DOI: 10.24352/UB.OVGU-2017-056

TECHNISCHE MECHANIK, 34, 2, (2014), 104 - 113
submitted: April 15, 2013

A Comparison between the 3D and the Kirchhoff-Love Solutions for Cylinders under Creep-Damage Conditions

A. Zolochevsky, S. Sklepus, A. Galishin, A. Kühhorn, M. Kober

Dedicated to Prof. Johannes Altenbach on the occasion of his 80th birthday

The 3D theory of creep deformation and creep damage growth in a cylindrical body of revolution is considered. Constitutive equations describing the creep deformation and unilateral creep damage in initially isotropic materials with characteristics dependent on the kind of the stress state are discussed. The numerical approach to obtain the 3D solution for a cylinder under creep-damage conditions is developed. The numerical results generated by the proposed 3D theory are compared with the analogous results based on the Kirchhoff-Love model. Thin and moderately thick cylindrical shells of revolution made from the material with the creep and creep damage characteristics dependent on the kind of the stress state are considered in this comparative analysis. The influence of tension–compression asymmetry on the stress–strain state and damage evolution with time in thin and moderately thick cylindrical shells is discussed. If it is assumed that the properties of the shell material do not depend on the kind of the stress state, this could lead to an overprediction of the creep damage growth and a significant underestimation of the failure initiation time.

1 Introduction

Creep and creep damage characteristics of initially isotropic polycrystalline materials depend strongly on the kind of the stress state (Altenbach et al., 1995b; Kletschkowski et al., 2004; Zolochevskij, 1988; Zolochevsky, 1982). In some cases, this dependence can be identified using the data of basic experiments like uniaxial tension, uniaxal compression and pure torsion (Altenbach et al., 1995b; Zolochevskij, 1988; Zolochevsky, 1982). In the past, a number of constitutive models coupled with three series of these basic experiments have been developed (Altenbach et al., 1995a, 1995b; Betten et al., 1998; Kawai, 2002; Mahnken, 2003; Rubanov, 1987; Zolochevskij, 1988; Zolochevsky, 1982; Zolochevsky et al., 2007). The effect of the kind of the stress state on the creep deformation and creep damage growth in thin shells using the Kirchhoff-Love hypotheses has been investigated by Altenbach and Zolochevsky (1991), Betten and Borrmann (1987), Zolochevsky (1982) and Zolochevsky et al. (2007). In the following, moderately thick shells have been considered in Galishin et al. (2009) and Zolochevsky et al. (2009a). Thin plates of complex shape under plane stress conditions and bending are considered by Zolochevsky et al. (2009b) and Zolochevsky et al. (2012), respectively. The 3D solutions related to the analysis of creep deformation and creep damage taking into account the effect of the kind of the stress state have been also obtained (Mahnken, 2003; Zolochevsky et al., 2012). The aim of the present paper is to compare the 3D and the Kirchhoff-Love solutions for thin and moderately thick shells of revolution made of a material with tension-compression asymmetry. This comparative analysis is important under theoretical and practical points of view (Ganczarski and Skrzypek, 2001, 2004). Furthermore, the obtained 3D solutions will serve as a background of the future research for moderately thick shells to compare the results within the 3D formulation and the results within the refined kinematic model proposed recently by Galishin et al. (2009) and Zolochevsky et al. (2009a). The main feature of this refined kinematic model is the consideration of transversal shear effect and of nonlinear distribution over shell thickness of the components of the strain tensor as well as of the angles of the normal element rotation. Thus, this future comparative analysis can be important taking into account the physical phenomena that can occur, for example, in a cylindrical shell on the micro- and nanoscale (Altenbach and Eremeyev, 2011; Duan et al., 2009).

The material under consideration in this paper is an aluminum alloy AK4-1T at the temperature of 473 K discussed broadly in the literature (Altenbach et al., 1995b; Betten, 2002; Betten et al., 1998; Galishin et al. 2009; Kawai, 2002; Mahnken, 2003; Rubanov, 1987; Zolochevskij, 1988; Zolochevsky, 1982; Zolochevsky et al., 2007, 2009a, 2009b, 2012). The creep curves of this light alloy up to rupture are analyzed in the coordinates ϕ - t. Here ϕ is the specific dissipation,

$$\phi = \int_0^t \sigma_{kl} \dot{p}_{kl} \mathrm{d}t \tag{1}$$

σ_{kl} is the Cauchy stress tensor, p_{kl} is the infinitesimal creep strain tensor, and the dot above the symbol denotes the derivative with respect to time *t*. It is interesting to note that the creep deformation and creep damage growth in this aluminum alloy under uniaxial tension, uniaxial compression, and pure shear realized under pure torsion conditions must be considered as independent phenomena. Furthermore, it is established experimentally (Zolochevsky et al., 2009a) that the dissipation defined by equation (1) and, therefore, the level of the creep damage in the light alloy under discussion are largest under pure torsion conditions. In other words, the intensity of dislocation creep, as well as the intensity of the material deterioration due to the growth of voids, microscopic cavities and microcracks at the boundary grains are largest under pure torsion and are smallest under uniaxial compression.

