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Development and analysis of radial force waves in electrical rotating machines

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Vibrations in electrical machines lead to undesired operating conditions and noise. The reasons lie in the design of the machine and the lack of precision in manufacturing. In order to avoid excessive vibrations, complex numerical analyses are carried out. This work deals with the development and analysis of electromechanical excitations in asynchronous machines with a short circuit rotor. The time-dependent electromagnetic forces acting on the stator bore are simulated with the method of finite elements. Subsequently, the force waves with respect to the frequencies and amplitudes are analyzed.

1 Introduction

Asynchronous machines are used as drive units in many industrial applications and therefore have great practical importance. The increasing electromobility also leads to an increase in production of electrical motors and to changes in the specifications regarding vibration and sound emission. Numerical tools based on a physical model are often used in order to predict the dynamic behavior of a machine during the development. For the calculation of velocities of surface structures in the context of a vibration analysis, detailed information about eigenfrequencies and eigenforms of the structure is necessary. This information is derived from structural stiffness, damping and mass distribution. For the asynchronous machine, the main focus is set on the stator structure. However, there are vibration modes, which are very different, especially in terms of their form, see e. g. Haas et al. (2016). The different excitation forces and moments have different effects on these modes. The excitation of the structure is important for numerical calculations, as eigenmodes become especially important when excited externally.

This article discusses the magnetic force excitation in asynchronous motors with squirrel cage rotor. It focuses on the radial forces acting in the interface between air gap and the laminated stator core stack. An overview about excitation in induction machines as well as their analytical and numerical calculations are given by Seinsch (1992) and Seinsch (1993). A technique for calculating forces in electromagnetic fields is the finite element method. An essential contribution to the formulation of this method for electromagnetic application can be found in the articles Biro and Preis (1989), Biro et al. (1991) and Biro et al. (1992). This method was implemented in commercial software. One application to electric drives is described in the books Aschendorf (2014a) and Aschendorf (2014b).

Details about the analysis of resulting force waves can be found in the articles by Van der Giet et al. (2008) and Weilharter et al. (2012).

2 Forces in asynchronous squirrel-cage induction motors

One of the most important effect of forces in electrical machines is generating a torque. This desired force component is due to a current-carrying conductor in a magnetic airspace field. It is calculated via the Lorentz force

$$\vec{f} = i \cdot \int_{l} d\vec{s} \times \vec{B} \tag{1}$$

where, in electrical machines, the current i flows in the axial direction and thus the force solely has a tangential component, which results from the magnetic flux density $\vec{B}$ along the length l of the rotor. A tangential vibration component has no influence on sound radiation of the surface of a stator core. Since the air gap torque is constant along the circumference of the stator bore and along the axial length, there is also no excitation through torsional vibration. In machines with skewed stator or rotor slots, axial forces can occur. However, they have only a small technical significance, since the rotor itself adjusts to his axial mean and the axial load on the bearing is very small.

Forces at the interface are much more important for vibrations in electrical machines. These occur in the transitions of materials with very different permeability values. In the case of electrical machines, this transition area is located in the air gap between stator and rotor. The field lines emerge almost perpendicularly from the iron surface (permeability $\approx \mu_0$). Therefore, these interfacial stresses are mainly directed radially. They are determined from the relations

$$\sigma_n = \frac{B_n^2 - B_t^2}{2\mu_0} \quad , \tag{2}$$

$$\sigma_t = \frac{B_n \cdot B_t}{\mu_0} \quad . \tag{3}$$

The indices n and t represent the normal and tangential component of the surface. In order to calculate these forces for an asynchronous machine, a finite element model is created. As mainly radial force waves are considered, a two-dimensional model of the electrical machine is sufficient.

3 Simulation model

Figure 1 shows schematically the structure of an asynchronous machine in a sectional view. In the stator plate, slots are punched out for the coils. The stator consists of two layers of short-pitched coils, where the end turns are connected to a triangular circuit. The squirrel-cage rotor consists of oval bars distributed symmetrically along the circumference. The ends are connected with a short-circuit ring. The air gap is located at the interface between stator and rotor. At the boundary surfaces between the iron and the air gap, Maxwell forces occur. For a numerical

