Modal Analysis of Rotors under Special Support Conditions

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Gyroscopic rotors in rolling element or tilting pad bearings assume a particular model structure if the bearing load directions coincide. For simple bearing models in suitable coordinates, the stiffness and damping matrix are free from cross-coupling terms. Under these conditions, a relationship between right and left eigenvectors is derived. The rows of the frequency response matrix are related to its columns, and the elements of the frequency response matrix are expressed in terms of eigenvalues and right eigenvectors. The results are illustrated by a numerical example. The theory can be used to facilitate advanced applications of modal testing for a special class of rotors.

1   Introduction

Modal testing of rotors has been complicated by the fact that the right and left eigenvectors do not coincide. If a rotor is excited in one degree of freedom and the vibration responses are measured in all degrees of freedom, the right eigenvectors can be obtained, which correspond to the vibration mode shapes (Nordmann, 1984). Advanced applications such as model correlation or modification prediction require a complete modal model; the left eigenvectors can be obtained by exciting the rotor in all degrees of freedom, but this is often too much effort and sometimes even impossible. It would be helpful if a relationship between right and left eigenvectors could be established in advance. Zhang et al. (1988) as well as Lee (1991) found such relationships for several cases of isotropic rotors. However, bearings are often anisotropic and thus do not meet the assumptions made by these authors. For undamped gyroscopic rotors, Meirovitch (1974) and Meirovitch and Ryland (1979) proved that the left eigenvectors are the complex conjugates of the right eigenvectors. Bucher and Ewins (2001) used a perturbation analysis for lightly damped systems that was developed by Wang and Kirkhope (1994a,b); for a special type of anisotropic bearings, they simplified the relationship between frequency response functions and modal parameters and concluded that a complete modal model can be obtained from an excitation at one point along the shaft in both the x- and y-directions. Nevertheless, a considerable amount of damping may be present in practice. For damped gyroscopic rotors, Gutierrez-Wing (2003) published a general method whereby left eigenvectors can be derived from right eigenvectors; however, this method involves some matrix algebra and the solution of two eigenvalue problems.

As an alternative, one can take advantage of the model structure that is assumed by special types of bearings. Simple models of rolling element and tilting pad bearings indicate that in a coordinate system aligned with the load, the stiffness and damping matrix are free from cross-coupling terms. If bearing load directions coincide, this also applies for the contributions of bearings to the rotor's overall stiffness and damping matrix. A multi-degrees-of-freedom model of the rotor itself is, for instance, described in Genta (2005), where Timoshenko beam elements are used. With respect to the coordinate axes, the symmetric mass, damping, and stiffness matrices appear in separate blocks. The skew-symmetric gyroscopic matrix only contains symmetric off-diagonal submatrices. This leads to a special case of the vibro-acoustical model structure, for which a simple relationship between right and left eigenvectors is known. The rotor model may further include a skew-symmetric circulatory matrix from internal damping, which only contains symmetric off-diagonal submatrices.

In this paper, the cross-coupling properties of rolling element and tilting pad bearings are extracted from literature. The vibro-acoustical model is expressed in velocity and pressure state variables in order to demonstrate the analogy to a special case of the rotordynamic model. This model is extended by a circulatory matrix to account for internal damping. Under these conditions, a simple relationship between right and left eigenvectors is derived. The rows of the frequency response matrix are related to its columns, and the elements of the frequency response matrix are expressed in terms of eigenvalues and right eigenvectors. The results are illustrated by a numerical example. The theory can be used to facilitate advanced applications of modal testing for a special class of rotors.

2   Cross-coupling Properties of Supports

In rotordynamic models, bearings are often considered by 2×2 stiffness and damping matrices which need not be symmetric. These matrices can be split into symmetric and skew-symmetric parts, and principal axes can be found where the symmetric parts are free from cross-coupling terms. The skew-symmetric parts do not change with the angular position of the coordinate system, are cross-coupling in nature, and thus contribute to the overall circulatory and gyroscopic matrices. For journal bearings in general, the principal axes of stiffness and damping may differ among the individual supports of a rotor; any choice of coordinate system may lead to cross-coupling terms in the overall stiffness or damping matrix. The overall stiffness matrix could be decoupled for equally loaded identical journal bearings, but this is a somewhat unrealistic configuration.

