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On the linear growth rate of disturbance in double-diffusive Navier–Stokes–Voigt fluid with uniform rotation

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Abstract: In the present paper linear mathematical analysis is performed for a rotatory incompressible Navier–Stokes–Voigt (NSV) fluid. It is analytically proved that the principle of the exchange of stabilities in a rotatory incompressible Navier–Stokes–Voigt fluid is valid in the regime $\frac{R_s \Pr Le}{2\pi^4} + \frac{T_a}{\pi^2}\left(\frac{1}{\pi^2} + \lambda\right) \leq 1$. Further, upper bounds for the linear growth rate of disturbance are also obtained. It is mathematically established that upper bounds for the linear growth rate $\sigma = \sigma_r + i\sigma_i$ (σ_r and $\sigma_i$are the real and imaginary parts of σ , respectively) of an arbitrary neutral or unstable oscillatory disturbance of growing amplitude, lies within a semicircle in the right half of the $\sigma_r\sigma_i-$ plane, whose centre is at the origin and radius = $max\left(\sqrt{\frac{R_s}{\Pr Le(1+2\pi^2\lambda)}}, \sqrt{\frac{T_a}{(1+\pi^2\lambda)}}\right)$, where R_s is concentration Rayleigh number, and and Le is the Lewis number, Pr is the thermal Prandtl number, T_a is the Taylor number and λ is the Navier-Stokes-Voigt parameter. The results derived herein are uniformly valid for any combination of rigid and free boundaries.

Keywords: Concentration Rayleigh Number, Navier-Stokes-Voigt Fluid, Prandtl Number, Taylor Number, Principle of the Exchange of Stabilities, Linear Growth Rate.

1 Introduction

Research on convective fluid motion, driven by the combined influence of a uniform vertical temperature and an opposing uniform vertical concentration gradient, referred to as double-diffusive convection, has been a highly active area of research due to its importance in predicting groundwater movement in aquifers, assessing the effectiveness of fibrous materials, and its applications in engineering geology and nuclear engineering (Turner (1974); Turner (1973); Huppert and Turner (1981)). Double-diffusive convection manifests in fluids when two stratifying agents with different molecular diffusivities act in opposing directions on the vertical density gradient. These agents can vary, such as heat and helium in astrophysical cases, heat and salt in oceanographic scenarios, or any two different solutes in chemical engineering. Therefore, this phenomenon is also thermohaline or thermosolutal convection. Despite these variations, the fundamental characteristics of double-diffusive convection remain consistent across all cases and are now widely recognized. This subject has been extensively researched and reviewed, and the findings are well-recorded in the literature (Schechter et al. (1974), Stern (1960), Chen and Johnson (1984) and Radko (2013)).
The influence of external rotation on double-diffusive convection has drawn significant experimental and theoretical interest. Due to its widespread presence in geophysical and oceanic flows, it is essential to comprehend how the Coriolis force impacts the structure and transport properties of double-diffusive convection. Schmitt and Lamber (1979) experimentally studied the effects of rotation on highly supercritical salt fingers. The numerical calculations of the influence of rotation on the stability of a binary fluid, in which diffusive and convective mass transfer is augmented by thermal diffusion, were reported by Antoranz et al. (1979). In his seminal paper, Pearlstein (1981) studied analytically double-diffusive convection in a rotating Newtonian fluid layer and uncovered several remarkable departures that the earlier investigators did not recognize. Duba et al. (2016) investigated Soret and Dufour's effects on thermohaline convection in the rotating fluid using linear and weakly non-linear stability theory. Babu et al. (2022) discussed the analytical study of linear and weakly nonlinear stability of thermohaline convection in a sparsely packed porous under the influence of rotation.
In recent years, the examination of non-Newtonian fluids has gained special attention due to their applications in various fields of science, engineering, and technology. Of particular interest are visco-elastic fluids, as they play a significant role among various types of non-Newtonian fluids in the study of fluid motion in dynamical systems as encountered in the industry such as the extrusion of polymer fluids, the solidification of liquid crystals, petroleum, and chemical processes, food processing and exotic and colloidal fluids, etc (Raghunatha and Shivakumara (2021)). Foundational models for viscoelastic fluids include the Oldroyd, Kelvin-Voigt and Maxwell models. Oskolkov (1988, 1995) introduced a model for the Kelvin-Voigt fluid which has been widely utilized to study the motion of fluids with very weak viscoelastic properties. The Kelvin-Voigt fluid of order zero is commonly referred to as the Navier-Stokes-Voigt (NSV) fluid. Thus, thermal Kelvin-Voigt fluids represent a more general class that includes Navier-Stokes-Voigt fluids as a special case. We are interested in Kelvin-Voigt fluid of order zero also referred to as

