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Dynamic Response of Heterogeneity and Reinforcement on the Propagation of Torsional Surface Waves

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This paper aims to investigate the effect of reinforcement and heterogeneity on the propagation of torsional surface waves. Geometry of the problem is consists of heterogeneous fibre-reinforced layer lying over a heterogeneous isotropic half-space. Heterogeneity in the layer is caused due to exponential variation of elastic parameters whereas quadratic variation in elastic parameters is considered for half-space. Dispersion relation for torsional surface waves has been obtained and matched with classical Love wave equation by taking an isotropic homogeneous layer lying over an isotropic homogeneous half-space. Some existing results have been deduced as particular case of the present study. Velocity profile of surface waves is compared for both, reinforced and reinforced free cases. Numerical examples have been discussed by taking steel fibre-reinforced material. Graphical representation has been made to exhibit the findings.

Nomenclature

τ_{ij}, Components of stress;
δ_{ij}, Kronecker delta;
e_{ij}, Components of strain;
α, Γ, λ, Elastic constants for fibre-reinforced material;
μ_L, Shear modulus in longitudinal direction;
μ_T, Shear modulus in transverse direction;
ξ, ζ, Heterogeneity parameters for the layer;
b_1, d_1, Heterogeneity parameters for the half-space;
v_1, v_2, Displacement of the wave;
$\frac{c}{\beta_1}$ Phase velocity of the wave in fibre-reinforced layer;

1 Introduction

The study of mechanical behavior of a self-reinforced material has great importance in geomechanics. Many elastic fibre-reinforced composite materials are strongly anisotropic in behavior. It is desirable to study the surface wave propagation in anisotropic media, as the propagation of elastic waves in anisotropic media is fundamentally different from their propagation in isotropic media. As the Earth's crust and mantle are not homogeneous, it is also interesting to know the propagation pattern of surface waves in heterogeneous medium. As a result of stress, developed within the earth crust, deformation takes place along with fracture. These fracture release large amount of energy which gives rise to elastic waves travelling through the materials beneath the earth surface and finally reaches to the surface. During the progress of the waves, different layers of different materials come in the way and characteristics of different wave fields are influenced by the elastic properties of the media through which they travel (Achenbach (1984)). The Earth's crust contains some hard and soft rocks or materials that may exhibit self-reinforcement property, and inhomogeneity is trivial characteristic of the Earth. These facts motivate us towards this study.

The idea of introducing a continuous reinforcement at every point of an elastic solid was given by Belfield et al. (1983). Later, Verma and Rana (1983) applied this model to the rotation of tube, illustrating its utility in strengthening the lateral surface of the tube. Verma (1986) also discussed the propagation of magneto-elastic shear waves in self-reinforced bodies. The problem of magneto-elastic transverse surface waves in self-reinforced elastic solids was studied by Verma et al. (1988). Chattopadhyay and Chaudhury (1990) studied the propagation, reflection and transmission of magneto-elastic shear waves in a self-reinforced elastic medium. Chattopadhyay and Chaudhury (1995) studied the propagation of magneto-elastic shear waves in an infinite self-reinforced plate. The materials, such as resin (reinforced by strong aligned fibres), exhibit highly anisotropic behavior in certain cases is discussed by Spencer (1974). Crampin (1987) proposed the theory of earthquake prediction using shear wave splitting measurements.

Some notable efforts have been made to discuss the propagation of surface waves in elastic medium with or without heterogeneity. The propagation of torsional surface waves in heterogeneous elastic media has been discussed by Vardoulakis (1984). Sengupta and Nath (2001) studied the propagation of surface waves in fibre-reinforced anisotropic elastic solid media leading to particular cases such as Rayleigh waves, Love waves and Stoneley waves. Some possible applications of surface wave theory have made the study useful to geoscientists and geophysicists is discussed by Chammas et al. (2003). Due to such applications the study of seismic wave behavior in reinforced medium has got remarkable attention in recent past by Chattopadhyay et al. (2009), Kumar and Gupta (2010), Sethi et al. (2012), Chattopadhyay et al. (2012), Abd-Alla et al. (2013), Chattopadhyay et al. (2014), and Kundu et al. (2014a,b).

Because of heterogeneous layered nature of Earth, it can be regarded as composed of different heterogeneous layers with certain variation in rigidity and density. In all technological applications, the problem on heterogeneity can be considered in unbounded space with variation in rigidity and density below the crust area can be approximated linearly (Meissner (2002), pp 33) with certain discontinuous jump. This is due not only to relatively small dimensions of the heterogeneity but also to the fact that the knowledge of the stress-strain state near the edge of the heterogeneity (which is a stress concentrator) is of special interest. But this situation is often seriously changed in Earth sciences: heterogeneity in the Earth crust may be rather close to the surface so that it is impossible to neglect the influence of the surface. Moreover, in many cases, the medium behavior just on the surface and at large distances (at dozens of the specific dimensions of the heterogeneity) is of special importance. For this reason, the problem on heterogeneity in a layered media over a half-space is also of interest in applications. It is the purpose of this contribution to discuss the qualitative features of the more generalized problem, together with some numerical results for dispersion of torsional wave under the assumed medium.