Models of rolling element bearings include further restrictions. Krämer (1993) used 2×2 stiffness and damping matrices which are diagonal in a coordinate system aligned with the load. For a rotor whose bearing load directions coincide, this model does not contribute any cross-coupling terms to the overall stiffness or damping matrix. Dietl (1997) assembled 5×5 stiffness and damping matrices of rolling element bearings based on the stiffness and damping coefficients of the individual elasto-hydrodynamic lubrication contacts. One of his models describes the dry Hertzian contact with respect to stiffness and damping and uses an empirical law for elasto-hydrodynamic oil-film damping. For radial, axial, and tilting loads in a plane through the rotor axis, this model is symmetric to that plane. If x denotes the radial load direction, it follows that the resulting stiffness and damping matrices do not include any coupling terms between the local displacement vectors

$$\mathbf{x}_l = \begin{bmatrix} x \\ \varphi_y \end{bmatrix} \quad \text{and} \quad \mathbf{y}_l = \begin{bmatrix} y \\ \varphi_x \end{bmatrix}. \tag{1}$$

This conclusion still holds for a rotor whose bearing loads are confined to the x-z-plane. Such conditions may be realistic, especially if the load is constituted by the rotor's own weight.

Similar models are obtained for tilting pad bearings. According to Someya (1989), the cross-coupling stiffness and damping terms of four pad and five pad bearings disappear in a coordinate system aligned with the load. Dimond et al. (2011) stated that in tilting pad bearings, the cross-coupled stiffness terms are generally three orders of magnitude less than the direct stiffness terms. Experimental results for five pad and four pad bearings were published by Childs et al. (2011); cross-coupled stiffness and damping coefficients were always much smaller than direct ones.

These considerations are summarized as follows. For 2×2 matrix models of rolling element or tilting pad bearings, cross-coupling can be avoided if the coordinate system is aligned with the load. If radial and tilting loads are confined to a plane through the rotor axis, this also holds for a 4×4 matrix model of rolling element bearings.

3   From Vibro-acoustics to Rotordynamics

According to Wyckaert et al. (1996), the vibro-acoustical model can be described by

$$\begin{bmatrix} \mathbf{M}_s & \mathbf{0} \\ \mathbf{K}_c^T & \mathbf{M}_f/\rho \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{s}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_f/\rho \end{bmatrix} \begin{bmatrix} \dot{\mathbf{s}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_s & -\mathbf{K}_c \\ \mathbf{0} & \mathbf{K}_f/\rho \end{bmatrix} \begin{bmatrix} \mathbf{s} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s \\ \dot{\mathbf{q}} \end{bmatrix} \tag{2}$$

with the structural displacement vector $\mathbf{s}$, the sound pressure vector $\mathbf{p}$, the vector of external force loading $\mathbf{f}_s$, the vector of acoustical source loading $\mathbf{q}$, the structural mass matrix $\mathbf{M}_s$, the structural damping matrix $\mathbf{C}_s$, the structural stiffness matrix $\mathbf{K}_s$, the fluid mass matrix $\mathbf{M}_f$, the fluid damping matrix $\mathbf{C}_f$, the fluid stiffness matrix $\mathbf{K}_f$, the coupling submatrix $\mathbf{K}_c$, and the fluid density ρ. Wyckaert et al. (1996) derived a simple relationship between right and left eigenvectors of such systems. If structural velocities $\mathbf{v}$ are chosen as state variables instead of structural displacements $\mathbf{s}$, the vibro-acoustical model (2) becomes

$$\begin{bmatrix} \mathbf{M}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_f/\rho \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{v}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_s & -\mathbf{K}_c \\ \mathbf{K}_c^T & \mathbf{C}_f/\rho \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_f/\rho \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s \\ \dot{\mathbf{q}} \end{bmatrix}. \tag{3}$$