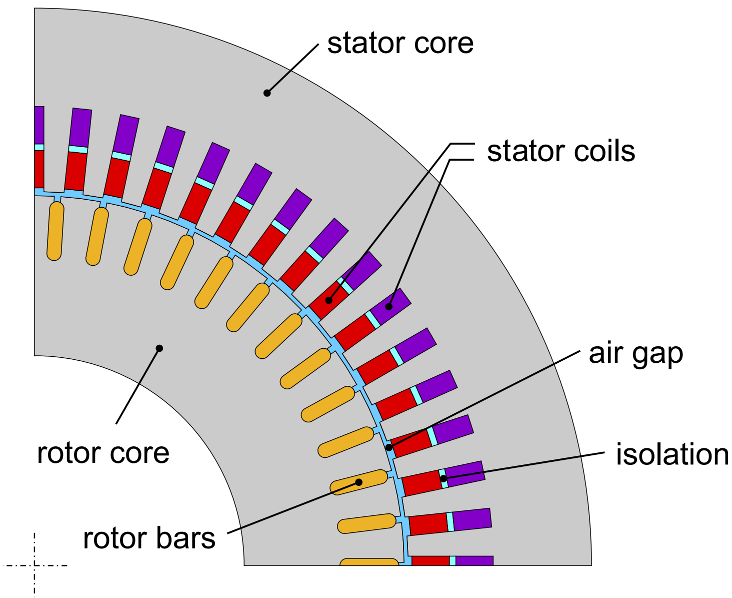

Figure 1: Sectional view of an asynchronous machine.

example, we assumed a parameter set for a machine, which is summarized in Table 1.

The geometry is meshed with a slice finite element for magnetic field calculations (see Figure 2). The element is defined by 8 nodes and has up to 3 degrees of freedom per node. It is based on a vector potential formulation with a magnetic vector potential A_z for each node. It can also be used for electrostatic field calculations with a scalar potential V or coupled to the magnetic field. The third degree of freedom EMF provides the coupling with electronic components, in order to be able and realize oscillating circuits or to interconnect the elements. These components were used for interconnecting the rotor.

The stator current is imposed onto the coils of the upper and lower rod by the specification of current densities. In the case of coils with several strands, a constant distribution of the current over the cross-section is assumed.

Table 1: Machine parameters of an asynchronous machine.

Symbol	Value	Unit	Description
m	3		number of strands
p	2		number of pole pairs
q	5		number of slots per pole and phase
f_n	50	Hz	mains frequency
J	1e6	A/m^2	current density
Q_s	60		number of stator slots
Q_r	50		number of rotor slots
d_{as}	0.85	m	outside diameter of stator
Δ	0.003	m	air gap

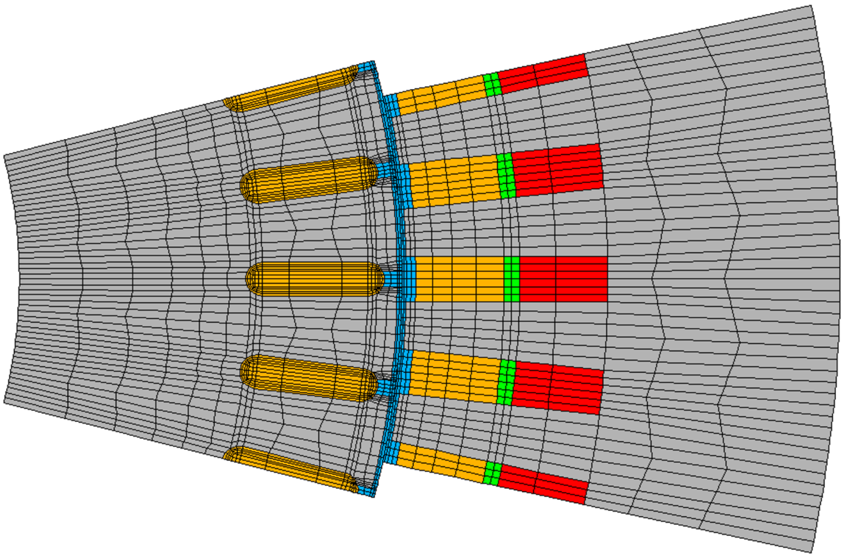

Figure 2: Mesh of the asynchronous machine – Partial segment of rotor and stator.

