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Stability Analysis of Double-diffusive Convection in a Couple Stress Nanofluid

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Stability of double-diffusive convection in a horizontal layer of nanofluid is studied. The couple-stress fluid model is employed to describe the rheological behavior of the nanofluid. Stability of nanofluid has been influenced by the features of couple-stress fluid, suspended nanoparticles and examined under the consideration of momentum and thermal slip boundary conditions. By applying normal mode analysis method and linear stability theory, the dispersion relation describing the effect of various parameters is derived. We have assumed that the nanoparticle concentration flux is zero on the boundaries which neutralizes the possibility of oscillatory convection and only stationary convection occurs. The impact of the physical parameters, like the couple stress parameter, solutal-Rayleigh Number, thermo-nanofluid Lewis number, thermo-solutal Lewis number, Soret parameter and Dufour parameter have also been observed and compared with the published work. A very good agreement is found between the present paper and earlier published results.

1 Introduction

The technology of nanofluid becomes a new challenge for the heat transfer fluid due to their higher thermal conductivity. The word nanofluid was first proposed by Choi (1995). The main important feature of nanofluid is the enhancement of thermal conductivity. Buongiorno (2006) proposed a mathematical model for nanofluid based on the effects of Brownian motion and thermophoresis of suspended nanoparticles after analyzing the effect of seven slips mechanism, he concluded that in the absence of turbulent eddies, Brownian diffusion and thermophoresis are the dominant slip mechanisms. Stokes (1966) proposed and postulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as lubricant. When fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded-bearing synovial joints of human body and these joints have low-friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is a viscous non- Newtonian fluid. According to the theory of Stokes, couple-stresses are found to appear in noticeable magnitude in fluids. Since the long chain hylauronic acid molecules are found as additives in synovial fluid. Walicki and Walicka (1999) modeled synovial fluid as couple-stress fluid in human joints. Sharma and Thakur (2000) studied the couple-stress fluid heated from below in hydromagnetics and found that couple stress parameter has stabilizing effect on the stationary convection.

Double-diffusive convection in nanofluid is an important phenomenon that has various applications in the fields of chemical science, food processing, engineering and nuclear industries, geophysics, bioengineering and cancer therapy, movement of biological fluid, oceanography etc. The base fluid of the nanofluid is a couple-stress elastico-viscous fluid. Nanoparticles do not affect the solute concentration. In nanofluids, the base fluid does not satisfy the properties of Newtonian fluids in the real situations. The onset of convection in a horizontal layer heated from below (Bénard problem) for a nanofluid was studied by Choi (1995), Tzou (2008), Alloui et al. (2011), Nield and Kuznetsov (2009, 2011), Wang and Tan (2011), Chand and Rana (2012) and Rana et al. (2014a, b). More realistic boundary conditions are used in this paper as discussed by Nield and Kuznetsov (2014). We assume that there is no flux at the plate and the nanoparticle flux value adjust accordingly. There is a need of changing the scale of dimensionless parameters. The basic solution of nanoparticle volume fraction is changed. The oscillatory convection does not exist and only stationary convection occurs. Shivakumara et al. (2013) studied the electrohydrodynamic instability of a rotating couple stress fluid and found that the rotating fluid layer becomes destabilizing in the presence of couple stress for all the boundary condions considered. Rana (2014) studied the thermal convection in couple-stress fluid in hydromagnetics saturating a porous medium and found that couple-stress parameter has stabilizing effect on the system.

Keeping in view of various applications of couple stress and nanofluid as mentioned above, our main aim in the present paper is to study the double-diffusive convection in a horizontal layer of couple-stress nanofluid.

2 Mathematical Model and Governing Equations

We consider an infinite horizontal layer of a couple stress elastico-viscous nanofluid of thickness *d*, bounded by the planes z = *0* and *z = d* as shown in Fig.1. The layer is heated and soluted from below, which is acted upon by a gravity force $\mathbf{g} = (0, 0, -g)$ aligned in the *z* direction. The temperature, *T*, concentration, *C* and the volumetric fraction of nanoparticles, φ, at the lower (upper) boundary is assumed to take constant values T_0, C_0 and φ_0 (T_1, C_1 and φ_1), respectively. We know that keeping a constant volume fraction of nanoparticles at the horizontal boundaries will be almost impossible in a realistic situation.

