})$ is the shape function representing the shear stress/strain distribution through the beam thickness which excludes the shear correction factor in view of trigonometric function for the current study (Arani et al. (2016)):

$$f(z_{ns}) = z_{ns} + h_0 - \tan[m(z_{ns} + h_0)],\ m = 0.03. \tag{6}$$

where m is the parameter of the shape function. The non-zero strains of the suggested beam model can be expressed as follows:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z_{ns}\frac{\partial^2 w_b}{\partial x^2} - f(z_{ns})\frac{\partial^2 w_s}{\partial x^2} \tag{7a}$$

$$\gamma_{xz} = g(z_{ns})\frac{\partial w_s}{\partial x} \tag{7b}$$

where $g(z_{ns}) = 1 - \frac{df(z_{ns})}{dz_{ns}}$. Also, the Hamilton's principle states that:

$$\int_0^t \delta(\Pi_S + \Pi_W) dt = 0 \tag{8}$$

where Π_S is the total strain energy and Π_W is the work done by external applied forces. The first variation of strain energy Π_S can be calculated as:

$$\delta\Pi_S = \int \sigma_{ij}\delta\varepsilon_{ij} dv = \int \sigma_x\delta\varepsilon_x + \sigma_{xz}\delta\gamma_{xz} + \sigma_{yz}\delta\gamma_{yz}. \tag{9}$$

Substituting Eqs.(5a) - (7b) into Eq.(9) yields:

$$\delta\Pi_S = \int_0^l \left(N\frac{\partial\delta u}{\partial x} - M_b\frac{\partial^2\delta w_b}{\partial x^2} - M_s\frac{\partial^2\delta w_s}{\partial x^2} + Q\frac{\partial\delta w_s}{\partial x}\right) dx. \tag{10}$$

In which the forces and moments expressed in the above equation are defined as follows:

$$(N, M_b, M_s) = \int_{-h/2-h_0}^{h/2-h_0} (1, z_{ns}, f)\sigma_{xx} dz_{ns}, \quad Q = \int_{-h/2-h_0}^{h/2-h_0} g\sigma_{xz} dA \tag{11}$$

In this study, the nanobeam is subjected to an in-plane axial magnetic field. Hence, to derive the exerted body force from longitudinal magnetic field $H = (H_x, 0, 0)$, the Maxwell relations are adopted:

$$f_{Iz} = \eta(\nabla \times (\nabla \times (\vec{u} \times \vec{H}))) \times \vec{H}. \tag{12}$$

where $\vec{u} = (u_x, 0, u_z)$ is displacement vector and η is magnetic permeability. For a planar beam deformation with the assumed displacement field, the resultant Lorentz force takes the form:

$$f_{Iz} = \eta \int_A f_z dA = \eta A H_x^2 \frac{\partial^2 w}{\partial x^2}. \tag{13}$$

The first variation of the work done by applied forces can be written in the form:

$$\delta\Pi_w = \int_0^l \left(-N_x^0 \frac{\partial w^b}{\partial x}\frac{\partial \delta w^b}{\partial x} - N_y^0 \frac{\partial w^s}{\partial y}\frac{\partial \delta w^s}{\partial y} + 2\delta N_{xy}^0 \frac{\partial w^b}{\partial x}\frac{\partial \delta w^s}{\partial y} - k_w \delta(w^b + w^s) + k_p \frac{\partial^2 (w^b + w^s)}{\partial x^2}\right) dxdy - c_d \frac{\partial \delta(w^b + w^s)}{\delta t} - \eta A H_x^2 \frac{\partial^2 \delta(w^b + w^s)}{\partial x^2} + f_{13}\delta(w^b + w^s) \tag{14}$$

where f_{13} is the flexoelectricity coefficient, N^T and N^H are applied forces due to variation of temperature and moisture and are defined as:

$$\begin{aligned} N^T &= \int_{-h/2-h_0}^{h/2-h_0} E(z_{ns})\alpha(z_{ns})(\Delta T) dz_{ns}, \\ N^H &= \int_{-h/2-h_0}^{h/2-h_0} E(z_{ns})\beta(z_{ns})(\Delta C) dz_{ns} \end{aligned} \tag{15}$$

where $\Delta T = T - T_0$ and $\Delta C = C - C_0$ in which T_0 and C_0 are the reference temperature and moisture concentration.

