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The onset of electrothermoconvection in a viscoelastic dielectric fluid layer with internal heat source: Navier-Stokes-Voigt model

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Abstract: In the present paper, the combined effect of a vertical alternating current (AC) electric field and a vertical temperature gradient on a horizontal layer of viscoelastic dielectric fluid with internal heat source/heat sink has been investigated using Navier-Stokes–Voigt model. A linear stability analysis employing the normal mode technique is conducted in the present problem for free boundaries, considering cases of heating the fluid layer from below and above separately. The effect of the electric field and internal heat source/heat sink is analysed numerically on the fluid system for stationary convection are using the software MATHEMATICA and also predicted graphically.

Keywords: electrothermoconvection; stationary convection; internal heat source; viscoelasticity; dielectric fluid; Navier-Stokes–Voigt model.

1 Introduction

The subject of thermal instability of a fluid layer heated from below, is of great importance in real world industrial and engineering applications. The problem of thermal instability also known as B$\acute{e}$nard problem has been exhaustively studied since the experiments of Bénard (1900). For the broader view of the subject one may be refered to Chandrasekhar (1981), Bodenschatz et al. (2000), Prakash et al. (2018) and Song et al. (2023).

Electrohydrodynamics (EHD) has manifold applications such as EHD enhanced heat transfer, EHD pump, micromechanic systems, drug delievery, microcooling systems, electrospray mass spectrometry, electrospray nano-technology, etc. (Chen and Cheng (2003)). The effect of electric field on convective instability field becomes significant if the fluid is dielectric. Several theoretical and experimental studies on the convective instabilities in a dielectric fluid layer in the presence of an electric field has been carried out in the recent past. In general, the convective instability in a fluid layer is reported when it is heated from below. The convection produced in a dielectric fluid layer heated from above were reported by Gross and Porter (1966) and **?** which was kept under the influence of a uniform electric field. Roberts (1969) studied electroconvection by considering the dielectric constant as a function of temperature. Castellanos and Velarde (1981) studied the effect of a temperature-dependent dielectric constant in the stability analysis of a fluid layer subjected to an electric field, weak unipolar injection and temperature gradient. For the broader view of the subject one may be referred to Bradely (1978), Takashima and Hamabata (1984), Pontiga and Castellanos (1994), Straughan (2013) and Nekrasov and Smorodin (2023).

The Navier-Stokes equations for the viscous, incompressible fluid flow, which are derived by using the linear relationship of stress and the velocity gradient, are well familier in the domains of applied mathematics. Many systems in real life situations, various industries, such as liquid crystal solidification, biomechanics, cosmetics, construction material, chemical and petroleum processes, farmaceuticals, electro-viscoelastic fluids etc. do not enjoy this special linear relationship. In the past, different mathematical models are developed to study the complex relationship between stress and the history of the velocity gradient in a viscoelastic fluid by incoorporating the time derivatives of the stress and/or velocity gradient of different orders which typically result in Maxwell fluid, Oldroyd fluid, Walters' fluid or Kelvin-Voigt fluid (Straughan (2021a); Oskolkov and Shadiev (1994)).

Recent studies are focussing on these very interesting complex (viscoelastic) fluids associated with the names of Kelvin and of Voigt (Straughan (2021b); Chiriţă and Zampoli (2015)) for which the mathematical model have been presented by (Oskolkov (1988, 1995)). Generalizations of these to incorporate temperature effects are given by Sukacheva and Matveeva (2010) and Matveeva (2013). The Kelvin-Voigt fluid of order zero is known as Navier-Stokes Voigt fluids. Recently, the Kelvin-Voigt model has been extensively used by many researchers to study the hydrodynamic stability problems of viscoelastic fluids. Straughan (2021b) studied competitive double diffusive convection in a Kelvin-Voigt fluid and showed that the Kelvin Voigt parameters play an important role in acting as a stablizing agents when the convection is of oscillatory type. Straughan (2022b) presented a numerical technique to calculate instability thresholds in model for thermal convection in Kelvin-Voigt fluid and showed that oscillatory instability may occur in such fluids. Sharma et al. (2024) presents the linear and nonlinear analyses of the rotating Navier-Stokes-Voigt fluid layer soluted and heated from below. For more studies on Kelvin-Voigt model, one may be referred to Straughan (2021b, 2023, 2022a).