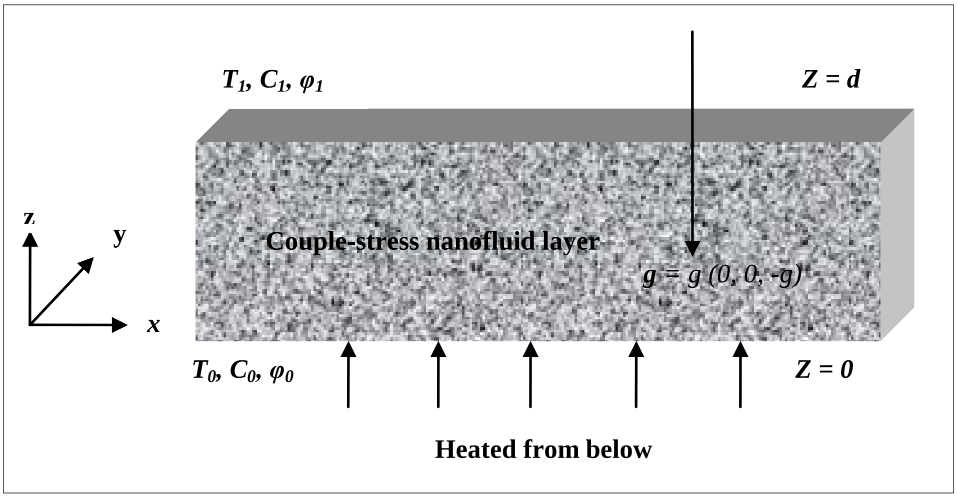

Fig.1. Physical configuration

By applying Boussinesq approximation, the equations of conservation of mass and momentum for couple-stress [Kuznetsov and Nield (2011), Shivakumara (2013), Chand and Rana (2012), Rana et al. (2014a, b)] nanofluid are

$$\nabla.\mathbf{q} = 0, \tag{1}$$

$$\rho\frac{d\mathbf{q}}{dt} = -\nabla p + \left(\mu - \mu_c\nabla^2\right)\nabla^2\mathbf{q} + g\left(\varphi\rho_p + \rho(1-\varphi)\{(1-\alpha_T(T-T_0)-\alpha_C(C-C_0))\}\right), \tag{2}$$

where ρ, μ, μ_c, p and $\mathbf{q}(u, v, w)$, denote respectively, the density, viscosity, the material constant responsible for couple stress property known as the couple stress viscosity, pressure, and Darcy velocity vector, where φ is the volume fraction of nano particles, ρ_p is the density of nanoparticles, α_T is the coefficient of thermal expansion and α_C is analogous to solute concentration.

The equation of conservation of mass for the nanoparticles [Buongiorno (2006)] is

$$\frac{d\varphi}{dt} = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T. \tag{3}$$

Let c, c_p , k and D_T be the fluid specific (at constant pressure), the heat capacity of the material constituting nanoparticles, the thermal conductivity and the diffusivity of Dufour type, respectively. Then the thermal energy equation for a nanofluid is

$$\rho c\frac{dT}{dt} = \mathrm{k}\nabla^2\mathrm{T} + \rho_p c_p\left(\mathrm{D_B}\nabla\varphi.\nabla\mathrm{T} + \frac{\mathrm{D_T}}{\mathrm{T}}\nabla\mathrm{T}.\nabla\mathrm{T}\right) + \rho c D_{TC}\nabla^2 C \tag{4}$$

The conservation equation for solute concentration [Nield and Kuznetsov (2011)] is

$$\frac{dC}{dt} = D_S \nabla^2 C + D_{CT} \nabla^2 T. \tag{5}$$

where D_S and D_{CT} are respectively, the solute diffusivity and diffusivity of Soret type.

Assuming the temperature to be constant and the thermophoretic nanoparticles flux to be zero at the boundaries [Nield and Kuznetsov (2014)]. Now the boundary conditions are

$$w = 0, \frac{\partial w}{\partial z} = 0, T = T_0, C = C_0, D_B \frac{\partial \varphi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0, \tag{6}$$

$$w = 0, T = T_1, C = C_1, \frac{\partial w}{\partial z} = 0, D_B \frac{\partial \varphi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = d. \tag{7}$$

We introduce non-dimensional variables as

$$(x', y', z',) = \left(\frac{x, y, z}{d}\right), \ (u', v', w',) = \left(\frac{u, v, w}{\kappa_f}\right)d, \ t' = \frac{t\kappa_f}{d^2}, \ p' = \frac{pd^2}{\mu\kappa_f}, \ \varphi' = \frac{(\varphi - \varphi_0)}{\varphi_0},$$

$$T' = \frac{(T - T_1)}{(T_0 - T_1)}, \ C' = \frac{(C - C_1)}{(C_0 - C_1)}, \tag{8}$$

where $\kappa_f = \frac{k}{(\rho c)_f}$ is the thermal diffusivity of the fluid. Thereafter dropping the dashes (') for convenience.

Equations (1) - (8) in non-dimensional form can be written as

$$\nabla.\mathbf{q} = 0, \tag{9}$$

$$\frac{1}{Pr}\left(\frac{\partial \mathbf{q}}{\partial t} + \mathbf{q}.\nabla\mathbf{q}\right) = -\nabla p + (1 - \eta\nabla^2)\nabla^2\mathbf{q} - Rm\,\hat{e}_z + RaT\,\hat{e}_z + \frac{Rs}{Le}C\hat{e}_z - Rn\varphi\,\hat{e}_z, \tag{10}$$

$$\frac{\partial \varphi}{\partial t} + \mathbf{q}.\nabla\varphi = \frac{1}{Ln}\nabla^2\varphi + \frac{N_A}{Ln}\nabla^2 T, \tag{11}$$

$$\frac{\partial T}{\partial t} + \mathbf{q}.\nabla T = \nabla^2 T + \frac{N_B}{Ln}\nabla\varphi.\nabla T + \frac{N_A N_B}{Ln}\nabla T.\nabla T + N_{TC}\nabla^2 C, \tag{12}$$

