TECHNISCHE MECHANIK
an open access journal

journal homepage: www.ovgu.de/techmech

Tech. Mech., Vol. 45, Is. 1, (2025), 63–77

Received: 22.11.2024
Accepted: 15.08.2025
Available online: 25.08.2025

Nonlinear wave motion in a magneto thermo piezoelectric FG nanobeam using nonlocal state-space strain gradient integral model

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Abstract: Vibration analysis of magneto thermo piezoelectric Functionally Graded Material (MTPFGM) nanobeam is studied using nonlinear state space nonlocal strain gradient integral model. Constituent characteristics of FGM are discussed by using the power law relations. Mathematical modelling of MTPFGM nanobeam is performed by considering the concept of refined higher order beam theory and Hamilton's principle. Besides, the governing equations of the MTPFGM nanobeam are addressed via Eringen-Boltzmann superposition integral model. And to solve the wave propagation problems, the analytical wave dispersion method is used. The analytical outcomes to the physical variables such as magnetic potential, temperature gradient, electric voltage and amplitude of nonlinear frequency are evaluated and presented in graph and tables.

Keywords: Nonlinear wave dispersion, MTPFGM, Eringen-Boltzmann superposition model, Piezo-magnetic nanobeam.

1 Introduction

.Functionally Graded Materials (FGMs), developed by Japanese scientists, are advanced composites with a gradual variation in composition, enabling tailored mechanical, thermal, and electrical properties. FGMs are widely applied in aerospace, biomedical devices, and particularly in MEMS/NEMS technologies. Piezoelectric nanobeams made of FGMs are crucial in smart structures, sensors, and actuators. At the nanoscale, classical theories fall short; hence, nonlocal elasticity theory is employed to capture size-dependent effects. Studies have explored the nonlinear vibration of such beams under thermal and electrical fields, highlighting the impact of nonlocal parameters Ke et al. (2012); Ke and Wang (2012) . A Study has been Ebrahim et al. (2021) reported the scattering of waves FG nano beam of viscoelastic nature. In the framework of third order shear deformation theory Ebrahimi and Barati (2018) the vibration characteristics of functionally graded (METE-FG) nanobeams was analyzed. And the free vibrations of FG nano plates resting on elastic foundation via Hamilton principle was dealt in detail Zaoui et al. (2019). One researcher [Alibeigi et al. (2018) introduced the buckling retliation of nanobeams on the basis of the Euler-Bernoulli beam model with the von Kármán geometrical nonlinearity. Shariati et al. (2020) Bending of flexo electric Magneto-Electro-Elastic (MEE) nanobeams lying over Winkler-Pasternak according to nonlocal elasticity theory has been studied. Several studies were conducted on Ebrahimi et al. (2020); Ebrahimi and Dabbagh (2020); Li et al. (2009) hygro-thermal loading, motion of magneto-piezoelectric nanobeams system, dynamic analysis of smart nanostructures and frequency analysis of thermally post buckled FGM thin beams. Stress driven versus strain driven for elastic nanobeams has been discussed via integral elasticity Romano and Barretta (2017); Barretta et al. (2018). Using the kinematic model Kiani and Eslami (2017) reported the buckling of beams made of FG under different types of thermal loading. The propagation of wave of infinite functionally graded plate in thermal environment was reported in Sun and Luo (2011). A consistent refined higher-order shear deformation theory is developed to investigate the free vibration behavior of glass fiber-reinforced plates resting on an elastic foundation, along with the influence of boundary conditions on their natural frequencies Thai and Choi (2012). The different working conditions of the nano sized elements were studied in Thai et al. (2013). By considering the nonlocal elasticity Shahsavari et al. (2018) a prediction has been made for the essential behaviors of the nanostructures cannot be same as the macro scale structures. The Euler Bernoulli beam theory were used to study the bending analysis of microtubules in Eringen (1983). Based on Euler Bernoulli beam theory, the bending analysis of microtubules (MT) was studied using the method of Differential Quadrature (DQ) in Civalek and Demir (2011). By finite element method (FEM) Arani and Shajari (2012), the nonlinear bending in nanobeams were discussed. Reddy and El-Borgi (2014) investigated on the dispersion of waves with the effects of surface stresses in smart piezoelectric nanoplates. Bi-directional FGM nanobeams with the characteristics of bending, buckling, and vibrational nonlocal elements were concentrated in Zhang and Fang (2014); Nejad and Hadi (2016b,a); Nejad and Rastgoo (2016). The natural frequency variation were surveyed by using the nonlocal theory which is located on a viscoelastic sheet Nejad and Rastgoo (2016).
The size-dependent elements of beam were analyzed in Ebrahimi and Barati (2017). Nonlocal elasticity and its running conditions are discussed in details in the literature Lim and Reddy (2015). Based on the nonlocal strain gradient theory (NSGT), Farajpour and Rastgoo (2016) the thermo-mechanical buckling problem of grapheme sheets was proposed. Stiffness – softening - hardening effect of FG beam were studied in Li and Hu (2016). To solve for the nanoplates having the wave dispersion problem was

* E-mail address: selvam1729@gmail.com doi: 10.24352/UB.OVGU-2025-041

accomplished in Ebrahimi and Dabbagh (2017) with the application of infusing NSGT and surface related elasticity for responsive piezoelectric materials. With the small scale effect, the free vibration of 3D FGM Euler Bernoulli nano beam was studied in Hadi and Hosseini (2018). In Alibeigi and Mehralian (2018), the buckling response of a nanobeam on the basis of the Euler Bernoulli beam model using a couple stress theory under the various types of thermal loading and an electrical and magnetic field has been exposed. Timoshenko beam theory were investigated in Ke and Wang (2014) with the rise in uniform temperature, magnetic potential and external electric potential via nonlocal form to magneto electro elastic vibrations. Bending of Magneto electro elastic nanobeam was studied in detail Ebrahimi and Singhal (2021). Along with that, Ebrahimi and Selvamani (2000)investigated the bending of magneto-electro-elastic (MEE) nanobeams relating the nonlocal elasticity theory under hygro-thermal loading embedded in Winkler-Pasternak foundation. The size dependent problems using nonlocal elasticity theory, nonlocal couple stress theory and shear deformation theory were reported Ramezani and Mojra.A (2020); Ebrahimi (2018). In Farzad Ebrahimi and Dabbagh. (2016), the effects of various parameters on the wave dispersion characteristics of size-dependent nanoplates were discussed. Ebrahimi (2016) The thermal effects on the buckling and free vibration of the FG nanobeams is documented well in the literature. In Ebrahimi and Barati. (2017), the damping vibration characteristics of the hygro thermally affected FG viscoelastic nanobeams were discussed. The thermal effect on buckling and free vibration characteristics of size-dependent Timoshenko nanobeams, the free vibration of curved FG nanosize beam in thermal environment been discussed in Ebrahimi and Salari (2015); Ebrahimi and Daman (2017). The buckling and vibration properties of sandwich FG beams were studied in Vo and Lee (2015). In Jalaei and Nguyen-Xuan (2019), the thermal and magnetic effects on the FG Timoshenko nanobeam have been studied. A study over the hygro thermal wave characteristic of nanobeam of an inhomogeneous material with porosity under magnetic field is notable Karami and Li (2019).