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the Navier-Stokes–Voigt fluids, which have been studied extensively by researchers Kalantarov and Titi (2009), Çelebı et al. (2009), Sukacheva and Matveeva (2010), Chiriţă and Zampoli (2015),Damázio et al. (2016), Straughan (2021, 2023, 2025), Sharma and Sunil (2024) and Kumari et al. (2024a).

The occurrence of stationary convection or, in particular, the validity of the principle of the exchange of stabilities implies that the oscillatory motions cannot manifest in a stability problem. Pellew and Southwell (1940) proved this principle for the classical Rayleigh-Benard problem. Later, Banerjee et al. (1985) derived a sufficient condition for its validity for magnetohydrodynamic Rayleigh- Benard problem which was further extended to different complex hydrodynamic configurations by Gupta et al. (1984, 1986), Prakash (2014), Prakash and Manan (2016), and Kumari et al. (2024b).

For case when both the bounding surfaces are not free, the exact solutions in closed form are not derivable. In that case the problem of obtaining the upper bounds for the complex rate of an arbitrary neutral or unstable oscillatory perturbation of growing amplitude in instability problems becomes important. Banerjee et al. (1981) formulated a method to combine the governing equations and boundary conditions for classic thermohaline instability problems, yielding the desired bounds. Their work is further extended to different hydrodynamic configurations by Gupta et al. (1983), Banerjee et al. (1995) Prakash (2013), Prakash and Gupta (2013), and Kumar et al. (2024).

In the present communication, an attempt has been made to derive sufficient conditions for the manifestation of stationary convection in terms of the parameters of the system. Since the complement of this condition implies the occurrence of oscillatory motions, therefore as a second problem we also derive the upper bounds for the complex growth rate of an arbitrary neutral or unstable oscillatory perturbation of growing amplitude, for the case of thermosolutal convection in a rotating Navier–Stokes–Voigt fluid layer, heated from below for the cases of free boundaries and rigid boundaries.

2 Formulation of the problem

We consider an infinite horizontal layer of incompressible NSV fluid confined between two horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and $T_1 (< T_0)$ and uniform concentrationS_0 and $S_1 (< S_0)$ by heating and salting from below. The layer is rotating with an angular velocity $\overrightarrow{\Omega} = (0, 0, \Omega)$ about the vertical axis, while the force of gravity $\overrightarrow{g} = (0, 0, -g)$ is acting in the vertical direction. (as shown in Fig 1)

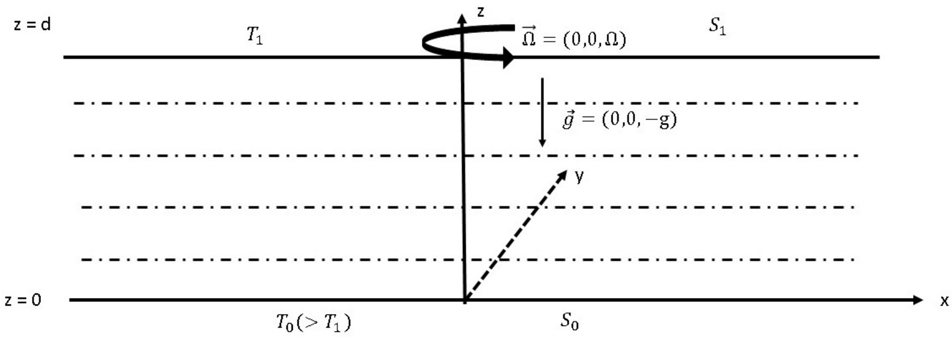

Fig. 1: Geometrical configuration of the problem

The equations that govern the system of a rotating incompressible NSV fluid heated and soluted from below are given by (Straughan (2021))