$$\frac{\partial C}{\partial t} + \mathbf{q}.\nabla C = \frac{1}{Le}\nabla^2\varphi + N_{CT}\nabla^2 T, \tag{13}$$

$$w = 0, \frac{\partial w}{\partial z} = 0, T = 1, C = 1, \frac{\partial \varphi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0, \tag{14}$$

$$w = 0, \frac{\partial w}{\partial z} = 0, T = 0, C = 0, \frac{\partial \varphi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 1. \tag{15}$$

where we have dimensionless parameters as:

$$Le = \frac{\kappa_f}{D_S}; \quad Ln = \frac{\kappa_f}{D_B}; \quad \eta = \frac{\mu_c\kappa_f}{\mu d^2}; \quad Ra = \frac{g\rho\alpha_T d^3(T_0 - T_1)}{\mu\kappa_f}; \ Rs = \frac{g\rho\alpha_C d^3(C_0 - C_1)}{\mu D_S};$$

$$Rm = \frac{\rho_p\varphi_0 + \rho(1 - \varphi_0)gd^3}{\mu\kappa_f}; \quad Rn = \frac{(\rho_p - \rho)\varphi_0 g d^3}{\mu\kappa_f}; \quad N_A = \frac{D_T(T_0 - T_1)}{D_B T_1\varphi_0}; \quad N_B = \frac{(\rho c)_p}{(\rho c)_f}\varphi_0;$$

$$N_{TC} = \frac{D_{TC}(C_0 - C_1)}{\kappa_f(T_0 - T_1)}; \quad N_{CT} = \frac{D_{CT}(T_0 - T_1)}{\kappa_f(C_0 - C_1)};$$

Here the parameter Le is a thermo-solutal Lewis number, Ln is a thermo-nanofluid Lewis number, η is the couple-stress parameter, *Ra* is the thermal Rayleigh Number, Rs is the solutal-Rayleigh Number, *Rm* Density Rayleigh number, *Rn* is the nanoparticle Rayleigh number, N_A is the modified diffusivity ratio, N_B is the modified particle-density ratio, D_B is the diffusion constant of nanoparticles, N_{TC} is the Dufour parameter and N_{CT} is the Soret parameter.

2.1 Basic Solutions

The time-independent quiescent solution of equations (11) – (15) with temperature, concentration and nanoparticle volume fraction varying in the z-direction only [Nield and Kuznetsov (2011, 2014)], and Sheu (2011) is

$$u = v = w = 0,\; p = p(z), \mathrm{C} = \mathrm{C_b}(\mathrm{z}), T = T_b(z), \varphi = \varphi_b(z). \tag{16}$$

Therefore, equations (10) – (13) reduce to

$$0 = -\frac{dp_b(z)}{dz} - Rm + R_D T_b(z) + \frac{Rs}{Le} C_b(z) - Rn\varphi_b(z), \tag{17}$$

$$\frac{d^2\varphi_b(z)}{dz^2} + N_A \frac{d^2T_b(z)}{dz^2} = 0, \tag{18}$$

$$\frac{d^2T_b(z)}{dz^2} + \frac{N_B}{Ln}\frac{d\varphi_b(z)}{dz}\frac{dT_b(z)}{dz} + \frac{N_A N_B}{Le}\left(\frac{dT_b(z)}{dz}\right)^2 + N_{TC}\frac{d^2C_b(z)}{dz^2} = 0, \tag{19}$$

$$\frac{1}{Le}\frac{d^2C_b(z)}{dz^2} + N_{CT}\frac{d^2T_b(z)}{dz^2} = 0, \tag{20}$$

Using boundary conditions (14) and (15), the solution of equation (18) is given by

$$\frac{d\varphi_b(z)}{dz} + N_A \frac{dT_b(z)}{dz} = 0, \tag{21}$$

On substituting this value in equation (19), we get

$$\frac{d^2T_b(z)}{dz^2} + N_{TC}\frac{d^2C_b(z)}{dz^2} = 0, \tag{22}$$

On solving equations (18)-(20) by applying boundary conditions (14) and (15), we obtain

$$T_b = 1 - z, C_b = 1 - \mathrm{z} \text{ and } \varphi_b = \varphi_0 + N_A z. \tag{23}$$

It is observed that the couple stress parameter has no effect on the basic solution. These results are identical with the results obtained by Nield and Kuznetsov (2014).