The study of surface wave in a half-space has their possible application in geophysical prospecting and in understanding the cause and estimation of damage due to earthquake. The present analytical study aims to investigate the effect of reinforcement and heterogeneity on the propagation of torsional surface waves. Solution part of the problem includes the use of Whittaker's function and the method of separation of variables. It is found that heterogeneity and reinforcement of the medium have significant influence on phase velocity of torsional surface waves.

2 Definition of the Problem

When the wave propagate through the Earth's interior different types of layers comes in the way of wave motion. The heterogeneity present in these layers affects the wave motion. Heterogeneity and reinforcement are one of the most important factors which influence the wave propagation. So the study of wave propagation through a heterogeneous layer got a remarkable attention. The procedure to study the wave propagation through a heterogeneous fiber-reinforced layer lying over a heterogeneous half-space is explained as follows.

i. The stresses present in the heterogeneous fiber-reinforced layer are calculated using the constitutive equations given by Belfield et al. (1983). Using variable separable method and stress equations, we obtain the displacement due to torsional surface waves in a fiber-reinforced heterogeneous layer.

ii. The displacement due to torsional wave in heterogeneous elastic half-space is obtained in terms of Whittaker and Bessel functions.

iii. Using the boundary conditions, we obtain the dispersion relation in closed form.

iv. Numerical examples are considered to compute the effect of heterogeneity and reinforcement on the propagation of torsional surface wave. Three different types of fiber-reinforced elastic materials are chosen to compare the effect of heterogeneity and reinforcement on torsional wave propagation.

3 Formulation and Solution for the Layer

We consider the cylindrical co-ordinate system, in which the origin is taken at the initial point of the interface between layer and half-space. The direction of wave propagation is along the r axis. The z axis is taken as positive downwards. The constitutive equation for self-reinforced linear elastic material with direction $\vec{a}$ is given by Belfield et al. (1983).

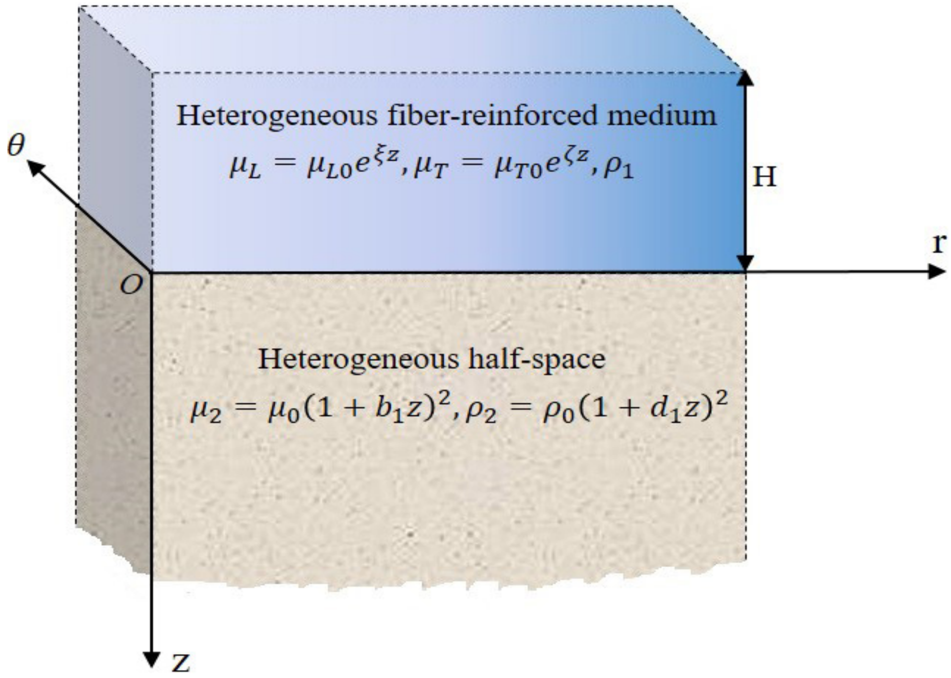

Figure 1: Geometry of the problem

$$\begin{aligned}\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j) \\ +2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \Gamma a_k a_m e_{km} a_i a_j,\end{aligned} \tag{1}$$