A suppression of the current can be expected in the massive conductor of the rotor. For verification of the model, static and harmonic analysis are carried out. In both analyses, the influence of the rotating rotor is neglected. In the static analysis, no current is induced in the short-circuit conductors. According to the induction law

$$u_i = \frac{d\Phi}{dt} = \frac{d(\vec{B}\cdot\vec{A})}{dt} = \vec{B}\frac{d\vec{A}}{dt} + \vec{A}\frac{d\vec{B}}{dt} \tag{4}$$

an induced voltage u_i is generated in a coil when a magnetic flux Φ changes in time. In this case, the cause of the flux changes has no influence: It can be either due to a moving coil in a stationary field that is equivalent to a shape change, or due to a time-changing magnetic field dB. Both are not present in a static analysis. In the case of a harmonic analysis, the magnetic field changes with time due to the supply frequency. Here, the massive conductor in the rotor is assumed not to change its position. This means, that the analysis refers to a steady state, where the rotor is not rotating. The frequency of the current, which flows in the rotor conductors, is the same as the supply frequency f_n. The rotor conductors are short-circuited in rings. Due to the Lorentz force according to Eq. (1), the so-called short-circuit torque acts on the rotor.

4 Transient analysis

In order to detect the influence of the rotating rotor field in an operating point of the machine, a transient analysis is performed. The difficulty is, that the finite elements for magnetic field calculations have no degrees of freedom for displacement and so they do not allow a time dependent rotation of the elements. This problem is solved by turning the elements of the rotating system into position in each time step and taking the conditions of the last time step into account. Therefore, the system matrices are rebuilt for the finite element formulation at every stage. As a result of the rotation of the elements, the mesh between the stator and the rotor does not coincide. At the interface in the air gap, coupling equations are formulated at the nodes located there. These equations ensure the correct transfer of the node results from the stator side to the rotor side. They must also be calculated for each step in the iteration.

In order to avoid excitations of all system eigenfrequencies, the external loads of the current densities to the stator coils are starting from zero and increase exponentially to their maximum amplitude. The time sequences of the load is shown in Figure 3. The current densities reach their maximum values after a mains period.

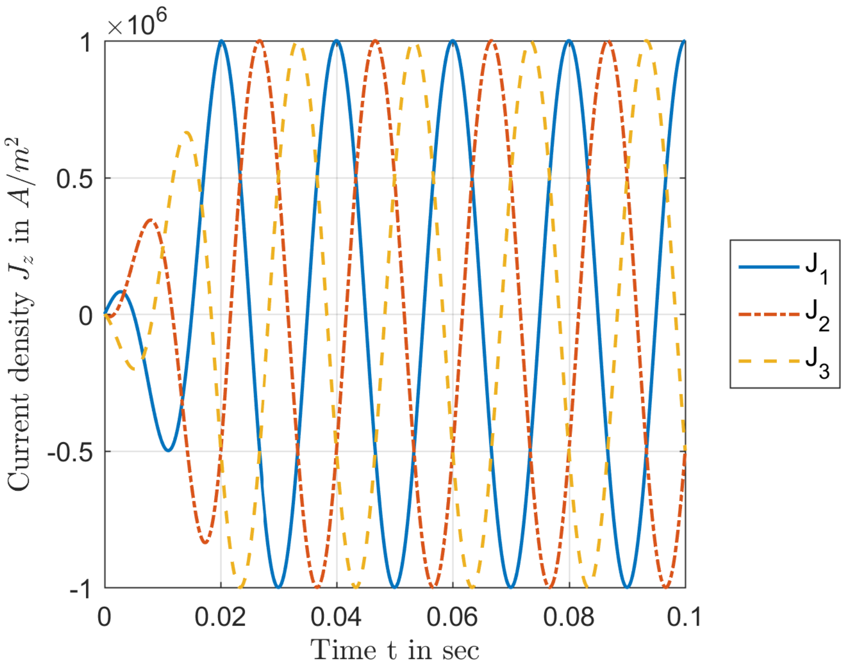

Figure 3: Chronological sequence of the current density.