The governing equations are obtained by inserting Eqs. (10)-(15) in Eq. (8) when the coefficients of δu, δw_b and δw_z are equal to zero:

$$\frac{\partial N}{\partial x} + \eta A H_x^2 (w^b + w^s) = 0 \tag{16}$$

$$\frac{\partial^2 M^b}{\partial x^2} + (-N^E - N^H - N^T + K_p)\nabla^2 (w^b + w^s) + k_w (w^b + w^s) + c_d \frac{\partial (w_b + w_s)}{\partial t} = 0 \tag{17}$$

$$\frac{\partial^2 M^b}{\partial x^2} + \frac{\partial Q}{\partial x} + (-N^E - N^H - N^T)\frac{\partial^2 (w^b + w^s)}{\partial x} - K_p \frac{\partial^2 (w^b + w^s)}{\partial x} + k_w (w^b + w^s) + c_d \frac{\partial (w_b + w_s)}{\partial t} - k_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} = 0 \tag{18}$$

2.3 Nonlocal elasticity theory for the hygro-thermo-magneto-electro-elastic materials

The main point of the nonlocal elasticity theory is that the nonlocal stress tensor at a reference point relies not only on the strain tensor of the same coordinate but also on other points in the solid.

The nonlocal theory can be extended for the piezoelectric nanoplates as:

$$\sigma_{ij} - (ea)^2 \nabla^2 \sigma_{ij} = [C_{ijkl}\varepsilon_{kl} - C_{ijkl}\alpha_{kl}\Delta T - C_{ijkl}\beta_{kl}\Delta C - e_{mij}E_m] \tag{19}$$

$$D_{ij} - (ea)^2 \nabla^2 D_{ij} = [e_{ikl}\varepsilon_{kl} - s_{im}E_m] \tag{20}$$

$$B_i - (ea)^2 \nabla^2 B_i = [q_{ikl}\varepsilon_{kl} + d_{im}E_m]. \tag{21}$$

where C_{ijkl} is the elastic moduli and ε_{kl} is strain tensor. The constitutive Eq. (19) can be developed to capture the influence of hygro-thermal loading as:

$$\sigma_{ij} - (ea)^2 \nabla^2 \sigma_{ij} = \left[C_{ijkl}\varepsilon_{kl} - f_{klij}\frac{\partial E_K}{\partial x_l} + C_{ijkl}\alpha_{kl} - C_{ijkl}\beta_{kl}\Delta C - \eta A H_x^2\right] \tag{22}$$

where α_{ij} and β_{ij} are thermal and moisture expansion coefficients respectively; ΔT and ΔC denote the temperature and moisture variation, respectively.

Where $\mu = (ea)^2$, inserting Eqs. (19)-(22) in Eq. (10) and considering Kelvin-Voit viscoelastic model(Hashemi et al. (2015)) and integrating Eq. (18) over the cross-section area of nanobeam provides the following nonlocal relations for a refined FGM beam model as

$$N - \mu \frac{\partial^2 N}{\partial x^2} = \left(A\frac{\partial u}{\partial x} - B\frac{\partial^2 w_b}{\partial x^2} - B_s\frac{\partial^2 w_s}{\partial x^2} \right) - N_x^T - N_x^H \tag{23}$$

$$M_b - \mu \frac{\partial^2 M_b}{\partial x^2} = \left(B\frac{\partial u}{\partial x} - D\frac{\partial^2 w_b}{\partial x^2} - D_s\frac{\partial^2 w_s}{\partial x^2} \right) - M_b^T - M_b^H \tag{24}$$

$$M_s - \mu \frac{\partial^2 M_s}{\partial x^2} = \left(B_s\frac{\partial u}{\partial x} - D_s\frac{\partial^2 w_b}{\partial x^2} - H_s\frac{\partial^2 w_s}{\partial x^2} \right) - M_b^T - M_b^H \tag{25}$$

$$Q - \mu \frac{\partial^2 Q}{\partial x^2} = 0 \tag{26}$$

where the cross-sectional rigidities are calculated as follows:

$$(A, B, B_s, D, D_s, H_s) = \int_{-h/2-h_0}^{h/2-h_0} E(z_{ns})(1, z_{ns}, f, z_{ns}^2, z_{ns}f, f^2)dz_{ns}, \tag{27}$$

$$A_s = \int_{-h/2-h_0}^{h/2-h_0} g^2 G(z_{ns})dz_{ns} \tag{28}$$

and

$$\begin{aligned} \{N_x^T, M_b^T, M_s^T\} &= \int_{-h/2-h_0}^{h/2-h_0} E(z_{ns})\alpha(z_{ns})(T - T_0)\{1, z_{ns}, f\}dz_{ns}, \\ \{N_x^H, M_b^H, M_s^H\} &= \int_{-h/2-h_0}^{h/2-h_0} E(z_{ns})\beta(z_{ns})(C - C_0)\{1, z_{ns}, f\}dz_{ns}. \end{aligned} \tag{29}$$

The governing equations of shear deformable viscoelastic FGM nanobeam resting on three-parameter viscoelastic medium in hygro-thermal environment in terms of displacements are obtained by inserting for N, M_b, M_s and Q from Eqs. (23)-(26), respectively into Eqs. (16)-(18) as follows:

$$A\left(\frac{\partial^2 u}{\partial x^2}\right) - B\left(\frac{\partial^3 w_b}{\partial x^3}\right) - B_s\left(\frac{\partial^3 w_s}{\partial x^3}\right) = 0 \tag{30}$$

$$\begin{aligned} &B\left(\frac{\partial^3 u}{\partial x^3}\right) - D\left(\frac{\partial^4 w_b}{\partial x^4}\right) - D_s\left(\frac{\partial^4 w_s}{\partial x^4}\right) - k_w(w_b + w_s) - \eta A H_x^2(w_b + w_s) + k_p\frac{\partial^2(w_b + w_s)}{\partial x^2} \\ &- c_d\frac{\partial(w_b + w_s)}{\partial t} + \mu((N^T + N^H)\frac{\partial^4(w_b + w_s)}{\partial x^4} + k_w\frac{\partial^2(w_b + w_s)}{\partial x^2} + k_p\frac{\partial^4(w_b + w_s)}{\partial x^4} + \eta A H_x^2\frac{\partial^2(w_b + w_s)}{\partial x^2} \\ &+ c_d\frac{\partial^3(w_b + w_s)}{\partial x^2 \partial t} - \frac{e_{31}}{2k_{33}}f_{13}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2}\right) \end{aligned} \tag{31}$$

$$\begin{aligned} &B_s\left(\frac{\partial^3 u}{\partial x^3}\right) - D_s\left(\frac{\partial^4 w_b}{\partial x^4}\right) - H_s\left(\frac{\partial^4 w_s}{\partial x^4}\right) + A_s\left(\frac{\partial^2 w_s}{\partial x^2}\right) - k_w(w_b + w_s) - \eta A H_x^2(w_b + w_s) + k_p\frac{\partial^2(w_b + w_s)}{\partial x^2} \\ &- c_d\frac{\partial(w_b + w_s)}{\partial t} + \mu((N^T + N^H)\frac{\partial^4(w_b + w_s)}{\partial x^4} + k_w\frac{\partial^2(w_b + w_s)}{\partial x^2} + k_p\frac{\partial^4(w_b + w_s)}{\partial x^4} + \eta A H_x^2\frac{\partial^2(w_b + w_s)}{\partial x^2} \\ &+ c_d\frac{\partial^3(w_b + w_s)}{\partial x^2 \partial t} - \frac{e_{31}}{2k_{33}}f_{13}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2}\right) \end{aligned} \tag{32}$$

3 Solution procedure

An analytical solution is implemented in this section for the generalized displacements by expanding a double Fourier series in terms of unknown parameters. The selection of the functions in these series is associated to those which satisfy the boundary edges of the nanoplate. These boundary edges are given as :

- **Simply-supported:**

$$w_b = w_s = N_x = M_x = 0 \text{ at } x = 0,\ a \tag{33}$$
$$w_b = w_s = N_y = M_y = 0 \text{ at } y = 0,\ b \tag{34}$$