2.2 Perturbation Solutions

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state. We write

$$\mathbf{q}(u,v,w) = q''(u,v,w), T = T_b + T'', \mathrm{C} = C_b + C'', \varphi = \varphi_b + \varphi'', p = p_b + p''. \tag{24}$$

Using equations given in (24) into equations (9) – (15), linearizing the resulting equations by neglecting nonlinear terms that are product of prime quantities and dropping the primes (") for convenience, we obtained

$$\nabla.\mathbf{q} = 0, \tag{25}$$

$$\frac{1}{\Pr}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \left(1 - \eta\nabla^2\right)\nabla^2\mathbf{q} + \mathrm{RaT}\,\hat{e}_z + \frac{Rs}{Le}C\hat{e}_z - \mathrm{Rn}\varphi\,\hat{e}_z, \tag{26}$$

$$\frac{\partial\varphi}{\partial t} + w = \frac{1}{Ln}\nabla^2\varphi + \frac{N_A}{Ln}\nabla^2 T, \tag{27}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{Ln}\left(\frac{\partial T}{\partial z} - \frac{\partial\varphi}{\partial z}\right) - \frac{2\mathrm{N}_A N_B}{Ln}\frac{\partial T}{\partial z} + \mathrm{N_{TC}}\nabla^2\mathrm{C}. \tag{28}$$

$$\frac{\partial C}{\partial t} - w = \frac{1}{Le}\nabla^2 C + N_{CT}\nabla^2 T. \tag{29}$$

$$w=0, T=0, C=0, \frac{\partial\varphi}{\partial z}+N_A\frac{\partial T}{\partial z}=0 \quad \text{at } z=0 \text{ and at } z=1. \tag{30}$$

Note that as the parameter *Rm* is not involved in equations (25)-(30) it is just a measure of the basic static pressure gradient.
The seven unknowns *u, v, w, p, T, C* and *φ* can be reduced to four by operating equation (26) with $\hat{e}_z \cdot curl\, curl$ and using the identity $curl\, curl \equiv grad\, div - \nabla^2$ together with equation (25) which yields

$$\left(\frac{1}{\Pr}\frac{\partial}{\partial t}-\nabla^2 w+\eta\nabla^4 w\right)\nabla^2 w = Ra\nabla_H^2 T+\frac{Rs}{Le}C\nabla_H^2 - Rn\nabla_H^2\varphi, \tag{31}$$

where ∇_H^2 is the two-dimensional Laplace operator on the horizontal plane, that is $\nabla_H^2 = \frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}$.

3 Normal Modes Analysis Method

Express the disturbances into normal modes of the form

$$[w, T, C, \varphi]=[W(z), \Theta(z), \Gamma(z), \Phi(z)]\exp(irx+isy+pt), \tag{32}$$

where r, s are the wave numbers in the *x* and *y* direction, respectively, and *p* is the growth rate of the disturbances.
Substituting equation (32) into equations (31) and (27)-(30), we obtain the following boundary-value problem

$$\left(\frac{p}{\Pr}-\left(D^2-\omega^2\right)+\eta\left(D^2-a^2\right)^2\right)\left(D^2-a^2\right)W+\omega^2 Ra\Theta+\frac{Rs}{Le}\omega^2\Gamma-\omega^2 Rn\Phi=0, \tag{33}$$

$$W+N_{CT}\left(D^2-\omega^2\right)\Theta+\frac{1}{Le}\left(D^2-\omega^2-p\right)\Gamma=0, \tag{34}$$

$$W+\left(D^2+\frac{N_B}{Ln}D-\frac{2N_AN_B}{Ln}D-\omega^2-p\right)\Theta-\frac{N_B}{Ln}D\Phi+N_{TC}\left(D^2-\omega^2\right)\Gamma=0, \tag{35}$$

$$W-\frac{N_A}{Ln}\left(D^2-\omega^2\right)\Theta-\left(\frac{1}{Ln}\left(D^2-\omega^2\right)-p\right)\Phi=0, \tag{36}$$

$$w=0, T=0, C=0, D\varphi+N_A D\Theta=0 \quad \text{at } z=0 \text{ and at } z=1. \tag{37}$$

where $D=\frac{d}{dz}$ and $\omega^2 = r^2+s^2$ is the dimensionless horizontal wave number.
Considering solutions W, Θ, Γ and Φ of the form

$$W=W_0\sin(\pi z), \Theta=\Theta_0\sin(\pi z), \Gamma=\Gamma_0\sin(\pi z), \Phi=\Phi_0\sin(\pi z). \tag{38}$$

Substituting (38) into equations (33) – (36) and integrating each equation from $z = 0$ to $z = 1$, we obtain the following matrix equations

$$\begin{bmatrix} \left(J^2+\frac{p}{\Pr}+\eta J^4\right)J^2 & -\omega^2 Ra & -\frac{Rs}{Le}\omega^2 & \omega^2 Rn \\ -1 & J^2 N_{CT} & \frac{J^2}{Le}+\frac{p}{Le} & 0 \\ -1 & J^2+p & -J^2N_{TC} & 0 \\ 1 & \frac{N_A}{Ln}J^2 & 0 & \frac{J^2}{Ln}+p \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \\ \Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{39}$$

where $J^2 = \pi^2 + a^2$ is the total wave number.
The linear system (37) has a non-trivial solution if and only if

$$Ra = \frac{1}{J^2 + p + N_{TC} J^2 Le} \times \left\{ \begin{array}{l} \frac{1}{\omega^2}\left(J^2 + \frac{p}{\Pr} + \eta J^4 \right) J^2 \left[\left(J^2 + p \right)^2 + LeN_{TC} N_{CT} J^4 \right] \\ + \mathrm{Rs}\left[\mathrm{N}_{CT} J^2 - \left(J^2 + \omega \right) \right] - \frac{\mathrm{Rn}}{J^2 + pLn} \times \\ \left(J^2 + p \right)\left(\left(\left(J^2 + p \right) Ln + N_A J^2 \right) + \mathrm{N}_{TC} J^2 Le \left(\mathrm{N}_A + LnN_{CT} \right) \right) \end{array} \right\}. \tag{40}$$

Equation (38) is the dispersion relation representing the effect of medium porosity, thermo-solutal Lewis number, thermo-nanofluid Lewis number, solutal Rayleigh Number, nanoparticle Rayleigh number, kinematic visco-elasticity parameter, modified diffusivity ratio, Soret and Dufour parameter on double-diffusive convection in a layer of couple stress nanofluid saturating a porous medium.

