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Enhanced Utilization of structural Damping of rotating Machines using impulsively shaped torsional Moments

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The reduction of torsional vibrations of rotating machines is an important issue, as they may lead to a decrease of the performance, or in the worst case, to a damage of the machinery. In particular, self-excited vibrations have to be suppressed in any case due to their hazardous nature.
In this contribution, a method is proposed, which allows utilizing the structural damping inherent to every rotating machine much more effectively by introducing impulsively shaped torsional moments, resulting in repeated modal transfers of vibration energy. Depending on the chosen impulsive strength, the energy transfers are accompanied by feeding external energy to, or extracting energy from the mechanical system. It is shown theoretically by approximating the impulses by Dirac-delta functions that an impulsive strength exists, where no energy crosses the system boundary, i.e. energy extracted from one mode is fed entirely to another mode of vibration. In the case of a conservative system, a repeated application of such impulses induces a periodic exchange of energy between lower and higher modes. Taking into account the structural damping reveals the advantages of transferring energy across modes. As higher modes possess higher damping ratios than lower ones, the structural damping of the rotating machine can be utilized much more effectively, which leads to a significant reduction of torsional vibrations. The underlying equations of motion of the impulsively excited system can be written as recursive difference equations with constant coefficients. Hence, the stability properties of the system can be investigated according to the Floquet theory. It is shown that the proposed concept is capable of suppressing self-excited vibrations. Stability charts are presented which allow to identify stable areas of operation. Finally, some numerical results of a test-rig are shown, underlining the effectivity of the proposed method.

1 Introduction

Torsional vibrations of rotating machines have been investigated for a long time. This is founded in the fact that they may decrease the performance of such machines, or in the worst case, result in a damage or breakdown. Especially, self-excited vibrations, due to several mechanisms, have led to catastrophic failures of rotating machines. Therefore, a lot of measures have been developed to suppress or at least to reduce such vibrations.
Reduction of vibrations of mechanical systems means to reduce the energy content. A natural approach is to introduce additional devices which allow a rapid dissipation of vibration energy. Within this context, an interesting concept are nonlinear energy sinks (NES), see Vakakis et al. (2009). They consist of a lightweight vibrating system which is coupled in an essential nonlinear manner to the primary structure. It is shown that, under certain conditions, an unidirectional energy flow from the primary structure to the NES occurs, where energy is dissipated effectively. Another method is the modal redistribution of energy. As higher modes usually possess enhanced damping properties compared to lower ones, it is beneficial to shift vibration energy to higher modes, where it can be dissipated more effectively. This is demonstrated in Al-Shudeifat et al. (2015), where vibro-impact NES are used to achieve a modal redistribution of energy of mechanical structures to utilize the damping properties of the structure more efficiently. Hence, transient vibrations decay much faster compared to the case where no NES is used. Methods for tracking energy flows in dynamical systems can be found in Quinn et al. (2012), for example.
A continuous and repeated transfer of energy from low to high modes and vice versa can also be achieved by a periodic variation of system stiffness parameters at certain frequencies, see Tondl (1998). To induce the modal energy transfer, external energy has to be fed to the system and energy has to be extracted from the system in a periodic manner. In the following years, a variety of investigations focused on this effect, see Ecker (2005); Tondl (2000); Dohnal (2008); Ecker and Pumhössel (2012), for example.
Dynamical systems subjected to periodic impulsive parametric excitation were investigated in Hsu (1972), wherein the question of stability of such systems has been addressed extensively. In Pumhössel et al. (2013) and Pumhössel (2016a), the effect of stiffness variation of impulsive type on the energy content of mechanical systems was inves-

tigated. It was shown that stiffness impulses, whose strength depends in a nonlinear manner on the state-vector, allow to transfer discrete amounts of energy from pre-defined low-modes, to a target set of higher modes. The existence of energy-neutral transfers, i.e. energy transfers where neither external energy is fed to the mechanical system, nor energy is extracted from the system while energy is transferred across modes, was discovered and reported. In Pumhössel (2016b), the general case of modal energy transfers using impulsive forcing of mechanical systems was investigated.
The present contribution addresses targeted modal energy transfers in rotor systems, induced by applying impulsive torsional moments. It is shown that energy can be transferred periodically from low to high modes and vice versa. In a further development of the work presented in Pumhössel (2016b), the stability of a rotor system subjected to self-excitation is addressed in a comprehensive manner. The proposed approach allows to write the equations of motion of the impulsively excited system as a set of difference equations with constant coefficients. Hence, the stability properties can be investigated easily. Stability charts demonstrate which combinations of system parameters allow to stabilize the self-excited rotor system. Finally, some numerical results of a test-rig under construction are shown.

2 Modal Energy Transfers - analytical Investigations

The n-dimensional equations of motion of a rotor system consisting of flexible shafts with isotropic properties, and rigid disks with zero unbalance eccentricity may be written as

$$\mathbf{I}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \sum_{k=1}^{N} \varepsilon_k \delta(t - t_k)\mathbf{f}, \tag{1}$$