The literature review indicates that the Nonlinear wave motion in a magneto thermo piezoelectric FG nanobeam using nonlocal state-space strain gradient integral model has not been extensively studied. The present study investigates wave propagation in functionally graded nanobeams using a nonlocal state-space strain gradient viscoelastic model. The magneto-thermo-electrical properties are graded using a power-law distribution, and the governing equations are derived via Hamilton's principle. The effects of external electric voltage, magnetic field, temperature gradient, and nonlinear frequency amplitude are analyzed and illustrated graphically. The analytical wave dispersion method is employed to solve the governing equations efficiently. This work incorporates multiphysical field interactions and scale-dependent effects within a unified framework. Additionally, internal material damping is considered to capture memory-dependent behavior, enhancing the model's applicability to real-world nanostructures. These contributions provide insights relevant for the design of advanced nano-scale electromechanical devices.

2 Problem formulation

Based on the state - space non local strain gradient theory, the length L, width b, and thickness h of a viscoelastic FG nanobeam has been investigated. The two parts of Constituent FGM composed of $BaTiO_3$ - ceramic part and $CoFe_2O_4$ - metallic part. The components of FGM are considered to be temperature dependent to evolve a realistic viscoelastic study.

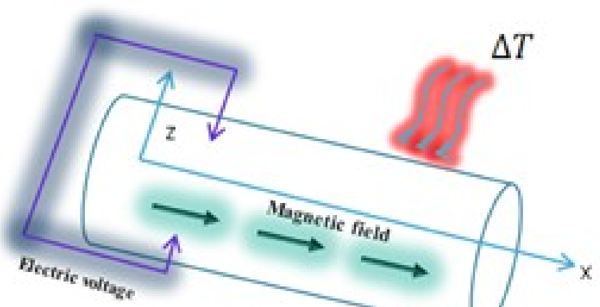

Fig. 1: Vibration analysis of FG Nanobeam

In this section, power-law relations been used to compute the properties. To calculate the variation of material properties along the thickness direction of the functionally graded nanobeam, the volume fraction of each constituent phase is determined using the power law distribution. Accordingly, the volume fraction of the ceramic phase is given by Ebrahimi (2016).

$$V_c = \left(\frac{Z}{h} + \frac{1}{2}\right)^2 \tag{1}$$

where $h$is the thickness coordinate, $p$is the total thickness of the beam, and is the power law index. This expression ensures a smooth and continuous gradation of material properties through the thickness, with controlling the degree of gradation. When , the material is fully metallic; when , it is fully ceramic. This distribution plays a key role in accurately modeling thermal and mechanical behavior under external fields. By considering any desired material property at the local temperature as Ebrahimi

(2018),

$$P = P_O(P_1T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \tag{2}$$

whereas, $P_O, P_1, P_2, P_3$are the coefficients of material phases. The volume fraction of the metallic phase gives the volume fraction of the ceramic phase by $V_m = 1 - V_c$. According to Eringen's nonlocal theory Eringen (1983), the stress state at a point inside a body is a function of the strains at all points in the neighbouring regions. The basic equations with zero body force can be defined as,

$$\sigma_{ij} = \int_v \alpha(\mid y' - y \mid, \tau)[C_{ijkl}\epsilon_{kl}(y') - e_{mij}E_m(y') - \Omega_n(y') - C_{ijkl}\alpha_{kl}\Delta T]dV(y')$$

$$D_i = \int_v \alpha(\mid y' - y \mid, \tau)[e_{ikl}\epsilon_{kl}(y') - E_m(y') - \Omega_n(y') - \Delta T]dV(y')$$

$$B_i = \int_v \alpha(\mid y' - y \mid, \tau)[\epsilon_{kl}(y') - E_m(x') - \chi_{ni}\Omega_n(y') - \lambda_i\Delta T]dV(y') \tag{3}$$

where $\sigma_{ij}, \epsilon_{ij}, D_i, E_i$ respectively, represents the stress, strain, electric displacement and electric field andB_i, Ω_i are the magnetic induction and magnetic field. α_{kl} and ΔT stands for thermal expansion and temperature difference. C_{ijkl}, e_{mij} andχ_{ni} are the elastic, piezoelectric and magnetic constants respectively and $\tau = \frac{e_o a}{l}$ defines the scale coefficient, e_o is material constant a and l, are characteristic length in the internal and external sides.
The governing equations of the nanobeams are obtained by an accurate kinematic theory. Higher order shear deformation theory also reveals stress-strain changes in solid bodies. From [8] we can take the refined shear deformable beam's displacement as

$$\Pi_x(x, z, l)\textperthousand = \Pi(x, t)\textperthousand - \frac{\partial W_b(x, t)\textperthousand}{\partial x} z - \frac{\partial W_s(x, t)\textperthousand}{\partial x} f^*(z)$$

$$\Pi_x(x, z, l)\textperthousand = W_b(x, t)\textperthousand + W_s(x, t)\textperthousand \tag{4}$$

Here,Π , $W_b$and W_s are the longitudinal displacement, bending and shear components of the transverse displacement. Furthermore, in order to distribute the shear strain, $f^*(z)$ is the shape function which is designed as Ebrahimi and Barati (2018).

$$f^*(z) = \frac{he^z}{h^2 + \pi^2}[\pi \sin\frac{\pi z}{h} + h\cos\frac{\pi z}{h}] - \frac{h^2}{h^2 + \pi^2}] \tag{5}$$

To capture the shear strain and stress, the deformed structural cross section is uncertain with this function. At free surfaces, it is required to satisfy the assumption of shear strain nonexistence. By continuum infinitesimal strain tensor, the nonzero strains can be,

$$\varepsilon_{xx} = \frac{\partial \Pi}{\partial x} - z\frac{\partial^2 W_b}{\partial x^2} - f^*(z)\frac{\partial^2 W_s}{\partial x^2}$$

$$\gamma_{xz} = g(z)\frac{\partial W_s}{\partial x} \tag{6}$$

where$g(z) = 1 - \frac{df^*(z)}{dz}$

2.1 Motion Equations

In accordance with Hamilton's principle, the extended Lagrangian can be,

$$L = \Pi - T + V \tag{7}$$

In accordance with Hamilton's principle, the extended Lagrangian can be Ebrahimi and Selvamani (2000),

$$\delta\int_{\tilde{t}_1}^{\tilde{t}_2}(\Pi - T + V)d\tilde{t} \tag{8}$$

In equation (8),Π,V variables T and strain energy, V work done and T is the kinetic energy. Hence, the virtual strain energy can be,

$$\delta\Pi = \int_\forall(\sigma_{ij}\delta\varepsilon_{ij}d\forall - D_xE_x - D_zE_z)dzdx \tag{9}$$

$$\phi(x, z, \tilde{t}) = -cos(\beta z)\phi(x, \tilde{t}) + \frac{2zV_O}{h}e^{-ik\tilde{t}} \tag{10}$$