2 Constitutive Model

The constitutive equation and creep damage evolution equation proposed by Zolochevsky (1982) at small strains for initially isotropic polycrystalline materials with characteristics dependent on the kind of the stress state have the following structure

$$\dot{p}_{kl} = \sigma_e^m \left(1 - \frac{\phi}{\phi_*}\right)^{-q} \left(\frac{AI_1\delta_{kl} + B\sigma_{kl}}{\sigma_2} + \alpha C\delta_{kl}\right), \quad \dot{\phi} = \sigma_e^{m+1}\left(1 - \frac{\phi}{\phi_*}\right)^{-q} \tag{2}$$

with

$$\sigma_e = \sigma_2 + \alpha\sigma_1 , \ \sigma_1 = CI_1 , \ \sigma_2 = \sqrt{AI_1^2 + BI_2} \ , \ I_1 = \sigma_{kl}\delta_{kl} \ , \ I_2 = \sigma_{kl}\sigma_{kl} \ , \ \phi \in [0, \phi_*]$$

Here σ_e is the equivalent stress, I_1 and I_2 are the first and the second invariants of the stress tensor, α is a weight coefficient, δ_{kl} is the Kronecker delta, and ϕ_* is a critical value of the specific dissipation that corresponds to the creep rupture time. Material parameters *m*, *q*, *A*, *B* and *C* can be found (Zolochevsky, 1982) using the results of the basic experiments, such as, $\dot{p}_{11} = K_+\sigma_{11}^m\left(1-\frac{\phi}{\phi_*}\right)^{-q}$, $\phi = \sigma_{11}p_{11}$ (in tension); $\dot{p}_{11} = -K_- \,|\,\sigma_{11}\,|^m \left(1-\frac{\phi}{\phi_*}\right)^{-q}$, $\phi = \sigma_{11}p_{11}$ (in compression) and $2\dot{p}_{12} = K_0\sigma_{12}^m\left(1-\frac{\phi}{\phi_*}\right)^{-q}$, $\phi = 2\sigma_{12}p_{12}$ (under pure torsion). The correlation for the aluminum alloy AK4-1T at the temperature of $473\ \mathrm{K}$ between the experimental data and approximations of the creep curves with the critical value $\phi_* = \frac{1}{2}\left(3I_2 - I_1^2\right)\left(a - bI_1\right)$ and with values of the material constants $K_+ = 55.0\,\mathrm{GPa}^{-m}\mathrm{h}^{-1}$, $K_- = 22.5\ \mathrm{GPa}^{-m}\mathrm{h}^{-1}$, $K_0 = 1.14\cdot 10^4\ \mathrm{GPa}^{-m}\mathrm{h}^{-1}$, $m = 8$, q = 3, a = 0.4 GPa^{-1}, b = 0.4 GPa^{-2} can be considered as satisfactory (Zolochevsky et al., 2009a). Parameters in equations (2) can be found as $B = 0.5K_0^{\frac{2}{m+1}}$, $A = 0.25\left(K_+^{\frac{1}{m+1}} + K_-^{\frac{1}{m+1}}\right)^2 - B$, $\alpha C = 0.5\left(K_+^{\frac{1}{m+1}} - K_-^{\frac{1}{m+1}}\right)$ and have the numerical values $\alpha\, C = 0.0738\ \mathrm{GPa}^{-\frac{m}{m+1}}\ \mathrm{h}^{-\frac{1}{m+1}}$, $B = 3.99\,\mathrm{GPa}^{-\frac{2m}{m+1}}\ \mathrm{h}^{-\frac{2}{m+1}}$, $A = -1.77\,\mathrm{GPa}^{-\frac{2m}{m+1}}\ \mathrm{h}^{-\frac{2}{m+1}}$. Note that equations (2) reflect the effect of the kind of the stress state on the creep deformation and the unilateral creep damage effect. The reader who is interested in detail, is refereed to Altenbach et al. (1995b) and Zolochevsky et al. (2011).

3 Basic Equations of the 3D Theory

A 3D body of revolution made from the material with the creep and creep damage characteristics dependent on the kind of the stress state is considered with reference to the cylindrical coordinate system r, z, φ with origin 0. Here z is directed along the axis of rotation, r is the distance from the axis (radius) and φ is the angular coordinate. Let the meridional sections of the cylindrical body in the plane $r0z$ have the shape of domain Ω with the boundary $\partial\Omega$. The 3D body is considered under the action of the surface forces, applied to a part of the surface $\partial\Omega_p$. The kinematic boundary conditions for rates of displacements $\dot{u}_r$ and $\dot{u}_z$ along the axes $0r$ and $0z$ are given on the remaining part of the body $\partial\Omega_u$. Let the body be subjected to axisymmetric loading, and volume forces are equal to zero. The components of the total infinitesimal strain tensor are assumed to be the sum of the elastic components, defined according to the generalized Hooke's law for the initially isotropic material, and the creep components given by equations (2).