Although, there are few studies predicting electrohydrodynamic instability in viscoelastic liquid layer using Oldroyd model

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(Takashima and Ghosh (1979)), Walters model (Othman and Sweilam (2002)) but to the best of authors' knowledge electrohydrodynamic analysis of Navier-Stokes-Voigt fluid has not been reported yet.
Further, there are many practical situations like nuclear reactions, geophysics, nuclear energy, fire and combustion studies and storage of radioactive materials etc.wherein any reacting material undergoes a weak exothermic reaction and heat is being generated internally through radioactive decay or through chemical reaction. The subsequent investigations on internal heat generation on the onset of convection in porous and non-porous medium has attracted the attention of many researchers (see Bhattacharyya and Jena (1984), Bhadauria et al. (2011), Shivakumara and Suma (2000), Othman and Sweilam (2002), Shivakumara et al. (2007), Bhadauria et al. (2013), Yadav et al. (2017) and Shivaraj et al. (2021)).
The present paper specifically focuses on the above mentioned specific class of viscoelastic fluids, known as Navier-Stokes-Voigt fluids. We investigate the stationary convection in a horizontal layer of viscoelastic dielectric fluid subjected to a simultaneous vertical AC electric field and a vertical temperature gradient in the presence of internal heat source/heat sink. The Navier-Stokes–Voigt model is employed to characterize the behaviour of a viscoelastic dielectric fluid layer. The critical Rayleigh numbers and electric Rayleigh numbers for linear instability are computed numerically for different values of internal Rayleigh number by using the software MATHEMATICA for free-free boundaries for the cases when fluid layer is heated from below and from above. Further, the destabilizing effect of AC electric field on thermal Rayleigh number is also predicted graphically.

2 Mathematical Formulation of the Problem

2.1 When fluid layer is heated from below

We consider a Boussinesq dielectric viscoelastic fluid layer (Navier-Stokes-Voigt model) of infinite horizontal extension and finite vertical thickness d subjected to a uniform vertical AC electric field, which is statically confined within the horizontal boundaries $z = 0$ and $z = d$, respectively maintained at uniform temperatures T_0 and T_1 ($< T_0$). It is further assumed that the fluid layer is heated internally with internal heat source Q (Fig. 1).

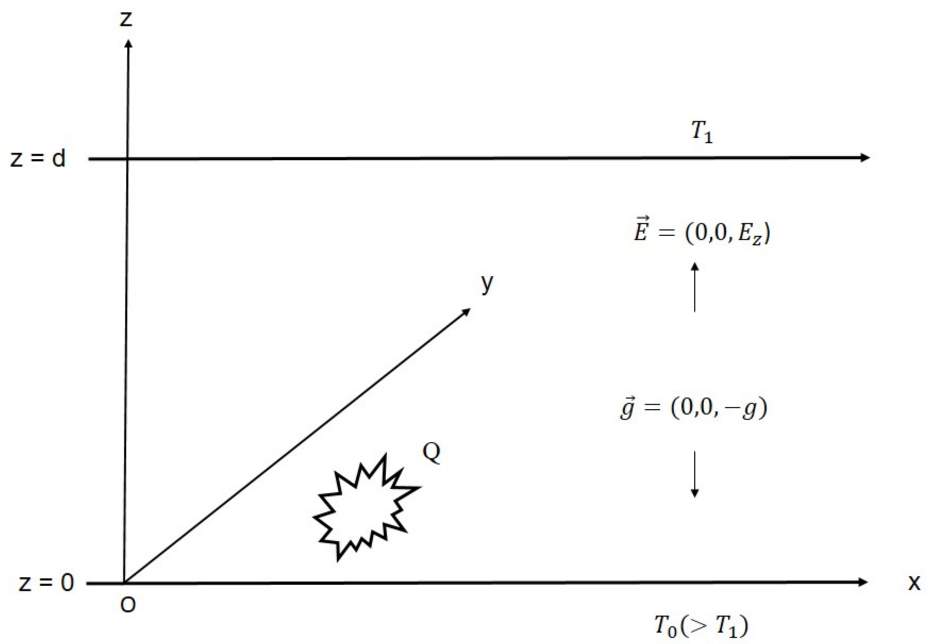

Fig. 1: Geometrical configuration of the problem

The mathematical equations governing the flow of dielectric viscoelastic fluid (Navier-Stokes–Voigt model) are given by (Shivaraj et al. (2021) and Sukacheva and Kondyukov (2014))

$$\nabla.\overrightarrow{q} = 0, \tag{1}$$

$$\rho_0 \left[\left(1 - \lambda \nabla^2\right) \frac{\partial}{\partial t} + \left(\overrightarrow{q}.\nabla\right) \right] \overrightarrow{q} = -\nabla P + \rho \overrightarrow{g} + \mu \nabla^2 \overrightarrow{q} + q_e \overrightarrow{E} - \frac{1}{2} \left(\overrightarrow{E}.\overrightarrow{E}\right) \nabla \varepsilon, \tag{2}$$

$$\frac{\partial T}{\partial t} + \left(\overrightarrow{q}.\nabla\right) T = \kappa \nabla^2 T + Q\left(T - T_1\right), \tag{3}$$