where $\mathbf{q} = [q_1, q_2, \ldots q_n]^T$ represents the vector of rotational degrees of freedom, and $\mathbf{I} = \text{diag}(I_1, I_2, \ldots I_n)$ and $\mathbf{K}$ denote the constant and symmetric mass-, and stiffness-matrices. The damping matrix $\mathbf{C}$ is assumed to be decomposable in a damping and a self-excitation part, according to $\mathbf{C} = \mathbf{C}_{damp} + \mathbf{C}_{self}$, where $\mathbf{C}_{damp}$ is assumed to be stiffness-proportional, i.e. $\mathbf{C}_{damp} = \beta\mathbf{K}$ holds. The rotor system is subjected to a sequence of N torsional moments of impulsive type, see right hand side of Eq. (1), where $\delta(t - t_k)$ represents the Dirac-delta function, and $\mathbf{f} = [f_1, f_2, \ldots f_n]^T$, $f_i = 1 \vee 0$, $i = 1, \ldots n$ denotes the constant vector, which allows to select specific disks. The scalar ε_k, which will be state-dependent, as described later, represents the strength of the impulses. To investigate the effect of the impulsive excitation to the rotor system, especially discrete modal energy transfer effects, the equations of motion are transformed to modal coordinates $\mathbf{u}$ according to $\mathbf{q} = \mathbf{\Phi}\mathbf{u}$, where $\mathbf{\Phi}$ represents the modal matrix of the undamped rotor system. This yields to the equations of motion in the modal form

$$\bar{\mathbf{I}}\ddot{\mathbf{u}} + (\beta\bar{\mathbf{K}} + \bar{\mathbf{C}}_{self})\dot{\mathbf{u}} + \bar{\mathbf{K}}\mathbf{u} = \sum_{k=1}^{N} \varepsilon_k \delta(t - t_k)\bar{\mathbf{f}}. \tag{2}$$

In the case of no self-excitation, i.e. $\bar{\mathbf{C}}_{self} = \mathbf{0}$, the left hand side of Eq. (2) is decoupled. Hence, modal energy transfers can occur only at the instants of time t_k, where an impulse is applied. In the following, it is assumed that the instants of time $t_1, t_2, \ldots$ where impulses are applied, are equidistant in time, i.e. $t_{k+1} - t_k = T_P$, $k = 1, \ldots N-1$, $T_P = \text{const.}$ holds. This assumption allows to relate the state-vector of the system at an instant of time t, to the state-vector at $t + T_P$ using a constant matrix-mapping. Therefore, the equations of motion (1) are written in first order form according to

$$\dot{\mathbf{x}} = \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{E} \\ -\mathbf{I}^{-1}\mathbf{K} & -\mathbf{I}^{-1}\mathbf{C} \end{bmatrix}}_{\mathbf{A}} \mathbf{x} + \begin{bmatrix} \mathbf{0} \\ \sum_{k=1}^{N} \varepsilon_k \delta(t - t_k)\mathbf{I}^{-1}\mathbf{f} \end{bmatrix}, \tag{3}$$

where the state-vector is given by $\mathbf{x} = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T$, and $\mathbf{E}$ denotes the unity-matrix. The solution of the autonomous set of differential equations $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ is of the well-known form

$$\mathbf{x}(t) = \mathbf{e}^{\mathbf{A}(t-t_0)}\mathbf{x}_0, \tag{4}$$

with an initial state-vector $\mathbf{x}_0$. Hence, the state-vectors at the beginning and at the end of an autonomous timespan T_P are related to each other by

$$\mathbf{x}(t_{k+1})_- = \mathbf{e}^{\mathbf{A}T_P}\mathbf{x}(t_k)_+ = \mathbf{D}\mathbf{x}(t_k)_+, \tag{5}$$

where the $\pm$ signs indicates instants of time just after and just before an impulse. At this point, the question about the effect of a force impulse to the state-vector arises. This was investigated extensively in the past, see Angeles

(2012), for example. Therefore, only a brief description is given in the following. The momentum balance for a single impulse is given by

$$\mathbf{I}[\dot{\mathbf{q}}(t_k)_+ - \dot{\mathbf{q}}(t_k)_-] = \varepsilon_k \mathbf{f}, \tag{6}$$

which can be written in the form

$$\dot{\mathbf{q}}(t_k)_+ = \dot{\mathbf{q}}(t_k)_- + \varepsilon_k \mathbf{I}^{-1}\mathbf{f}, \tag{7}$$

and hence, provides a relation of the velocities $\dot{\mathbf{q}}$ just before and just after an impulse. From Eq. (7) one can easily see that the velocities exhibit a jump at the instant of time the impulse is applied. By contrast, the displacements remain continuous, see Angeles (2012), for example, i.e.

$$\mathbf{q}(t_k)_+ = \mathbf{q}(t_k)_-. \tag{8}$$

Summarizing Eqs. (7) and (8) yields to

$$\begin{bmatrix} \mathbf{q}(t_k)_+ \\ \dot{\mathbf{q}}(t_k)_+ \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{q}(t_k)_- \\ \dot{\mathbf{q}}(t_k)_- \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \varepsilon_k \mathbf{I}^{-1}\mathbf{f} \end{bmatrix}. \tag{9}$$

The equation above holds for any arbitrary impulsive strength ε_k. In the following, the special case, where an impulse neither extracts energy from the mechanical system, nor adds external energy to the system, which means that the variation of the kinetic energy ΔT_k is equal to zero, i.e. $\Delta T_k = T_{k+} - T_{k-} = 0$, is investigated. With Eq. (7) the total kinetic energy of the mechanical system after an impulse is given by

$$T_{k+} = \underbrace{\frac{1}{2}\left[\varepsilon_k^2 \mathbf{f}^T\mathbf{I}^{-1}\mathbf{f} + 2\varepsilon_k \mathbf{f}^T \dot{\mathbf{q}}(t_k)_-\right]}_{\Delta T_k} + \underbrace{\frac{1}{2}\dot{\mathbf{q}}^T(t_k)_- \mathbf{I}\dot{\mathbf{q}}(t_k)_-}_{T_{k-}}, \tag{10}$$

which leads to the variation ΔT_k in the form

$$\Delta T_k = \frac{1}{2}\varepsilon_k \left[\varepsilon_k \mathbf{f}^T\mathbf{I}^{-1}\mathbf{f} + 2\mathbf{f}^T\dot{\mathbf{q}}(t_k)_-\right]. \tag{11}$$