The variational problem for the determination of the stress-strain state in the 3D body under study with axisymmetric loading consists of minimizing the following functional of Lagrange (Zolochevsky et al., 2011)

$$\Lambda(\dot{u}_r, \dot{u}_z) = 0.5\iint_{\Omega}\left[\lambda_1\left(\dot{u}_{r,r}^2 + \dot{u}_{z,z}^2 + \dot{u}_r^2 r^{-2}\right) + G\left(\dot{u}_{r,z} + \dot{u}_{z,r}\right)^2 + 2\lambda\left(\dot{u}_{r,r}\dot{u}_{z,z} + \dot{u}_r\left(\dot{u}_{r,r} + \dot{u}_{z,z}\right)r^{-1}\right)\right] r\mathrm{d}r\mathrm{d}z -$$

$$- \iint_{\Omega}\left[\dot{u}_{r,r}\dot{N}_r^f + \dot{u}_{z,z}\dot{N}_z^f + \dot{u}_r\dot{N}_\varphi^f r^{-1} + \dot{N}_{rz}^f\left(\dot{u}_{r,z} + \dot{u}_{z,r}\right)\right] r\mathrm{d}r\mathrm{d}z - \int_{\partial\Omega_p}\left(\dot{P}_n^0\dot{u}_n + \dot{P}_\tau^0\dot{u}_\tau\right)\mathrm{d}\partial\Omega \tag{3}$$

Here $(\ldots)_{,r} = \dfrac{\partial(\ldots)}{\partial r}$, $(\ldots)_{,z} = \dfrac{\partial(\ldots)}{\partial z}$; n is the direction of the external normal to the boundary $\partial\Omega$ at each point, τ is the direction of the tangent to the boundary at each point; $n_r = \cos(n, r)$, $n_z = \cos(n, z)$; $\dot{u}_n = \dot{u}_r n_r + \dot{u}_z n_z$, $\dot{u}_\tau = \dot{u}_z n_r - \dot{u}_r n_z$; $\lambda_1 = \lambda + 2G$, $\lambda = \dfrac{E\nu}{(1-2\nu)(1+\nu)}$, $G = \dfrac{E}{2(1+\nu)}$; E is the Young's modulus, ν is the Poisson's ratio; P_n^0 and P_τ^0 are the normal and tangential components of the contour forces. The rates of "fictitious forces" due to creep are given by

$$\dot{N}_r^f = \left[\lambda_1\dot{p}_r + \lambda\left(\dot{p}_z + \dot{p}_\varphi\right)\right],\ \dot{N}_z^f = \left[\lambda_1\dot{p}_z + \lambda\left(\dot{p}_r + \dot{p}_\varphi\right)\right],\ \dot{N}_\varphi^f = \left[\lambda_1\dot{p}_\varphi + \lambda\left(\dot{p}_r + \dot{p}_z\right)\right],\ \dot{N}_{rz}^f = 2G\dot{p}_{rz} \tag{4}$$

The problem is related to that of finding the displacement rates $\dot{u}_r$, $\dot{u}_z$ which yield to an extreme value of the functional in equation (3), assuming that components of the creep strain rate tensor $\dot{p}_r$, $\dot{p}_z$, $\dot{p}_\varphi$, $\dot{p}_{rz}$ are given by equations (2) as some known functions of the coordinates r, z at each fixed instant of time.

It is clear that the analysis of the stress-strain state under creep conditions is that of determining the solution to a physically nonlinear initial/variational problem. Minimizing the functional given by equation (3) must be considered simultaneously with the initial value problem (with respect to time) for the ordinary differential equations (2), as well as, for the following equations

$$\frac{\mathrm{d}u_r}{\mathrm{d}t} = \dot{u}_r,\ \frac{\mathrm{d}u_z}{\mathrm{d}t} = \dot{u}_z,$$

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}t} = \lambda\left(\dot{\varepsilon}_z + \dot{\varepsilon}_\varphi - \dot{p}_z - \dot{p}_\varphi\right) + \lambda_1\left(\dot{p}_r - \dot{p}_r\right),\ \frac{\mathrm{d}\sigma_z}{\mathrm{d}t} = \lambda\left(\dot{\varepsilon}_r + \dot{\varepsilon}_\varphi - \dot{p}_r - \dot{p}_\varphi\right) + \lambda_1\left(\dot{p}_z - \dot{p}_z\right), \tag{5}$$

$$\frac{\mathrm{d}\sigma_{\varphi}}{\mathrm{d}t}=\lambda\left(\dot{\varepsilon}_r+\dot{\varepsilon}_z-\dot{p}_r-\dot{p}_z\right)+\lambda_1\left(\dot{\varepsilon}_{\varphi}-\dot{p}_{\varphi}\right),\ \frac{\mathrm{d}\sigma_{rz}}{\mathrm{d}t}=G\left(\dot{\gamma}_{rz}-2\dot{p}_{rz}\right)$$

which are complemented by the relationship between strain rates and displacement rates, i.e.