$$\nabla . q = 0, \tag{1}$$

$$\left(\left(1 - \widehat{\lambda}\nabla^2\right) \frac{\partial}{\partial t} + (q . \nabla)\right) q = -\frac{1}{\rho_o}\nabla p + \nu\nabla^2 q + \left(1 + \frac{\delta\rho}{\rho_o}\right) g + 2 (q \times \Omega), \tag{2}$$

$$\frac{\partial T}{\partial t} + (q . \nabla) T = \kappa\nabla^2 T, \tag{3}$$

$$\frac{\partial S}{\partial t} + (q . \nabla) S = \kappa_1\nabla^2 S, \tag{4}$$

together with the equation of state given by

$$\rho = \rho_o (1 - \alpha (T - T_0) + \alpha_1 (S - S_0)), \tag{5}$$

where $q, p, \rho, \rho_o, \delta\rho, \kappa, \kappa_1, \widehat{\lambda}, \nu, \mu, t, \alpha,$ and α_1 are the velocity, reduced pressure, density, reference density, change in density, thermal diffusivity, solute diffusivity, Kelvin-Voigt coefficient, Kinematic viscosity, coefficient of viscosity, time, thermal and solutal coefficients of expansion respectively. Further, $2 (\Omega \times q)$ is the Coriolis acceleration and the reduced pressure is given by

$$p = p_f - \frac{\rho_o}{2} grad \left(|\Omega \times r|^2\right), \tag{6}$$

where p_f is fluid pressure and $-\frac{1}{2}grad|\Omega \times r|^2$ is the centrifugal force due to rotation applied.
Now following Chandrasekhar (1981), using linear stability theory and normal mode technique, we can easily derive the governing equations for the rotatory Incompressible NSV fluid instability problem in non-dimensional form given by

$$\left[\sigma - (1+\lambda\sigma)\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W = -a^2 {R_t}^{1/2}\Theta + a^2 R_s^{1/2}\Phi - Ta^{1/2}DZ, \tag{7}$$

$$\left((1+\lambda\sigma)\left(D^2-a^2\right)-\sigma\right)Z = -Ta^{1/2}DW, \tag{8}$$

$$\left(D^2-a^2-\Pr\sigma\right)\Theta = -{R_t}^{1/2}W, \tag{9}$$

$$\left(D^2-a^2-\Pr Le\sigma\right)\Phi = -R_s^{1/2}W, \tag{10}$$

The horizontal boundaries are considered to be either free or rigid. Thus, the boundary conditions are given by

$$W = D^2W = \Theta = \Phi = DZ = 0, \tag{11}$$

for a free boundary

$$W = DW = \Theta = \Phi = Z = 0, \tag{12}$$

for a rigid boundary
where z is the vertical coordinate such that $0 \le z \le 1$, $D = \frac{d}{dz}$ is the differentiation w.r.t z, a^2 is the square of the wave number, $W(z)$ is the vertical velocity, $\Pr = \frac{\nu}{\kappa}$ is the Prandtl number, $Ta = \frac{4\Omega^2 d^4}{\nu^2}$ is the Taylor number, $R_t = \frac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number, $R_s = \frac{g\alpha_1\beta_1 d^4}{\nu\kappa_1}$ is the concentration Rayleigh number, $\lambda = \frac{\widehat{\lambda}}{d^2}$ is the Navier-Stokes-Voigt parameter , $\sigma(=\sigma_r + i\sigma_i)$ is the complex growth rate, $\Theta(z)$ is the perturbation temperature, $Le = \frac{\kappa}{\kappa_1}$ is the Lewis number, $\Phi(z)$ is the perturbation concentration and $Z(z)$ is z-component of the perturbation vorticity. It may further be noted that Eqs. (7)-(12) describe an eigenvalue problem of the instability of the double diffusive rotatory NSV fluid for the complex growth rate $\sigma(=\sigma_r + i\sigma_i)$, where σ_r is the amplification factor and σ_i is the circular frequency. Further, if $\sigma_r = 0$ implies $\sigma_i = 0$ in which case stationary pattern of motions prevails at neutral stability and we say that the principle of the exchange of stabilities is valid. On the other hand if $\sigma_r \ge 0$ implies $\sigma_i \ne 0$ in which case oscillatory motions prevails on the onset of instability and we say that we have a case of ovestability occurs.