where $\overrightarrow{q} = (u, v, w)$, λ, t, ρ, P, $\overrightarrow{g} = (0, 0, -g)$,μ, q_e, $\overrightarrow{E}$, κ, ε, T, T_1 and Q respectively denote the velocity, Navier-Stokes–Voigt parameter, time, fluid density, pressure, acceleration due to gravity, coefficient of viscosity, free charge density, electric field, thermal diffusivity, dielectric constant, temperature, temperature at upper plate and uniformly distributed volumetric heat generation within the fluid layer. In addition, ρ_0 is the reference fluid density. Further, a positive value of Q will give rise to an increase in the temperature of the layer and therefore is an internal heat source. Thus, a negative value of Q represents an internal heat sink. Also, in the right hand side of Eq. (2), $P = p - \frac{1}{2}\left(\rho \frac{\partial \varepsilon}{\partial \rho} \overrightarrow{E}.\overrightarrow{E}\right)$ is the modified pressure. The Coulomb force term $q_e \overrightarrow{E}$ is usually insignificant when compared to the fourth term known as dielectrophoretic force term for most dielectric fluids in a 60-Hz AC electric field. Therefore, in Eq. (2), the Coulomb force term has been disregarded, and only the dielectrophoretic force term has been retained. Thus Eq. (2) modifies to

$$\rho_0 \left(\left(1 - \lambda \nabla^2\right) \frac{\partial \overrightarrow{q}}{\partial t} + \left(\overrightarrow{q}.\nabla\right) \overrightarrow{q} \right) = -\nabla P + \rho \overrightarrow{g} + \mu \nabla^2 \overrightarrow{q} - \frac{1}{2} \left(\overrightarrow{E}.\overrightarrow{E}\right) \nabla \varepsilon, \tag{4}$$

The equation of state is given by

$$\rho = \rho_0 \left[1 - \alpha \left(T - T_1\right)\right], \tag{5}$$

where α is the coefficient of volume expansion.
The Maxwell's equations relevant to the present problem are

$$\nabla \times \overrightarrow{E} = 0, \tag{6}$$

$$\nabla . \left(\varepsilon \overrightarrow{E}\right) = 0. \tag{7}$$

In the light of the Eq. (6), $\overrightarrow{E}$ can be expressed as

$$\overrightarrow{E} = -\nabla V, \tag{8}$$

where V is root mean square value of the electric potential.
The dielectric constant can be defined as

$$\varepsilon = \varepsilon_0 \left[1 - \gamma \left(T - T_1\right)\right], \tag{9}$$

where ε_0 is the reference value of dielectric constant and γ ($\gamma > 0$) is the thermal expansion coefficient of dielectric constant.
The initial state is assumed to be quiescent and is given by

$$\begin{aligned} &\overrightarrow{q} = \overrightarrow{q_b} = 0, T = T_b(z), P = P_b(z), \overrightarrow{E} = \overrightarrow{E_b}(z), \varepsilon = \varepsilon_b(z), T_b = T_1 + \Delta T G(z), \\ &\varepsilon_b = \varepsilon_0 \left(1 - \gamma \Delta T G(z)\right), \rho_b = \rho_0 \left(1 - \alpha \Delta T G(z)\right), \end{aligned} \tag{10}$$

where the subscript b denotes the initial state and

$$G(z) = \frac{\sin \sqrt{R_i} \left(1 - \frac{z}{d}\right)}{\sin \sqrt{R_i}}.$$

where $R_i = \frac{Qd^2}{\kappa}$ is known as internal Rayleigh number.

Now, Eq. (7) implies that

$$\varepsilon_b E_{bz} = \varepsilon_0 E_0 = \text{constant(say)}, \tag{11}$$

and hence

$$E_{bz} = \frac{E_0}{1 - \gamma \Delta T G(z)} = E_b(z). \tag{12}$$

In order to examine the stability of the initial state, we introduce infinitesimally small perturbations $\left(\overrightarrow{q'}, P', \overrightarrow{E'}, T', \rho', \varepsilon'\right)$ on the initial state in the form

$$\overrightarrow{q} = 0 + \overrightarrow{q'}, P = P_b + P', \overrightarrow{E} = \overrightarrow{E_b} + \overrightarrow{E'}, T = T_b + T', \rho = \rho_b + \rho', \varepsilon = \varepsilon_b + \varepsilon', \tag{13}$$

Now, substituting Eq. (13) in Eq. (1) and Eqs. (3)-(9), utilizing the initial state solutions, neglecting the non-linear terms from the resulting equations and eliminating the pressure term by applying curl twice, we obtain the following equations (removing the primes for simplicity)

$$\left[\left(1-\lambda\nabla^2\right)\frac{\partial}{\partial t}-\nu\nabla^2\right]\nabla^2 w=\alpha g\nabla_h^2 T-\nabla_h^2\left[\frac{\varepsilon_0 E_0\gamma\Delta T G'(z)}{\rho_0}\left(E_0\gamma T+\frac{\partial V}{\partial Z}\right)\right], \tag{14}$$

$$\left[\frac{\partial}{\partial t}-Q-\kappa\nabla^2\right]T=-w\frac{dT_b}{dz}, \tag{15}$$

$$\nabla^2 V=-\gamma E_0\frac{\partial T}{\partial z}, \tag{16}$$

where $\nu=\frac{\mu}{\rho_0}$, is the kinematic viscosity and $\nabla_h^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$.