- **Clamped:**

$$u = v = w_b = w_s = 0 \text{ at } x = 0,\ a \text{ and } y = 0,\ b \tag{35}$$

To satisfy the above-mentioned boundary conditions, the displacement quantities are presented in the following form:

$$u = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn} \frac{\partial X_m(x)}{\partial x} Y_n(y) \tag{36}$$
$$w_b = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{b_{mn}} X_m(x) Y_n(y) \tag{37}$$
$$w_s = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{s_{mn}} X_m(x) Y_n(y) \tag{38}$$

where $(U_{mn},\ W_{b_{mn}},\ W_{s_{mn}})$ are the unknown coefficients and for different boundary conditions $(\alpha = m\pi/a,\ \beta = n\pi/b)$, where:

$$([\mathscr{K}] + [\mathscr{C}]) \begin{Bmatrix} U \\ W_b \\ W_s \end{Bmatrix} = 0 \tag{39}$$

Where $[\mathscr{K}]$, $[\mathscr{C}]$ are the stiffness, damping matrixes for FG nanobeam respectively.

$$\begin{gathered}
k_{1,1} = A(\alpha_3 - \eta\alpha_{11}), \quad k_{1,2} = B(\alpha_9 - \eta\alpha_{13}), \quad k_{1,3} = B_s(\alpha_9 - \eta\alpha_{13}) \\
k_{2,3} = (N^H + N^T - k_p)(-\alpha_7 + \mu\alpha_9) - k_w(\alpha_5 - \mu\alpha_7) - f_{lz}(\alpha_5 - \mu\alpha_7) - D_s(\alpha_9 - \eta\alpha_{13}) \\
k_{2,2} = (N^H + N^T - k_p)(-\alpha_7 + \mu\alpha_9) - k_w(\alpha_5 - \mu\alpha_7) - f_{lz}(\alpha_5 - \mu\alpha_7) - D(\alpha_9 - \eta\alpha_{13}) \\
k_{3,3} = (N^H + N^T - k_p)(-\alpha_7 + \mu\alpha_9) - k_w(\alpha_5 - \mu\alpha_7) - A_s(\alpha_5 - \mu\alpha_7) - \frac{e_{31}}{2k_{33}} f_{13}(\alpha_5 - \mu\alpha_7) - H_s(\alpha_9 - \eta\alpha_{13}) \\
c_{1,1} = A\,\Im(\alpha_3 - \eta\alpha_{11}), \quad c_{1,2} = B\,\Im(\alpha_9 - \eta\alpha_{13}), \quad c_{1,3} = B_s\,\Im(\alpha_9 - \eta\alpha_{13}) \\
c_{3,3} = -c_d i(\alpha_5 - \mu\alpha_7) + A_s\,\Im(\alpha_7 - \mu\alpha_9) - H_s(\alpha_9 - \eta\alpha_{13})
\end{gathered} \tag{40}$$

in which

$$\alpha_1 = \int_0^L X'_m X'_m dx, \quad \alpha_3 = \int_0^L X'''_m X'_m dx \tag{41}$$
$$\alpha_5 = \int_0^L X_m X_m dx, \quad \alpha_7 = \int_0^L X''_m X_m dx, \quad \alpha_9 = \int_0^L X''''_m X_m dx \tag{42}$$
$$\alpha_{11} = \int_0^L X'''''_m X'_m dx, \quad \alpha_{13} = \int_0^L X''''''_m X_m dx \tag{43}$$

4 Numerical Results and Discussion

In this section, we studied the buckling critical loading behavior of nonlocal strain gradient magneto-piezoelectric FG on viscoelastic substrate. The model introduces two scale coefficients related to nonlocal and strain gradient effects for more accurate analysis of smart FG nanobeams under various boundary conditions (Simply-supported and Clamped). Material properties of the FG nanobeams are presented in Table 1. Configuration of FG nanobeams on viscoelastic medium is shown in Fig. 1. Temperature-dependent material properties are shown in Table 1 for a P-FGM nanobeam which include of metal (SUS304) at $z = -h/2$ with $\beta_m = 0.0005$ and ceramic (Si_3N_4) at $z = +h/2$ with $\beta_c = 0$. The length of nanobeam is considered to be $L = 10$nm. Also, the dimensionless buckling and dimensionless form of viscoelastic parameters are adopted as:

$$\bar{N} = N^b \frac{a^2}{D_c}, \quad K_w = \frac{k_w a^4}{D_c}, \quad K_p = \frac{k_p a^2}{D_c}, \quad D_c = c^u_{11} h^3 \tag{44}$$