where τ_{ij} are the components of stress, δ_{ij} are correlated to the Kronecker delta and e_{ij} are the components of infinitesimal strain. a_i defines the components of $\vec{a}$ in different direction whereas the indices take the values $1, 2, 3$ and summation convention is employed. $\alpha, \Gamma, \lambda, \mu_L, \mu_T$ are elastic constants. μ_T is assumed to be the shear modulus in transverse direction and μ_L the shear modulus in longitudinal direction. Different homogenization methods may also be used to determine the elastic coefficients in different fibre-reinforced materials. The asymptotic homogenization method to determine the analytical formulas for the elastic effective coefficients of a two phase fibrous composite material and the full-wave homogenization method for the calculation of the effective permittivity of steel fibre-reinforced concrete are given by Diaz et al. (2002) and Damme and Franchois (2006), respectively. In Eq.(1) the unit vector $\vec{a} = (a_1, a_2, a_3)$ gives the orientation of the family of fibres in axial (z), azimuthal (θ) and radial (r) directions, respectively. Choosing the component $a_2 = 0$ gives the orientation of our choice. The fibres initially lie in the surface for some constant value of θ and are inclined at an angle φ to the r axis. so the components of $\vec{a}$ in cylindrical polar co-ordinate system are $\vec{a} = (\sin\varphi, 0, \cos\varphi)$. Now, for the propagation of torsional waves in radial direction and causing displacement in azimuthal direction only, we have the following displacement components

$$u_r = 0, \qquad u_z = 0, \qquad u_\theta = v_1(r, z, t) \tag{2}$$

which gives

$$e_{rr} = 0, \quad e_{\theta\theta} = 0, \quad e_{zz} = 0, \quad e_{zr} = 0, \quad 2e_{\theta z} = \frac{\partial v_1}{\partial z}, \quad 2e_{r\theta} = \frac{\partial v_1}{\partial r} - \frac{v_1}{r}. \tag{3}$$

By using Eqs.(2) and (3) in Eq.(1), we have the following stress components

$$\tau_{\theta z} = R\frac{\partial v_1}{\partial z} + Q(\frac{\partial v_1}{\partial r} - \frac{v_1}{r}), \tag{4}$$

and

$$\tau_{r\theta} = P(\frac{\partial v_1}{\partial r} - \frac{v_1}{r}) + Q\frac{\partial v_1}{\partial z}, \tag{5}$$

where

$$\begin{aligned} P &= \mu_{T0}e^{\zeta z} + (\mu_{L0}e^{\xi z} - \mu_{T0}e^{\zeta z})a_1^2, \\ Q &= (\mu_{L0}e^{\xi z} - \mu_{T0}e^{\zeta z})a_1 a_3, \\ R &= \mu_{T0}e^{\zeta z} + (\mu_{L0}e^{\xi z} - \mu_{T0}e^{\zeta z})a_3^2. \end{aligned}$$

where μ_{L0} and μ_{T0} are shear modulus in longitudinal and transverse direction, respectively.
The governing equation of motion is given by

$$\tau_{ij,j} = \rho_1\frac{\partial^2 u_i}{\partial t^2}. \tag{6}$$

By the characteristic of torsional surface waves, we have the only non vanishing equation of motion as

$$\frac{\partial}{\partial r}\tau_{r\theta} + \frac{\partial}{\partial z}\tau_{\theta z} + \frac{2}{r}\tau_{r\theta} = \rho_1\frac{\partial^2 v_1}{\partial t^2}. \tag{7}$$

Eqs.(4),(5) and (7) gives

$$\begin{aligned} P\frac{\partial^2 v_1}{\partial r^2} + 2Q\frac{\partial^2 v_1}{\partial r\partial z} + R\frac{\partial^2 v_1}{\partial z^2} + \frac{P}{r}\frac{\partial v_1}{\partial r} + \frac{Q}{r}\frac{\partial v_1}{\partial z} - \frac{P}{r^2}v_1 \\ + \dot{R}\frac{\partial v_1}{\partial z} + \dot{Q}\left(\frac{\partial v_1}{\partial r} - \frac{v_1}{r}\right) = \rho_1\frac{\partial^2 v_1}{\partial t^2} \end{aligned} \tag{8}$$

where

$$\begin{aligned} \dot{R} &= \frac{\partial R}{\partial z} = \zeta\mu_{T_0}e^{\zeta z} + (\xi\mu_{L_0}e^{\xi z} - \zeta\mu_{T_0}e^{\zeta z})a_3^2, \\ \dot{Q} &= \frac{\partial Q}{\partial z} = (\xi\mu_{L_0}e^{\xi z} - \zeta\mu_{T_0}e^{\zeta z})a_1 a_3. \end{aligned}$$

A harmonic wave in radial direction can be written in the form

$$v_1(z) = V_1(z)J_1(kr)e^{i\omega t} \tag{9}$$

where $k, c, \omega(= kc)$ and J_1 are wave number, wave velocity, circular frequency and Bessel's function of first order and first kind, respectively.
Using Eq.(9) in Eq.(8), we have

$$\frac{d^2 V_1}{dz^2} + L\frac{dV_1}{dz} + MV_1 = 0 \tag{10}$$

where the expressions for L and M are given as

$$\begin{aligned} L &= \frac{2QkJ_1'(kr)}{RJ_1(kr)} + \frac{Q}{Rr} + \frac{\dot{R}}{R}, \\ M &= \frac{Pk^2J_1''(kr)}{RJ_1(kr)} + \frac{PkJ_1'(kr)}{RrJ_1(kr)} + \frac{\rho\omega^2}{R} - \frac{P}{Rr^2} + \frac{\dot{Q}kJ_1'(kr)}{RJ_1(kr)} - \frac{\dot{Q}}{Rr}, \\ J_1' &= \frac{\partial J_1}{\partial r}, \quad J_1'' = \frac{\partial^2 J_1}{\partial r^2}. \end{aligned}$$