Now, non-dimensionalizing the Eqs. (14)-(16) by scaling $(x,y,z),\lambda,t,w,T$ and V respectively by $d,d^2,\frac{d^2}{\kappa},\frac{\kappa}{d},T\Delta T$ and $E_0\gamma\Delta T d$, we have the following equations

$$\left(\frac{1}{\sigma}\left(1-\lambda\nabla^2\right)\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2 w=R\nabla_h^2 T-R_e\frac{\partial G(z)}{\partial z}\nabla_h^2\left[T+\frac{\partial V}{\partial Z}\right], \tag{17}$$

$$\left(\frac{\partial}{\partial t}-R_i-\nabla^2\right)T=-wg_2(z), \tag{18}$$

$$\nabla^2 V=-\frac{\partial T}{\partial z}, \tag{19}$$

where $\sigma=\frac{\nu}{\kappa}$ is the Prandtl number, $R=\frac{\alpha g(\Delta T)d^3}{\kappa\nu}$ is the thermal Rayleigh number, $R_e=\frac{\gamma^2\varepsilon_0 {E_0}^2(\Delta T)^2 d^2}{\mu\kappa}$ is the electric Rayleigh number and $g_2(z)=\frac{dT_b}{dz}=-\sqrt{R_i}\frac{\cos\sqrt{R_i}(1-z)}{\sin\sqrt{R_i}}$.

Now, using the normal mode technique, we ascribe a dependence on x, y and t of the quantities w, T and V, of the following form

$$(w,T,V)(x,y,z,t)=(W(z),\Theta(z),\Phi(z))\times e^{\left(ik_x x+ik_y y+\omega t\right)}, \tag{20}$$

where k_x and k_y are the wave numbers along x and y directions, respectively, such that $a=\sqrt{{k_x}^2+{k_y}^2}$ is the resultant horizontal wave number, $\omega\,(=\omega_r+i\omega_i)$ is the complex growth rate while W,Θ and Φ are the amplitudes of perturbed velocity, temperature and electric potential respectively.

On using Eq. (20) in Eqs. (17)-(19), we get the governing equations in the following non-dimensional form

$$\left(\frac{\omega}{\sigma}\left(1-\lambda\left(D^2-a^2\right)\right)-\left(D^2-a^2\right)\right)\left(D^2-a^2\right)W=-Ra^2\Theta+R_e a^2 DG(z)\left(\Theta+D\Phi\right), \tag{21}$$

$$\left(D^2-a^2+R_i-\omega\right)\Theta=Wg_2(z), \tag{22}$$

$$\left(D^2-a^2\right)\Phi=-D\Theta. \tag{23}$$

The bounding surfaces are considered as free. Hence the boundary conditions for the free boundaries are

$$W=D^2W=\Theta=D\Phi=0, \tag{24}$$

at $z=0$ and $z=1$.

Though, this case is admittedly, an artificial case to consider, mathematically it is of importance, since its analytical solutions can readily be obtained and thereby the necessary physical features of the eigen value problem can be understood.

2.1.1 Linear stability analysis

To satisfy boundary conditions (24), we assume the solution for W, Θ and Φ in the form

$$W = W_0 Sin\pi z, \Theta = \Theta_0 Sin\pi z \text{ and } \Phi = \Phi_0 Cos\pi z \quad , \tag{25}$$

where W_0, Θ_0 and Φ_0 are constants. Substituting equation (25) into equations (21)-(23), multiplying the resulting equations by $\sin\pi z$ and integrating each equations from $z = 0$ and $z = 1$, we obtain the following matrix equation

$$\begin{bmatrix} -\delta^2\left(\delta^2 + \frac{\omega}{\sigma}\left(1+\lambda\delta^2\right)\right) & a^2\left(R - 2FR_e\right) & 2a^2\pi FR_e \\ 2F & \delta^2 - R_i + \omega & 0 \\ 0 & \pi & -\delta^2 \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{26}$$

where $\delta^2 = \pi^2 + a^2$ and $\mathrm{F} = \int_0^1 \mathrm{g}_2(\mathrm{z}) \sin^2 \pi \mathrm{z} \mathrm{d}\mathrm{z} = -\frac{2\pi^2}{4\pi^2 - \mathrm{R_i}}$.
The above system of homogeneous equations admits a non-trivial solution only if its determinant is equal to zero which on solving yields the characteristic equation of the system. This characteristic equation gives the following expression for the thermal Rayleigh number

$$R = -\frac{\delta^2}{2Fa^2}\left(\frac{\omega}{\sigma}\left(1+\lambda\delta^2\right)+\delta^2\right)\left(\delta^2 - R_i + \omega\right) + \frac{2a^2F}{\delta^2}R_e \tag{27}$$