Equation (3) is recognized as a special case of the linear time-invariant rotor model

$$\mathbf{M}\ddot{\mathbf{u}} + (\mathbf{C}+\mathbf{G})\dot{\mathbf{u}} + (\mathbf{K}+\mathbf{N})\mathbf{u} = \mathbf{f}, \qquad (4)$$

which reads

$$\begin{bmatrix} \mathbf{M}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_y \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}} \\ \ddot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_x & \mathbf{G}_0 \\ -\mathbf{G}_0^T & \mathbf{C}_y \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_y \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_x \\ \mathbf{f}_y \end{bmatrix}. \qquad (5)$$

The mass, damping, and stiffness matrix are partitioned into separate symmetric blocks $\mathbf{M}_x$, $\mathbf{M}_y$, $\mathbf{C}_x$, $\mathbf{C}_y$, $\mathbf{K}_x$, and $\mathbf{K}_y$, respectively; if bearing load directions coincide, this applies for the configurations summarized in Section 2. A skew-symmetric gyroscopic matrix $\mathbf{G}$ is taken into account; in the following, $\mathbf{G}_0$ need not be symmetric. The rotordynamic model (5) is extended to include a skew-symmetric matrix

$$\mathbf{N} = \begin{bmatrix} \mathbf{0} & \mathbf{N}_0 \\ -\mathbf{N}_0^T & \mathbf{0} \end{bmatrix}, \qquad (6)$$

where $\mathbf{N}_0$ need not be symmetric; this results in the model

$$\begin{bmatrix} \mathbf{M}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_y \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}} \\ \ddot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_x & \mathbf{G}_0 \\ -\mathbf{G}_0^T & \mathbf{C}_y \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_x & \mathbf{N}_0 \\ -\mathbf{N}_0^T & \mathbf{K}_y \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_x \\ \mathbf{f}_y \end{bmatrix} \qquad (7)$$

under consideration.

4   Eigenvector relations

The right eigenvectors

$$\boldsymbol{\theta}_{rn} = \begin{bmatrix} \boldsymbol{\theta}_{xrn} \\ \boldsymbol{\theta}_{yrn} \end{bmatrix} \qquad (8)$$

of equation (7) satisfy the equation

$$\begin{bmatrix} \lambda_n^2 \mathbf{M}_x + \lambda_n \mathbf{C}_x + \mathbf{K}_x & \lambda_n \mathbf{G}_0 + \mathbf{N}_0 \\ -\lambda_n \mathbf{G}_0^T - \mathbf{N}_0^T & \lambda_n^2 \mathbf{M}_y + \lambda_n \mathbf{C}_y + \mathbf{K}_y \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}_{xrn} \\ \boldsymbol{\theta}_{yrn} \end{bmatrix} = \mathbf{0} \qquad (9)$$

with the n-th eigenvalue λ_n. This is equivalent to

$$\begin{bmatrix} \lambda_n^2 \mathbf{M}_x + \lambda_n \mathbf{C}_x + \mathbf{K}_x & -\lambda_n \mathbf{G}_0 - \mathbf{N}_0 \\ \lambda_n \mathbf{G}_0^T + \mathbf{N}_0^T & \lambda_n^2 \mathbf{M}_y + \lambda_n \mathbf{C}_y + \mathbf{K}_y \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}_{xrn} \\ -\boldsymbol{\theta}_{yrn} \end{bmatrix} = \mathbf{0}. \qquad (10)$$