Where, $D_x = e_{15}\nu_{xz} + e_{11}E_x$ and $D_z = e_{31}\epsilon_{xx} + e_{33}E_z$.The variation of electric potential in the x direction is $\beta = \frac{\pi}{h}; \phi(x, \tilde{t}); V_O$ and Ω is the is the external electric voltage and natural frequency of the piezoelectric nanobeam.
By infusing equation (10) in equation (9),

$$\delta\Pi = \int_0^L (N\frac{\partial\delta\Pi}{\partial x} - M_b\frac{\partial^2\partial W_b}{\partial x^2} - M_s\frac{\partial^2\delta W_s}{\partial x^2} + Q\frac{\partial\delta W_s}{\partial x} - D_xE_x - D_zE_z)dzdx \tag{11}$$

the stress resultants can be obtained,

$$[N, M_b, M_s] = \int_A [1, z, f^*(z)]\sigma_{xx}dA \tag{12}$$

$$Q = \int_A g(z)\sigma_{xx}dA \tag{13}$$

The kinetic energy of the system can be arrived as,

$$\delta T = \int_\forall \rho(z)[\frac{\partial\Pi_x}{\partial\tilde{t}}\frac{\partial\delta\Pi_x}{\partial\tilde{t}} + \frac{\partial\Pi_z}{\partial\tilde{t}}\frac{\partial\delta\Pi_z}{\partial\tilde{t}}]dV \tag{14}$$

Infusion of equation (4) in the equation (14),

$$\delta T = \int_0^L \begin{bmatrix} I_0^*(\frac{\partial\Pi}{\partial\tilde{t}}\frac{\partial\delta\Pi}{\partial\tilde{t}} + \frac{\partial(W_b+W_s)}{\partial\tilde{t}}\frac{\partial\delta(W_b+W_s)}{\partial\tilde{t}}) \\ -I_1^*(\frac{\partial\Pi}{\partial\tilde{t}}\frac{\partial^2\delta W_b}{\partial x\partial\tilde{t}} + \frac{\partial^2 W_b}{\partial x\partial\tilde{t}}\frac{\partial\delta\Pi}{\partial\tilde{t}}) \\ -J_1^*(\frac{\partial\Pi}{\partial\tilde{t}}\frac{\partial^2\delta W_s}{\partial x\partial\tilde{t}} + \frac{\partial^2 W_s}{\partial x\partial\tilde{t}}\frac{\partial\delta\Pi}{\partial\tilde{t}}) \\ I_2^*\frac{\partial_b^W}{\partial x\partial\tilde{t}}\frac{\partial^2\delta W_b}{\partial x\partial\tilde{t}} + K_2^*\frac{\partial^2 W_s}{\partial x\partial\tilde{t}}\frac{\partial\delta\ W_s}{\partial x\partial\tilde{t}} \\ J_2^*(\frac{\partial_b^W}{\partial x\partial\tilde{t}}\frac{\partial^2\delta W_s}{\partial x\partial\tilde{t}} + \frac{\partial^2 W_s}{\partial x\partial\tilde{t}}\frac{\partial\delta\ W_b}{\partial x\partial\tilde{t}}) \end{bmatrix} \tag{15}$$

In accordance with the magnetic and temperature effect Ebrahimi and Selvamani (2000),

$$f_B = \eta A\Omega_x^2\frac{\partial^2 W}{\partial x^2}$$

$$N_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)\lambda(z)\Delta T dz$$

Where f_B ,η ,A and Ω_x stands for magnetic force, magnetic field permeability, cross sectional area of the nanobeam and the magnetic potential of the longitudinal magnetic field. For a FG nanobeam, it is assumed that the temperature can be distributed uniformly across its thickness and the temperature gradient at stress free state is ΔT.
In the above definition the inertia of mass moments can be defined,

$$[I_0^*, I_1^*, I_2^*, J_1^*, J_2^*, K_2^*] = \int_A [1, z, z^2, f^*(z), zf^*(z), f^*2(z)]\rho(Z)dA$$

and the work done N_x with a temperature gradient of thermal effect can be Ebrahimi (2018) ,

$$\delta V = \frac{1}{2}\int_0^1 N_x + N_T(\frac{\partial W}{\partial x}\frac{\partial\delta W}{\partial x} + \eta A\Omega_x^2\frac{\partial^2 W}{\partial x^2})dx \tag{16}$$

by inserting the equations (11) and (15) in (8), the equation of beam in Euler-Lagrange can be derived and the outcome can be coupled as,

$$\frac{\partial N}{\partial x} = I_0^*\frac{\partial^2\Pi}{\partial\tilde{t}^2} - I_1^*\frac{\partial^3 W_b}{\partial x\partial\tilde{t}^2} - J_1^*\frac{\partial^3 W_s}{\partial x\partial\tilde{t}^2} \tag{17}$$

$$\frac{\partial^2 M_b}{\partial x^2} = I_0^*\frac{\partial^2(W_b + W_s)}{\partial\tilde{t}^2} - I_1^*\frac{\partial^3\Pi}{\partial x\partial\tilde{t}^2} - I_2^*\frac{\partial^4 W_b}{\partial x^2\partial\tilde{t}^2} - J_2^*\frac{\partial^4 W_s}{\partial x^2\partial\tilde{t}^2} - (N_x + N_T\frac{\partial W}{\partial x}\frac{\partial\delta W}{\partial x} + \eta A\Omega_x^2\frac{\partial^2 W}{\partial x^2}) \tag{18}$$

$$\frac{\partial^2 M_b}{\partial x^2} + \frac{Q}{\partial x} - N_x\frac{\partial^2 w}{\partial x^2} = I_0^*\frac{\partial^2(W_b + W_s)}{\partial\tilde{t}^2} - J_1^*\frac{\partial^3\Pi}{\partial x\partial\tilde{t}^2} - J_2^*\frac{\partial^4 W_b}{\partial x^2\partial\tilde{t}^2} - K_2^*\frac{\partial^4 W_s}{\partial x^2\partial\tilde{t}^2} - (N_x + N_T\frac{\partial W}{\partial x}\frac{\partial\delta W}{\partial x} + \eta A\Omega_x^2\frac{\partial^2 W}{\partial x^2}) \tag{19}$$

$$\delta\Phi = \int_{-\frac{h}{2}}^{\frac{h}{2}} [cos(\beta z)(\frac{\partial Dx}{\partial x})] + \beta sin(\beta z)Dz \tag{20}$$