4.1 Force calculation

The calculation of the force in the finite element method can be carried out in two ways: the first possibility is the calculation by means of a stress tensor

$$\mathbf{T} = \begin{bmatrix} H_x B_x - \frac{1}{2}|\mathbf{H}\,\mathbf{B}| & H_x B_y & H_x B_z \\ H_y B_x & H_y B_y - \frac{1}{2}|\mathbf{H}\,\mathbf{B}| & H_y B_z \\ H_z B_x & H_z B_y & H_z B_z - \frac{1}{2}|\mathbf{H}\,\mathbf{B}| \end{bmatrix} \tag{5}$$

which is also called Maxwell's stress tensor. The force calculation can be applied to any area which is enclosed by a surface Γ and is determined from the equation

$$\vec{f} = \int\limits_{\Omega} \nabla \cdot \mathbf{T} \cdot d\Omega = \oint\limits_{\Gamma} \mathbf{T} \cdot d\Gamma. \tag{6}$$

Another method of calculating the force is developed from the principle of virtual work. The external work of a small element, which is moved in an electromagnetic field, has to equal the internal work. This formulation is based on the law of conservation of energy,

$$\vec{f} \cdot d\vec{s} + dW = 0 \Rightarrow \vec{f} = -\mathrm{grad}(\vec{W}). \tag{7}$$

Both methods are integrated in the FEM software "ANSYS ©". The calculation of the force is activated by setting a flag. The relevant areas for electrical machines are the closed curves along the stator iron and the rotor. In the case of the rotor, the focus of interest is mainly in the calculation of the effective torque, while on the stator side the radial force wave is important for the oscillation excitation.

4.2 Results of transient analysis

In the investigation, the rotor speed is set and kept constant. Due to the supply frequency f_n and the number of pole pairs p of the machine, the synchronous speed of the rotor is obtained from

$$n_1 = \frac{f_n \cdot 60}{p} = \frac{50 \cdot 60}{2} = 1500\text{rpm}. \tag{8}$$

The slip

$$s = \frac{n_1 - n}{n_1} \tag{9}$$

is used as the deviation from synchronous speed.

The calculations for the transient behavior were carried out without slippage. As a first plausibility test and for a qualitative result of the numerical simulation, the time profile of the rotor currents in Figure 4 is considered. When synchronized, the rotor rotates at the same frequency as the magnetic rotating field. According to the induction law in Equation 4, only a voltage in a conductor is induced, when the conductor moves in a stationary field or the magnetic field changes in time. Both conditions are not met by synchronous speed. Therefore, no voltage is induced in the conductors and for this reason no current can flow into them. The decay of the current is also evident in the simulation results. The fluctuations are attributed to the counter-induction on the conductors. In order to validate this statement, studies on the model itself still have to be carried out. When the rotor is not turning, the same frequency of the winding currents is imposed onto the rotor rods by the stator field. This frequency is also clearly visible in the harmonic analysis.

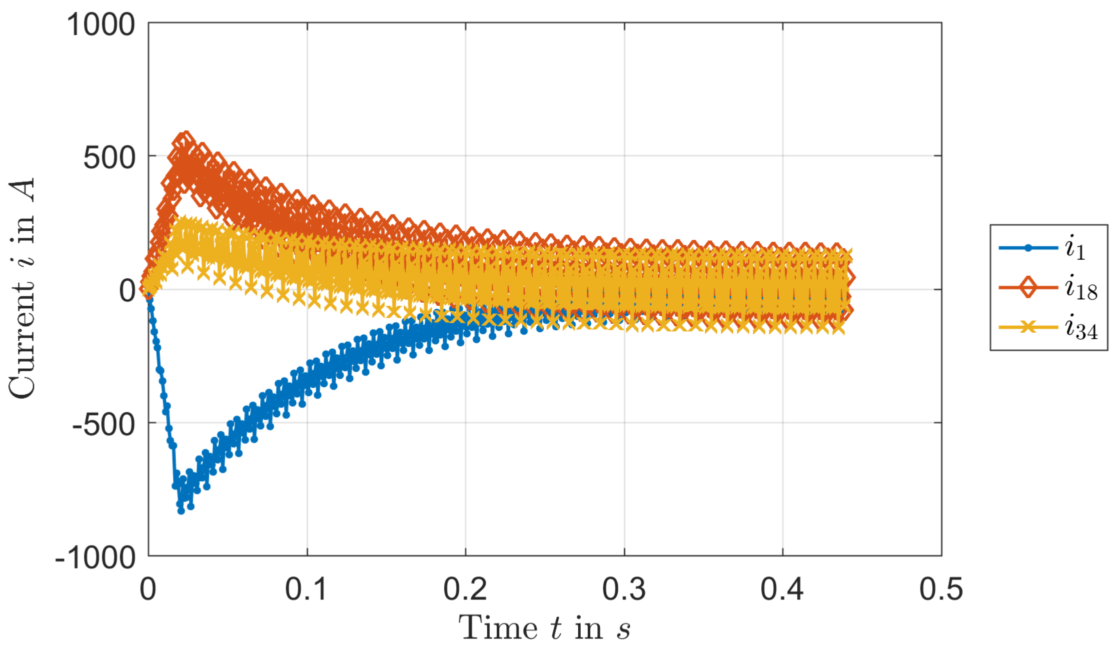

Figure 4: Chronological sequence of the rotor currents.