The zeros of the equation $\Delta T_k = 0$ are

$$\varepsilon_{k,1} = 0 \qquad \text{and} \qquad \varepsilon_{k,2} = -2\frac{\mathbf{f}^T}{\mathbf{f}^T\mathbf{I}^{-1}\mathbf{f}}\dot{\mathbf{q}}(t_k)_-. \tag{12}$$

If an impulse with $\varepsilon_k = \varepsilon_{k,2}$ is applied, the overall energy content of the mechanical system remains unchanged. However, this does not prevent energy transfers from one mode to another. It has to be pointed out that $\varepsilon_{k,2}$ remains bounded, as the denominator of $\varepsilon_{k,2}$ in Eq. (12) is positive definite. For the following investigations, the impulsive strength ε_k is rewritten in the form $\varepsilon_k = \vartheta\varepsilon_{k,2}$, with the scalar ϑ. Therewith, ΔT_k is given by

$$\Delta T_k(\vartheta) = 2\vartheta(\vartheta - 1)\frac{(\mathbf{f}^T\dot{\mathbf{q}})^2}{\mathbf{f}^T\mathbf{I}^{-1}\mathbf{f}} = \begin{cases} < 0 & \text{if } 0 < \vartheta < 1 \\ = 0 & \text{if } \vartheta = 0 \vee \vartheta = 1 \\ > 0 & \text{if } \vartheta < 0 \vee \vartheta > 1 \end{cases}. \tag{13}$$

The sign of ΔT_k depends only on the selected value for ϑ, as the fraction in Eq. (13) is always positive. If $0 < \vartheta < 1$, ΔT_k is negative, which means that kinetic energy is extracted from the mechanical system by the impulse. Energy is fed into the mechanical system, i.e. ΔT_k is positive, if $\vartheta < 0$ or $\vartheta > 1$ holds. No energy crosses the system boundary, if $\vartheta = 1$, denoted as the energy-neutral case introduced in Pumhössel (2016a). Inserting $\varepsilon_k = \vartheta\varepsilon_{k,2}$ into the expression for the velocity just after an impulse, Eq. (7) leads to

$$\dot{\mathbf{q}}(t_k)_+ = \dot{\mathbf{q}}(t_k)_- - 2\vartheta \underbrace{\frac{\mathbf{I}^{-1}\mathbf{f}\,\mathbf{f}^T}{\mathbf{f}^T\mathbf{I}^{-1}\mathbf{f}}}_{\mathbf{G}} \dot{\mathbf{q}}(t_k)_- = \dot{\mathbf{q}}(t_k)_- - 2\vartheta\mathbf{G}\dot{\mathbf{q}}(t_k)_- = [\mathbf{E} - 2\vartheta\mathbf{G}]\dot{\mathbf{q}}(t_k)_-. \tag{14}$$

The relation between the state-vector just before and just after an impulse can now be written in the form

$$\begin{bmatrix} \mathbf{q}(t_k)_+ \\ \dot{\mathbf{q}}(t_k)_+ \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} - 2\vartheta\mathbf{G} \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} \mathbf{q}(t_k)_- \\ \dot{\mathbf{q}}(t_k)_- \end{bmatrix}, \tag{15}$$

where the matrix $\mathbf{J}$ is denoted as *jump transfer matrix*, according to the notation introduced by C.S. Hsu, see Hsu (1972). With Eqs. (5) and (15) the state-vector after one period T_P is given by

$$\begin{bmatrix} \mathbf{q}(t_{k+1})_+ \\ \dot{\mathbf{q}}(t_{k+1})_+ \end{bmatrix} = \mathbf{J}\begin{bmatrix} \mathbf{q}(t_{k+1})_- \\ \dot{\mathbf{q}}(t_{k+1})_- \end{bmatrix} = \mathbf{J}\mathbf{D}\begin{bmatrix} \mathbf{q}(t_k)_+ \\ \dot{\mathbf{q}}(t_k)_+ \end{bmatrix}, \tag{16}$$

which is a set of linear difference equations with constant coefficients. For the following investigations, the abbreviation $\mathbf{Q} = \mathbf{JD}$ is used. Equation (16) allows to easily calculate the state-vector at instants of time $\ldots t_{k-1}, t_k, t_{k+1} \ldots$, and hence, the matrix $\mathbf{Q}$ describes the growth of the state-vector. For this reason, the eigenvalues of $\mathbf{Q}$ decide about the stability of the trivial solution of Eq. (3) for $N \to \infty$. The trivial solution is asymptotically stable if for all eigenvalues Λ_i, $i = 1 \ldots n$, of $\mathbf{Q}$ holds

$$|\Lambda_i| = |\mathrm{eig}(\mathbf{Q})| < 1. \tag{17}$$

3 Example - 2 DOF System

A schematic of the investigated mechanical system is depicted in Fig. (1). It consists of two disks with inertias I_1 and I_2 as well as rotary degrees of freedom q_1 and q_2. The disks are connected to each other and to the inertial

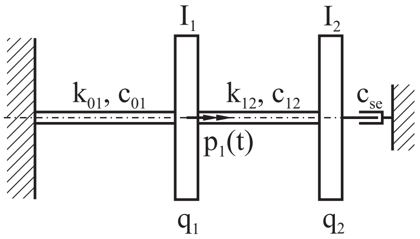

Figure 1: Schematic of the investigated mechanical system.