3 Nonlocal state-space model

This section demonstrates the nonlocality stress and strain effect in the time-space domains, when the wave length or excitation frequency interferes with time and intrinsic characteristic length. Nonlocal time-space viscoelasticity problems are based on the combination of the Boltzmann superposition integral and the Eringen concept of nonlocal elasticity. Accordingly, integral stress in nonlinear state and strain equations are stated as,

$$\int_{-\infty}^{t}\int_{v} K_{\sigma}(\tilde{t}-\tau, \mid r-r' \mid)\sigma_{ij}(r',\tau)dr'd\tau = \int_{-\infty}^{t}\int_{v} K_{\epsilon}(\tilde{t}-\tau, \mid r-r' \mid)\sigma_{ij}(r',\tau)C_{ijkl}\epsilon_{kl}dr'd\tau \tag{21}$$

where $\sigma_{ijkl}$and ϵ_{kl} are stress and strain tensors array and the nonlocal kernel functions are $K_{\sigma}(\tilde{t}-\tau, \mid r-r' \mid)$ and $K_{\epsilon}(\tilde{t}-\tau, \mid r-r' \mid)$. Equation (27) can be read in the following form via Fourier, inverse Fourier and Taylor series,

$$(1 - l_{\sigma}^2\nabla^2 + \tau_{\sigma}\frac{\partial}{\partial\tilde{t}})\sigma_{ij} = C_{ijkl}(1 - l_{\sigma}^2\nabla^2 + \tau_{\epsilon}\frac{\partial}{\partial\tilde{t}})\epsilon_{kl} \tag{22}$$

To balance the absence of stiffness-hardening behavior, the nonlocal strain gradient elasticity must be incorporated in the equation. The following relation can be used to derive the nonlocal strain gradient viscoelasticity with fraction.

$$(1 - l_{\sigma}^2\nabla^2 + \tau_{\sigma}\frac{\partial}{\partial\tilde{t}})\sigma_{ij} = C_{ijkl}(1 - l_{\sigma}^2\nabla^2 + \tau_{\epsilon}\frac{\partial}{\partial\tilde{t}})\epsilon_{kl} \tag{23}$$

The Kelvin – Voigt relation of viscoelastic material with three parameter in a solid state is given by,

$$(1 - \mu^2\nabla^2 +)\sigma_{ij} = C_{ijkl}(1 - \lambda^2\nabla^2 + \tau_{\epsilon}\frac{\partial}{\partial\tilde{t}})\epsilon_{kl} \tag{24}$$

where $\lambda = l_{\epsilon}$ and $\mu = l_{\sigma}$ are length scale and nonlocal parameters. The relation between rheological character and spatial nonlocality of the system is designed as$\tau = \mu\sqrt{\frac{\tau_c}{E_c}}$. From equation (2)-(3), the displacement field having resultant stress over the area of cross section of the beam can be computed as

$$(1 - \mu^2\nabla^2 +)N = C_{ijkl}(1 - \lambda^2\nabla^2 + \tau_{\epsilon}\frac{\partial}{\partial\tilde{t}})(A_{xx}\frac{\partial u}{\partial x} - B_{xx}\frac{\partial^2 W_b}{\partial x^2} - B_{xx}^{s}\frac{\partial^2 W_s}{\partial x^2}) - e_{31}E_z \tag{25}$$

$$(1 - \mu^2\nabla^2 +)M^b = (1 - \lambda^2\nabla^2 + \tau_{\epsilon}\frac{\partial}{\partial\tilde{t}})(B_{xx}\frac{\partial u}{\partial x} - D_{xx}\frac{\partial^2 W_b}{\partial x^2} - D_{xx}^{s}\frac{\partial^2 W_s}{\partial x^2}) - (N_x + N_T + \eta A\Omega_x^2\frac{\partial^2 W}{\partial x^2}) - e_{31}E_z \tag{26}$$

$$(1 - \mu^2\nabla^2 +)M^S = (1 - \lambda^2\nabla^2 + \tau_{\epsilon}\frac{\partial}{\partial\tilde{t}})(B_{xx}^{s}\frac{\partial u}{\partial x} - D_{xx}^{s}\frac{\partial^2 W_b}{\partial x^2} - H_{xx}^{s}\frac{\partial^2 W_s}{\partial x^2}) - (N_x + N_T + \eta A\Omega_x^2\frac{\partial^2 W}{\partial x^2}) - e_{31}E_z \tag{27}$$

$$(1 - \mu^2\nabla^2 +)Q^{xz} = (1 - \lambda^2\nabla^2 + \tau_{\epsilon}\frac{\partial}{\partial\tilde{t}})(A_{xx}^{s}\frac{\partial W_s}{\partial x}) - (N_x + N_T + \eta A\Omega_x^2\frac{\partial^2 W}{\partial x^2}) - e_{15}E_z \tag{28}$$

$$(1 - \mu^2\nabla^2 +)D_x = e_{15}\gamma_{xz} - e_{33}E_z \tag{29}$$

$$(1 - \mu^2\nabla^2 +)D_z = e_{15}\epsilon_{xx} - e_{33}E_z \tag{30}$$

Where cross sectional rigidities are,

$$[A_{xx}, B_{xx}, B_{xx}^{s}, D_{xx}, D_{xx}^{s}, H_{xx}^{s},] = \int_A [1, z, z^2, f^*(z), zf^*(z), f^*2(z)] \tag{31}$$

$$A^s = \int_A g^2(z)G(z)dA \tag{32}$$

Here $e_{31}E_x = e_{31}cos(\beta z)\frac{\partial\phi}{\partial x}$ and $e_{15}E_x = e_{15} - \beta sin(\beta z)\phi - \frac{2V_0 e^{ikt}}{h}$

4 Governing Equations

$$(1-\lambda^2\nabla^2+\tau\frac{\partial}{\partial\tilde{t}})(A_{xx}\frac{\partial^2\Pi}{\partial x^2}-B_{xx}\frac{\partial^3 W_b}{\partial x^3}-B^s_{xx}\frac{\partial^3 W_s}{\partial x^3})+(1-\mu^2\nabla^2)(-I_0\ddot{\Pi}+I_1\frac{\partial\ddot{W}_b}{\partial x}+J_1\frac{\partial\ddot{W}_s}{\partial x})-(N_x+N_T+\eta\Omega_x^2\frac{\partial^2 W}{\partial x^2})-e_{31}E_z=0 \quad (33)$$

$$(1-\lambda^2\nabla^2+\tau\frac{\partial}{\partial\tilde{t}})(B_{xx}\frac{\partial^3\Pi}{\partial x^3}-D_{xx}\frac{\partial^4 W_b}{\partial x^4}-D^s_{xx}\frac{\partial^4 W_s}{\partial x^4})+(1-\mu^2\nabla^2)(-I_0(\ddot{w_b}+\ddot{W}_s) \\ -I_1\frac{\partial\ddot{\Pi}}{\partial x}+I_2\frac{\partial^2\ddot{W}_b}{\partial x^2}+J_2\frac{\partial^2\ddot{W}_s}{\partial x^2})-(N_x+N_T+\eta\Omega_x^2\frac{\partial^2 W}{\partial x^2})-e_{31}E_z=0 \quad (34)$$

$$(1-\lambda^2\nabla^2+\tau\frac{\partial}{\partial\tilde{t}})(B^s_{xx}\frac{\partial^3\Pi}{\partial x^3}-D^s_{xx}\frac{\partial^4 W_b}{\partial x^4}-H^s_{xx}\frac{\partial^4 W_s}{\partial x^4}+A_s\frac{\partial^2 W_s}{\partial x^2})+(1-\mu^2\nabla^2)(-I_0(\ddot{w_b}+\ddot{W}_s)-J_1\frac{\partial\ddot{\Pi}}{\partial x} \\ +J_2\frac{\partial^2\ddot{W}_b}{\partial x^2}+K_2\frac{\partial^2\ddot{W}_s}{\partial x^2})-(N_x+N_T+\eta\Omega_x^2\frac{\partial^2 W}{\partial x^2})-e_{31}E_z-e_{15}E_z=0 \quad (35)$$