3 Mathematical analysis

Now we derive a sufficient condition for the occurrence of stationary convection in a rotatory incompressible Navier–Stokes–Voigt (NSV) fluid layer when heated and salted from below in the form of following theorem:

Theorem1. If(W,Θ,Z,Φ,σ) , $R_t > 0$, $R_s > 0, \Pr > 0$, $T_a > 0, \lambda > 0, Le > 0$, $\sigma = \sigma_r + i\sigma_i$, $\sigma_r \ge 0$ is a solution of Eqs. (7)-(10) together with the boundary conditions (11)-(12) and $\frac{R_s \Pr Le}{2\pi^4} + \frac{T_a}{\pi^2}\left(\frac{1}{\pi^2}+\lambda\right) \le 1$.then we must have $\sigma_i = 0$. In particular, $\sigma_r = 0$ implies $\sigma_i = 0$ if $\frac{R_s \Pr Le}{2\pi^4} + \frac{T_a}{\pi^2}\left(\frac{1}{\pi^2}+\lambda\right) \le 1$.

Proof. Multiplying Eq. (7) by W^* (the superscript * henceforth denotes the complex conjugation) and integrating the resulting equation from $z = 0$ to $z = 1$, we get

$$\int_0^1 W^*\left[\sigma - (1+\lambda\sigma)\left(D^2-a^2\right)\right]\left(D^2-a^2\right)Wdz = -a^2{R_t}^{1/2}\int_0^1 W^*\Theta dz + a^2R_s^{1/2}\int_0^1 W^*\Phi dz - T_a^{1/2}\int_0^1 W^*DZdz. \tag{13}$$

Using Eqs. (8), (9), (10) and the boundary conditions (11) or (12), we can write

$$a^2{R_t}^{1/2}\int_0^1 W^*\Theta dz = -a^2\int_0^1 \Theta\left(D^2-a^2-\Pr\sigma^*\right)\Theta^* dz, \tag{14}$$

$$a^2R_s^{1/2}\int_0^1 W^*\Phi dz = -a^2\int_0^1 \Phi\left(D^2-a^2-\Pr Le\sigma^*\right)\Phi^* dz, \tag{15}$$

and

$$T_a^{1/2}\int_0^1 W^* DZdz = -T_a^{1/2}\int_0^1 ZDW^* dz = \int_0^1 Z\left((1+\lambda\sigma^*)\left(D^2-a^2\right)-\sigma^*\right)Z^* dz. \tag{16}$$

Combining Eqs. (13)-(16), we get

$$\begin{aligned}\int_0^1 W^*\left[\sigma-(1+\lambda\sigma)\left(D^2-a^2\right)\right]\left(D^2-a^2\right)Wdz =& a^2\int_0^1 \Theta\left(D^2-a^2-\Pr\sigma^*\right)\Theta^* dz\\ -a^2\int_0^1 \Phi\left(D^2-a^2-\Pr Le\sigma^*\right)\Phi^* dz -& \int_0^1 Z\left((1+\lambda\sigma^*)\left(D^2-a^2\right)-\sigma^*\right)Z^* dz,\end{aligned} \tag{17}$$