Transposing equation (10) leads to

$$\begin{bmatrix} \boldsymbol{\theta}_{xrn}^T & -\boldsymbol{\theta}_{yrn}^T \end{bmatrix} \begin{bmatrix} \lambda_n^2 \mathbf{M}_x + \lambda_n \mathbf{C}_x + \mathbf{K}_x & \lambda_n \mathbf{G}_0 + \mathbf{N}_0 \\ -\lambda_n \mathbf{G}_0^T - \mathbf{N}_0^T & \lambda_n^2 \mathbf{M}_y + \lambda_n \mathbf{C}_y + \mathbf{K}_y \end{bmatrix} = \mathbf{0}; \qquad (11)$$

it follows that the left eigenvectors $\boldsymbol{\theta}_{ln}$ of equation (7) are related to the right eigenvectors by

$$\boldsymbol{\theta}_{ln} = \begin{bmatrix} \boldsymbol{\theta}_{xln} \\ \boldsymbol{\theta}_{yln} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}_{xrn} \\ -\boldsymbol{\theta}_{yrn} \end{bmatrix}. \qquad (12)$$

5 Properties of the Frequency Response Matrix

The rotor model (7) is transferred to the frequency domain by taking the Laplace transform and setting the Laplace variable $s=i\omega$. The Laplace transforms of the vectors $\mathbf{x}$, $\mathbf{y}$, $\mathbf{f}_x$, and $\mathbf{f}_y$ are denoted as $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{F}_x$, and $\mathbf{F}_y$, respectively. From the frequency domain model

$$\begin{bmatrix} -\omega^2\mathbf{M}_x + i\omega\,\mathbf{C}_x + \mathbf{K}_x & i\omega\,\mathbf{G}_0 + \mathbf{N}_0 \\ -i\omega\,\mathbf{G}_0^T - \mathbf{N}_0^T & -\omega^2\mathbf{M}_y + i\omega\,\mathbf{C}_y + \mathbf{K}_y \end{bmatrix} \begin{bmatrix} \mathbf{X}(i\omega) \\ \mathbf{Y}(i\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{F}_x(i\omega) \\ \mathbf{F}_y(i\omega) \end{bmatrix} \tag{13}$$

it can be seen that the displacement vector is related to the force vector by

$$\begin{bmatrix} \mathbf{X}(i\omega) \\ \mathbf{Y}(i\omega) \end{bmatrix} = \mathbf{H}(i\omega) \begin{bmatrix} \mathbf{F}_x(i\omega) \\ \mathbf{F}_y(i\omega) \end{bmatrix} \tag{14}$$

with the frequency response matrix

$$\mathbf{H}(i\omega) = \begin{bmatrix} \mathbf{H}_{xx}(i\omega) & \mathbf{H}_{xy}(i\omega) \\ \mathbf{H}_{yx}(i\omega) & \mathbf{H}_{yy}(i\omega) \end{bmatrix} = \begin{bmatrix} -\omega^2\mathbf{M}_x + i\omega\,\mathbf{C}_x + \mathbf{K}_x & i\omega\,\mathbf{G}_0 + \mathbf{N}_0 \\ -i\omega\,\mathbf{G}_0^T - \mathbf{N}_0^T & -\omega^2\mathbf{M}_y + i\omega\,\mathbf{C}_y + \mathbf{K}_y \end{bmatrix}^{-1}. \tag{15}$$

Equation (14) is equivalent to

$$\begin{bmatrix} \mathbf{X}(i\omega) \\ -\mathbf{Y}(i\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{xx}(i\omega) & \mathbf{H}_{xy}(i\omega) \\ -\mathbf{H}_{yx}(i\omega) & -\mathbf{H}_{yy}(i\omega) \end{bmatrix} \begin{bmatrix} \mathbf{F}_x(i\omega) \\ \mathbf{F}_y(i\omega) \end{bmatrix}. \tag{16}$$