5 Analytical solution

The governing equation obtained in the preceding section will be solved in this section. The wave harmonic method is used in this case to solve the proposed problem of this study:

$$\begin{bmatrix}\Pi\\ W_b\\ W_s\end{bmatrix}=\begin{bmatrix}\Pi e^{i\beta x-\omega\tilde{t}}\\ w_b e^{i\beta x-\omega\tilde{t}}\\ w_s e^{i\beta x-\omega\tilde{t}}\end{bmatrix} \quad (36)$$

Where Π, W_b and $W_s$are the anonymous amplitudes of propagating waves. Here ω and β are frequency and wave number. By substituting the above expression in equations (33) – (37), then the achieved form will be:

$$([K]+[C]\omega+[M]\omega^2)\begin{bmatrix}\Pi\\ W_b\\ w_s\end{bmatrix}=0 \quad (37)$$

$$k_{11}=-(1+\lambda^2\beta^2)A_{xx}\beta^2$$
$$k_{12}=i(N_x+\eta\Omega_x^2)(1+\lambda^2\beta^2)B_{xx}\beta^5$$
$$k_{13}=i(N_x+\eta\Omega_x^2)(1+\lambda^2\beta^2)B^s_{xx}\beta^5$$
$$k_{14}=e_{31}(1-\lambda^2\beta^2)\beta sin(\beta z)\phi-\frac{2V_0e^ikt}{h}$$
$$k_{22}=-(N_x+\eta\Omega_x^2)N_T(1+\lambda^2\beta^2)D_{xx}\beta^6$$
$$k_{23}=-(N_x+\eta\Omega_x^2)N_T(1+\lambda^2\beta^2)D^s_{xx}\beta^6$$
$$k_{24}=e_{31}(1-\lambda^2\beta^2)\beta sin(\beta z)\phi-\frac{2V_0e^ikt}{h}$$
$$k_{33}=-(N_x+\eta\Omega_x^2)N_T(1+\lambda^2\beta^2)(H^s_{xx}\beta^6+A^s\beta^4)$$
$$k_{34}=e_{31}(1-\lambda^2\beta^2)\beta^2 sin(\beta z)\phi-\frac{2V_0e^ikt}{h}$$
$$c_{11}=-A_{xx}\tau\beta^2$$
$$c_{12}=i(N_x+\eta\Omega_x^2)B_{xx}\tau\beta^5$$
$$c_{13}=i(N_x+\eta\Omega_x^2)B^s_{xx}\tau\beta^5$$
$$c_{14}=\tau e_{31}(\beta^2cos(\beta z)\frac{dz}{dt}\phi-(\frac{2V_0e^ikt}{h}(ik)))$$
$$c_{22}=-(N_x+\eta\Omega_x^2)D_{xx}\tau\beta^6$$
$$c_{23}=-(N_x+\eta\Omega_x^2)D^s_{xx}\tau\beta^6$$
$$c_{24}=\tau e_{31}(\beta^2cos(\beta z)\frac{dz}{dt}\phi-(\frac{2V_0e^ikt}{h}(ik)))$$
$$c_{33}=-(N_x+\eta\Omega_x^2)$$

Tab. 1: Comparison of FGM beam non-dimensional buckling for various power - law exponents.

L/h		p=0	p=0.5	p=1	p=2	p=5	p=10
5	Nguyen et al. (2015)	48.8406	32.0013	24.6894	19.1577	15.7355	14.1448
	Present	48.835	31.967	24.687	19.1605	15.7401	14.13
10	Nguyen et al. (2015)	52.3083	34.0002	26.1707	20.3909	17.1091	15.5278
	Present	52.3082	34.0087	26.1727	20.3936	17.1118	15.5291

Tab. 2: Dimensionless frequency of a FG nanobeam varies with nonlocal parameters, electric voltages and magnetic potentials.

μ		p=0.2				p=1				p=5		
		V=-5	V=0	V=+5		V=-5	V=0	V=+5		V=-5	V=0	V=+5
0	Ω=-0.05	8.34927	7.68726	7.24086		8.3637	7.69636	7.24398		8.3781	7.70545	7.24711
v	Ω=0	8.34708	7.68034	7.22898		8.36151	7.68946	7.23212		8.37592	7.69856	7.23525
	Ω=-0.05	8.34489	7.67343	7.21709		8.35933	7.68255	7.22023		8.37373	7.69166	7.22337
1	Ω=-0.05	7.96429	7.33365	6.9088		7.97941	7.34319	6.91208		7.9945	7.35273	6.91536
	Ω=0	7.96199	7.32641	6.89636		7.97712	7.33596	6.89964		7.99222	7.3455	6.90293
	Ω=-0.05	7.95969	7.31915	6.88389		7.97483	7.32872	6.88718		7.98993	7.33827	6.89047
2	Ω=-0.05	7.62789	7.02471	6.61874		7.64368	7.03468	6.62216		7.65944	7.04462	6.62558
	Ω=0	7.6255	7.01715	6.60575		7.64129	7.02712	6.60917		7.65705	7.03708	6.6126
	Ω=-0.05	7.6231	7.00958	6.59273		7.6389	7.01956	6.59617		7.65466	7.02953	6.5996
3	Ω=-0.05	7.33065	6.75176	6.3625		7.34708	6.76213	6.36606		7.36347	6.77248	6.36962
	Ω=0	7.32816	6.74389	6.34898		7.34459	6.75427	6.35255		7.36099	6.76463	6.35612
	Ω=-0.05	7.32566	6.73601	6.33544		7.3421	6.7464	6.33902		7.3585	6.75678	6.34259
0	Ω=-0.05	8.55703	8.31260	8.37748		9.43205	8.84645	8.55259		10.2325	9.34987	8.72418
	Ω=0	8.41640	7.88376	7.67745		9.30465	8.44476	7.86815		10.1152	8.97075	8.05433
	Ω=-0.05	8.27337	7.43021	6.90683		9.17548	8.02299	7.11820		9.99651	8.57489	7.32348
1	Ω=-0.05	8.08984	7.91790	8.03872		9.01035	8.47664	8.22104		9.84516	9.00077	8.39941
	Ω=0	7.94094	7.46642	7.30630		8.87690	8.05654	7.50644		9.72317	8.60629	7.70137
	Ω=-0.05	7.78919	6.98583	6.49177		8.74141	7.61329	6.71622		9.59964	8.19284	6.93341
2	Ω=-0.05	7.67792	7.57252	7.74446		8.64241	8.15496	7.93354		9.50958	8.69849	8.11823
	Ω = 0	7.52087	7.09911	6.98123		8.50319	7.71737	7.19042		9.38323	8.28964	7.39369
	Ω=-0.05	7.36046	6.59180	6.12362		8.36165	7.25342	6.36107		9.25516	7.85955	6.58997
3	Ω=-0.05	7.31058	7.26690	7.48595		8.31777	7.87198	7.68141		9.21554	8.43377	7.87201
	Ω=0	7.14545	6.77216	6.69332		8.17302	7.41771	6.91123		9.08510	8.01142	7.12247
	Ω=-0.05	6.97642	6.23831	5.79324		8.02566	6.93375	6.04369		8.95277	7.56553	6.28416