The solution of Eq.(10) is

$$V_1(z) = e^{\frac{-Lz}{2}}(c_1 \sin\sqrt{N}z + c_2 \cos\sqrt{N}z), \tag{11}$$

Hence the solution for the layer can be written as

$$v_1(z) = e^{\frac{-Lz}{2}}(c_1 \sin\sqrt{N}z + c_2 \cos\sqrt{N}z)J_1(kr)e^{i\omega t}, \tag{12}$$

where $N = M - \frac{L^2}{4}$ and L, M, c_1, c_2 are constants.

4 Formulation and Solution for Half-Space

The heterogeneity has been considered in both density and shear moduli in the following manner

$$\mu_2 = \mu_0(1 + b_1 z)^2 \tag{13}$$
$$\rho_2 = \rho_0(1 + d_1 z)^2 \tag{14}$$

where μ_2 and ρ_2 are the shear moduli and density for half-space, respectively; b_1 and d_1 are the constants having dimension inverse of the length.

We have the relationship between stress and shear moduli for an elastic medium as

$$\tau_{r\theta} = \mu_2(z)(\frac{\partial v_2}{\partial r} - \frac{v_2}{r}) \tag{15}$$

and

$$\tau_{z\theta} = \mu_2(z)(\frac{\partial v_2}{\partial z}) \tag{16}$$

which gives the non vanishing equation of motion for half-space as

$$\mu_2(z)(\frac{\partial^2}{\partial r^2} + \frac{\partial}{r\partial r} - \frac{1}{r^2})v_2 + \frac{\partial}{\partial z}(\mu_2(z)\frac{\partial v_2}{\partial z}) = \rho_2(z)\frac{\partial^2 v_2}{\partial t^2}. \tag{17}$$

Now again consider the solution of Eq.(17) in the form

$$v_2(z) = V_2(z)J_1(kr)e^{i\omega t} \tag{18}$$

where $V_2(z)$ is the solution to

$$V_2''(z) + \frac{\mu_2'(z)}{\mu_2(z)}V_2'(z) - k^2(1 - \frac{c^2}{\chi^2})V_2(z) = 0 \tag{19}$$

with $\chi = \sqrt{\frac{\mu_2}{\rho_2}}$.
Now putting $V_2(z) = \frac{u(z)}{(1+b_1 z)^2}$ in Eq.(19), we get

$$u''(z) + \left[-k^2(1 - \frac{c^2(1 + d_1 z)^2}{\chi_2^2(1 + b_1 z)^2})\right]u(z) = 0. \tag{20}$$

with $\chi_2 = \sqrt{\frac{\mu_0}{\rho_0}}$.
Using the dimensionless quantities $\gamma = 1 - \frac{c^2 d_1^2}{\chi_2^2 b_1^2}$ and $\eta = \frac{2\sqrt{\gamma}k(1+b_1 z)}{b_1}$ in Eq.(20), we have

$$u''(\eta) + \left[\frac{G}{\eta} - \frac{1}{4} + \frac{\frac{1}{4} - m^2}{\eta^2}\right]u(\eta) = 0, \tag{21}$$

where $m^2 = \frac{1}{4} - \frac{\omega^2(d_1-b_1)^2}{\chi_2^2 b_1^4}$ and $G = \frac{\omega^2(b_1-d_1)}{k\chi_2^2 b_1^3\sqrt{\gamma}}$.
Solution of Eq.(21) satisfying the condition $\lim_{z\to\infty} V_2(z) \to 0$ i.e. $\lim_{\eta\to\infty} u(\eta) \to 0$ may be written as

$$u(\eta) = AW_{G,m}(\eta) \tag{22}$$

where $W_{G,m}$ is the Whittaker function (Whittaker and Watson (1927)).
Thus the displacement component in heterogeneous half-space is given by

$$v_2(z) = \frac{AW_{G,m}(\frac{2\sqrt{\gamma}k(1+b_1 z)}{b_1})}{1+b_1 z} J_1(kr)e^{i\omega t}. \tag{23}$$

5 Boundary Conditions

The boundary conditions are the continuities of the displacement components and stress at $z = 0$ and also stress vanishes at $z = -H$. Mathematically, these conditions can be written as