frame by shafts with stiffness k_{01} and k_{12} and damping coefficients c_{01} and c_{12}. A self-excitation mechanism is modelled by an element connected to disk two, which is capable of possessing a negative damping coefficient. Disk one is subjected to a sequence of impulsively shaped torsional moments $p_1(t)$. The following system-parameters are used for the numerical simulations $I_1 = I_2 = 1\,\mathrm{kgm}^2$, $k_{01} = k_{12} = 1\,\mathrm{Nm}$, $\beta = 0.01\,\mathrm{s}$, $c_{se} = -0.02\,\mathrm{Nsm}$. The periods of first and second mode vibration are $T_1 = 10.1664\,\mathrm{s}$ and $T_2 = 3.3882\,\mathrm{s}$. Two different initial conditions (ICs) are used for the numerical investigations, the scaled first eigenvector (IC 1) and an initial velocity of disk two (IC 2), representing a shock load, see Eq. (18).

$$\mathbf{x}_0 = \begin{cases} [0.6180, 1, 0, 0]^T & \text{IC 1} \\ [0, 0, 0, 1]^T & \text{IC 2} \end{cases} \tag{18}$$

Figure (2) depicts first numerical results of the undamped system without self-excitation to show the basic mechanism of modal energy transfer in the energy-neutral case, i.e. $\vartheta = 1$. As initial condition, a first mode deflection according to IC1 is used. Impulses are applied to disk one. By contrast to the physical coordinates q_1 and q_2 (see left column), the modal coordinates u_1 and u_2 (right column) clearly indicate the occurring modal energy transfer. A (local) minimum of u_1 is accompanied by a (local) maximum of u_2 and vice versa. Correspondingly, the energy content E_1 of the first mode minimizes, when the energy content E_2 of the second mode peaks. It has to be pointed out that the overall energy content E_{tot} of the mechanical system remains unchanged, i.e. neither energy is extracted from the system, nor external energy is fed to the system by the impulsive excitation, as the energy-neutral case $\vartheta = 1$ is investigated and natural damping is neglected.

Exemplary time-series for extracting energy from, or feeding external energy into the mechanical system are depicted in Fig. (3). Also in these cases, natural damping as well as self-excitation are switched off. In the left column, $\vartheta = 0.8$ and hence, the torsional moments of impulsive type extract energy from the mechanical system. The modal energy transfer is accompanied by a decreasing of the overall energy content E_{tot}. The right column shows results for $\vartheta = 1.02$, which means that external energy is fed into the system by the impulsively shaped torsional moments. As a consequence, the overall energy E_{tot} of the system increases beyond all limits, hence, the mechanical system becomes unstable.

The advantages of transferring energy across modes in a recurring manner becomes clear if natural damping is introduced. Figure (4) shows some corresponding results for $\vartheta = 1$, where as initial condition IC2 is used. Results, where no impulsive excitation is present, are depicted grey-colored. One clearly observes that the modal energy transfer induced by the impulsive excitation leads to a faster decrease of torsional vibrations q_1 and q_2. The envelope of the modal coordinate u_1 is much smaller and that of u_2 is larger compared to the case, where no impulsive excitation is present. This means that impulsive excitation in the proposed manner decreases/increases the level of first/second mode vibrations. Hence, the damping properties, inherent to the mechanical structure due to the natural damping are utilized in a more efficient way. This is underlined by the time-series of the total energy content E_{tot}, which decreases much faster with impulsive excitation than without.

Figure (5) depicts the maximum absolute value of the eigenvalues of the matrix $\mathbf{Q}$, max$|\Lambda|$, which decides about the stability of the trivial solution of the impulsively excited system. For the sake of comparison, the maximum absolute eigenvalue of the matrix $\mathbf{D}$, max$|\rho|$, describing the system without impulsive excitation is shown as well. The dotted lines represent the case, where no self-excitation is present. With increasing pulse-pause T_P, max$|\rho|$ decreases linearly and is always below the stability threshold. By contrast, max$|\Lambda|$ of the system with impulsive excitation shows some minima and maxima and is below max$|\rho|$, i.e. has a larger distance from the stability limit. Two of the maxima of max$|\Lambda|$ coincide with max$|\rho|$ at a pulse-pause of $T_P = 0.5T_1$ and $T_P = T_1$. If self-excitation is introduced, i.e. $c_{se} = -0.02$, the system without impulsive excitation becomes unstable, see solid, grey-colored line in Fig. (5). For almost all values of T_P, also max$|\Lambda|$ is above the stability limit. However, there exist some small intervals around $T_P = 5.5/8.785/10.98/14.29$ where the impulsively excited system is asymptotically stable. Hence, the modal energy transfer induced by the impulsive excitation is capable of stabilizing the otherwise unstable system.

In the following, the effect of a variation of system parameters on the stability is investigated. It has to be pointed out that for all values of system parameters, the mechanical system without impulsive excitation is unstable. Figure (6) depicts max$|\Lambda|$ for different values of the pulse-pause T_P and the inertia I_1 of disk one. The stability threshold max$|\Lambda| = 1$ is indicated by a black, solid line. A variety of areas is observed, where max$|\Lambda| < 1$, and hence, the trivial solution is asymptotically stable for the corresponding values of T_P and I_1. Besides, characteristic resonances occur if the pulse-pause T_P is close to $T_P = nT_1/2$, $n \in \mathbb{N}$, see Fig. (7) (left). Therein, grey/white colored areas denote unstable/asymptotically stable trivial solutions. Moreover, the values for $nT_1/2$ (red dashed line) and nT_1 (red solid line), $n \in \mathbb{N}$, indicating resonance areas, are depicted as well. One notes that the stable areas are located close to $nT_1/2$ and nT_1. Moreover, a kind of self-similarity with increasing T_P is observed. The effect of a variation of the inertia I_2 of disk two is shown in Fig. (7) (right). Large areas of stability are observed with increasing I_2. As previously shown, they are mainly located near $nT_1/2$ and nT_1. The results for a variation of the stiffness k_{01} are shown in Fig. (8) (left). Only small tongues of stability are observed. By contrast, the stiffness k_{12}, see Fig. (8) (right), has a large effect on the stability. For almost all values of $k_{12} >\approx 2.3$, the trivial solution is asymptotically stable, except for small areas near $nT_1/2$ and nT_1. The previous results were based on using impulsive excitation of Dirac-delta type, which allows to give a clear insight into the physical mechanism behind modal energy transfer effects.