$$c_{34} = \tau e_{31}(\beta^2 cos(\beta z)\frac{dz}{dt}\phi - (\frac{2V_0 e^i kt}{h}(ik)))$$

$$m_{11} = -(1+\mu^2\beta^2)I_0^*$$

$$m_{12} = i(N_x + \eta\Omega_x^2)\beta^2(1+\mu^2\beta^2)I_1^*$$

$$m_{13} = i(N_x + \eta\Omega_x^2)\beta^2(1+\mu^2\beta^2)J_0^*$$

$$m_{22} = -(1+\mu^2\beta^2)(I_0^* + I_2^*(N_x + \eta\Omega_x^2)\beta^4)$$

$$m_{23} = -(1+\mu^2\beta^2)(I_0^* + J_2^*(N_x + \eta\Omega_x^2)\beta^4)$$

$$m_{33} = -(1+\mu^2\beta^2)(I_0^* + k_2^*(N_x + \eta\Omega_x^2)\beta^2)$$

$$m_{34} = -(1+\mu^2\beta^2)e_{31}(\beta^2 cos(\beta z)\frac{dz}{dt}\phi - (\frac{2V_0 e^i kt}{h}(ik)))$$

6 Results and Discussion

This section illustrates the magneto – thermo vibration of FG nanobeam with the numerical examples. The material properties are composed of $BaTiO_3$, $CoFe_2O_4$ which is presented in Table 1. Table 2 shows the comparison of the buckling load for the various power law exponent values. Whereas, Table.3 gives the variation of the magnetic potential and electric voltages with the varying and increasing non local parameter. Since an increasing nonlocal parameter shows a decrement with the magnetic potential Ω and the electric voltage V.
Now, the Fig. 1 and Fig. 2 are presented to highlight the effect of external voltage (V) with the variation of the rising temperature ΔT and the gradient index (p) for $\mu = 1 and 1.5$. Whereas, with the increasing external voltage (V= 0, 0.5, 1) the temperature gradually decreases with respect to the gradient index (p) and nonlocal parameter. As the external voltage increases, the internal temperature distribution shows a gradual reduction due to enhanced electrothermal coupling. This behavior is significant in high-temperature microelectronic applications where thermal regulation through electric fields is desired. Fig. 3 shows the decrement of the buckling load with an increase in the magnetic effect Ωand the nonlocal value (μ) . It is observed that the buckling load decreases with a stronger magnetic field due to magneto-mechanical softening effects. In the absence of magnetic influence, the buckling load still shows a reduction trend with an increasing nonlocal parameter, confirming the size-dependent mechanical weakening at nanoscale dimensions. Fig. 4 presents the variation of the dimensionless buckling load with the nonlocal parameter (μ) in the effect of the external voltage (V) have been shown. As voltage increases, the buckling load decreases and reaches a neutralization point beyond which the nonlocal effect dominates. This interaction is critical in electrically actuated nanostructures where stability thresholds depend on both scale effects and applied potential. Fig. 5 and Fig. 6 depicts the effect of gradient index with respect to the presence of the dimensionless buckling load and magnetic potential for different nonlocal values. Therefore, with the surge effect of the gradient index (p), there is an increses in magnetic potential with the dimensionless buckling decreases and ampilifies over the nonlocal parameter (μ). Fig. 7 and Fig. 8 represents the impact of the electric voltage and dimensionless buckling load in the presence of the gradient index p = 0, 1, 5 over nonlocal parameter (μ). Hence when the gradient index is null then there is no fluctuation in the buckling load and forms a linear effect and nonlocal values ampilifies the buckling load. So, if the gradient index rises then the electric voltage increases with the buckling load decreases in gradual. Fig. 9 and Fig. 10interprets that temperature ΔT stabilizes with an increase in the gradient index and magnetic potential. Thus, when the magnetic potential ($\Omega = 0$) , the temperature calms down and stabilizes at some point with respect to an increasing gradient index (p) and nonlocal values. Fig. 11 and Fig. 12 states that the effect of an internal damping factor (τ) in the existence of wave number and wave frequency over nonlocal parameter (μ) . In these figures, the damping factor with shows a raising linear value and when ($\tau \leq 0.5$)there is an oscillation in the wave number and wave frequency Finally, Fig. 13 and Fig. 14 reveal the effect of magnetic potential and electric voltage on nonlinear frequency–amplitude behavior. The results show that increasing either field parameter leads to an increase in the nonlinear natural frequency. This amplification behavior is critical for the frequency tuning of nanodevices in magneto-electro-mechanical environments.

7 Conclusion

This work presents a nonlinear wave propagation analysis of magneto-thermo-piezoelectric functionally graded nanobeams using a nonlocal strain gradient viscoelastic model. The model effectively captures small-scale effects, multiphysical couplings, and viscoelastic behavior, offering a more realistic analytical approach compared to conventional methods. The governing equations were derived using higher- order beam theory and Hamilton's principle, and solved via an analytical wave dispersion method. Results show that electric voltage, magnetic field, and gradient index significantly influence buckling load, wave frequency, and thermal stability. The study addresses key gaps in earlier models by unifying multiple field interactions within a size-dependent framework, thus contributing to the reliable design of NEMS/MEMS devices under real-world operating conditions. Hence the upshots of the work are bolded as,

- The stability behaviours of FGM nanobeam are affected by magneto thermo piezo electricity and nonlocal values.
- Physical variants could be controlled via applying a suitable value of damping factor.
- Natural frequency reduces while the nonlocal parameter and gradient index of the FG nanobeam amplifies.
- The increase in power-law index softens the volume fraction.
- The bending rigidity and phase velocities are high in amplified wave numbers and getting reversed in low wave numbers.
- Nonlinear frequency attains higher magnitude in positive voltage and magnetic potentials.