The consideration of the flux lines in the left part of Figure 5 ensures the functionality of the coupling equations for the interface at the air gap. The lines spread over the entire cross-section of the stator plate and the rotor plate. In addition, the number of poles is clearly shown by the closed curves. The right part of the figure shows the magnitude of the magnetic flux density. It provides information on the validity of the assumption of a linear model. The relationship between the magnetic field strength H and the flux density B is given by the material constant of the permeability μ

$$\vec{B} = \mu \cdot \vec{H}. \tag{10}$$

For dynamo sheets, the iron is saturated from a certain value of the flux density and the initial linear slope in the BH characteristic falls off. This area cannot be expected to be linear. The increased peak values in the sharp corners of the windings are due to constriction effects conditions.

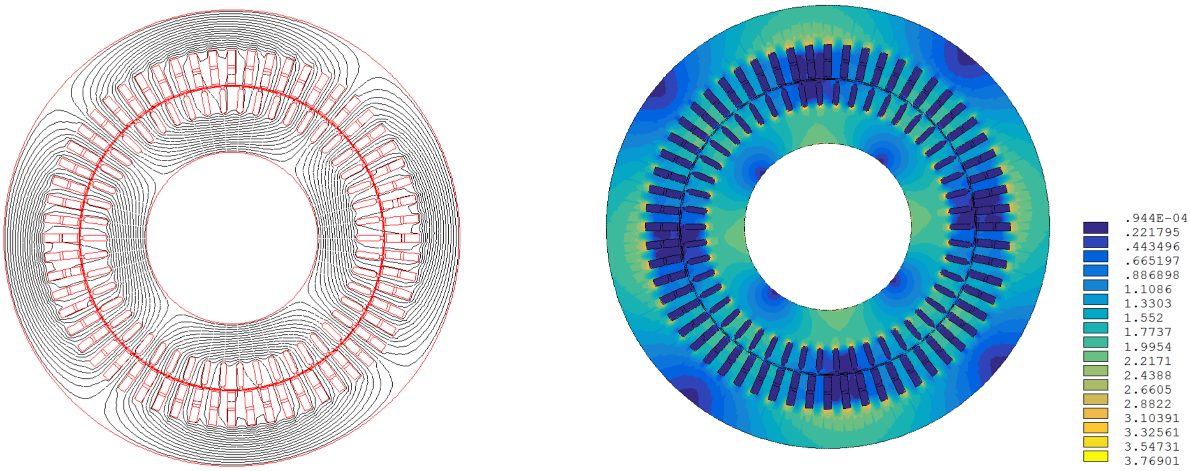

Figure 5: Results of a transient field calculation at stationary speed. Left: flux lines, right: magnetic flux density.

5 Analysis of force waves

The calculated forces are spatially distributed on the stator inner contour. Figure 6 shows the radial forces as vector plots and as quantities developed over the circumference. The force is directed radially inwards. The effect of the forces on the stator therefore corresponds to that of a tensile component. The discontinuity of the course of forces is due to the slots of the stator plate, since forces can only occur at the sudden transitions of strongly changing material properties. This condition is strongly present in electrical machines, especially at the boundary layers between air and iron. The force profile also rotates with the rotating field of the machine.