1. Continuity of displacement components and stress at $z = 0$ provide

$$v_1 = v_2 \tag{24}$$

and

$$R\frac{\partial v_1}{\partial z} + Q\left(\frac{\partial v_1}{\partial r} - \frac{v_1}{r}\right) = \mu_2 \frac{\partial v_2}{\partial z}. \tag{25}$$

2. Stress will vanish at $z = -H$ .i.e.

$$R\frac{\partial v_1}{\partial z} + Q\left(\frac{\partial v_1}{\partial r} - \frac{v_1}{r}\right) = 0. \tag{26}$$

Taking the expansion of Whittaker function upto the linear term and substituting in Eqs.(25) and (26), we get

$$c_2 = A(\frac{2\sqrt{\gamma}k}{b_1})^{\frac{1}{2}+m} e^{-\frac{\sqrt{\gamma}k}{b_1}}(1 + \frac{2\sqrt{\gamma}k}{b_1}s) \tag{27}$$

and

$$\begin{aligned}&\left(c_1\sqrt{N_1} - \frac{L_1}{2}c_2\right)R_1 + Q_1 c_2\left(\frac{kJ_1'(kr)}{J_1(kr)} - \frac{1}{r}\right)\\ &\quad = \mu_1 A\left(\frac{2\sqrt{\gamma}k}{b_1}\right)^{\frac{1}{2}+m} e^{\frac{-k\sqrt{\gamma}}{b_1}}\left(-\frac{1}{2}b_1 + b_1 m - \sqrt{\gamma}k\right)\left(1 + s\frac{2k\sqrt{\gamma}}{b_1}\right)\end{aligned} \tag{28}$$

where $s = \frac{\frac{1}{2}+m-G}{2m+1}$, P_1, Q_1, R_1, L_1, M_1 and N_1 are the values of P, Q, R, L, M and N at $z = 0$, respectively.
Similarly, from Eq.(27), we have the following condition

$$\begin{aligned}&R_2\left[\sqrt{N_2}(c_1\cos\sqrt{N_2}H + c_2\sin\sqrt{N_2}H) - \frac{L_2}{2}(c_2\cos\sqrt{N_2}H - c_1\sin\sqrt{N_2}H)\right]\\ &\quad + Q_2\left[(c_2\cos\sqrt{N_2}H - c_1\sin\sqrt{N_2}H)\frac{kJ_1'(kr)}{J_1(kr)} - \frac{1}{r}(c_2\cos\sqrt{N_2}H - c_1\sin\sqrt{N_2}H)\right] = 0\end{aligned} \tag{29}$$

where P_2, Q_2, R_2, L_2, M_2 and N_2 are the values of P, Q, R, L, M and N at $z = -H$, respectively.
Eliminating c_1, c_2 and A from Eqs.(27), (28) and (29), we have the dispersion relation as

$$\tan(kHT_2) = \frac{S_1 + S_2}{X_1 + X_2} \tag{30}$$

where the expressions for S_1, S_2, X_1 and X_2 are given in Appendix and T_2 is the value of T at $z = -H$.
Eq.(30) is the dispersion relation for torsional surface waves in a heterogeneous self-reinforced layer lying over a heterogeneous half-space. Furthermore, we shall use this relation to obtain the variation of phase velocity of torsional waves with respect to the wave number.

6 Special Cases

The previous article which have already been done by various authors will be discussed as particular case of the present study.

Case(1): when the upper layer is isotropic homogeneous i.e. $\xi = 0, \zeta = 0$ and $\mu_L = \mu_T = \mu$ then Eq.(30) becomes

$$tan\left[kH\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}\right] = -\frac{\mu_1}{\mu}\frac{\left(-\frac{1}{2}B_1 + B_1 m - \sqrt{\gamma}\right)}{\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}} \tag{31}$$

with $B_1 = \frac{b_1}{k}$ and $\beta_1 = \sqrt{\frac{\mu}{\rho_1}}$. This is the dispersion equation of torsional waves in homogeneous isotropic layer over a heterogeneous half-space.

Case(2): when half space is of constant density i.e. $d_1 = 0$, then Eq.(30) becomes

$$\tan(kHT_2) = \frac{S_1 + S_2}{X_1 + X_2} \tag{32}$$

where

$$C = \left(\frac{kJ_1'(kr)}{J_1(kr)} - \frac{1}{r}\right)Q_1 a_1 a_3 + \frac{1}{2}\left(\frac{2kJ_1'(kr)}{J_1(kr)} + \frac{1}{r}\right)Q_1 + \mu_1\left(\frac{-b_1}{2} + b_1 m - k\right).$$

Eq.(32) is the dispersion relation of torsional waves propagating in heterogeneous fibre-reinforced medium lying over a half-space of constant density.