4 The Stationary Convection

Due to the absence of opposing buoyancy forces, the oscillatory convection does not exist. So we consider only the case of stationary convection. For stationary convection, putting p = 0 in equation (40) reduces it to

$$Ra = \frac{1}{1 + N_{TC} Le} \times \left\{ \begin{array}{l} \frac{1}{\omega^2}\left(\pi^2 + \omega^2 \right)^3 \left(1 + \eta\left(\pi^2 + \omega^2 \right) \right)\left(1 + LeN_{TC} N_{CT} \right) + Rs\left(N_{CT} - 1 \right) \\ - Rn\left[Ln + N_A + \mathrm{N}_{TC} Le\left(\mathrm{N}_A + LnN_{CT} \right) \right] \end{array} \right\}. \tag{41}$$

Equation (41) represents the thermal Rayleigh number as a function of the non-dimensional wave number ω corresponding to the parameters F, N_{TC}, N_{CT}, Rs, Ln Rn, Le, N_A. Since Pr vanishes with p, thus, the stationary convection does not depend on the value of Prandtl number Pr. Equation (41) is identical to that obtained by Nield and Kuznetsov (2011), Chand and Rana (2012) and Rana et al. (2014a, b). Also in equation (37) the particle increment parameter N_B does not appear and the diffusivity ratio parameter N_A appears only in association with the nanoparticle Rayleigh number Rn. This implies that the nanofluid cross-diffusion terms approach to be dominated by the regular cross-diffusion term.

In the absence of the Dufour and Soret parameters N_{TC} and N_{CT} equation (39) reduces to

$$Ra = \frac{1}{\omega^2}\left(\pi^2 + \omega^2 \right)^3 \left(1 + \eta\left(\pi^2 + \omega^2 \right) \right) - Rs - \left(Ln + N_A \right) Rn, \tag{41a}$$

which is identical with the result derived by Kuznetsov and Nield and Rana et al. In the absence of the stable solute gradient parameter Rs, equation (40) reduces to

$$Ra = \frac{1}{\omega^2}\left(\pi^2 + \omega^2 \right)^3 \left(1 + \eta\left(\pi^2 + \omega^2 \right) \right) - \left(Ln + N_A \right) Rn, \tag{42}$$

Equation (42) is identical with the results derived by Sheu (2011), Chand and Rana (2012) and Rana et al. (2014a, b).

The critical cell size at the onset of instability is obtained by minimizing Ra with respect to a. Thus, the critical cell size must satisfy

$$\left(\frac{\partial Ra}{\partial \omega} \right)_{\omega = \omega_c} = 0,$$

Equation (39) which gives

$$\omega_c = \frac{\pi}{\sqrt{2}} \cong 2.2223. \tag{43}$$

5 Results and Discussions

The dispersion relation (41) is analyzed numerically and graphs have been plotted to depict the stability characteristics. According to the definition of nanoparticle Rayleigh number Rn, this corresponds to negative value of Rn for heavy nanoparticles $(\rho_p > \rho)$. In the following discussion, negative values of Rn (indicates a bottom heavy case) are presented.

The variations of thermal Rayleigh number Ra with the wave number ω for different values of the couple stress parameter $\eta = 0.2$, $\eta = 0.4$ and $\eta = 0.6$ is plotted in Fig. 2 and it is noticed that the thermal Rayleigh number Ra

increases with the increase of couple stress parameter. Thus couple stress parameter stabilizes the stationary convection.

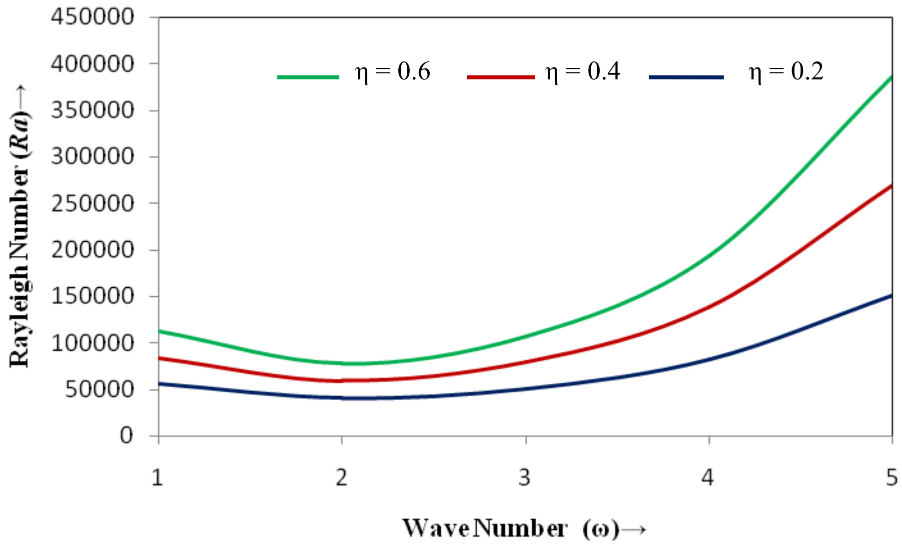

Fig. 2. The variations of Rayleigh number *Ra* with the wave number ω for different values of the couple stress parameter η = 0.2, η = 0.4 and η = 0.6