From equations (13) and (16), it follows that

$$\begin{bmatrix} \mathbf{H}_{xx}(i\omega) & \mathbf{H}_{xy}(i\omega) \\ -\mathbf{H}_{yx}(i\omega) & -\mathbf{H}_{yy}(i\omega) \end{bmatrix} = \begin{bmatrix} -\omega^2\mathbf{M}_x + i\omega\,\mathbf{C}_x + \mathbf{K}_x & -i\omega\,\mathbf{G}_0 - \mathbf{N}_0 \\ -i\omega\,\mathbf{G}_0^T - \mathbf{N}_0^T & \omega^2\mathbf{M}_y - i\omega\,\mathbf{C}_y - \mathbf{K}_y \end{bmatrix}^{-1}, \tag{17}$$

and this matrix is symmetric; this leads to the properties

$$\mathbf{H}_{xx}(i\omega) = \mathbf{H}_{xx}^T(i\omega), \tag{18}$$

$$\mathbf{H}_{yy}(i\omega) = \mathbf{H}_{yy}^T(i\omega), \tag{19}$$

and

$$\mathbf{H}_{yx}(i\omega) = -\mathbf{H}_{xy}^T(i\omega) \tag{20}$$

of the frequency response matrix $\mathbf{H}(i\omega)$.

Using equations (18), (19), and (20), each column of the frequency response matrix can be related to a row. For a column located in the left half of $\mathbf{H}(i\omega)$, the corresponding row is obtained from the transpose after changing the sign in the lower half of the column. For a column located in the right half of $\mathbf{H}(i\omega)$, the corresponding row is obtained from the transpose after changing the sign in the upper half of the column.

6 Modal Analysis

In terms of eigenvalues and eigenvectors, the elements of the frequency response matrix $\mathbf{H}(i\omega)$ are expressed as

$$H_{jk}(i\omega) = \sum_{n=1}^{N} \left(\frac{\theta_{rnj}\theta_{lnk}}{a_n(i\omega - \lambda_n)} + \frac{\bar{\theta}_{rnj}\bar{\theta}_{lnk}}{\bar{a}_n(i\omega - \bar{\lambda}_n)} \right), \qquad j,k = 1,\ldots,N, \tag{21}$$

in which θ_{rnj} is the j-th component of the right eigenvector $\mathbf{\theta}_{rn}$, θ_{lnk} is the k-th component of the left eigenvector $\mathbf{\theta}_{ln}$, a_n is a constant for each mode, and N is the number of degrees of freedom (Irretier, 1999); the bar above a symbol denotes the complex conjugate. Equation (21) shows that the right eigenvectors $\mathbf{\theta}_{rn}$ can be identified from the k-th column of $\mathbf{H}(i\omega)$ if the values of θ_{lnk}/a_n are selected in advance; the left eigenvectors $\mathbf{\theta}_{ln}$ can be identified from the j-th row of $\mathbf{H}(i\omega)$ after selecting the values of θ_{rnj}/a_n.

For rotors modelled by equation (7), the j-th row of $\mathbf{H}(i\omega)$ is obtained from the j-th column using the properties (18), (19), and (20). This means that the left eigenvectors can be identified from a column of the frequency response matrix, which only requires an excitation in one degree of freedom.

Alternatively, the left eigenvectors can be determined from the right eigenvectors using the relationship (12). If equation (12) is inserted into equation (21), the elements of $\mathbf{H}(i\omega)$ become

$$H_{abjk}(i\omega) = \sigma \sum_{n=1}^{N} \left(\frac{\theta_{arnj}\theta_{brnk}}{a_n(i\omega - \lambda_n)} + \frac{\bar{\theta}_{arnj}\bar{\theta}_{brnk}}{\bar{a}_n(i\omega - \bar{\lambda}_n)} \right), \qquad a \in \{x,y\}, \;\; b \in \{x,y\}, \;\; j,k = 1,\ldots,N/2, \tag{22}$$

$\sigma=1$ for $b=x$ and $\sigma=-1$ for $b=y$; θ_{xrnj}, θ_{xrnk}, θ_{yrnj}, and θ_{yrnk} are the j-th and k-th components of $\mathbf{\theta}_{xrn}$ and $\mathbf{\theta}_{yrn}$, respectively.

