Propagation of Plane Waves in a Generalized Thermo-magneto-electro-elastic Medium

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Abstract. In the present paper, the governing equations of a generalized thermo-magneto-electro-elastic medium are formulated in the x-z plane. The plane wave solution of these equations indicates the existence of three quasi plane waves, namely, quasi-P, quasi-T and quasi-SV waves. The thermo-magneto-electro-elastic medium is modeled with $LiNbO_3$ for computing the speeds of these plane waves. Effects of the frequency, thermal relaxation time, electric coupling coefficient, magnetic coupling coefficient and angle of propagation on the speeds of these plane waves are observed and shown graphically.

1 Introduction

Layered materials and composites have components which display some degree of piezoelectric coupling effects. Piezoelectric materials have been applied in various fields including geophysics, electronics, communication, instrumentation and nondestructive evaluation and testing of materials. For general discussions of piezoelectric effects, the standard books by Cady (1964), Tiersten (1969), Auld (1973) and Rosenbaum (1988) are referred. Piezoelectric guided waves have been studied by Gazis (1960), Bleustein (1968), Nasser (1983), Nayfeh and Chien (1992 a, b) and Nie et al (2009). Piezoelectric materials are integrated with the structural systems to form a class of smart structures and are embedded as layers or fibres into multi functional composites. Various novel and practical applications in many fields of technology have been promoted due to the better recognition and knowledge of the complex phenomena behind the propagation of elastic waves in structural elements. For example, diagnostics of structural elements is one such field, where the use of elastic waves increases rapidly every year. Applications of elastic waves in a global sense for diagnosing structural elements are still at an early stage. Piezoelectric materials constitute another important research topic with respect to SHM applications. Piezoelectric transducers are often used to excite and receive ultrasonic guided waves due to their low cost and an easy integration into existing structures. Important contributions are listed in Nayfeh (1995), Liu and Xi (2002), Ostachowicz et al. (2012), Giurgiutiu (2014) and Willberg et al. (2015).

In the classical theory of thermoelasticity given by Biot (1956), the thermal wave propagates with an infinite speed. The non-classical theories of generalized thermoelasticity were introduced in the literature in an attempt to eliminate the shortcomings of the classical dynamical thermoelasticity. For example, Lord and Shulman (1967), by incorporating a flux-rate term into Fourier's law of heat conduction, formulated a generalized theory which involves a hyperbolic heat transport equation admitting a finite speed for thermal signals. Green and Lindsay (1972), by including temperature rate among the constitutive variables, developed a temperature-rate-dependent thermoelasticity that does not violate the classical Fourier law of heat conduction, when the body under consideration has a centre of symmetry and this theory also predicts a finite speed for the heat propagation. Chandrasekharaiah (1986) referred to this wavelike thermal disturbance as the 'second sound'. The Lord and Shulman theory of generalized thermoelasticity was further extended by Dhaliwal and Sherief (1980) to include the anisotropic case. Hetnarski and Ignaczak (1999) presented a survey on the representative theories in the range of generalized thermoelasticity.

Magneto-electro-elastic materials display coupling behavior among electric, magnetic and mechanical fields. Magneto-electro-elastic materials have various applications due to their ability of converting energy from one kind to the other. These materials have been used in lasers, supersonic devices, microwave and infrared applications. Problems related to the wave propagation in thermo-elastic or magneto-thermo-elastic solids using these generalized

theories have been studied by several authors (Paria, 1962; Nayfeh and Nemat-Nasser, 1971, 1972;, Roychoudhuri and Chatterjee, 1990; Hsieh, 1990; Ezzat, 1997; Sherief and Yossef, 2004; Baksi and Bera, 2005; and Das and Kanoria, 2009).

Thermo-magneto-electro-elastic materials are extensively used as electric packaging, sensors and actuators. Wave propagation in thermo-magneto-electro-elastic solid is of much importance due to wide use of piezoelectric and piezomagnetic materials in aerospace, automobile industries, etc. The theory of thermo-magneto-electro-elasticity was developed due to the significant contributions by various authors, for example, Kaliski (1965), Coleman and Dill (1971), Amendola (2000), Li (2003) and Aouadi (2007). This paper is motivated by the linear theory of thermo-magneto-electro-elasticity developed by Aouadi (2007). In this paper, the time-harmonic plane waves in a generalized thermo-magneto-electro-elastic medium are investigated. The speeds of these plane waves are computed numerically by taking $LiNbO_3$ as an example of the model. The dependence of wave speeds of these plane waves on the frequency, thermal relaxation time, electric coupling coefficient, magnetic coupling coefficients and angle of propagation is shown graphically.