Integrating the various terms of Eq. (17), by parts, from $z = 0$ to $z = 1$ for an appropriate number of times with the help of the boundary conditions (11) or (12), we get

$$\begin{aligned}&\sigma\int_0^1\left(|DW|^2+a^2|W|^2\right)dz+(1+\sigma\lambda)\int_0^1\left(\left|D^2W^2\right|+2a^2|DW|^2+a^4|W|^2dz=\right.\\ &a^2\int_0^1\left(|D\Theta|^2+\left(a^2+\Pr\sigma^*\right)|\Theta|^2\right)dz-a^2\int_0^1\left(|D\Phi|^2+\left(a^2+\Pr Le\sigma^*\right)|\Phi|^2\right)dz\\ &\qquad -(1+\sigma^*\lambda)\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz-\sigma^*\int_0^1|Z|^2dz.\end{aligned} \tag{18}$$

Now equating imaginary parts of both sides of Eq. (7) and cancelling σ_i $(\neq 0)$ throughout, we obtain

$$\begin{aligned}&\int_0^1\left(|DW|^2+a^2|W|^2\right)dz+\lambda\int_0^1\left(\left|D^2W\right|^2+2a^2|DW|^2+a^4|W|^2\right)dz=\\ &-a^2\Pr\int_0^1|\Theta|^2dz+a^2\Pr Le\int_0^1|\Phi|^2dz+\lambda\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz+\int_0^1|Z|^2dz.\end{aligned} \tag{19}$$

Multiplying Eq. (8) by Z^* and integrating the resulting equation, by parts, by making use of boundary conditions (11)-(12) on Z and W, we have from real part of the final equation

$$\begin{aligned}(1+\sigma_r\lambda)\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz+\sigma_r\int_0^1|Z|^2dz &= Real\ part\ of\ T_a^{1/2}\int_0^1 Z^* DWdz\\ &\leq T_a^{1/2}\int_0^1|DW|\,|Z|dz\\ &\leq T_a^{1/2}\left(\int_0^1|DW|^2dz\right)^{1/2}\left(\int_0^1|Z|^2dz\right)^{1/2}.\end{aligned} \tag{20}$$

(using Schwarz inequality).

Also, since for free boundaries, we have by Rayleigh-Ritz inequality (Schultz (1973))

$$\pi^2\int_0^1|Z|^2dz\leq\int_0^1|DZ|^2dz, \tag{21}$$

and for rigid boundaries we have (Banerjee et al. (1995))

$$\int_0^1|DZ|^2dz=\pi^2\int_0^1|Z|^2dz. \tag{22}$$

Combining (20), (21), and (22) and using the fact that $\sigma_r \geq 0$, we obtain

$$\left(\int_0^1 |Z|^2 dz\right)^{1/2} \leq \frac{T_a^{1/2}}{\pi^2}\left(\int_0^1 |DW|^2 dz\right)^{1/2}. \tag{23}$$

Using inequality (23) in inequality (20), we get

$$\int_0^1 \left(|DZ|^2 + a^2|Z|^2\right) dz \leq \frac{T_a}{\pi^2}\int_0^1 |DW|^2 dz. \tag{24}$$

Inequality (23), can also be expressed as

$$\int_0^1 |Z|^2 dz \leq \frac{T_a}{\pi^4}\int_0^1 |DW|^2 dz. \tag{25}$$

Multiplying Eq. (10) by its complex conjugate and integrating over the vertical range of z for a suitable number of times and using boundary conditions (11)-(12), we get

$$\int_0^1 \left(\left|D^2\Phi\right|^2 + 2a^2|D\Phi|^2 + a^4|\Phi|^2\right) dz + 2\,\mathrm{Pr}\,Le\sigma_r \int_0^1 \left(|D\Phi|^2 + a^2|\Phi|^2\right) dz + \mathrm{Pr}^2 Le^2|\sigma|^2 \int_0^1 |\Phi|^2 dz + = R_s \int_0^1 |W|^2 dz. \tag{26}$$