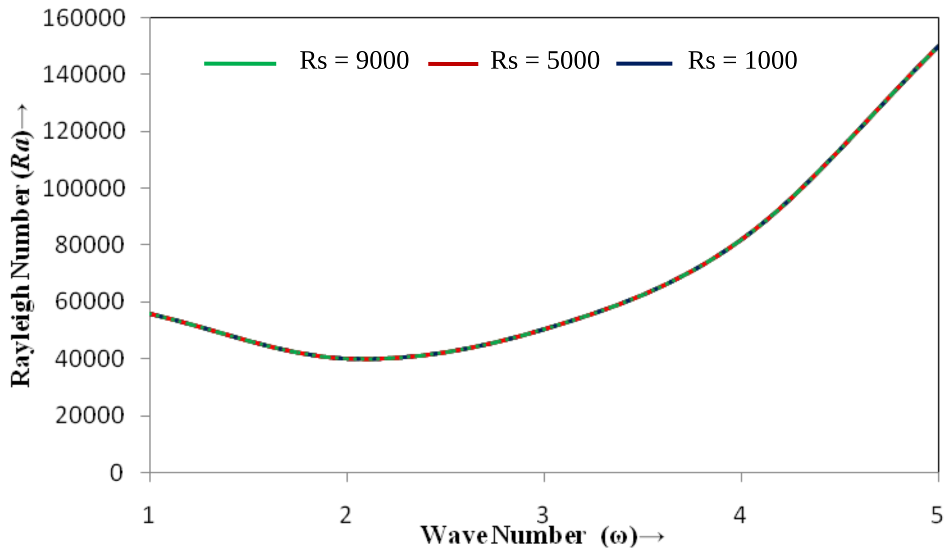

Fig. 3. The variations of Rayleigh number *Ra* with the wave number a for different values of the solute concentration Rs = 1000, Rs = 5000 and Rs = 9000

In Fig. 3, the variations of thermal Rayleigh number *Ra* with the wave number ω for three different values of the solutal-Rayleigh number, namely, *Rs = 1000, 5000* and 9*000* is plotted and it is observed that the thermal Rayleigh number slightly increases with the increase in solutal Rayleigh number so the solutal Rayleigh number stabilizes the system slightly. In Fig. 4, the variations of thermal Rayleigh number Ra with the wave number a for three different values of the thermo-nanofluid Lewis number, namely, *Ln = 300, 600* and *900* which shows that thermal Rayleigh number increases with the increase in thermo-nanofluid Lewis number. Thus thermo-nanofluid Lewis number has stabilizing effect on the system.

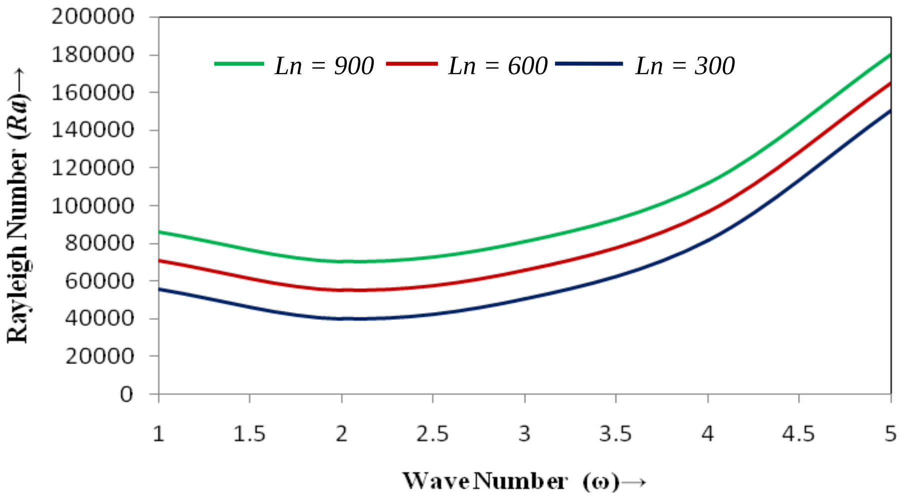

Fig. 4. The variations of Rayleigh number *Ra* with the wave number a for different values of the thermo-nanofluid Lewis number *Ln* = 300, *Ln* = 600 and *Ln* = 900