$$\frac{\mathrm{d}\varepsilon_r}{\mathrm{d}t}=\dot{u}_{r,r},\ \frac{\mathrm{d}\varepsilon_z}{\mathrm{d}t}=\dot{u}_{z,z},\ \frac{\mathrm{d}\varepsilon_{\varphi}}{\mathrm{d}t}=\dot{u}_r r^{-1},\ \frac{\mathrm{d}\gamma_{rz}}{\mathrm{d}t}=\dot{u}_{r,z}+\dot{u}_{z,r} \tag{6}$$

Initial conditions at the reference state $t=0$ include natural conditions $p_r=p_z=p_{\varphi}=p_{rz}=\phi=0$ as well as the solution of the elastic variational problem of minimizing the functional that can be obtained from equation (3), considering the displacements u_r, u_z and surface forces P_n^0, P_τ^0 instead of its rates and putting the creep terms $N_r^f=N_z^f=N_{\varphi}^f=N_{rz}^f=0$.

For the solution of a 3D problem formulated above, the fourth-order Runge-Kutta-Merson method of time integration with automatic time step control, combined with the Ritz method and R-functions theory (Rvachev, 1963) has been used.

4 Numerical Example of a Thin Shell

A cylinder of revolution with the radius of the middle surface $R_0=0.1\ \mathrm{m}$, thickness $h=0.01\ \mathrm{m}$ and length $L=0.2$ m is considered. The material of the cylinder is an aluminum alloy AK4-1T at the temperature of $473\ \mathrm{K}$. The elastic constants are $E=60\ \mathrm{GPa}$ and $\nu=0.35$. Both ends of the cylinder are free from external loads, but they are fixed in such a manner that the edges are restrained from radial displacements. The inner surface $r=r_{inn}$ of the cylinder is free of loads, $r_{inn}=R_0-h/2$. The outer surface $r=r_{out}$ of the cylinder is loaded by a normal pressure $q_r=14.2857\ \mathrm{MPa}$, $r_{out}=R_0+h/2$. Taking into account the symmetry of the cylinder, only one half of the cylinder will be considered with the following boundary conditions

$$\dot{u}_z=0,\ \frac{\partial\dot{u}_r}{\partial z}=0\ \text{ for } z=0, \tag{7}$$

$$\dot{u}_r=0,\ \dot{\sigma}_z=0\ \text{ for } z=L/2, \tag{8}$$

$$\dot{\sigma}_r=\dot{\sigma}_{rz}=0\ \text{ for } r=r_{inn}, \tag{9}$$

$$\dot{\sigma}_r=-\dot{q}_r=0,\ \dot{\sigma}_{rz}=0\ \text{ for } r=r_{out} \tag{10}$$

The particular structure of solutions that satisfy the main boundary conditions for the displacement rates in the linearized variational problem under consideration at each instant of time can be presented in the form

$$\dot{u}_r=\omega_1\Phi_1-\omega_0 D_1^{(\omega_0)}\left(\omega_1\Phi_1\right), \tag{11}$$

$$\dot{u}_z=\omega_2\Phi_2, \tag{12}$$

where $\omega_0=\frac{2}{L}z\left(\frac{L}{2}-z\right)\geq 0$ is a horizontal band Ω_0 between the lines $z=0$ and $z=\frac{L}{2}$ ($\omega_0=0$, $\omega_{0,n}=-1$ on the border $\partial\Omega_0$, $\omega_0>0$ inside the band); $\omega_1=\frac{L}{2}-z\geq 0$ is a half-plane Ω_1 lying below the line $z=\frac{L}{2}$ ($\omega_1=0$, $\omega_{1,n}=-1$ on the border $\partial\Omega_1$ and $\omega_1>0$ inside of the $\partial\Omega_1$); $\omega_2=z\geq 0$ is a half-plane Ω_2 lying above the line $z=0$ ($\omega_2=0$, $\omega_{2,n}=-1$ on the border $\partial\Omega_2$ and $\omega_2>0$ inside of $\partial\Omega_2$); $D_1^{(\omega_0)}=\frac{\partial\omega_0}{\partial r}\frac{\partial}{\partial r}+\frac{\partial\omega_0}{\partial z}\frac{\partial}{\partial z}$ is the differential operator introduced by Rvachev (Rvachev and Sheiko, 1995). The indefinite components Φ_1, Φ_2 in the solution structure given by equations (11), (12) can be approximated by the finite series

$$\Phi_1(r,z)=\sum_{n=1}^{N_1}C_n^{(1)}f_n^{(1)}(r,z),\ \Phi_2(r,z)=\sum_{n=1}^{N_2}C_n^{(2)}f_n^{(2)}(r,z) \tag{13}$$

where $C_n^{(1)}$, $C_n^{(2)}$ are indefinite coefficients, and $\left\{f_n^{(1)}, f_n^{(2)}\right\}$ is a known system of the linear-independent basis functions taken at fixed time instant *t*. In the present paper, bicubic Schönberg splines are used for this purpose. In this case, the systems of splines are built on a uniform rectangular mesh $N_r\times N_z$, where N_r, N_z are the number of segments along the axis $0r$ and $0z$, respectively.

