Study of the Influence of a Delayed Yielding Phenomenon in Magnetorheological Damping Devices on the Vibration Attenuation of a Jeffcott Rotor

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A frequently used technological solution for attenuation of lateral oscillations of rotating machines consists in inserting damping devices between the rotor and its stationary part. To achieve their optimum performance, their damping effect must be controllable. This is enabled by magnetorheological squeeze film dampers. As resistance against the flow of magnetorheological liquids depends on magnetic induction, the change of magnetic flux passing through the lubricating film enables to control the damping effect. The developed mathematical model of a magnetorheological squeeze film damper is based on assumptions of the classical theory of lubrication. The oil is represented by bilinear material. The pressure distribution in the damper gap is described by the modified Reynolds equation. The dependence of the stationary value of the yielding shear stress on magnetic induction is approximated by a power function. Its dependence on time is governed by a convolution integral, which enables to take into account that the yielding shear stress depends not only on the instant value of magnetic induction but also on its history in the past. In cavitated regions it is considered that the yielding shear stress drops to zero. The developed mathematical model of the damper was implemented in the computational procedures for analysis of lateral vibrations of a flexibly supported Jeffcott rotor loaded by the disc unbalance. The carried out simulations showed that the rising value of the delayed yielding time constant reduces the damping effect. The development of a novel model of a magnetorheological squeeze film damper based on representing the lubricating oil by bilinear material taking into account the delayed yielding phenomenon, its implementation into the procedures for analysis of oscillations of rotating machines, increasing their computational stability, and learning more on the effect of magnetorheological damping devices on behaviour of flexible rotors are the principal contributions of this article.

1   Introduction

The unbalance of rotating machines produces their lateral oscillation. A frequently used technological solution for its attenuation consists in inserting damping devices between the rotor and its stationary part. To achieve their optimum performance, their damping effect must be controllable. This is enabled by magnetorheological squeeze film dampers. The magnetorheological oils belong to the category of fluids with a yielding shear stress. The flow occurs only in those areas in which the shear stress exceeds a limit value - the yielding shear stress. In regions, called a core, where the shear stress is lower the magnetorheological oil behaves as solid matter.

The principles of work of magnetorheological dampers and practical experience with their applications are reported in a number of publications (Gong et al, 2014; Aravindhan and Gupta, 2006; Carmignani et al., 2006). The mathematical model of a squeeze film magnetorheological damper, in which the lubricant is represented by Bingham material, is reported in (Zapoměl et al., 2012; Zapoměl and Ferfecki, 2010). The modelling of magnetorheological oil by bilinear material (Zapoměl et al., 2016b) arrives at increase of stability of the computational procedures, in which the mathematical model of the magnetorheological squeeze film damper is implemented.

In this paper, the mathematical model of a magnetorheological squeeze film damper developed in Zapoměl et al., 2016b has been extended. The model was completed with the phenomenon of the delayed yielding which takes into account the time history of magnetic induction on the yielding shear stress magnitude. Unlike of Zapoměl and Ferfecki, 2016a, the new mathematical model was implemented into computational procedures for investigation of lateral vibrations of flexible rotors.

The implementation of the delayed yielding phenomenon in the mathematical model of a magnetorheological squeeze film damper, its application into the computational procedures for analysis of vibrations of flexible rotors, increase of their numerical stability, and learning more on the effect of magnetorheological damping devices on the oscillations attenuation of flexible rotors are the principal contributions of this article.

2   A Novel Model of a Magnetorheological Squeeze Film Damper

The main parts of a magnetorheological squeeze film damper (Figure 1) are two concentric rings between which there is a layer of magnetorheological oil. The inner ring is coupled with the shaft through a rolling element bearing and with the damper housing by a squirrel cage spring. Lateral vibration of the shaft squeezes the oil film, which produces the damping effect. Magnetic flux generated in the damper coils passes through the lubricant and as its resistance against the flow depends on magnetic induction the change of the applied current changes the damping force.
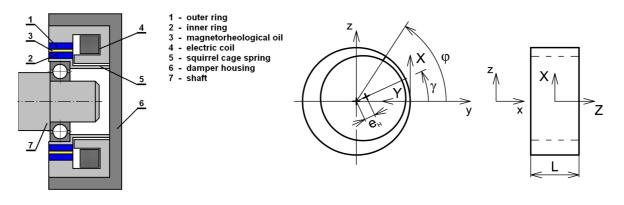

Figure 1. MR squeeze film damper	Figure 2. The damper coordinate system

The developed mathematical model of the damper is based on assumptions of the classical theory of lubrication. The magnetorheological oil is represented by bilinear material the yielding shear stress of which is a function of magnetic induction. In addition it is assumed that both the geometric and design parameters of the damper make it possible to consider it as short (Childs, 1993; Hori, 2006).