Case(3): when $\xi = 0, \zeta = 0, \mu_L = \mu_T = \mu$ and the variation in half space is taken as $\mu_2 = \mu_0(1 + b_1 z), \rho_2 = \rho_0(1 + d_1 z)$ then Eq.(30) becomes

$$\tan\left[kH\left(\frac{c^2}{\beta_1^2}-1\right)^{\frac{1}{2}}\right] = \frac{\mu_1\gamma}{\mu}\left[1 - \frac{2s}{\left(1+\frac{2\gamma ks}{b_1}\right)}\right]\frac{1}{(\frac{c^2}{\beta_1^2}-1)^{\frac{1}{2}}}. \tag{33}$$

Eq.(33) is the dispersion relation for torsional waves propagating in a homogeneous isotropic substratum over a heterogeneous half-space (Dey et al. (1996)).

Case(4): when $\xi = 0, \zeta = 0, \mu_2 = \mu_0(1 + b_1 z), \rho_2 = \rho_0(1 + d_1 z)$, then dispersion relation becomes

$$\tan(kHT_2) = \frac{\mu_1\gamma kr\left[1 - \frac{2s}{1+\frac{2\gamma ks}{b_1}}\right]}{\frac{3Q_2}{2R_2krT_2}\left[\mu_1\left(\frac{2b_1\gamma krs}{b_1+2\gamma ks} - \gamma kr\right) + \frac{3Q_2}{2}\right] + R_2krT_2}. \tag{34}$$

This is the dispersion relation for torsional surface waves propagating in a self reinforced layer lying over a half-space with linearly varying rigidity and density (Chattopadhyay et al. (2012)).

Case(5): when the variation of half-space is taken as $\mu_2 = \mu_1(1 + b_1 z)$ and $\rho_2 = \rho_0$ i.e. $(d_1 = 0)$ then the dispersion relation (30) reduces to

$$\tan(kHT_2) = \frac{\mu_1 kr \left[1 - \frac{2s}{1+\frac{2ks}{b_1}}\right]}{\frac{3Q_2}{2R_2krT_2}\left[\mu_1\left(\frac{2b_1krs}{b_1+2ks} - kr\right) + \frac{3Q_2}{2}\right] + R_2krT_2}. \tag{35}$$

which is the dispersion relation of torsional surface waves in heterogeneous fibre-reinforced medium lying over a Gibson half-space.

Case(6): when the layer and half-space both are isotropic homogeneous i.e. $a = 0, b = 0, \mu_L = \mu_T = \mu, b_1 = 0, d_1 = 0$, then Eq.$(30)$ reduces to

$$tan\left[kH\left(\frac{c^2}{\beta_1^2} - 1\right)^{\frac{1}{2}}\right] = \frac{\mu_1}{\mu}\frac{\left(1 - \frac{c^2}{\chi_2^2}\right)^{\frac{1}{2}}}{\left(\frac{c^2}{\beta_1^2} - 1\right)^{\frac{1}{2}}}. \tag{36}$$

This is the classical Love wave equation for propagation of surface wave in homogeneous isotropic layer over a homogeneous isotropic half-space.

7 Numerical Examples and Discussions

Since the dispersion relation is obtained in closed form. All the quantities presented in this relation are in implicit form. So in order to discuss the effect of different parameters present in the study, numerical examples are required. Here we have considered three different types of fiber-reinforced elastic materials to discuss the effect of parameters on the propagation of torsional waves present in the study. The data for three different materials are as follows.

1. For heterogeneous steel fiber-reinforced layer (Hool et al. (1944)).

$$\mu_L = 7.07 \times 10^9 \quad N/m^2,$$
$$\mu_T = 3.5 \times 10^9 \quad N/m^2,$$
$$\rho_1 = 1,600 \quad Kg/m^3.$$

   where μ_L and μ_T are the shear moduli in longitudinal and transverse direction, respectively.

2. For heterogeneous half-space (Gubbins (1990)).

$$\mu_2 = 6.34 \times 10^{10} \quad N/m^2,$$
$$\rho_2 = 3,364 \quad Kg/m^3.$$

where μ_2 and ρ_2 are the shear modulus and density for the half-space.
Also, we have used the following data: $a_1 = 0.866, a_3 = 0.5, kr = 5.4, \varphi = 60\,^{\circ}$.