The effect of magnetic field length scale parameter on vibration behavior of FG nanobeams at $L/h = 30$, $K_p = K_w = 0$ is presented in Fig. 2. It is visible from this figure that the magnitudes of natural frequencies become larger as the parameter increases for all values of magnetic field intensity. Magnetic field shows a significant effect on vibration behavior of FG nanobeams. Increasing magnetic field intensity leads to larger frequencies.

Tab. 1: Temperature-dependent material properties of FGMs (Sobhy (2015))

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E(Pa)	348.43e+9	0	−3.070e−4	2.160e−7	−8.946e−11
	$\alpha(K^{-1})$	5.8723e−6	0	9.095e−4	0	0
	$\rho(Kg/m^3)$	2370	0	0	0	0
	ν	0.24	0	0	0	0
SUS304	E(Pa)	201.04e+9	0	3.079e−4	−6.534e−7	0
	$\alpha(K^{-1})$	12.330e−6	0	8.086e−4	0	0
	$\rho(Kg/m^3)$	8166	0	0	0	0
	ν	0.3262	0	−2.002e−4	3.797e−7	0

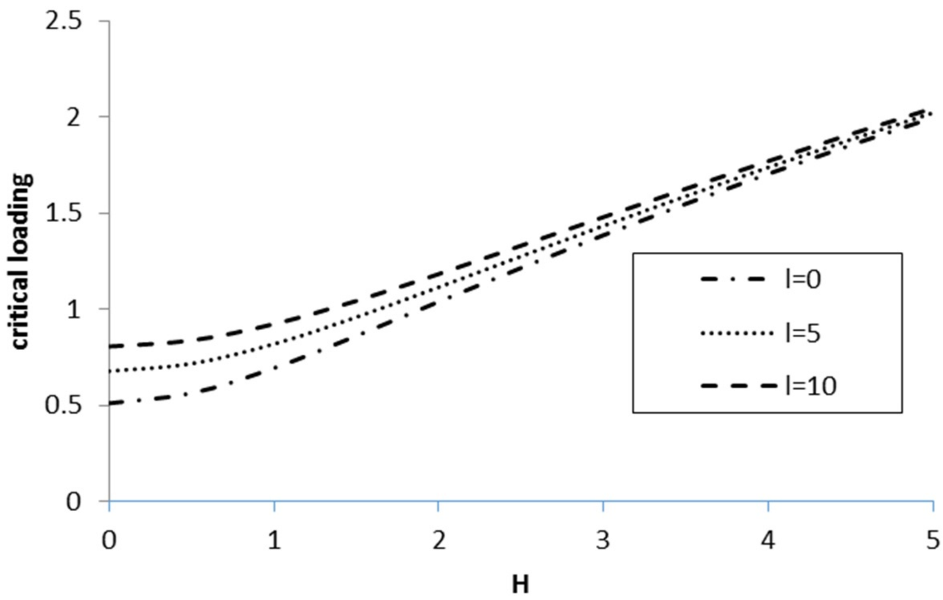

Fig. 2: Critical loading versus magnetic field intensity(H) for different length scale parameters under flexoelectric effect ($L/h = 30$, $\mu = 2$, $K_p = K_w = 0$)

Figs. 3 and 4 demonstrate the effect of Winkler and Pasternak coefficients on the critical buckling moisture with changing of side-to-thickness ratio (a/h) when $\mu = 2\text{nm}^2$ and $V = 0$, $\eta = 0.01$. It can be deduced that effect of foundation coefficients is more sensible at lower side-to-thickness ratios for all thermal environments. In fact, effect of foundation parameters vanishes at larger side-to-thickness ratios. Thus, it can be concluded that effect of Winkler-Pasternak foundation is negligible for thinner piezo nanoplates.