In the following, some numerical results of a test rig are presented using half-sine shaped impulses. A sketch of the designed test-rig consisting of two connected disks is shown in Fig. (9). Impulsively shaped torsional moments are applied to the disks using permanent excited synchronous electrical engines (PSM 1 and PSM 2), where the

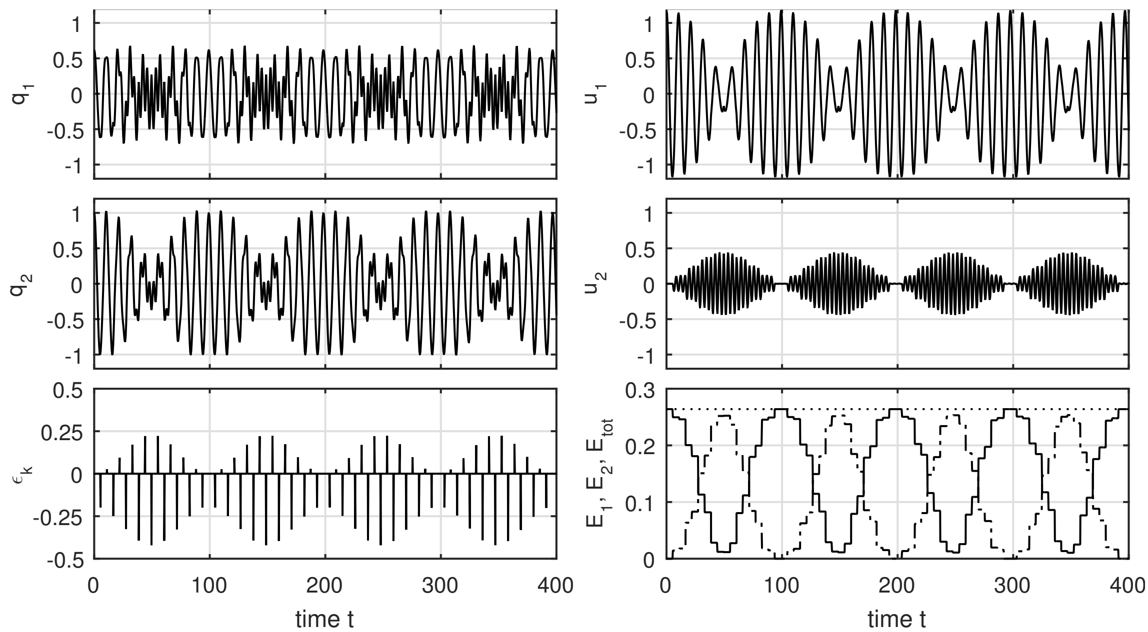

Figure 2: Time-series of physical coordinates q_1 and q_1, impulsive strength ε_k (left column), modal coordinates u_1 and u_2, modal energy contents E_1 (solid), E_2 (dashed-dotted) and overall energy content E_{tot} (dotted), (right column). Undamped system without self-excitation. Impulses applied to disk one. Initial condition: IC1. Energy-neutral case $\vartheta = 1$.

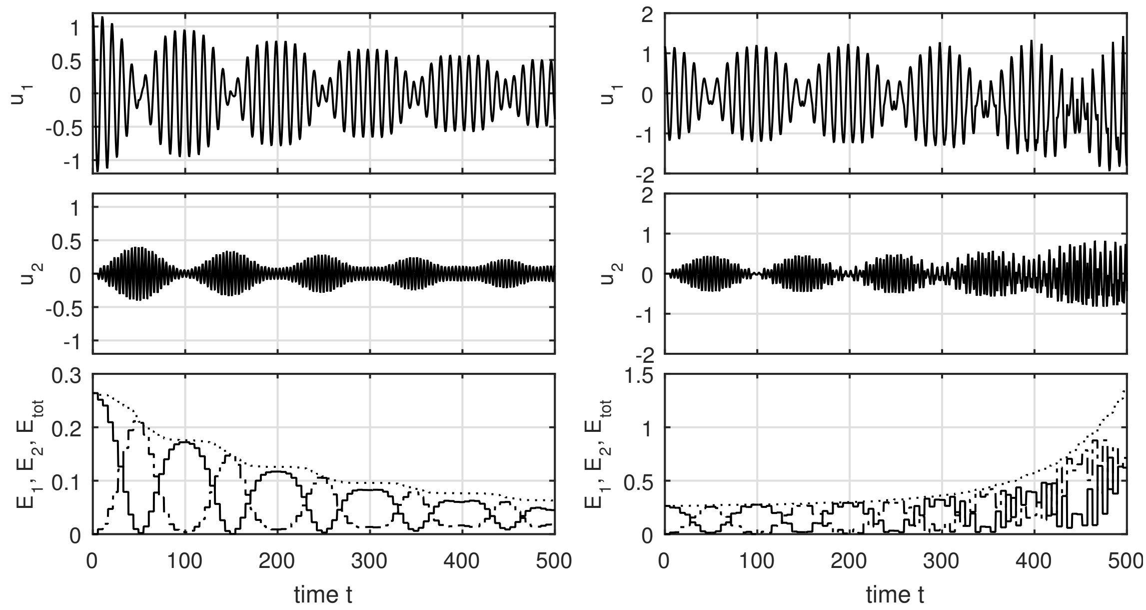

Figure 3: Modal coordinates u_1 and u_2, modal energy contents E_1 (solid), E_2 (dashed-dotted) and overall energy content E_{tot} (dotted), for extracting energy from the system, $\vartheta = 0.8$, (left column) and feeding energy to the system, $\vartheta = 1.02$, (right column). Undamped system without self-excitation. Impulses applied to disk one. Initial condition: IC1.