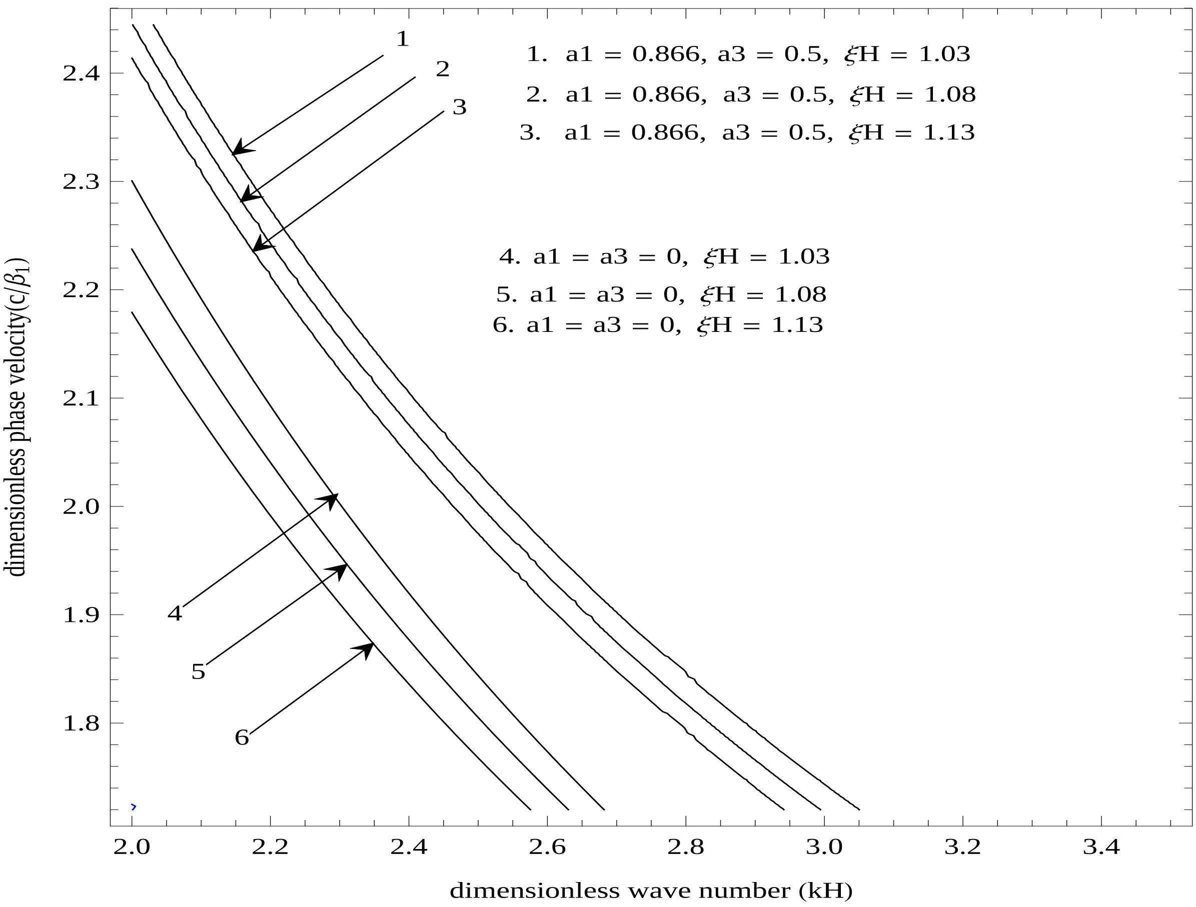

Figure 2: Variation of phase velocity (c/β_1) against wave number kH for different values of heterogeneity parameter (ξH).