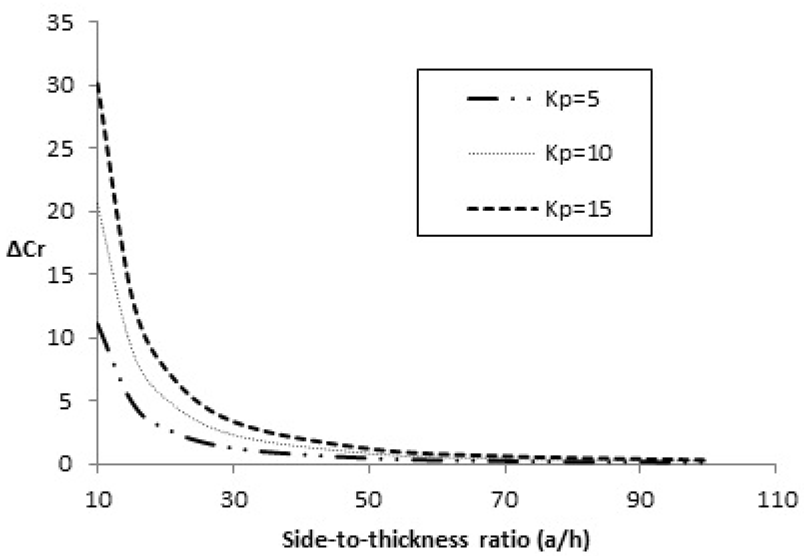

Fig. 3: Critical buckling of moisture versus side-to-thickness ratio for various Pasternak parameters ($\eta = 0.01$, $\mu = 2$, $\Omega = V = 0$)

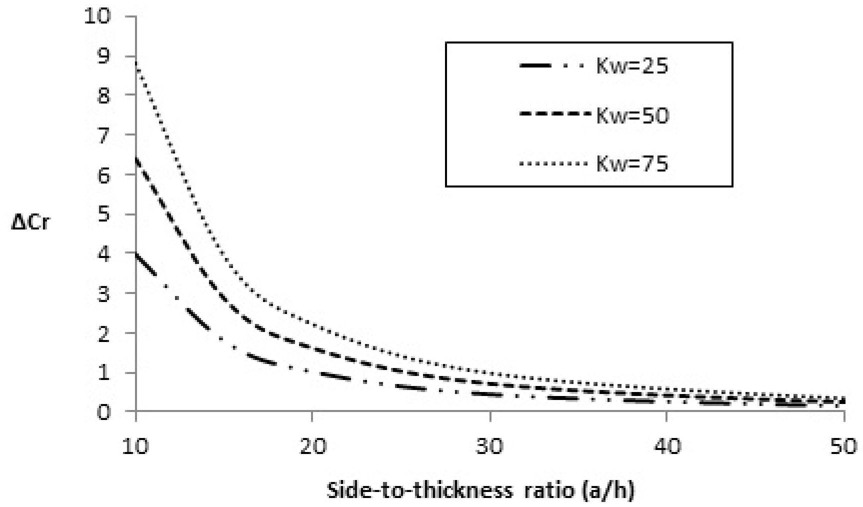

Fig. 4: Critical buckling moisture versus side-to-thickness ratio for various Winkler parameters (μ = 2, V = 0, η = 0.01)