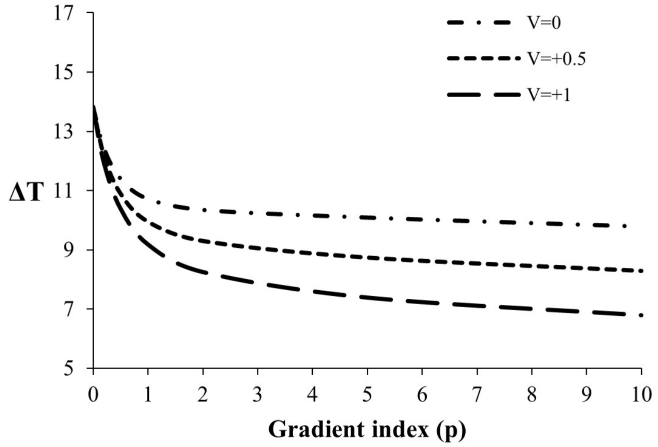

Fig. 2: External voltage with the presence of rising temperature to gradient index for μ = 1.5

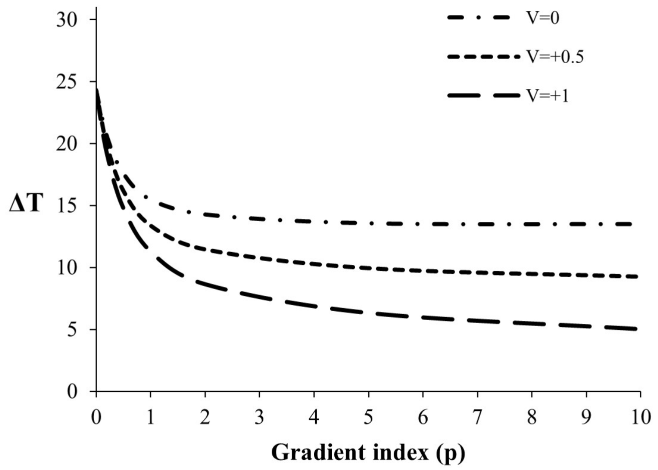

Fig. 3: External voltage with the presence of rising temperature to gradient index for μ = 1.5

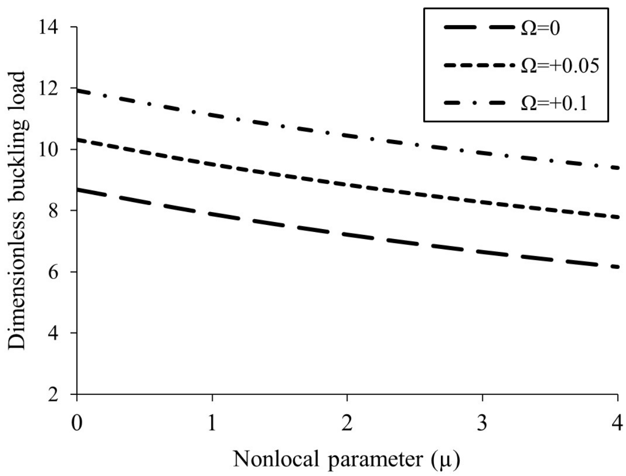

Fig. 4: External voltage in the presence of temperature with respect to gradient index for μ = 1.5

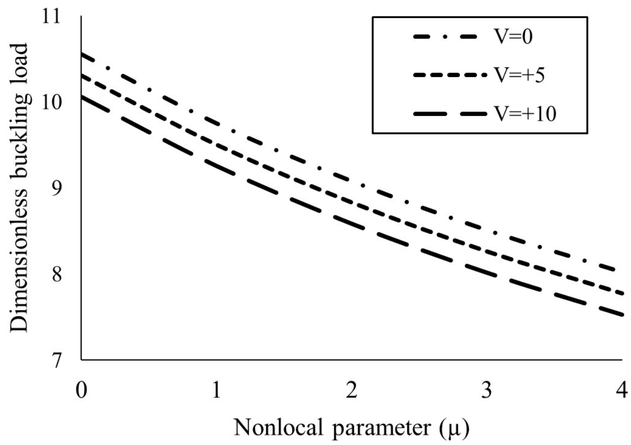

Fig. 5: Magnetic potential on the dimensionless buckling with respect to nonlocal parameter.

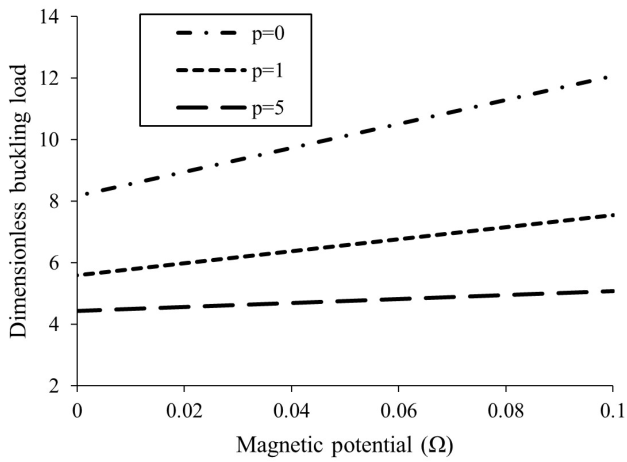

Fig. 6: External voltage on the dimensionless buckling load with respect to nonlocal parameter.

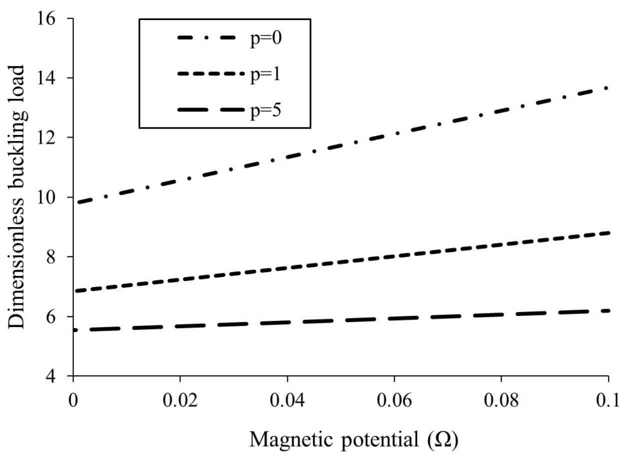

Fig. 7: Gradient index with the presence of rising dimensionless buckling load to magnetic potential for $\mu = 1$.

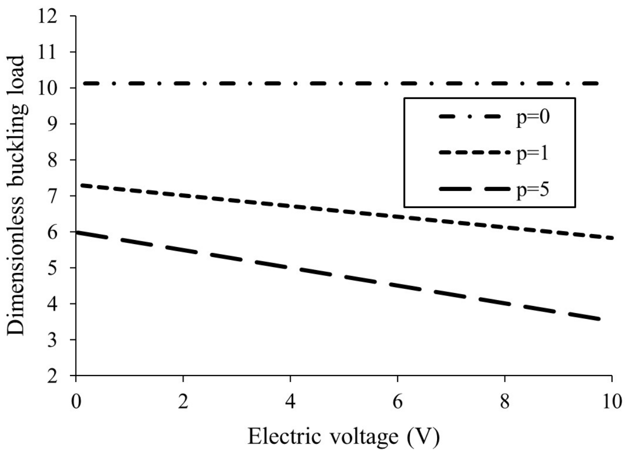

Fig. 8: Gradient index in the presence of dimensionless buckling load to Magnetic potential μ = 1.5