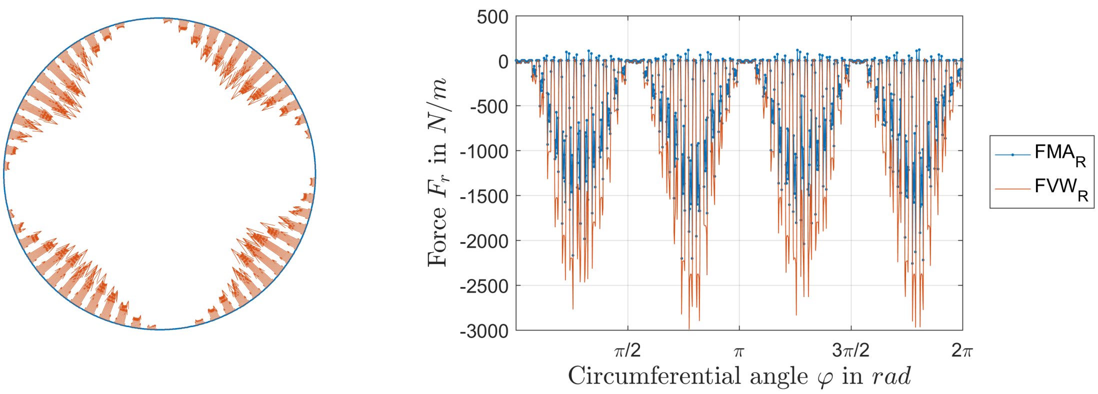

Figure 6: Radial force wave on the stator lamination stack.

A common method for analyzing the force waves is based on the conversion of the signal from time domain to a representation in the frequency domain, performed by a Fast Fourier Transformation. In this way, the temporally harmonic components of the signal are visualized in the form of amplitudes and frequencies (see Figure 7, left picture). This method is applied once more along the circumferential direction to the obtained Fourier spectrum. The result is a spatially harmonic ordinal number. It indicates the order of the occurring force wave for a certain frequency. The force wave is determined from equation

$$F_\nu = \bar{F}_\nu \cos(\nu \cdot \varphi - \omega_\nu t - \psi_n) \tag{11}$$

and is composed of the magnitude of the force $\bar{F}_\nu$, the ordinal number ν, the spatial angle φ, the phase angle ψ_ν and the local angular frequency ω_ν.

From the graphs in Figure 7, it is to be recognized that the dominant forces occur at a frequency of $f = 100Hz$. This corresponds to the double of the mains frequency f_n. The spectrum also clearly shows an equal proportion. This acts constantly as a pulling force. The harmonic components oscillate around this force component. The force wave at this frequency has a dominant spatial order number of $\nu = -4$. This reflects the number of poles of the machine. The negative sign of the order means that the force wave rotates in the opposite direction to the magnetic rotating field. The other dominant orders occur at a distance of $\Delta\nu = 60$. This number coincides with the number of slots of the stator Q_s. The fundamental wave occurs with approximately twice the amplitude $\bar{F}_{-4} \approx 2 \cdot \bar{F}_{-64,56}$ opposite to the other two force waves.

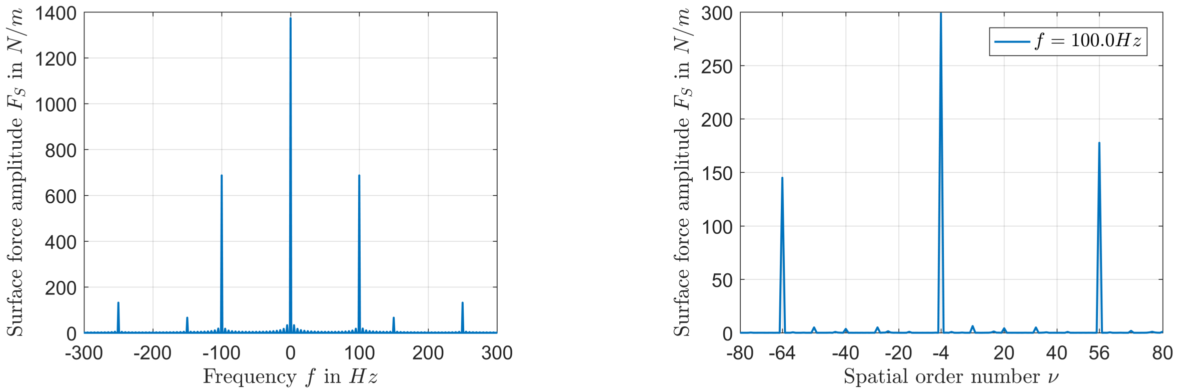

Figure 7: Frequency range and order number of the force wave. Left: the spectrum, right: the spatial order number.

6 Summary

By using the finite element method on an asynchronous machine, its electromagnetic rotating field was calculated at synchronous speed. Subsequently, the results were validated and a force calculation was carried out in the area of interest. These forces were analyzed with regard to their temporal as well as spatial development.

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