Since $W(0) = 0 = W(1)$ and $\Phi(0) = 0 = \Phi(1)$, therefore by Rayleigh-Ritz inequality (Schultz 1973), we get

$$\pi^2 \int_0^1 |W|^2 dz \leq \int_0^1 |DW|^2 dz, \tag{27}$$

and

$$\pi^2 \int_0^1 |\Phi|^2 dz \leq \int_0^1 |D\Phi|^2 dz. \tag{28}$$

Upon using inequalities (27) and (28) in Eq. (26), we have

$$a^2 \int_0^1 |\Phi|^2 dz \leq \frac{R_s}{2\pi^4}\int_0^1 |DW|^2 dz. \tag{29}$$

Utilizing inequalities (24), (25) and (29) in Eq. (19), we get

$$\left(1 - \frac{\mathrm{Pr}\,LeR_s}{2\pi^4} - \frac{T_a}{\pi^2}\left(\lambda + \frac{1}{\pi^2}\right)\right)\int_0^1 |DW|^2 dz + a^2 \int_0^1 |W|^2 dz + \lambda \int_0^1 \left(\left|D^2W\right|^2 + 2a^2|DW|^2 + a^4|W|^2\right) dz + a^2\,\mathrm{Pr}\int_0^1 |\Theta|^2 dz \leq 0, \tag{30}$$

and thus for $\sigma_r \geq 0,\ \sigma_i \neq 0$, we necessarily have

$$\frac{R_s\,\mathrm{Pr}\,Le}{2\pi^4} + \frac{T_a}{\pi^2}\left(\frac{1}{\pi^2} + \lambda\right) > 1, \tag{31}$$

which is a necessary condition for oscillatory motions to manifest.
Hence by contradiction if $\frac{R_s\,\mathrm{Pr}\,Le}{2\pi^4} + \frac{T_a}{\pi^2}\left(\frac{1}{\pi^2} + \lambda\right) \leq 1$, we must have $\sigma_i = 0$.
This proves the theorem.
Theorem1, from physical point of view, may be stated in an equivalent form as, for rotatory NSV fluid layer heated and salted from below, an arbitrary neutral or unstable mode of the system is definitely non-oscillatory in character and in particular, the principle of the exchange of stabilities is valid if $\frac{R_s\,\mathrm{Pr}\,Le}{2\pi^4} + \frac{T_a}{\pi^2}\left(\frac{1}{\pi^2} + \lambda\right) \leq 1$. Further, this result is uniformly valid for the quite general nature of the bounding surfaces.

Special Cases: The above theorem yields that the principle of the exchange of stabilities (PES) (i.e. non-occurrence of oscillatory motions) is valid for:

- Rayleigh-Benard convection ($R_s = T_a = \lambda = 0$) (Pellew and Southwell (1940))
- Rotatory Rayleigh-Benard convection ($R_s = \lambda = 0$) if $\frac{T_a}{\pi^4} \leq 1$. (Gupta et al. (1986))
- Thermohaline convection ($T_a = \lambda = 0$) if $\frac{R_s \Pr Le}{2\pi^4} \leq 1$. (Gupta et al. (1986))
- Rotatory convection in NSV fluid ($R_s = 0, \lambda \neq 0$) if $\frac{T_a}{\pi^2}\left(\frac{1}{\pi^2} + \lambda\right) \leq 1$.
- Thermohaline convection in NSV fluid ($T_a = 0, \lambda \neq 0$) if $\frac{R_s \Pr Le}{2\pi^4} \leq 1$.

Since the complementary of the above condition means the manifestation of oscillatory convection, thus it is important to derive upper bounds for the linear growth rate of an arbitrary oscillatory disturbance. This is done in the form of following theorem:

Theorem2. If$(W, \Theta, Z, \Phi, \sigma)$, $R_t > 0$, $R_s > 0, \Pr > 0$, $T_a > 0, \lambda > 0, Le > 0$, $\sigma = \sigma_r + i\sigma_i$, $\sigma_r \geq 0$then a necessary condition for the existence of a non-trivial solution of Eqns. (7)–(10) together with the boundary conditions (11)-(12) is that $|\sigma|^2 < max\left(\frac{R_s}{\Pr Le\left(1+2\pi^2\lambda\right)}\ ,\ \frac{T_a}{\left(1+\pi^2\lambda\right)}\right)$.