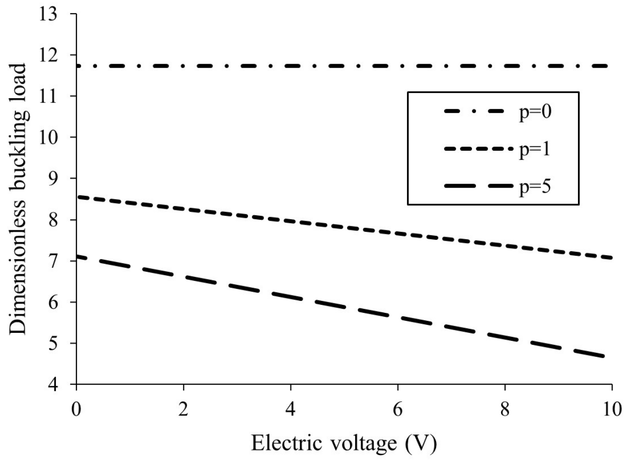

Fig. 9: Gradient index in the presence of dimensionless buckling load to Electric voltage when μ = 1

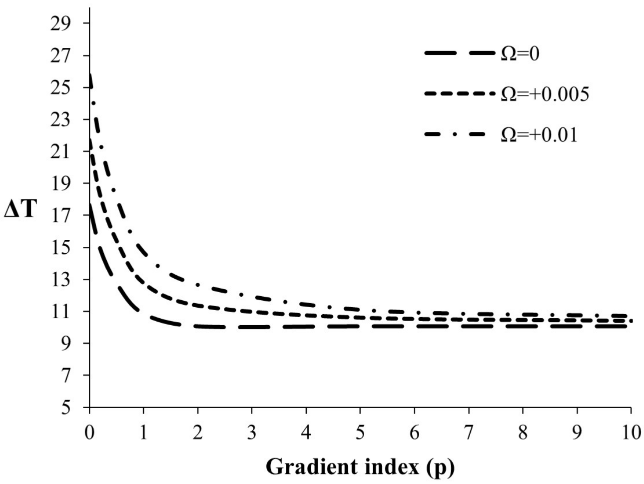

Fig. 10: Magnetic potential in the presence of temperature with respect to gradient index for μ = 1.0

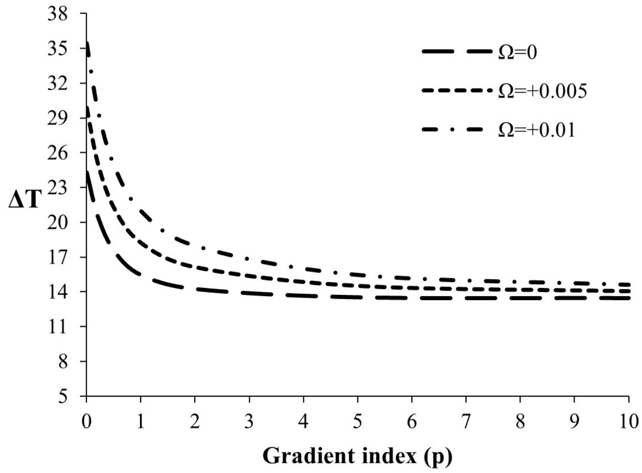

Fig. 11: Magnetic potential in the presence of temperature with respect to gradient index for μ = 1.5

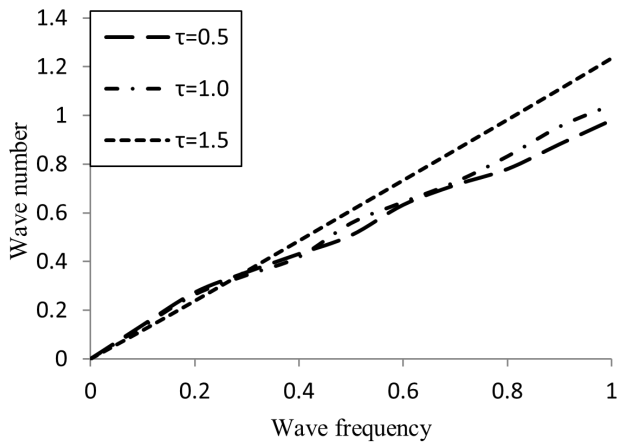

Fig. 12: Damping factor in the presence of wave number with respect to wave frequency for μ = 1.

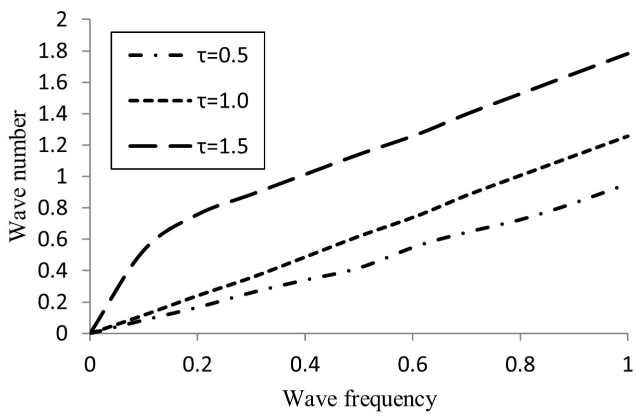

Fig. 13: Damping factor in the presence of wave number with respect to wave frequency for μ = 1.5

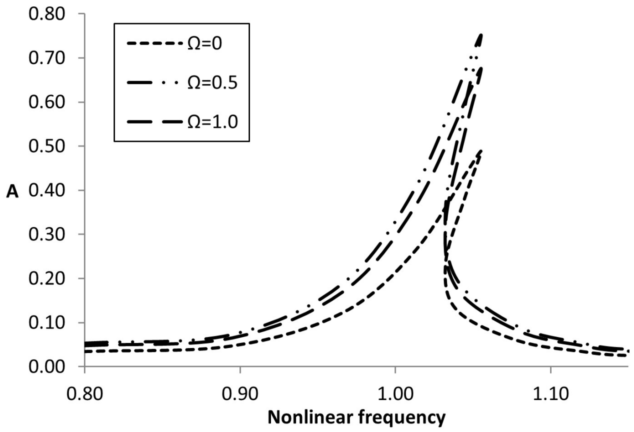

Fig. 14: Magnetic potential in the presence of amplitude with respect to nonlinear frequency for $\mu = 1.5$

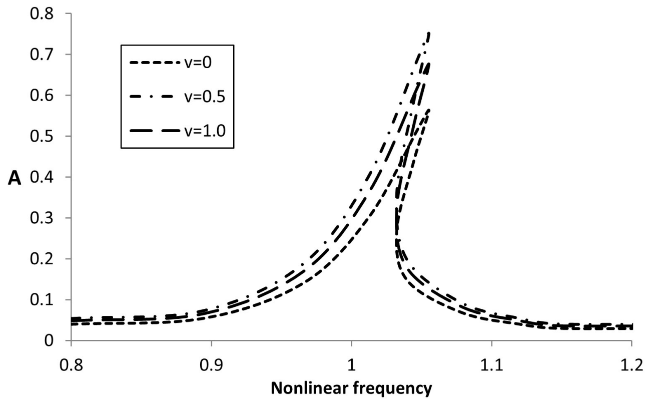

Fig. 15: External voltage in the presence of amplitude with respect to nonlinear frequency for $\mu = 1.5$

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