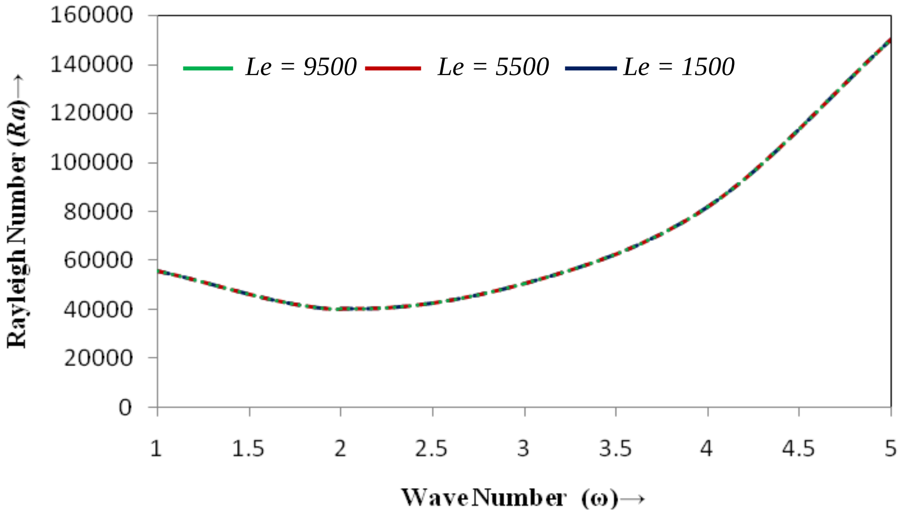

Fig. 5. The variations of Rayleigh number *Ra* with the wave number a for different values of the thermosolutal Lewis number *Le* = 1500, *Le* = 5500 and *Le* = 9500

The variations of thermal Rayleigh number *Ra* with the wave number ω for three different values of the thermosolutal Lewis number, namely, *Le = 1500*, *5500* and *9000* is plotted in Fig. 5 and it is noticed that thermal Rayleigh- Darcy number increases slightly with the increase in thermosolutal Lewis number so the thermosolutal Lewis number has slight stabilizing effect on the system.

In Fig. 6, the variations of thermal Rayleigh number *Ra* with the wave number ω for three different values of the Soret parameter, namely N_{CT} = 10, 20, 30 which shows that thermal Rayleigh number increases with the increase in Soret parameter. Thus Soret parameter has stabilizing effect on the system.

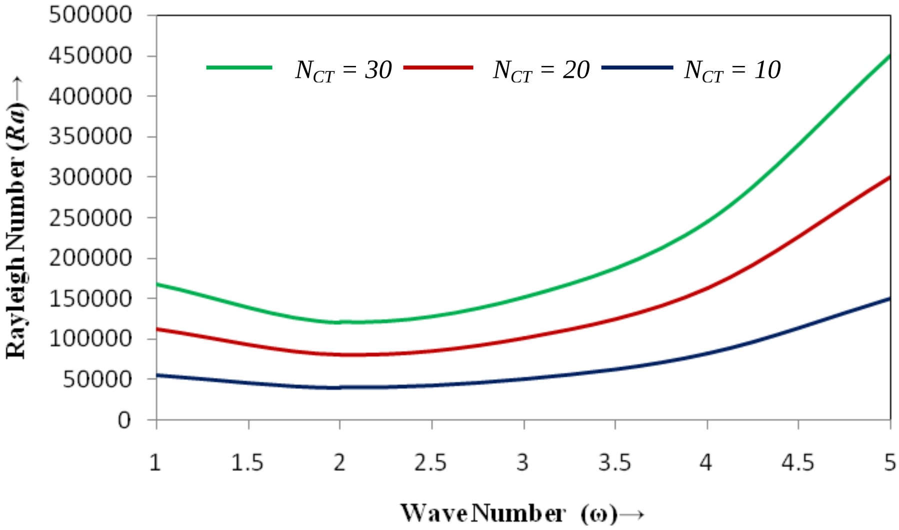

Fig. 6. The variations of Rayleigh number Ra with the wave number a for different values of the Soret parameter $N_{CT} = 10$, $N_{CT} = 20$ and $N_{CT} = 30$

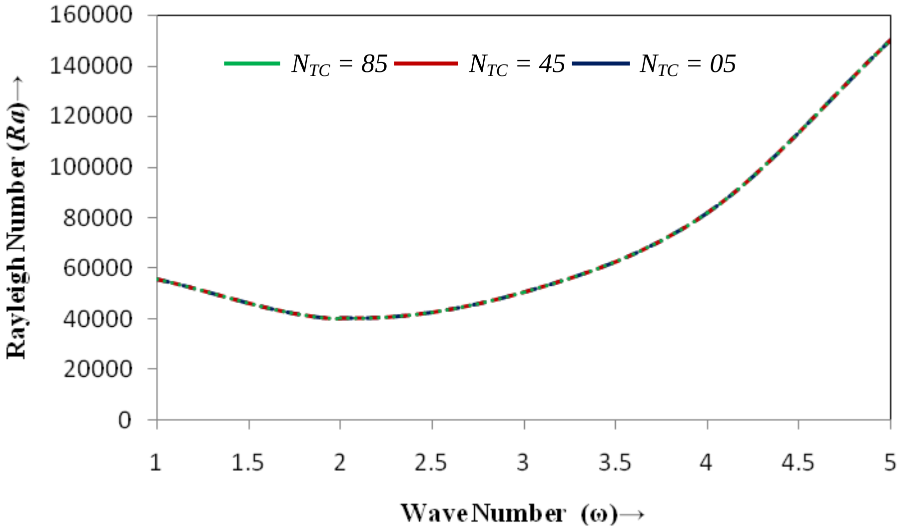

Fig. 7. The variations of Rayleigh number Ra with the wave number a for different values of the Dufour parameter $N_{TC} = 5$, $N_{TC} = 45$ and $N_{TC} = 85$