The 3D solution for the cylinder under consideration has been obtained with the initial value of the time step $\Delta t=10^{-3}$ h and with the accuracy $\delta=10^{-4}$ while $\phi\le\phi_*$. A mesh is checked and accepted finally as $N_r=10$, $N_z=40$. The failure initiation time is found to be $t_{*1}=2050$ h. The damage variable reaches its critical value near the inner surface of the cylinder at the point with the coordinates $r_*=9.50469\cdot10^{-2}$ m, $z_*=0.0117275\cdot10^{-2}$ m.

The numerical results generated by the proposed 3D theory have been compared for this example with the analogous results of the thin shell theory based on the Kirchhoff-Love hypotheses. According to the recommendations by Boyarshinov (1973), the pressure at the middle surface of a shell is found as $q_0=q_r\ r_{out}/R_0=14.3571$ MPa. Both ends of the cylindrical shell are simply-supported. The symmetry of the thin cylindrical shell is taken into account, and boundary conditions for kinematic and force parameters at the middle surface of a cylinder are introduced by the analogy with equations (7) and (8). The one-dimensional initial/boundary-value problem of the Kirchhoff-Love theory formulated in Altenbach and Zolochevsky (1991), Zolochevsky (1982) and Zolochevsky et al. (2007) is solved using the fourth-order Runge-Kutta-Merson method of time integration with the combination of the discrete orthogonal shooting method of Godunov. Calculations are performed for discretization with 101 points in the axial direction and 11 points along the thickness of the shell with the initial value of the time step $\Delta t=10^{-3}$ h and with accuracy $\delta=10^{-8}$ while $\phi\le0.99\phi_*$. The failure initiation time is found as $t_{*2}=1948$ h. The damage variable reaches its critical value at the inner surface of a shell in its central part at the point with the coordinates $r_*=9.5\cdot10^{-2}$ m, $z_*=0$. It is not difficult to calculate that the relative error for the failure initiation time that occurs using the Kirchhoff-Love model is equal in the present case to 4.9%. Thus, the correlation between the lifetime predictions based on the 3D and the Kirchhoff-Love solutions can be considered as satisfactory for thin shells. In this case the relative error for the failure initiation time does not exceed the thin shell characteristic ratio (thickness/ radius of the middle surface) multiplied by 100, $h/R_0\cdot100=10\%$. Furthermore, even keeping in the mind the large natural scatter of creep test data (more than 20%) which is, however, of the same order in both numerical cases under study, it is possible to make the conclusion that the large scatter of creep test data does not restrict the conclusion given above for thin shells.

Tables 1- 3 show other results of a comparison between the 3D and the Kirchhoff-Love solutions for a thin cylindrical shell of revolution made from the material with the creep and creep damage characteristics dependent on the kind of the stress state. The numerical results are given for the central part of the shell. It is seen that in the elastic case as well as in the first and second stages of creep the maximum relative error that occurs using the Kirchhoff-Love theory for a thin shell under study does not exceed 10%, i.e. the estimation of this error can be related to the magnitude of the shell characteristic ratio (multiplied by 100) $h/R_0\cdot100$ (%). However, this disagreement is growing with time in the third stage of the creep process with accumulation of the creep damage.

Table 1. A comparison between the 3D (above the line) and the Kirchhoff-Love (below the line) solutions on the inner surface of a thin cylindrical shell at different instants of time

t, h	σ_θ, MPa	ϕ, $\mathrm{MJ/m^3}$	$p_\theta \cdot 10^2$
0.0	-163.5 -164.7	0.0 0.0	0.0 0.0
100.0	-163.0 -171.0	0.099 0.115	-0.065 -0.074
500.0	-154.2 -161.1	0.513 0.718	-0.337 -0.456
1000.0	-138.1 -145.0	1.074 1.581	-0.712 -1.009
1500.0	-124.9 -132.0	1.680 2.621	-1.122 -1.691
t_{*1} t_{*2}	-114.7 -88.8	2.440 3.918	-1.6508 –

Table 2. A comparison between the 3D (above the line) and the Kirchhoff-Love (below the line) solutions on the outer surface of a thin cylindrical shell at different instants of time

t, h	σ_θ, MPa	ϕ, $\mathrm{MJ/m^3}$	$p_\theta \cdot 10^2$
0.0	-144.4 -142.4	0.0 0.0	0.0 0.0
100.0	-146.1 -141.0	0.063 0.110	-0.043 -0.054
500.0	-156.0 -152.1	0.372 0.616	-0.251 -0.398
1000.0	-169.0 -164.0	0.783 1.345	-0.532 -0.870
1500.0	-179.9 -174.0	1.241 2.251	-0.847 -1.461
t_{*1} t_{*2}	-191.3 -189.8	1.841 3.437	-1.26 –

Table 3. A comparison between the 3D (above the line) and the Kirchhoff-Love (below the line) solutions for the radial displacements in the middle surface of a thin cylindrical shell at different instants of time

t, h	0.0	100.0	500.0	1000.0	1500.0	t_{*1} t_{*2}
$u_r \cdot 10^4$, m	-2.51 -2.55	-3.05 -3.30	-5.44 -6.90	-8.68 -11.90	-12.28 -18.29	-16.97 -26.26