From equation (22), it follows that

$$H_{xxjk}(i\omega) = H_{xxkj}(i\omega), \tag{23}$$

$$H_{yyjk}(i\omega) = H_{yykj}(i\omega), \tag{24}$$

and

$$H_{yxjk}(i\omega) = -H_{xykj}(i\omega). \tag{25}$$

Thus, the properties (18), (19), and (20) of the frequency response matrix have been rederived, and it is obvious that they are also valid separately for each mode.

7 Numerical Example

To illustrate the theory, the example of a rigid rotor supported by two identical anisotropic bearings is taken from Lee and Joh (1993) and described by the coordinates used in this paper. A schematic view of the rotor is given in Figure 1.

The system submatrices appearing in equation (7) are given as

$$\mathbf{M}_x = \mathbf{M}_y = \frac{1}{L^2} \begin{bmatrix} mL_2^2 + I_t & mL_1L_2 - I_t \\ mL_1L_2 - I_t & mL_1^2 + I_t \end{bmatrix}, \tag{26}$$
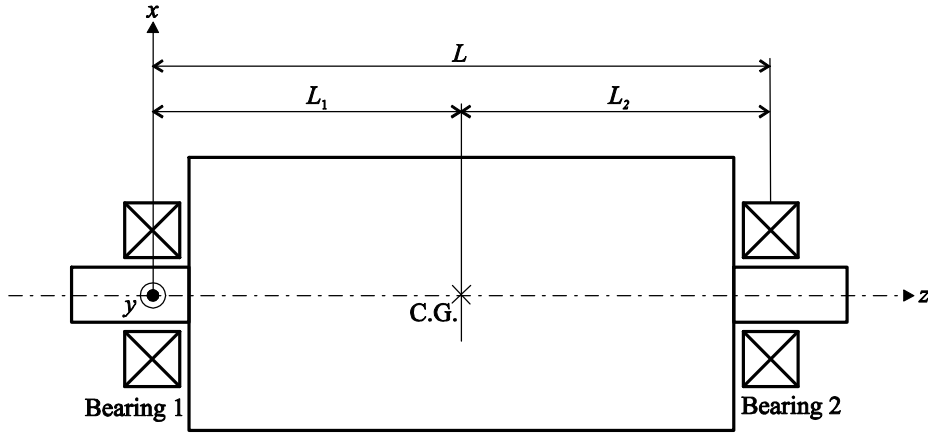
Figure 1. Rigid rotor-bearing system

$$\mathbf{C}_x = \begin{bmatrix} c_{xx} & 0 \\ 0 & c_{xx} \end{bmatrix}, \qquad \mathbf{C}_y = \begin{bmatrix} c_{yy} & 0 \\ 0 & c_{yy} \end{bmatrix}, \tag{27}$$

$$\mathbf{K}_x = \begin{bmatrix} k_{xx} & 0 \\ 0 & k_{xx} \end{bmatrix}, \qquad \mathbf{K}_y = \begin{bmatrix} k_{yy} & 0 \\ 0 & k_{yy} \end{bmatrix}, \tag{28}$$

$$\mathbf{G}_0 = \frac{\Omega}{L^2} \begin{bmatrix} I_p & -I_p \\ -I_p & I_p \end{bmatrix} + \begin{bmatrix} c_{xy} & 0 \\ 0 & c_{xy} \end{bmatrix}, \quad \text{and} \quad \mathbf{N}_0 = \begin{bmatrix} k_{xy} & 0 \\ 0 & k_{xy} \end{bmatrix}, \tag{29}$$