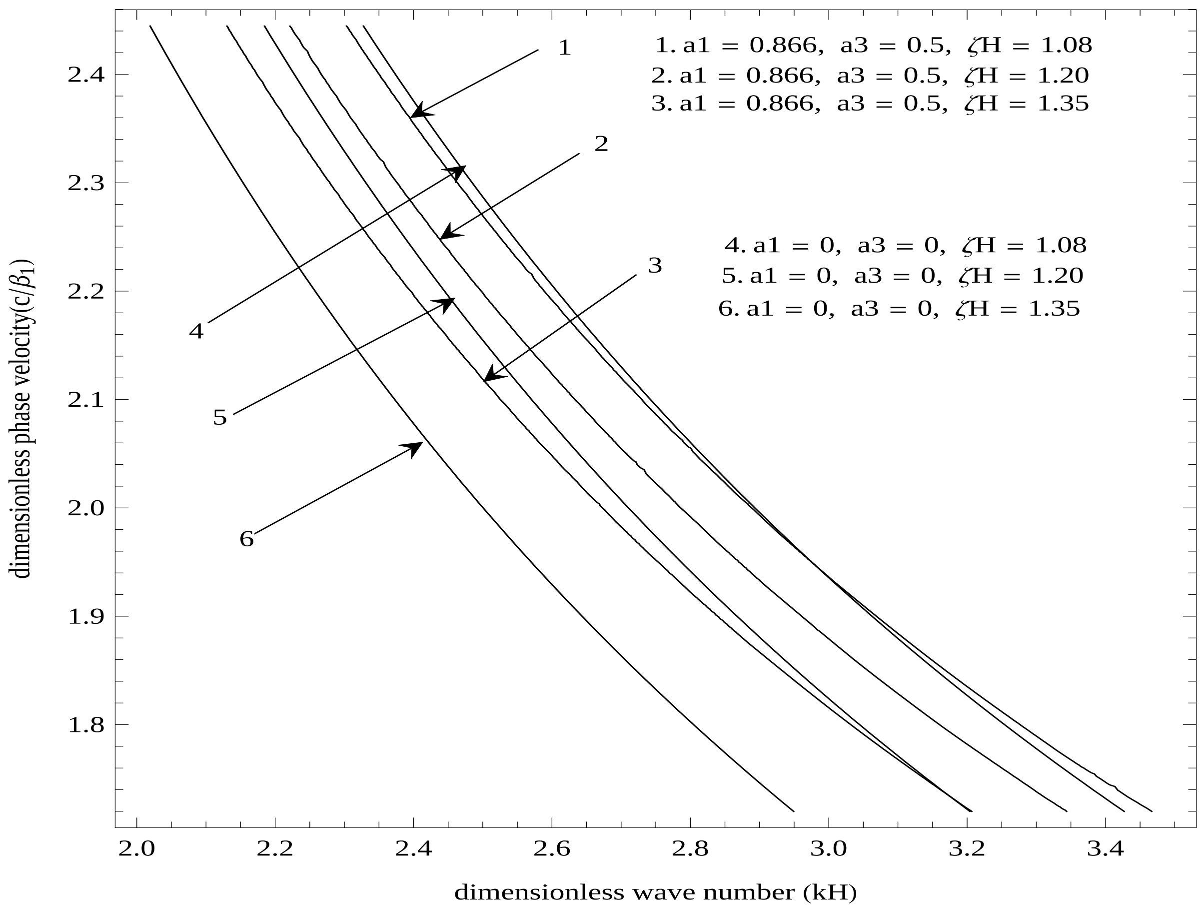

Figure 3: Variation of phase velocity (c/β_1) against wave number kH for different values of heterogeneity parameter (ζH).

Figs.(2) and (3) represents the variation of phase velocity $\frac{c}{\beta_1}$ against wave number kH for different values of heterogeneity parameters ξH and ζH. In both the figs.(2) and (3), curves $1, 2$ and 3 are in presence of reinforcement whereas the curves $4, 5$ and 6 are in absence of reinforcement for steel material. In figs. (2) and (3), it is clear that the phase velocity of the torsional surface waves decreases as the wave number increases. Fig.(1) shows that the phase velocity of the wave decreases as the values of heterogeneity parameters ξH increases in both reinforced and reinforced free cases but the decrement in phase velocity is more in reinforced free case than presence of reinforcement. In a similar manner the effect of heterogeneity parameter ζH has been shown in fig.(3) and again the decrement in phase velocity is more in case of reinforced free case than presence of reinforcement.

8 Conclusions

The effect of heterogeneity and reinforcement on propagation of torsional surface waves has been investigated. Dispersion relation is obtained in closed form and matched with classical Love wave equation. Also, some existing results have been deduced as particular case of the present study. Numerical results have been shown for steel fibre-reinforced materials. Some salient outcomes of the present study may be listed as

1. Heterogeneity and reinforcement of the medium are found to have significant effect on velocity profile of torsional surface waves.

2. Reinforced and reinforced free cases are compared for steel materials and it is found that the presence of reinforcement increases the phase velocity of torsional waves.

3. Heterogeneity of the layer is found to have reverse effect on phase velocity of surface waves (i.e. phase velocity decreases for increasing values of heterogeneity parameters).

4. Dispersion relation for propagation of torsional surface waves in heterogeneous reinforced layer over a Gibson half-space has been obtained.

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Appendix

$$T = \left[\left(\frac{PJ_1''(kr)}{RJ_1(kr)} + \frac{PJ_1'(kr)}{RkrJ_1(kr)} - \frac{P}{Rk^2r^2} + \frac{\rho c^2}{R} + \frac{\dot{Q}J_1'(kr)}{kRJ_1(kr)} - \frac{\dot{Q}}{k^2Rr} \right) \right.$$
$$\left. -\frac{1}{4}\left(\frac{2QJ_1'(kr)}{RJ_1(kr)} + \frac{Q}{Rkr} + \frac{\dot{R}}{kR} \right)^2 \right]^{\frac{1}{2}},$$
$$S_1 = \frac{Q_2}{kr} - \frac{Q_2kJ_1'(kr)}{J_1(kr)},$$
$$S_2 = \frac{Q_2J_1'(kr)}{R_2J_1(kr)} + \frac{Q_2}{2R_2kr} - CR_2T_2,$$
$$X_1 = C\left(\frac{Q_2J_1'(kr)}{R_2J_1(kr)} + \frac{Q_2}{2R_2kr} \right),$$
$$X_2 = C\left(\frac{Q_2}{kr} - \frac{Q_2J_1'(kr)}{J_1(kr)} \right) + R_2T_2,$$
$$C = \left(\frac{kJ_1'(kr)}{J_1(kr)} - \frac{1}{r} \right) Q_1a_1a_3 + \frac{1}{2}\left(\frac{2kJ_1'(kr)}{J_1(kr)} + \frac{1}{r} \right) Q_1 + \mu_1\left(\frac{-b_1}{2} + b_1m - \sqrt{\gamma}k \right).$$

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