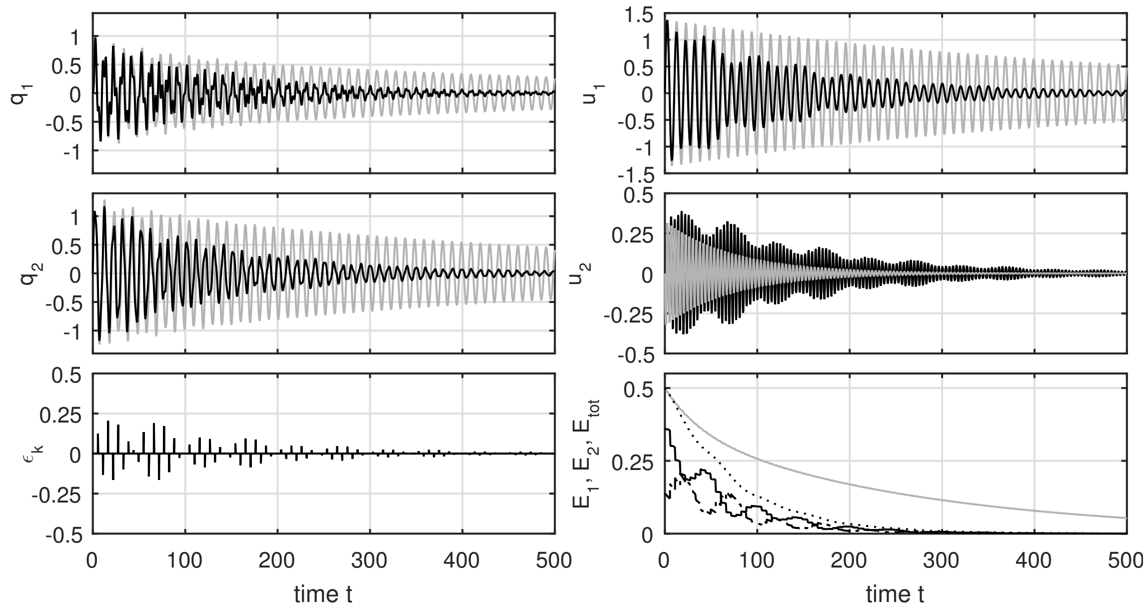

Figure 4: Time-series of physical coordinates q_1 and q_1, impulsive strength ε_k (left column), modal coordinates u_1 and u_2, modal energy contents E_1 (solid), E_2 (dashed-dotted) and overall energy content E_{tot} (dotted), (right column). Damped system without self-excitation. Impulses applied to disk one. Comparison with results where no impulsive excitation is present (grey colored). Energy-neutral case $\vartheta = 1$. Initial condition: IC2.

shaft of the first engine connects the part of the rotor to the left and to the right. In the general case, where impulses are applied to both disks, PSM1 and PSM2 require the actual values of the velocities $\dot{q}_1$ and $\dot{q}_2$ to apply impulses with the correct strength. The impulses are chosen to be of half-sine shape with a duration of 36 ms, which is equal to about 10% of the period of the second mode. Contrary to the previous investigations, the damping matrix is assumed to be of the form $\mathbf{C}_{damp} = \alpha\mathbf{I} + \beta\mathbf{K}$. The mass-proportional part allows to take the absolute damping, introduced by the electrical engines, into account. The following parameters are used for the numerical investigations: $I_1 = I_2 = 0.0065$ kgm^2, $k_{01} = k_{12} = 0.75$ Nm, $\alpha = 0.02$, $\beta = 0.0002$.

Figure (10) depicts the physical coordinates q_1 and q_2, the torsional moment M (applied to disk 1), the modal

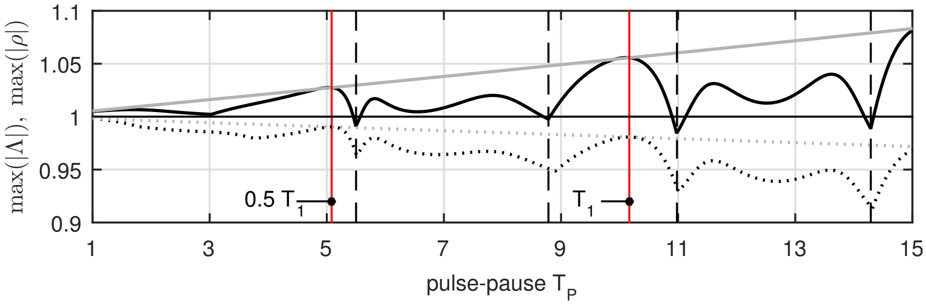

Figure 5: Maximum absolute value max$|\Lambda|$ of the eigenvalues of the matrix $\mathbf{Q}$ without (dotted, black) and with (solid, black) self-excitation for different values of the timespan T_P between impulses. Grey-colored lines depict the maximum absolute eigenvalue max$|\rho|$ of the matrix $\mathbf{D}$. Energy-neutral case $\vartheta = 1$.