To check the stability of the system at marginal state ($\omega_r = 0$), we put $\omega = i\omega_i$, in the Eq. (27), we get

$$R = A_1 + i\omega_i A_2, \tag{28}$$

where

$$A_1 = \frac{\omega_i^2\delta^2}{2F\sigma a^2}\left(1+\lambda\delta^2\right) - \frac{\delta^4}{2Fa^2}\left(\delta^2 - R_i\right) + \frac{2Fa^2}{\delta^2}R_e, \tag{29}$$

and

$$A_2 = -\frac{\delta^2}{2Fa^2}\left(\frac{1}{\sigma}\left(1+\lambda\delta^2\right)\left(\delta^2 - R_i\right) + \delta^2\right). \tag{30}$$

Since R is real quantity. Therefore, it follows from Eq. (28) that either $\omega_i = 0$ (stationary convection) or $A_2 = 0$ (oscillatory convection). For the validity of the principle of the exchange of stabilities ($\omega_r = 0$ implies that $\omega_i = 0$) we have, $\omega = 0$. If $\omega_i = 0$, then Eq. (28) reduces to

$$R^s = -\frac{\delta^4}{2Fa^2}\left(\delta^2 - R_i\right) + \frac{2Fa^2}{\delta^2}R_e, \tag{31}$$

where R^s is the Rayleigh number for stationary convection.
Thus, in the absence of internal heat source i.e. $R_i = 0$ and $F = -\frac{1}{2}$, we have the thermal Rayleigh number, as a function of the parameter R_e and the wave no. a, given by

$$R^s = \frac{\delta^6}{a^2} - \frac{a^2}{\delta^2}R_e. \tag{32}$$

It is clear from Eqs. (28) and (29) that Navier-Stokes-Voigt parameter λ vanishes with ω and thus Navier-Stokes-Voigt dielectric fluid behaves like an ordinary Newtonian dielectric fluid. Further, the Rayleigh number R^s obtained in Eq. (32) is in good agreement with Roberts (1969).
In the absence of electric Rayleigh number i.e. $R_e = 0$, we obtain

$$R^s = \frac{\delta^6}{a^2}. \tag{33}$$

From the above equation the critical wave number a_c can be easily obtained by differentiating R^s with respect to $a^2$and equating to zero, we have

$${a_c}^2 = \frac{\pi^2}{2} \tag{34}$$

and thus, the corresponding critical Rayleigh number from Eq. (32) is given by

$$R_c^s = \frac{27\pi^4}{4}, \tag{35}$$

a result derived by Chandrasekhar (1981) for the classical ordinary viscous fluid.
To find the critical value of R^s for present configuration, Eq. (31) is differentiated with respect to a^2 and equated to zero. A polynomial in ${a_c}^2$ is obtained in the form

$$2\left({a_c}^2\right)^5-\left(R_i-7\pi^2\right)\left({a_c}^2\right)^4-\left(2\pi^2R_i-8\pi^4\right)\left({a_c}^2\right)^3-\left(4\pi^2F^2R_e-2\pi^6\right)\left({a_c}^2\right)^2-\left(2\pi^8-2\pi^6R_i\right)\left({a_c}^2\right)-\left(\pi^{10}-\pi^8R_i\right)=0\ . \tag{36}$$

The numerical solution for the above equation is obtained for various values of R_e and the minimum value of ${a_c}^2$ is obtained every time. By putting this into Eq. (31), the critical Rayleigh number, predicting the onset of electrothermal instability, is calculated.

2.2 When fluid layer is heated from above

For the case when fluid layer is heated from above, then we have $R<0$. Therefore, substituting $R=-|R|$, in Eq. (27) and following the same procedure as is used to derived the Eq. (31), we obtain the expression for the thermal Rayleigh number for stationary convection, given by

$$|R^s|=\frac{\delta^4}{2Fa^2}\left(\delta^2-R_i\right)-\frac{2Fa^2}{\delta^2}R_e \tag{37}$$

For the numerical analysis of the present configuration, we again use the same procedure as is performed in case 2.1. we obtain a polynomial in a_c^2, given by

$$2\left({a_c}^2\right)^5-\left(R_i-7\pi^2\right)\left({a_c}^2\right)^4-\left(2\pi^2R_i-8\pi^4\right)\left({a_c}^2\right)^3-\left(4\pi^2F^2R_e-2\pi^6\right)\left({a_c}^2\right)^2-\left(2\pi^8-2\pi^6R_i\right)\left({a_c}^2\right)-\left(\pi^{10}-\pi^8R_i\right)=0 \tag{38}$$