Proof. Multiplying Eq. (8) by its complex conjugate and integrating over the vertical range of z for a suitable number of times and using boundary conditions (11)-(12), we get

$$\left(1 + 2\sigma_r\lambda + |\sigma|^2\lambda^2\right)\int_0^1 \left(\left|D^2Z\right|^2 + 2a^2|DZ|^2 + a^4|Z|^2\right)dz + 2\sigma_r\int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz + 2|\sigma|^2\lambda\int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz + |\sigma|^2\int_0^1 |Z|^2 dz = T_a\int_0^1 |DW|^2 dz. \tag{32}$$

Since $\sigma_r \geq 0$, therefore the above inequality, yields

$$\lambda\int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz + \int_0^1 |Z|^2 dz \leq \frac{T_a}{|\sigma|^2}\int_0^1 |DW|^2 dz. \tag{33}$$

We also have (Banerjee et al. [34])

$$\pi^2\int_0^1 |DW|^2 dz \leq \int_0^1 \left|D^2W\right|^2 dz. \tag{34}$$

Further, we have from Eq. (26) that

$$\int_0^1 |\Phi|^2 dz \leq \frac{R_s}{\Pr^2 Le^2|\sigma|^2}\int_0^1 |W|^2 dz. \tag{35}$$

Now using inequalities (33), (34) and (35) in Eq. (19), we get

$$\left(1 + \pi^2\lambda - \frac{T_a}{|\sigma|^2}\right)\int_0^1 |DW|^2 dz + a^2\left(1 + 2\pi^2\lambda - \frac{R_s}{\Pr Le|\sigma|^2}\right)\int_0^1 |W|^2 dz + \lambda a^4\int_0^1 |W|^2 dz + a^2\Pr\int_0^1 |\Theta|^2 dz < 0, \tag{36}$$

which implies that we must have

$$|\sigma|^2 < max\left(\frac{R_s}{\Pr Le\left(1 + 2\pi^2\lambda\right)}\ ,\ \frac{T_a}{\left(1 + \pi^2\lambda\right)}\right). \tag{37}$$

This establishes the theorem.

Theorem 2, from physical point of view, can be stated as: for rotatory NSV fluid layer heated and salted from below, the complex growth rate of an arbitrary oscillatory disturbance of neutral or growing amplitude, lies inside a semicircle in the right half of the $\sigma_r\sigma_i$–plane whose center is at the origin and radius = $max\left(\sqrt{\frac{R_s}{\Pr Le(1+2\pi^2\lambda)}}\ ,\ \sqrt{\frac{T_a}{(1+\pi^2\lambda)}}\right)$ (see Fig 2). Further, this result is uniformly valid for any combination of rigid and free bounding surfaces.