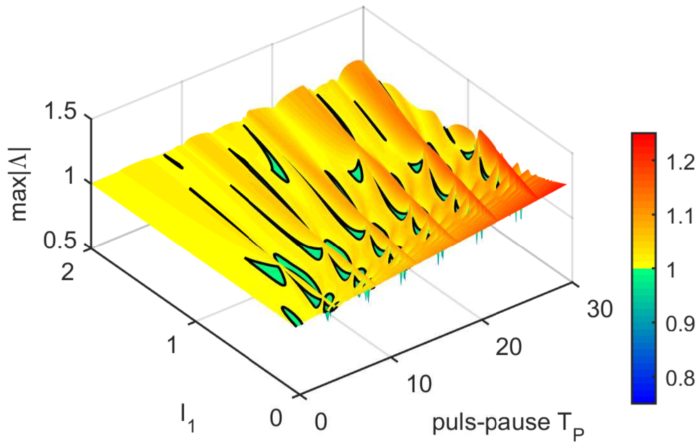

Figure 6: Maximum absolute value max$|\Lambda|$ of the eigenvalues of the matrix $\mathbf{Q}$ of the damped system with impulsive excitation for different values of the pulse-pause T_P and the moment of inertia I_1. Energy-neutral case $\vartheta = 1$.

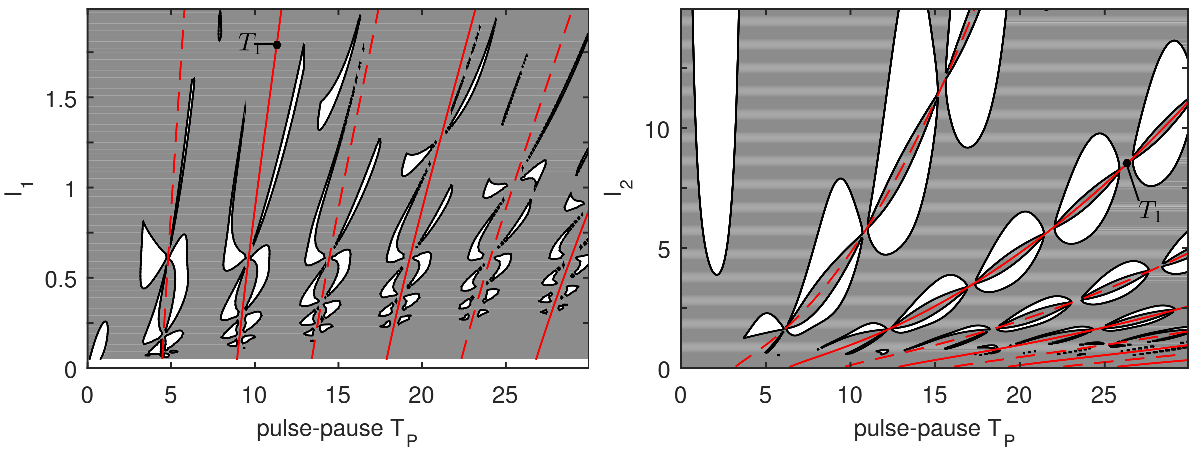

Figure 7: Stability chart for variation of system parameters I_1 (left) and I_2 (right) and T_P. Grey/white colored areas denote unstable/asymptotically stable trivial solutions. Energy-neutral case $\vartheta = 1$.

energy contents E_1 (solid) and E_2 (dashed-dotted) and the overall energy content E_{tot} (dotted). The selected pulse-pause $T_P = 2.30$ s corresponds to a local minimum of the maximum absolute value of the eigenvalues of the matrix $\mathbf{Q}$. One clearly observes the modal energy exchange between first and second mode. The overall energy content of the mechanical system E_{tot} decreases faster than the energy content of the corresponding system without impulsive excitation (see grey-colored line). The maximum required torque is about $M_{max} \approx 0.1$ Nm.
Some results of the self-excited case are shown in Fig. (11). Therein, a self-excitation mechanism is introduced by a negative damping coefficient $c_{se} = -0.0005$ Nsm between disk two and the inertial frame, where c_{se} is tuned

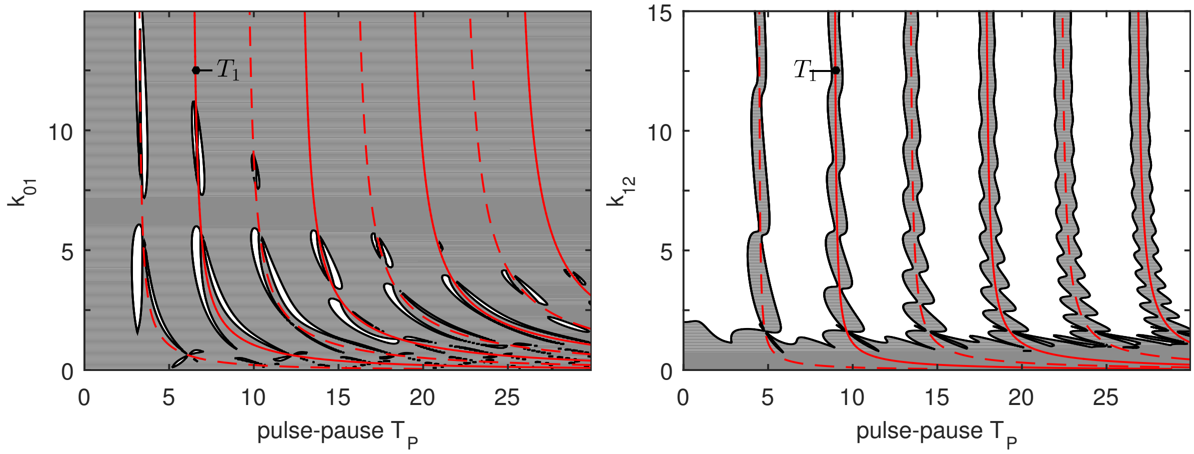

Figure 8: Stability chart for variation of system parameters k_{01} (left) and k_{12} (right), and T_P. Grey/white colored areas denote unstable/asymptotically stable trivial solutions. Energy-neutral case $\vartheta = 1$.