where m is the rotor mass, I_t and I_p are the transverse and polar mass moments of inertia about the centre of gravity of the rotor, Ω is the rotational speed, and the distances L, L_1, and L_2 are depicted in Figure 1; c_{xx}, c_{xy}, c_{yx}, c_{yy} and k_{xx}, k_{xy}, k_{yx}, k_{yy} are the damping and stiffness coefficients of the bearings. Due to the fact that $c_{yx}=-c_{xy}$ and $k_{yx}=-k_{xy}$, the rotor under consideration complies with the model structure (7). For model parameters according to Table 1, the right and left eigenvalue problems are solved, and the resulting eigenvalues and eigenvectors are listed in Table 2. It is obvious that each pair of corresponding right and left eigenvectors satisfies the relationship (12). Moreover, the frequency response functions $H_{xy11}(i\omega)$ and $H_{yx11}(i\omega)$ are depicted in Figure 2 and apparently satisfy the property (25). For the rotor under consideration, all modal parameters can be identified exactly from one row or column of the frequency response matrix even though the bearings are not isotropic.

Table 1. Model parameters of the rigid rotor-bearing system.

m	8 kg
L_1/L	0.5
L_2/L	0.5
I_t/L^2	0.45 kg
I_p/L^2	0.15 kg
Ω	10 000 rpm
c_{xx}	300 Ns/m
c_{yy}	300 Ns/m
c_{xy}	20 Ns/m
k_{xx}	3 900 000 N/m
k_{yy}	4 100 000 N/m
k_{xy}	50 000 N/m

Table 2. Modal parameters of the rigid rotor-bearing system (complex conjugates not included).

| mode	eigenvalue λ_n(s^{-1})	right eigenvector $\boldsymbol{\theta}_{rn}$	left eigenvector $\boldsymbol{\theta}_{ln}$
1 backward	−38.9+988.0i	1 1 0.2524+0.1184i 0.2524+0.1184i	1 1 −0.2524−0.1184i −0.2524−0.1184i
1 forward	−36.1+1010.5i	1 1 3.2220−1.5459i 3.2220−1.5459i	1 1 −3.2220+1.5459i −3.2220+1.5459i
2 backward	−165.2+1922.1i	1 −1 −0.0011+0.8564i 0.0011−0.8564i	1 −1 0.0011−0.8564i −0.0011+0.8564i
2 forward	−168.2+2297.1i	1 −1 −0.0012−1.1387i 0.0012+1.1387i	1 −1 0.0012+1.1387i −0.0012−1.1387i
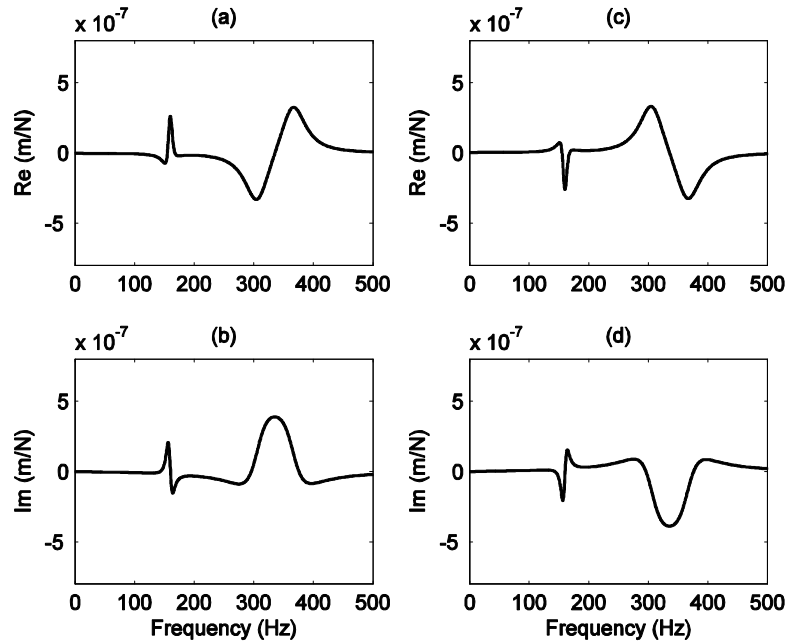
Figure 2. Selected frequency response functions of the rigid rotor-bearing system. (a): real part of $H_{xy11}(i\omega)$, (b): imaginary part of $H_{xy11}(i\omega)$, (c): real part of $H_{yx11}(i\omega)$, (d): imaginary part of $H_{yx11}(i\omega)$.