3 Results and Discussions

In the present section we discuss the quantitative results of our study. We have illustrated the graphs depicting the variation of the critical thermal Rayleigh number R_c^s with wave number a in Fig. 2. These plots illustrate the variation for different values of the electric Rayleigh number R_e, specifically at $R_e=0$ (solid lines) and $R_e=400$ (dashed lines) when the internal Rayleigh number $R_i=-1,0,1,$ and 3. In Fig. 2, it is predicted that with an increase in the electric Rayleigh number R_e, there is a corresponding decrease in the critical thermal Rayleigh number R_c^s for stationary convection which clearly implies that the presence of an AC electric field is found to induce a destabilizing effect on stationary convection. Also, it has been concluded that, the increase in positive value of internal Rayleigh number R_i also gives destabilizing effect to stationary convection. On the other hand, the negative value of internal Rayleigh number R_i gives the stabilizing effect. Fig. 3 depicted the variation of R^s with electric Rayleigh number R_e for different values of internal Rayleigh number $R_i=-1,0,1$ and 3 which also confirms the results predicted in Fig.2. In Fig.4, it is clear that applying heat from above ($R^s=-100$) significantly delays the onset of instability in the fluid system in comparison to the case 2.1 when heated from below ($R^s=100$) or the case when no heat supplied, that is, a case of simple electroconvection ($R^s=0$).

Tab. 1: The variation of critical thermal Rayleigh number R_c^s as a function of wave number a for various values of internal Rayleigh number R_i and electric Rayleigh number R_e.

R_i	R_e	a_c	${R_c}^s$
-1	0	2.257	657.733
	400	2.475	514.884
0	0	2.221	657.511
	400	2.461	514.856
1	0	2.182	597.309
	400	2.447	453.165
2	0	2.137	538.794
	400	2.433	393.333
3	0	2.086	481.851
	400	2.417	335.351
4	0	2.027	426.448
	400	2.402	279.211
5	0	1.957	372.31
	400	2.386	224.903

4 Closing Remarks

The linear stability analysis of a viscoelastic dielectric fluid layer for Navier-Stokes-Voigt model, in the presence of internal heat source/heat sink, heated from below and from above has examined for free boundaries. It is observed that the application of an AC electric field and presence of internal heat source have destabilizing effects on stationary convection. Further, the effect of heat sink is stabilizing to the stationary convection. It is also observed that the onset of instability in the fluid system is significantly delayed when the dielectric fluid layer is heated from above as compared to heated from the below case.