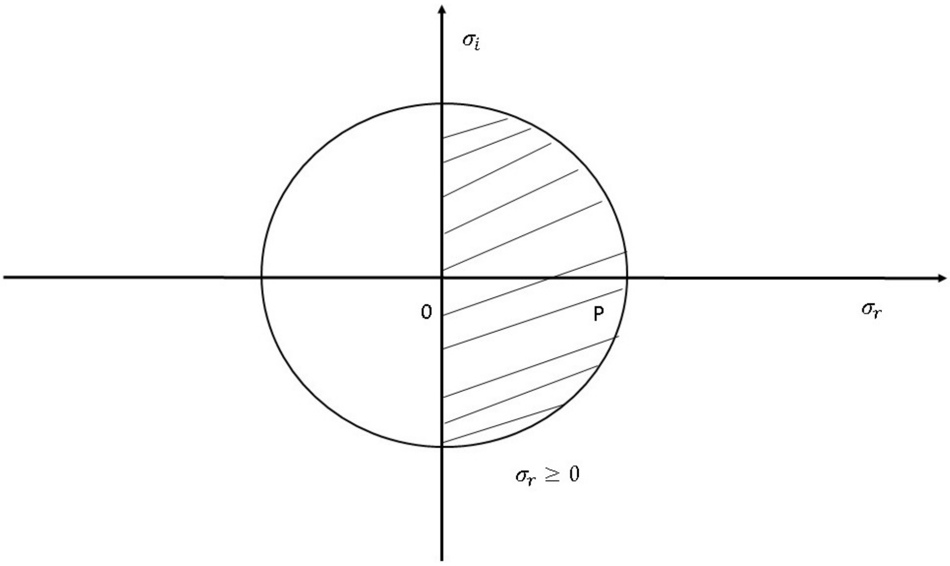

Fig. 2: Shaded region shows the region of complex growth rate in the $\sigma_r\sigma_i-$ plane.

Special Cases: The above theorem yields the following special cases:

- For rotatory Rayleigh-Benard convection ($R_s = \lambda = 0$) if $|\sigma|^2 < Ta$. (Banerjee et al. (1981))
- For thermohaline convection ($T_a = \lambda = 0$) if $|\sigma|^2 < \frac{R_s}{\Pr Le}$. (Banerjee et al. (1981))
- For rotatory thermohaline convection ($\lambda = 0$) if $|\sigma|^2 < max\left(\frac{R_s}{\Pr Le}, Ta\right)$. (Gupta et al. (1983)).
- For rotatory convection in NSV fluid ($R_s = 0, \lambda \neq 0$) if $|\sigma|^2 < \frac{T_a}{(1+\pi^2\lambda)}$.
- For thermohaline convection in NSV fluid ($T_a = 0, \lambda \neq 0$) if $|\sigma|^2 < \frac{R_s}{\Pr Le\left(1+2\pi^2\lambda\right)}$.

4 Conclusion

The linear stability theory is used to derive sufficient conditions for the validity of the principle of the exchange of stabilities which disallows the existence of oscillatory motions of growing amplitude in NSV fluid layer heated and salted from below. Further, since the complement of this condition implies the occurrence of oscillatory motions, therefore as a second problem we obtain the upper bounds for the complex growth rate of an arbitrary oscillatory amplitude in rotatory incompressible NSV thermosolutal fluid. The results for Rayleigh-Benard convection, thermohaline convection, rotatory thermohaline convection, rotatory NSV fluid layer, and thermohaline NSV fluid layer are also derived as special cases. It is further proved that these results are uniformly valid for any combination of rigid and/ or free boundaries.

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List of Symbols

Non-dimensional numbers

λ Navier-Stokes-Voigt parameter

Le Lewis number

Pr Prandtl number

T_a Taylor number

R_s concentration Rayleigh number

R_t thermal Rayleigh number

Greek Symbols

α coefficient of volume expansion due to temperature variation

α_1 coefficient of volume expansion due to concentration variation

κ thermal diffusivity

κ_1 solute diffusivity

ν kinematic viscosity

Ω angular velocity

Φ perturbation concentration

ρ density

σ complex growth rate

Θ perturbation temperature

$\widehat{\lambda}$ Kelvin-Voigt coefficient

ρ_0 reference density

σ_i imaginary part of complex growth rate

σ_r real part of complex growth rate

$*$ complex conjugation

Latin Symbols

(u, v, w) velocity components

(x, y, z) Cartesian co-ordinates

$\vec{g}$ acceleration due to gravity

$\vec{q}$ Velocity

a^2 square of the wave number

D differential operator

d thickness of the layer

p reduced pressure

p_f fluid pressure

S Solute concentration

T temperature

t time

W vertical velocity

Z $z-$component of perturbation vorticity

z vertical coordinate

S_0 Solute concentration at the lower boundary

S_1 Solute concentration at the upper boundary

T_0 temperature at the lower boundary

T_1 temperature at the upper boundary

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