The pressure distribution in the full oil film is governed by the Reynolds equation (1) - (2) adapted for bilinear material (Zapoměl et al., 2016b)

$$\frac{\partial}{\partial Z}\left(\frac{1}{\eta_C}h^3 p'\right)=12\dot{h}, \quad \text{for} \quad 0 \le Z \le Z_C, \tag{1}$$

$$\frac{\partial}{\partial Z}\left[\frac{1}{\eta}\left(h^3 p' + 3h^2\tau_y + 8\frac{\tau_C^3}{p'^2} - 12\frac{\tau_y\tau_C^2}{p'^2}\right) - \frac{8}{\eta_C}\frac{\tau_C^3}{p'^2}\right]=12\dot{h}, \quad \text{for} \quad \dot{h}<0, \quad Z>Z_C, \tag{2}$$

$$Z_C = -\frac{\tau_C h^2}{6\eta_C \dot{h}}, \tag{3}$$

$$p'_C = -\frac{2\tau_C}{h}. \tag{4}$$

p is the pressure, p' stands for the pressure gradient in the axial direction, Z is the axial coordinate perpendicular to the axes X, Y and defining position in the oil film (Figure 2), h is the film thickness, τ_y is the yielding shear stress, τ_C is the shear stress at the core border, η_C, η are the dynamic viscosities of the oil inside and outside the core area, respectively, Z_C defines the axial coordinate of the location where the core touches the rings surfaces, p'_C denotes the pressure gradient in the axial direction at that location Z_C, and (˙) denotes the first derivative with respect to time.

The thickness of the lubricating film depends on the position of the inner damper ring relative to the outer one Hori, 2006).

$$h = c - e_H \cos(\varphi - \gamma).$$ (5)

c is the width of the gap between the inner and outer rings of the damper, e_H is eccentricity of the rotor journal centre, φ is the circumferential coordinate, and γ is the position angle of the line of centres (Figure 2).

In that part of the damper gap where the oil film thickness rises with time a cavitation is assumed. The pressure of the medium in cavitated areas is considered to remain constant and equal to the pressure in the ambient space.

The y and z components of the magnetorheological damping forces F_{mry}, F_{mrz} are calculated by integration of the pressure distribution p_d which takes into account different pressure profiles in noncavitated and cavitated regions

$$F_{mry} = -2R \int_0^{\frac{L}{2}} \int_0^{2\pi} p_d \cos\varphi \, \mathrm{d}\varphi \, \mathrm{d}Z,$$ (6)

$$F_{mrz} = -2R \int_0^{\frac{L}{2}} \int_0^{2\pi} p_d \sin\varphi \, \mathrm{d}\varphi \, \mathrm{d}Z.$$ (7)

R is the mean radius of the damper gap, L is the damper length and φ is the circumferential coordinate (Figure 2).

Based on experiments, dependence of the stationary value of the yielding shear stress on magnetic induction is approximated by a power function

$$\tau_y = k_y B^{n_y}.$$ (8)

B is magnetic induction and k_y and n_y are material constants of the magnetorheological oil.

Due to the physical substance, the yielding shear stress of magnetorheological fluids depends not only on the instant value of magnetic induction but also on its history in the past. This time dependence is described by a convolution integral, which is consequently transformed to the differential form

$$T_y \dot{\tau}_y + \tau_y = k_y B^{n_y}.$$ (9)

T_y is the delayed yielding time constant, which expresses the rapidity of the change of the yielding shear stress on the change of magnetic induction. In cavitated regions it is considered that the yielding shear stress becomes zero there.

In the developed mathematical model the damper housing is considered to be composed of a series of meridian segments and each segment is considered to be a divided core of an electromagnet with the gap filled by magnetorheological oil. This enables to determine magnetic induction as a function of the applied current and thickness of the oil film at any location in the oil film around the damper circumference

$$B = k_B \mu_0 \mu_r \frac{I}{h}.$$ (10)

μ_0, μ_r are the vacuum and relative permeabilities of the magnetorheological oil, respectively, I is the applied current, k_B is the design parameter that is defined as a product of the number of the coil turns and the magnetic efficiency. More details on its determination can be found in Ferfecki et al., 2017.