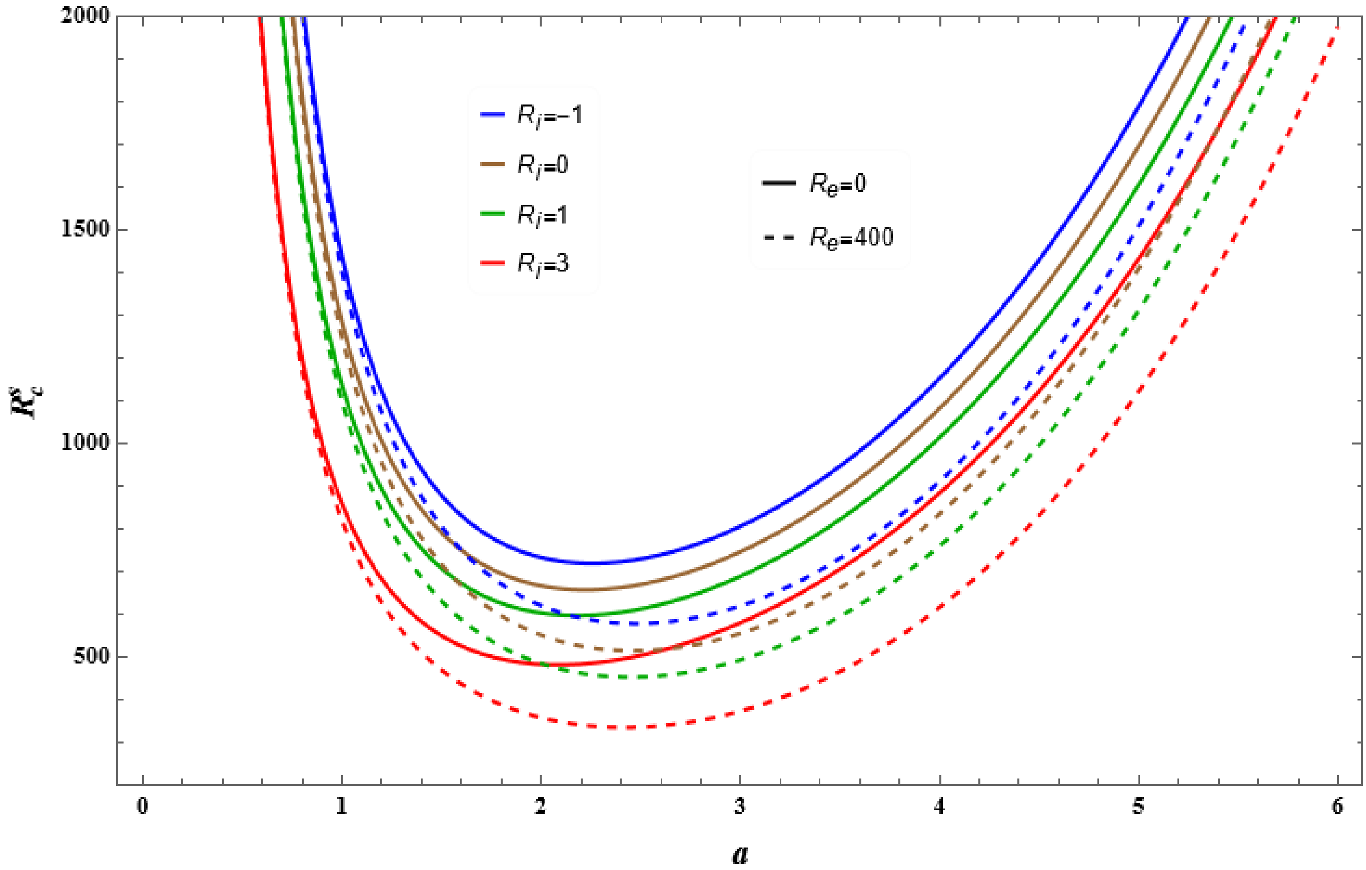

Fig. 2: The variation of critical thermal Rayleigh number R_c^s as a function of wave number a for various values of internal Rayleigh number R_i and electric Rayleigh number R_e.