5 Numerical Example of a Moderately Thick Shell

The next comparison is related to the numerical analysis of the creep deformation of a moderately thick cylindrical shell of revolution from the aluminum alloy AK4-1T at the temperature of 473 K. The geometrical parameters of the shell are $R_0 = 0.1$ m, $h = 0.02$ m and $L = 0.2$ m. Thus, the shell characteristic ratio h/R_0 in this example is equal to 1/5. The elastic constants are $E = 60$ GPa and $\nu = 0.35$. Both ends of the cylinder are free of external loads, but they are fixed in such a manner that the edges are restrained from radial displacements. The inner surface of the cylinder is free of loads, while the outer surface is loaded by a normal pressure $q_r = 25.4545$ MPa . According to the recommendations by Boyarshinov (1973), the pressure at the middle surface of a shell is found as $q_0 = q_r \, r_{out} / R_0 = 28.0$ MPa .

The failure initiation time found using the proposed 3D theory is equal to $t_{*1} = 3736$ h . It has been established numerically that the damage variable reaches its critical value near the inner surface of the moderately thick cylindrical shell at the point with the coordinates $r_* = 9.04615 \cdot 10^{-2}$ m, $z_* = 0.0576913 \cdot 10^{-2}$ m . On the other hand, the failure initiation time found for the moderately thick cylindrical shell using the Kirchhoff-Love theory occurs as $t_{*2} = 2490$ h . Furthermore, in this case the damage variable reaches its critical value at the inner surface of the moderately thick shell in its central part at the point with the coordinates $r_* = 9.0 \cdot 10^{-2}$ m, $z_* = 0$. Note also that all the calculations are performed in this example with the discretization described in the previous section. It is not difficult to conclude that the application of the Kirchhoff-Love theory for describing the creep deformation of the moderately thick shell made from the aluminum alloy AK4-1T leads to the essential underestimation of the failure initiation time. In this case the relative error for the failure initiation time is equal to 33%.

Table 4. A comparison between the 3D (above the line) and the Kirchhoff-Love (below the line) solutions on the inner surface of a moderately thick cylindrical shell at different instants of time

t, h	σ_θ, MPa	ϕ, MJ/m^3	$p_\theta \cdot 10^2$
0.0	-165.7 / -172.0	0.0 / 0.0	0.0 / 0.0
500.0	-129.7 / -131.1	0.380 / 0.599	-0.257 / -0.388
1000.0	-113.9 / -114.0	0.632 / 1.075	-0.440 / -0.711
2000.0	-93.6 / -92.0	1.090 / 2.105	-0.780 / -1.437
t_{*1} / t_{*2}	- / -61.0	1.856 / 2.739	-1.350 / -1.931

Table 5. A comparison between the 3D (above the line) and the Kirchhoff-Love (below the line) solutions on the outer surface of a moderately thick cylindrical shell at different instants of time

t, h	σ_θ, MPa	ϕ, MJ/m^3	$p_\theta \cdot 10^2$
0.0	-143.6 / -142.0	0.0 / 0.0	0.0 / 0.0
500.0	-162.9 / -163.1	0.135 / 0.359	-0.102 / -0.245
1000.0	-172.4 / -175.0	0.265 / 0.704	-0.204 / -0.482
2000.0	-187.5 / -190.9	0.515 / 1.518	-0.405 / -1.035
t_{*1} / t_{*2}	-209.2 / -201.0	1.012 / 2.041	-0.811 / -1.378

Tables 4 - 6 demonstrate other results of a comparison between the 3D and the Kirchhoff-Love solutions for the moderately thick cylindrical shell under study. The numerical results are shown for the central part of this shell. It is seen that disagreement in the third stage of the creep process can reach 300%. Thus, the comparison between the results within the 3D formulation and the results within the refined kinematic model proposed recently by Galishin et al. (2009) and Zolochevsky et al. (2009a) is necessary. In this regard, the 3D solution obtained for the moderately thick cylindrical shell under consideration will serve as a background of this comparative analysis. A comparison between the results generated from this refined kinematic model of a shell and the results within the 3D formulation will be a subject of the future paper.

Table 6. A comparison between the 3D (above the line) and the Kirchhoff-Love (below the line) solutions for the radial displacements in the middle surface of a moderately thick cylindrical shell at different instants of time

t, h	0.0	500.0	1000.0	2000.0	$\frac{t_{*1}}{t_{*2}}$
$u_r \cdot 10^4$, m	$\frac{-2.51}{-2.60}$	$\frac{-3.05}{-5.60}$	$\frac{-5.41}{-8.29}$	$\frac{-8.05}{-14.50}$	$\frac{-13.29}{-18.39}$

6 Effect of the Kind of Stress State

All the numerical results discussed below are obtained for the thin and moderately thick cylindrical shells taken from Sections 4 and 5, respectively, under assumption that these shells are composed of material with characteristics which are independent from the kind of the stress state during the first and second stages of the creep process but with the critical value of the specific dissipation energy being dependent on the kind of loading. For this purpose, the creep curves of an aluminum alloy AK4-1T at the temperature of T=473 K under uniaxial tension are taken into consideration, as well as, traditional equations of continuum creep damage mechanics (Zolochevsky, 1982) for initially isotropic materials with the same behavior in tension and compression during the first and second creep stages but with the critical value of the damage variable ϕ_* being dependent on the kind of the stress state, $\phi_* = \frac{1}{2}\left(3I_2 - I_1^2\right)\left(a - bI_1\right)$.