3   The Computational Model of the Studied Rotor

The investigated rotor (Figure 3) consists of a flexible shaft and of one rigid disc. At both its ends it is coupled with the stationary part by two magnetorheological squeeze film dampers. The rotor rotates at a constant angular speed, is loaded by its weight and excited by the disc unbalance. The squirrel cage springs of both dampers are pre-bent to be eliminated their deflection caused by the rotor weight. The whole system can be considered as symmetric relative to the disc middle plane.

The task was to study the effect of the delayed yielding phenomenon on performance of magnetorheological damping devices and on attenuation of vibrations of flexible rotors.
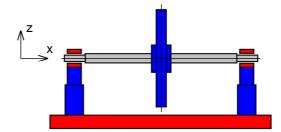

Figure 3. Scheme of the investigated rotor system

The rotor is implemented in the computational model by a Jeffcott one and the magnetorheological squeeze film dampers are represented by linear springs and nonlinear force couplings. With respect to the system symmetry, the lateral vibration of the rotor is governed by a set of four differential equations

$$m\ddot{y} + (b_P + b_M)\dot{y} - b_M\dot{y}_B + k_S y - k_S y_B + \dot{\vartheta}b_M z - \dot{\vartheta}b_M z_B = me_T\dot{\vartheta}^2 \cos\vartheta , \tag{11}$$

$$m\ddot{z} + (b_P + b_M)\dot{z} - b_M\dot{z}_B + k_S z - k_S z_B - \dot{\vartheta}b_M y + \dot{\vartheta}b_M y_B = me_T\dot{\vartheta}^2 \sin\vartheta - mg , \tag{12}$$

$$-b_M\dot{y} + b_M\dot{y}_B - k_S y + (k_S + 2k_B)y_B - \dot{\vartheta}b_M z + \dot{\vartheta}b_M z_B = 2F_{mry} , \tag{13}$$

$$-b_M\dot{z} + b_M\dot{z}_B - k_S z + (k_S + 2k_B)z_B + \dot{\vartheta}b_M y - \dot{\vartheta}b_M y_B = 2F_{mrz} . \tag{14}$$

m is the disc mass, k_S is the stiffness of the shaft, k_B is the stiffness of each squirrel cage spring, b_P and b_M are the linear viscous coefficients related to the rotor environmental and material damping, e_T is the eccentricity of the disc centre of gravity, g is the gravity acceleration, y, z, y_B, z_B are displacements of the disc and shaft journal centres in the horizontal and vertical directions, ϑ is the angle of the rotor rotation and ($\ddot{\ }$) denotes the second derivative with respect to time.

The solution of the governing equations was obtained by application of a numerical time integration method based on the Adams-Moulton one.

4   The Results of the Computational Simulations

The main technological parameters of the investigated rotor are: 250 kg the mass of the disc, 20 MN/m the bending stiffness of the shaft, 600 Ns/m is the shaft material viscous damping coefficient (material damping), 10 Ns/m the disc viscous damping coefficient (external damping), 15 kgmm the disc unbalance, 5 MN/m the stiffness of each squirrel cage spring, 0.3 Pas the oil viscosity (if not effected by a magnetic field), 150 mm the mean diameter of the damper gap, 50 mm the damper land length, 0.8 mm the width of damper clearance, 60 the damper design parameter, 5, 10 000 PaT$^{-1.1}$, 1.1 the oil relative permeability and the proportional and exponential constants, respectively. The rotor turns at constant angular speed of 150 rad/s.