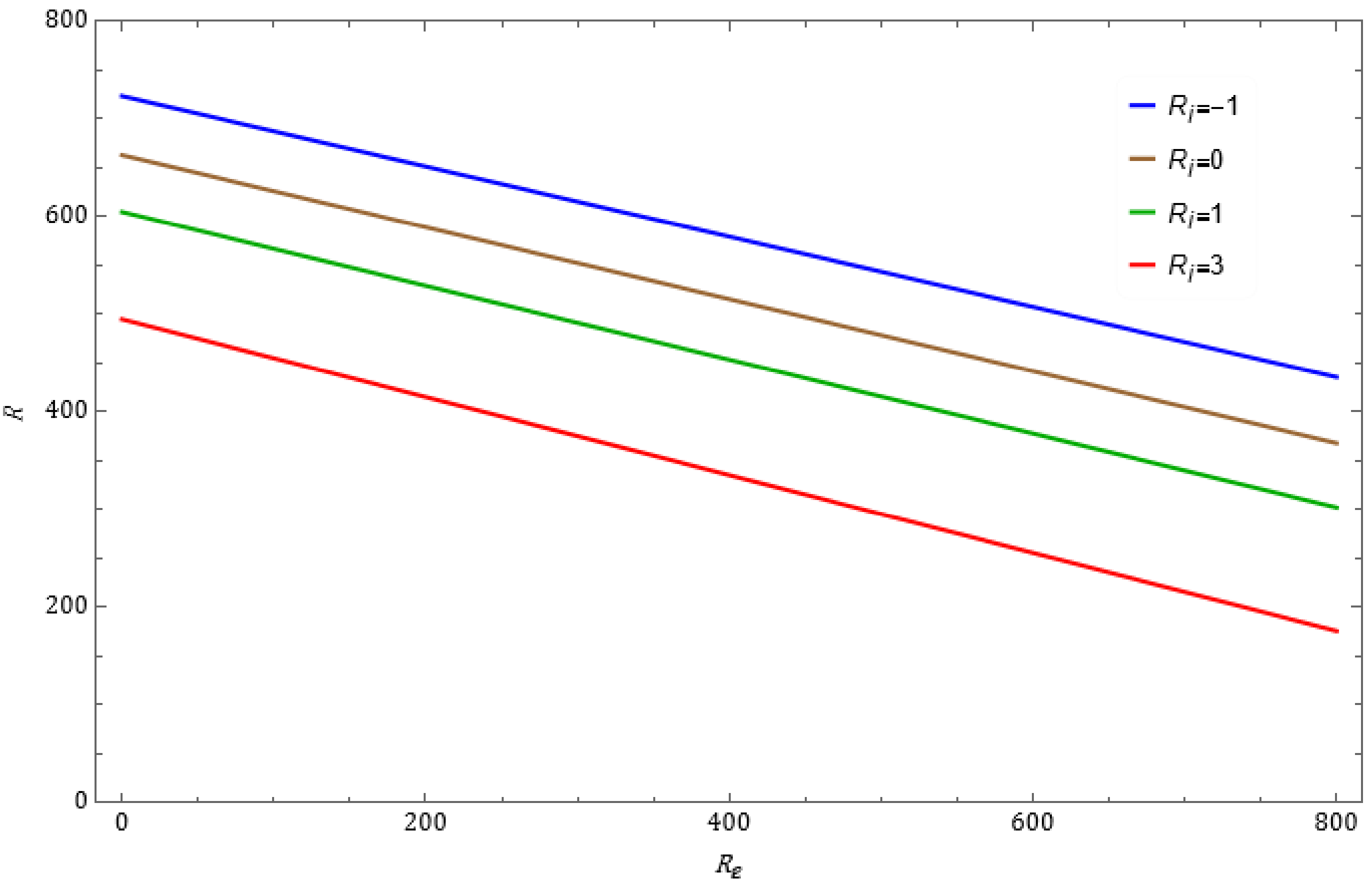

Fig. 3: The variation of thermal Rayleigh number R^s as a function of electric Rayleigh number R_e for various values of internal Rayleigh number R_i.

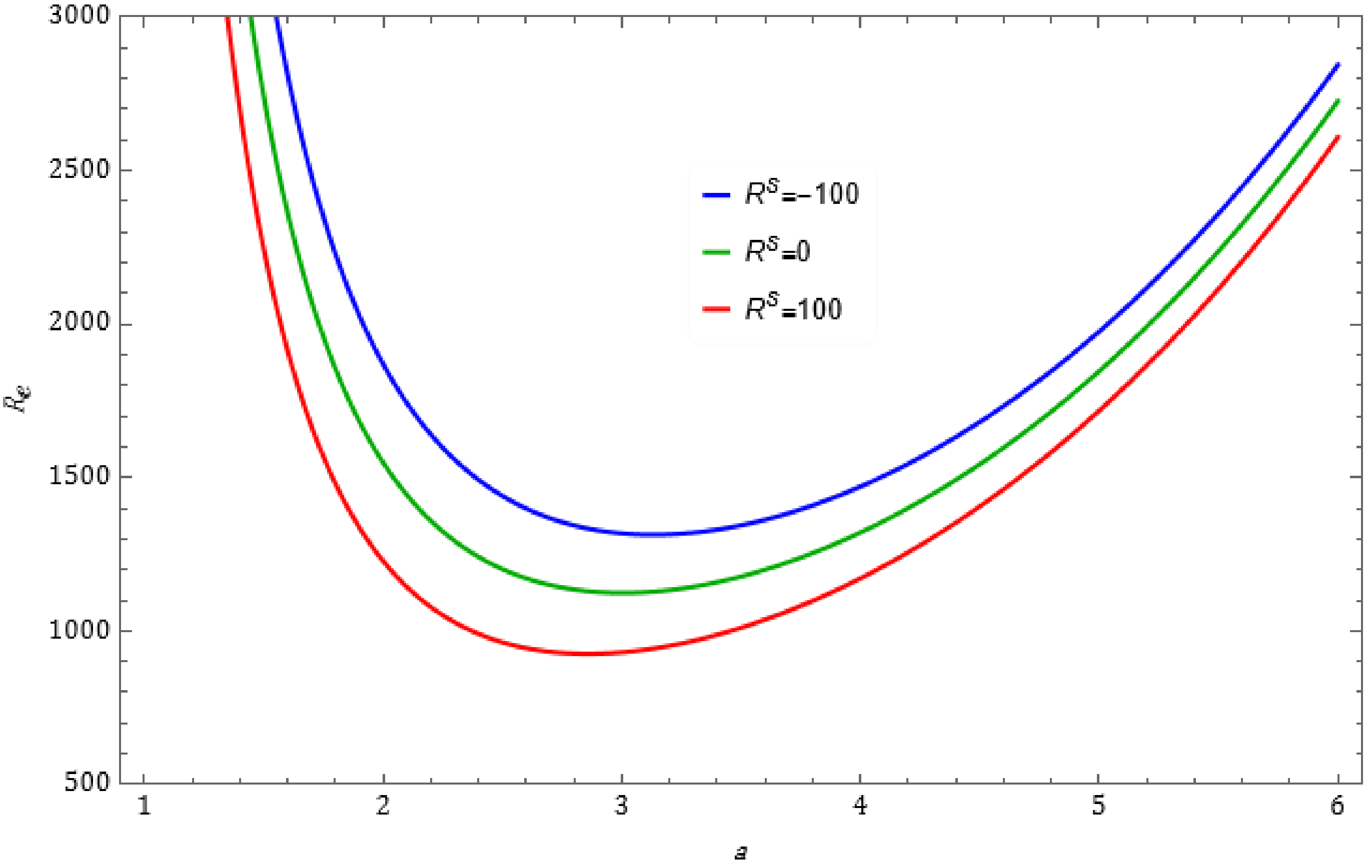

Fig. 4: The variation of electric Rayleigh number R_e as a function of wave number a for different values of thermal Rayleigh number R^s for the case when fluid layer is heated from above/no heat supplied/below.

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