The variations of thermal Rayleigh number Ra with the wave number ω for three different values of Dufour parameter, namely $N_{TC} = 5, 45$ and 85 is plotted in Fig. 7 and it is observed that thermal Rayleigh number increases with the increase in Dufour parameter so the Dufour parameter has stabilizing effect on the onset of stationary convection in a layer of couple stress nanofluid. The system becomes more stable when the values of Soret and Dufour parameters are equal. The results obtained in figures 2 to 7 are in good agreement with the result obtained by Nield and kuznetsov (2011, 2014) Chand and Rana (2012), Rana et al. (2014a, b), and Sheu (2011).

6 Conclusions

The onset of double-diffusive convection in a layer of couple stress Nanofluid in a more realistic boundary conditions has been investigated which comprises the effects of thermophoresis and Brownian motion. We have assumed that there is no flux at the boundary and the nanoparticle flux value adjust accordingly. It is found that the couple stress parameter has no effect on the basic solution. The couple stress parameter, solutal Rayleigh Number, thermo-nanofluid Lewis number, thermosolutal Lewis number, Soret parameter and Dufour parameter have stabilizing effects on the stationary convection as shown in figures 2, 3, 4, 5, 6 and 7 respectively. Oscillatory convection does not exist under the more realistic boundary conditions.

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References

Alloui, Z.; Vasseur, P.; Reggio, M.: Natural convection of nanofluids in a shallow cavity heated from below. *Int. J. of Thermal Science*, 50, (2011), 385-393.

J. Buongiorno: Convective transport in nanofluids. *ASME J. of Heat Transfer*, 128, (2006), 240-250.

R. Chand, G. C. Rana: Dufour and Soret effects on the thermosolutal instability of Rivlin-Ericksen elastico-viscous fluid in porous medium. *Zeitschrift für Naturforschung*, A 67a, (2012), 685 – 691.

S. Choi: Enhancing thermal conductivity of f luids with nanoparticles. In: D. A. Siginer and H. P. Wang (Eds), Developments and Applications of Non-Newtonian Flows, *ASME FED*, 231/MD, 66, (1995), 99-105.

D. A. Nield, A. V. Kuznetsov: Thermal instability in a porous medium layer saturated by a nanofluid. *Int. J. Heat Mass Transf.*, 52, (2009), 5796–5801.

D. A. Nield, A. V. Kuznetsov: The onset of double-diffusive convection in a nanofluid layer. *Int. J. of Heat and Fluid Flow*, 32, (2011), 771-776.

D. A. Nield, A. V. Kuznetsov: Thermal instability in a porous medium layer saturated by a nanofluid: A revised mode. *Int. J. heat Mass Transfer*, 68, (2014), 211-214.

G. C. Rana: The onset of thermal convection in couple-stress fluid in hydromagnetics saturating a porous medium. *Bulletin of the Polish Academy of sciences-Technical sciences*, 62, (2014), 357-362.

G. C. Rana, R .C. Thakur and S. K. Kango: On the onset of thermosolutal instability in a layer of an elastico-viscous nanofluid in porous medium. *FME Transactions*, 42, (2014), 1-9.

G. C. Rana, R. C. Thakur and S. K. Kango: On the onset of double-diffusive convection in a layer of nanofluid under rotation saturating a porous medium. *Journal of Porous Media*, 17, (2014), 657–667.

R. C. Sharma, K. D. Thakur: Couple-stress fluid heated from below in hydromagnetics. Czech. J. Phys., 50, (2000), 753-758.

L. J. Sheu: Thermal instability in a porous medium layer saturated with a viscoelastic nanofluid. *Transp Porous Med*. 88, (2011), 461-477.

I. S. Shivakumara, M. Akkanagamma, Chiu-On Ng: Electrohydrodynamic instability of a rotating couple stress dielectric fluid layer. *Int. J. Heat and Mass Transfer*, 62, (2013), 761-771.

V. K. Stokes: Couple-stress in fluids. Phys. Fluids, 9, (1966), 1709-1715.

D. Y. Tzou: Thermal instability of nanofluids in natural convection. *Int. J. of Heat and Mass Transfer*, 51, (2008), 2967-2979.

E. Walicki, A. Walicka: Inertial effect in the squeeze film of couple-stress fluids in biological bearings. *Int. J. Appl. Mech. Engg.*, 4, (1999), 363-373.

S. Wang, W. Tan: Stability analysis of double-diffusive convection of Maxwell fluid in a porous medium studied. *Int. J. of Heat and Fluid Flow*, 33, (2011), 88-94.

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