A simple dynamical analysis shows that the resonance frequencies of the rotor system related to the cases when the dampers exhibit no damping (e.g. no magnetorheological oil is supplied to the dampers) and when they work

in the overdamped regime are 163 and 283 rad/s, respectively. It implies the rotor operates below the first critical speed.
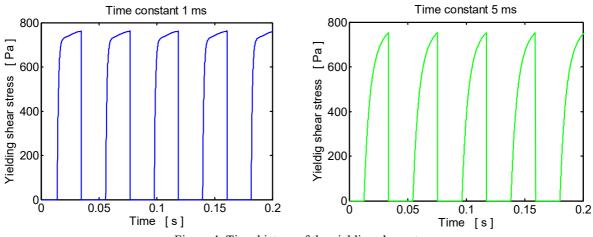

Figure 4. Time history of the yielding shear stress

Time history of the yielding shear stress for the current of 0.2 A and two delayed yielding time constants (1 ms and 5 ms) referred to the specified location on the damper circumference are depicted in Figure 4. The results show that rising magnitude of the delayed yielding time constant makes the response of the oil on the change of the magnetic field slower and slightly reduces the maximum value of the yielding shear stress. During the time periods when the cavitation takes place at the investigated location the yielding shear stress drops to zero.
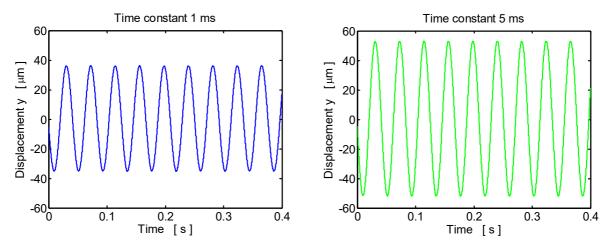

Figure 5. Time history of the disc displacement in the horizontal direction (time constants 1 ms, 5 ms)

Time histories of the disc centre displacement in the horizontal direction are depicted in Figure 5 for two magnitudes of the time constant (1 ms, 5 ms). The analysis of the results shows that higher value of the time constant increases amplitude of the oscillations. It implies it reduces the damping effect.

The steady state orbits of the disc centre for the applied current of 0.2 A and for two values of the time constant 1 ms and 5 ms are drawn in Figure 6. The trajectories are circular which is caused by prestressing the squirrel cage springs. The weight of the disc shifts the orbits in the vertical direction. The results show that higher value of the time constant reduces the damping effect and increases amplitude of the disc vibration.

The steady state trajectories of the rotor journal centre are drawn in Figure 7 for two values of the delayed yielding time constants of 1 ms and 5 ms and for the applied current of 0.2 A. Rising magnitude of the time constant arrives at increase of the orbit size.
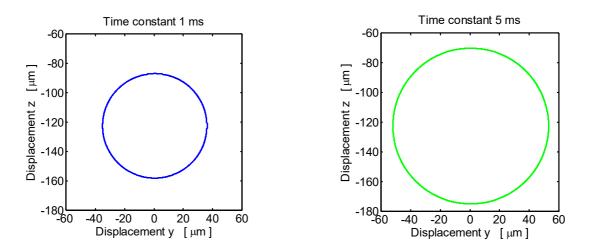
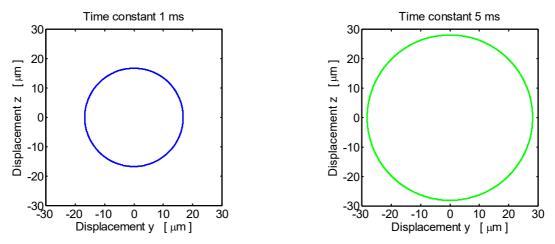Figure 6. Steady state orbits of the disc centre (time constants 1 ms, 6 ms)

Figure 7. Steady state orbits of the rotor journal centre (time constants 1 ms, 5 ms)

5   Conclusion

This paper presents a new mathematical model of a short magnetorheological squeeze film damper for rotordynamic applications in which the delayed yielding phenomenon is implemented. The model is based on assumptions of the classical theory of lubrication. The oil is represented by bilinear material. The pressure distribution in the full oil film in the damper gap is governed by the adapted Reynolds equation. The dependence of the stationary value of the yielding shear stress on magnetic induction is approximated by a power function and its dependence on time by a convolution integral. The developed mathematical model of the magnetorheological damper was implemented in the computational procedures for investigating lateral vibrations of flexible rotors. The results of the simulations show that rising magnitude of the delayed yielding time constant reduces rapidity of the response of the magnetorheological oil on the change of a magnetic field which arrives at reduction of the damping effect. The implementation of the delayed yielding phenomenon in the mathematical model of a magnetorheological squeeze film damper, its implementation into the computational procedures for analysis of lateral vibrations of flexible rotors, the increase of their numerical stability due to representing the magnetorheological oil by bilinear material, and learning more on the effect of magnetorheological damping devices on behaviour of flexible rotors are the principal contributions of this article.

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