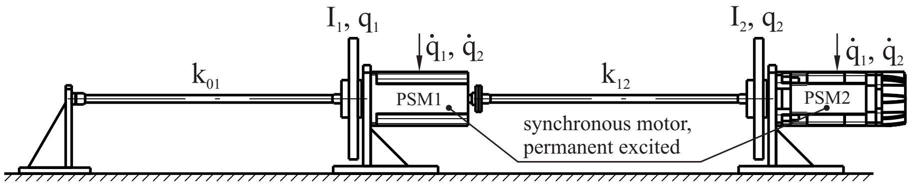

Figure 9: Test rig for investigating modal energy transfer effects in rotor systems.

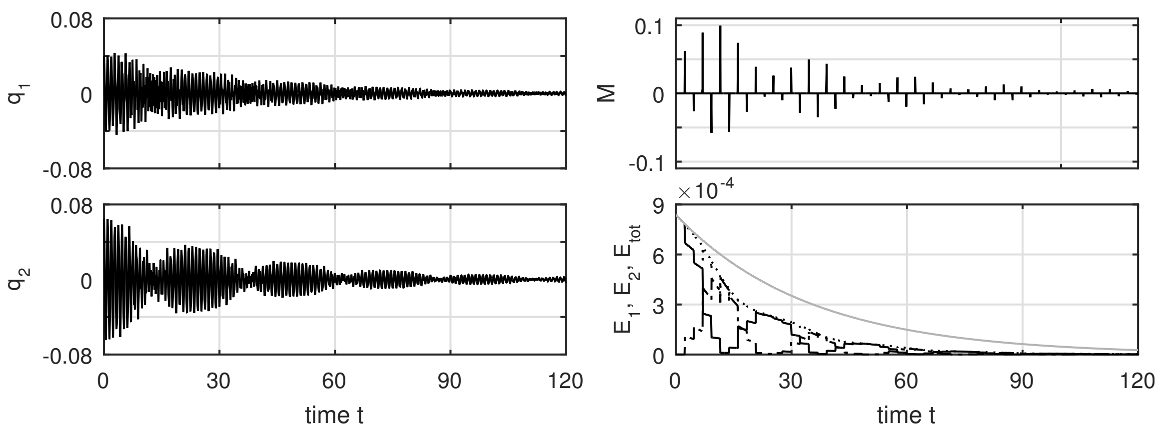

Figure 10: Physical coordinates q_1 and q_2 (left column), torsional moment M, modal energy contents E_1 (solid), E_2 (dashed-dotted) and overall energy content E_{tot} (dotted), (right column). Damped system without self-excitation. E_{tot} of corresponding system without impulsive excitation (grey-colored). Initial condition: IC 1. Sine-shaped impulses applied to disk one. Pulse-pause $T_P = 2.30$ s. Energy-neutral case $\vartheta = 1$.

in a way that the absence of impulsive excitation results in an unstable system. Introducing impulsive excitation to disk one according to the proposed approach causes a repeated modal energy exchange between modes one and two and, hence, allows to stabilize the mechanical system.

4 Conclusion

It was shown in this contribution that impulsively shaped torsional moments are capable of introducing modal energy transfers between low and high modes of vibration in a recurring manner. This allows to utilize the damping properties of the mechanical structure more efficiently and results in a faster decrease of transient vibrations. Moreover, it was shown that the proposed concept is capable of suppressing self-excited vibrations of rotor systems.

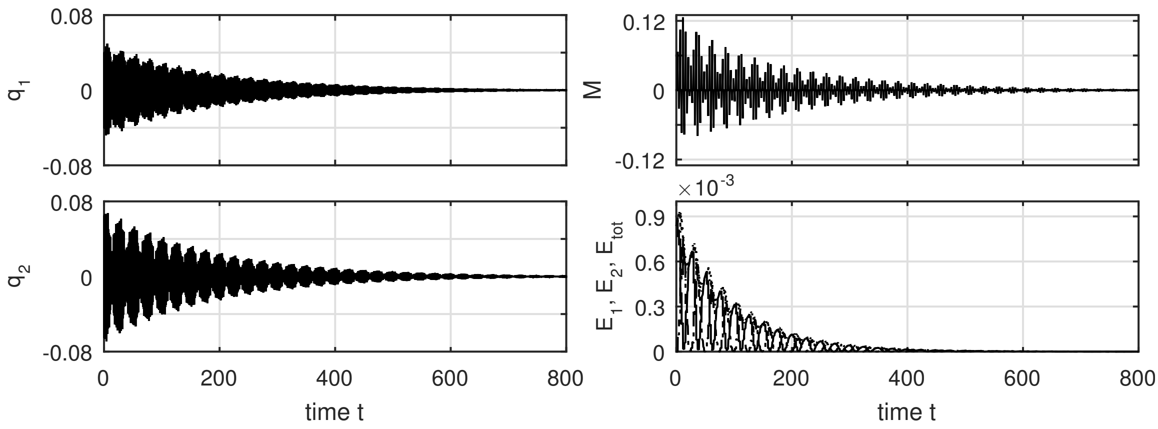

Figure 11: Physical coordinates q_1 and q_2 (left column), torsional moment M, modal energy contents E_1 (solid), E_2 (dashed-dotted) and overall energy content E_{tot} (dotted), (right column). E_{tot} of corresponding system without impulsive excitation (grey-colored). Damped system with self-excitation. Initial condition: IC 1. Sine-shaped impulses applied to disk one. Pulse-pause $T_P = 2.30$ s. Energy-neutral case $\vartheta = 1$.

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