It is established using the 3D theory that the failure initiation time for the thin shell under study is equal to $t_{*1} = 1079$ h . As an analogous Kirchhoff-Love solution $t_{*2} = 921$ h is found. Additionally, the failure initiation time for the moderately thick cylindrical shell under consideration is found using the 3D theory as $t_{*1} = 1803$ h while the analogous Kirchhoff-Love solution gives $t_{*2} = 1338$ h . The creep failure initiates always in the central part of shells on their inner surfaces. It is not difficult to calculate that in the case of the tension – compression material symmetry the relative error for the failure initiation time related to the use of the Kirchhoff-Love model is equal to 14.6% for the thin shell and 25.7% for the moderately thick shell. Thus, the relative error under discussion exceeds the shell characteristic ratio (multiplied by 100) of 10% and 20%, respectively.

Comparing the numerical values given above with the analogues values of the failure initiation time discussed in Sections 4 and 5, it is not difficult to see significant differences. Thus, the effect of the kind of the stress state must be included if accurate results are to be obtained from the numerical analyses of the creep deformation and the creep damage growth in thin and moderately thick shells. If it is assumed that the properties of the shell material are the same in tension and compression, this could lead to the overprediction of the creep damage accumulation and a significant underestimation of the failure initiation time.

7 Conclusions

A constitutive model for describing the creep deformation and creep damage development in initially isotropic materials with characteristics dependent on the kind of the stress state is used in the 3D theory of creep and creep damage growth in a cylindrical body of revolution. The 3D initial/ boundary value problem under creep-damage conditions is formulated, and the numerical approach to obtain a 3D solution is introduced. Thin ($h/R_0 = 1/10$) and moderately thick ($h/R_0 = 1/5$) cylindrical shells are considered, and a comparison between the 3D and the Kirchhoff-Love solutions is given. In the case of the tension – compression material asymmetry the relative error for the failure initiation time for a thin shell that occurs using the Kirchhoff-Love model does not exceed the shell characteristic ratio. On the other hand, the analogues error for a thin shell found in the case of the tension – compression symmetry exceeds the shell characteristic ratio. Thus, thin shells made of a material with tension-compression symmetry should be considered as those with $h/R_0 = 1/20$. In the case of materials with characteristics dependent on the kind of the stress state this estimation can be accepted as $h/R_0 = 1/10$. The obtained 3D solutions should be considered as a background of the future research for moderately thick shells to compare the results within the 3D formulation and the results within the refined kinematic model proposed recently by Galishin et al. (2009) and Zolochevsky et al. (2009a).

Acknowledgments

The authors gratefully acknowledge the support of this work by the German Academic Exchange Service (DAAD) and the Alexander von Humboldt Foundation, Germany.

References

Altenbach, H.; Altenbach, J.; Zolochevsky, A.: A generalized constitutive equation for creep of polymers at multiaxial loading. *Mechanics of Composite Materials*, 31, 6, (1995a), 511-518.

Altenbach, H.; Altenbach, J.; Zolochevsky, A.: *Erweiterte Deformationsmodelle und Versagenskriterien der Werkstoffmechanik*. Deutscher Verlag für Grundstoffindustrie, Stuttgart (1995b).

Altenbach, H.; Eremeyev, V. A.: On the shell theory on the nanoscale with surface stresses. *International Journal of Engineering Science*, 49, 12, (2011), 1294-1301.

Altenbach, H.; Zolochevsky, A.: Kriechen dünner Schalen aus anisotropen Werkstoffen mit unterschiedlichem Zug-Druck-Verhalten. *Forsch. Ingenieurwesen*, 57, 6, (1991), 172-179.

Betten, J.: *Creep Mechanics.* Springer-Verlag, Berlin (2002).

Betten, J.; Borrmann, M.: Stationäres Kriechverhalten innendruckbelasteter dünnwandiger Kreiszylinderschalten unter Berücksichtigung des orthotropen Werkstoffverhaltens und des CSD-Effects. *Forsch. Ingenieurwesen*, 53, 3, (1987), 75-82.

Betten, J.; Sklepus, S.; Zolochevsky, A.: A creep damage model for initially isotropic materials with different properties in tension and compression. *Engng. Fracture Mech.*, 59, (1998), 623-641.

Boyarshinov, S. V.: *Fundamentals of the Structural Mechanics of Machines*. Engineering, Moscow (1973).