8   Conclusion

For rolling element and tilting pad bearings, the stiffness and damping matrix of the rotor are free from cross-coupling terms if bearing loads are confined to a coordinate plane through the rotor axis. Under these conditions, a simple relationship between right and left eigenvectors has been derived; the rows of the frequency response matrix have been related to its columns, and the elements of the frequency response matrix have been expressed in terms of eigenvalues and right eigenvectors. The results allow for advanced applications of modal testing even if the rotor can only be excited in one degree of freedom.

Acknowledgement

References

Bucher, I.; Ewins, D.-J.: Modal analysis and testing of rotating structures. *Phil. Trans. R. Soc. Lond. A*, 359, (2001), 61-96.

Childs, D.-W.; Vannini, G.; Delgado, A.: Tilting-pad bearings: measured frequency characteristics of their rotordynamic coefficients. In: *Proc. Fortieth Turbomachinery Symposium* Houston, Texas, Sept. 12-15 (2011).

Dietl, P.: *Damping and Stiffness Characteristics of Rolling Element Bearings.* PhD Thesis, Vienna University of Technology, Vienna (1997).

Dimond, T.; Younan, A.; Allaire, P.: A review of tilting pad bearing theory. *International Journal of Rotating Machinery*, (2011), Article ID 908469, 23 pages, doi:10.1155/2011/908469.

Genta, G.: *Dynamics of rotating systems.* Springer, New York (2005).

Gutierrez-Wing, E.-S.: *Modal analysis of rotating machinery structures.* PhD Thesis, Imperial College, London (2003).

Irretier, H.: Mathematical foundations of experimental modal analysis in rotor dynamics. *Mechanical Systems and Signal Processing*, 13(2), (1999), 183–191.

Krämer, E.: *Dynamics of rotors and foundations*. Springer, Berlin (1993).

Lee, C.-W.: A complex modal testing theory for rotating machinery. *Mechanical Systems and Signal Processing*, 5(2), (1991), 119–137.

Lee, C.-W.; Joh, Y-D.: Theory of excitation methods and estimation of frequency response functions in complex modal testing of rotating machinery. *Mechanical Systems and Signal Processing*, 7(1), (1993), 57–74.

Meirovitch, L.: A new method for the solution of the eigenvalue problem for gyroscopic systems. *AIAA Journal*, 12(10), (1974), 1337-1342.

Meirovitch, L.; Ryland, G.: Response of slightly damped gyroscopic systems. *JSV*, 67(1), (1979), 1-19.

Nordmann, R.: Identification of the modal parameters of an elastic rotor with oil film bearings. *Transactions of the ASME – Journal of Vibration, Acoustics, Stress, and Reliability in Design* 106, (1984), 107–112.

Someya, T.: *Journal-Bearing Databook.* Springer, Berlin, Heidelberg (1989).

Wang, W.; Kirkhope, J.: New eigensolutions and modal analysis for gyroscopic/rotor systems, Part 1: Undamped systems. *JSV*, 175(2), (1994a), 159–170.

Wang, W.; Kirkhope, J.: New eigensolutions and modal analysis for gyroscopic/rotor systems, Part 2: Perturbation analysis for damped systems. *JSV*, 175(2), (1994b), 171–183.

Wyckaert, K.; Augusztinovicz, F.; Sas, P.: Vibro-acoustical modal analysis: reciprocity, model symmetry, and model validity. *The Journal of the Acoustical Society of America*, 100(5), (1996), 3172–3181.

Zhang, Q.; Lallement, G.; Fillod, R.: Relations between the right and left eigenvectors of non-symmetric structural models. Applications to rotors. *Mechanical Systems and Signal Processing*, 2(1), (1988), 97–103.

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