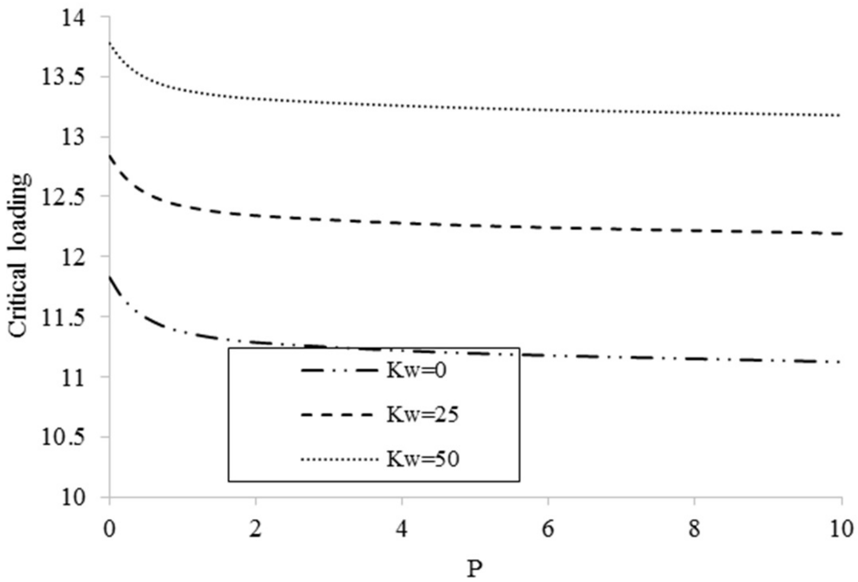

Fig. 5: Critical loading versus power law index for various Winkler parameter (L/h = 20, K_p = 5, μ = 2)

The effects of Winkler and Pasternak parameters on the variations of the first non-dimensional buckling FG nanobeams versus power-law exponent for different electric voltages at L/h = 10 and p = 1 are presented in Figs. 5, 6. It is seen that for all values of elastic foundation constants the non-dimensional buckling reduces with the increase of power-law exponent.

5 Conclusion

The stability analysis of hygro-magneto-flexo electricity on functionally graded (FG) viscoelastic nanobeam accommodate in viscoelastic foundation is verified via nonlocal elasticity theory. The displacement exposition is modelled via higher order refined beam theory. The bedded viscoelastic foundation is considered as Winkler-Pasternak layer. The governing equations of nonlocal gradient viscoelastic FG nanobeam are obtained by Hamilton's principle and solved by administrating an analytical solution for different boundary conditions. Power-law model is adopted to describe continuous variation of temperature-dependent material properties of FG nanobeam. A parametric study is presented to inquire the effect of the nonlocal parameter on various physical variables of HMFE nanoplates. The bolded points are

- The critical loading of piezoelectric FG nanobeam reduces by inclusion of nonlocal parameter.
- The buckling of piezoelectric FG nanobeam increases with increase of length scale parameter.
- Frequency of piezoelectric FG nanobeam increases with increase of length scale parameter which highlights the thermal loading effect due to the strain gradients.
- The impact of flexoelectricity on critical loading increases, with its effect being contingent on nonlocality.
- Pasternak layer provides a continuous interaction with piezoelectric sheet, while Winkler layer has a discontinuous interaction with the FG nanobeam.

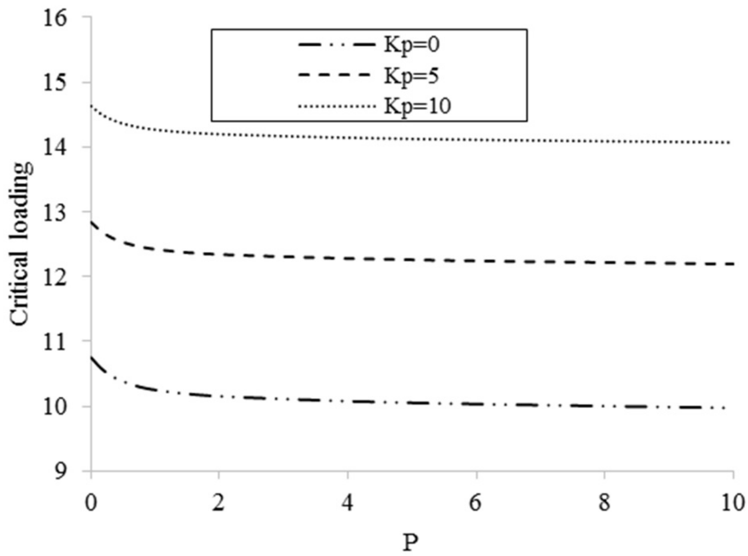

Fig. 6: Critical loading versus power law index for various Winkler parameter ($L/h = 20$, $K_w = 25$, $\mu = 2$)

- Nonlocal parameter exerts a stiffness-softening effect which leads to lower critical loading.
- The magnitude of both real and imaginary eigen frequencies reduces by increasing gradient index (p), especially at lower gradient indexes.
- This study can be extended to the nonlinear vibration of FG nano beam by including proper geometric relation.

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