Duan, H. L.; Wang, J.; Karihaloo, B. L.: Theory of elasticity at the nanoscale. *Advances in Applied Mechanics*, 42, (2009), 1-68.

Galishin, A.; Zolochevsky, A.; Kühhorn, A.; Springmann, M.: Transversal shear effect in moderately thick shells from materials with characteristics dependent on the kind of stress state under creep-damage conditions: Numerical modeling. *Technische Mechanik*, 29, 1, (2009), 48-59.

Ganczarski, A.; Skrzypek, J.: Anisotropic thermo-creep-damage in 3D thick plate vs Reissner's approach. In: R. Kienzler, H. Altenbach, I. Ott, eds., *Theories of Plates and Shells, EUROMECH Colloquium 444, Lecture Notes in Applied and Computational Mechanics*, 16, p. 39-44, Springer-Verlag, Berlin (2004).

Ganczarski, A.; Skrzypek, J.: Application of the modified Murakami's anisotropic creep-damage model to 3D rotationally-symmetric problem. *Technische Mechanik* 21, (2001), 251–260.

Kawai, M.: Constitutive modeling of creep and damage behaviors of the non-Mises type for a class of polycrystalline metals. *Int. J. Damage Mech.*, 11, (2002), 223-246.

Kletschkowski, T.; Schomburg, U.; Bertram, A.: An endochronic viscoplastic approach for materials with different behavior in tension and compression. *Mechanics of Time-dependent Materials*, 8, (2004), 119 – 135.

Mahnken, R.: Creep simulation of asymmetric effects by use of stress mode dependent weighting functions. *Int. J. Solids Structures*, 40, (2003), 6189-6209.

Rvachev, V. L.: Analytical description of some geometric objects. *Doklady Akademii Nauk USSR*, 153, (1963), 765-768.

Rvachev, V. L., Sheiko, T. I.: R-functions in boundary value problems in mechanics. *Applied Mechanics Reviews*, 48, (1995), 151-188.

Rubanov, V.V.: *Experimental Foundation of the Constitutive Equations of Creep for Materials with Different Behaviour in Tension and Compression*. Ph.D. thesis, Siberian Branch of the Russian Academy of Sciences, Institute of Hydrodynamics, Novosibirsk (1987).

Zolochevskij, A. A.: Kriechen von Konstruktionselementen aus Materialien mit von der Belastung abhängigen Charakteristiken. *Technische Mechanik*, 9, (1988), 177–184.

Zolochevsky, A. A. : *Creep of Thin Shells for Materials with Different Behavior in Tension and Compression*. Ph.D. thesis, National Academy of Sciences of Ukraine, Institute of Mechanical Engineering Problems, Kharkov (1982).

Zolochevsky, A.; Galishin, A.; Kühhorn, A.; Springmann, M.: Transversal shear effect in moderately thick shells from materials with characteristics dependent on the kind of stress state under creep-damage conditions: Theoretical framework. Technische Mechanik, 29, 1, (2009a), 38-47.

Zolochevsky, A.; Galishin, A.; Sklepus, S.; Voyiadjis, G.Z.: Analysis of creep deformation and creep damage in thin-walled branched shells from materials with different behavior in tension and compression. Int. J. Solids Structures, 44, (2007), 5075-5100.

Zolochevsky, A.; Martynenko, A.; Kühhorn, A.: Structural benchmark creep and creep damage testing for finite element analysis with material tension-compression asymmetry and symmetry. Computers and Structures, 100-101, (2012), 27-38.

Zolochevsky, A.; Sklepus, S.; Hyde, T.H.; Becker, A.A.; Peravali, S.: Numerical modeling of creep and creep damage in thin plates of arbitrary shape from materials with different behavior in tension and compression under plane stress conditions. Int. J. Numer. Eng., 80, (2009b), 1406-1436.

Zolochevsky, A.A.; Sklepus, A.N.; Sklepus, S.N.: *Nonlinear Solid Mechanics*. Business Investor Group, Kharkov (2011).

Addresses: Assoc. Prof. Dr.-Ing. habil. Alexander Zolochevsky, Department of Theory of Mechanisms and Machines, National Technical University "Kharkov Polytechnic Institute", Frunze 21, Kharkov, 61002, Ukraine; Dr.-Ing. Sergiy Sklepus, Department of Computational Mechanics, Institute of Mechanical Engineering Problems, Pozharskoho 2/10, Kharkov, 61046, Ukraine; Dr.-Ing. habil. Alexander Galishin, Department of Thermoplasticity, the Timoshenko Institute of Mechanics, Nesterova 3, Kiev, 03057, Ukraine; Prof. Dr.-Ing. Arnold Kühhorn and Dr.-Ing. Markus Kober, Lehrstuhl für Strukturmechanik und Fahrzeugschwingungen (SMF), Brandenburgische Technische Universität Cottbus, D-03046 Cottbus;
e-mail: azol@rambler.ru; ssklepus@rambler.ru; galishin55@mail.ru; kuehhorn@tu-cottbus.de; markus.kober@tu-cottbus.de