2 Basic Equations

A body is considered that occupies the region V of the Euclidean three-dimensional space at some instant and is bounded by the piece-wise smooth surface V. The reference configuration V and a fixed system of rectangular Cartesian axes $Ox_i(i = 1, 2, 3)$ are taken to describe the motion of the body. Aouadi (2007) developed the governing equations for thermo-magneto-electro-elasticity for heat-flux dependent theory of Lebon (1982). Following Coleman and Dill (1971), Amendola (2000) and Li (2003), he formulated linearized constitutive equations. The field equations of thermo-magneto-electro-elasticity are generalized in context of Lord and Shulman (1967) theory as
the equations of motion

$$\sigma_{ji,j} + F_i = \rho \ddot{u}_i, \tag{1}$$

the equations of the electric and magnetic fields

$$D_{i,i} = \rho_0, \quad B_{i,i} = \sigma, \tag{2}$$

the energy equation

$$\rho T_0 \dot{\eta} = q_{i,i} + \rho h, \tag{3}$$

the constitutive equations

$$\sigma_{ij} = c_{ijkl}e_{kl} + F_{ijk}\zeta_k + \lambda_{ijk}E_k - a_{ij}T, \tag{4}$$

$$D_k = -\lambda_{kij}e_{ij} + \alpha_{ki}\zeta_i + \gamma_{ki}E_i + p_k T, \tag{5}$$

$$B_k = -F_{kij}e_{ij} + A_{ki}\zeta_i + \alpha_{ki}E_i + m_k T, \tag{6}$$

$$\rho\eta = a_{ij}e_{ij} + m_k\zeta_k + p_k E_k + cT, \tag{7}$$

$$K_{ij}T_{,j} = q_i + \tau_0 \dot{q}_i, \tag{8}$$

and the geometrical equations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\psi_{,i}, \quad \zeta_i = -\phi_{,i} \tag{9}$$

where F_i, ρ_0 and σ are the body force, electric charge density, and electric current density, respectively; ρ is the mass density; h is the heat supply; u_i, ψ and ϕ are the displacement vector, the electric potential, and the magnetic potential, respectively; σ_{ij}, D_k, B_k and η are stress tensor, the dielectric displacement vector, the magnetic intensity, and the entropy density, respectively; e_{ij}, E_i, ζ_i and T are strain tensor, electric field, magnetic field, and temperature change to a reference temperature T_0, respectively; K_{ij} is the conductivity tensor; c_{ijkl}, γ_{kj}, A_{kj}, c, λ_{ijk}, F_{ijk}, α_{kj}, a_{ij}, p_i and m_i are constitutive coefficients connecting various fields like mechanical, magnetic,

thermal and electric fields and τ_0 is relaxation time. Latin subscripts range over the integers $(1, 2, 3)$ summation over and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. The superposed dot denotes the partial differentiation with respect to time t. The constitutive parameters satisfy the following symmetry conditions

$c_{ijkl} = c_{klij} = c_{jikl}, \; \lambda_{ijk} = \lambda_{kij} = \lambda_{kji}, \; F_{ijk} = F_{kij} = F_{kji},$

$$a_{ij} = a_{ji}, \; \gamma_{ij} = \gamma_{ji}, \; \alpha_{ij} = \alpha_{ji}, \; A_{ij} = A_{ji}, \; K_{ij} = K_{ji}. \tag{10}$$

3 Governing Equations in the *x-z* Plane

We consider a homogeneous and transversely isotropic thermo-magneto-electro-elastic medium of an infinite extent with Cartesian coordinate system (x, y, z), which is previously at a uniform temperature. We assume that the medium is transversely isotropic in such a way that the planes of isotropy are perpendicular to the z-axis. The origin is taken on the plane surface and z-axis is taken normally into the medium $(z \geq 0)$. The present study is restricted to the plane strain parallel to the x-z plane with the displacement vector $\mathbf{u} = (u_1, 0, u_3)$. In this section, the equations of a transversely isotropic, homogeneous and linear thermo-magneto-electro-elastic medium in the x-z plane are formulated after canceling the dependency of the y-direction as well as the derivative with respect to y. With the help of the symmetry conditions (10) and the equations (4) to (9), the equations (1) to (3) reduce to the following system of five partial differential equations in u_1, u_3, ϕ, ψ and T in absence of body forces, electric charge density, electric current density and heat supply

$$c_{11}\frac{\partial^2 u_1}{\partial x^2} + c_{31}\frac{\partial^2 u_3}{\partial x \partial z} - F_{11}\frac{\partial^2 \phi}{\partial x^2} - 2F_{31}\frac{\partial^2 \phi}{\partial x \partial z} - \lambda_{11}\frac{\partial^2 \psi}{\partial x^2}$$

$$-2\lambda_{31}\frac{\partial^2 \psi}{\partial x \partial z} - a_1\frac{\partial T}{\partial x} + c_{55}(\frac{\partial^2 u_1}{\partial z^2} + \frac{\partial^2 u_3}{\partial x \partial z}) = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{11}$$

$$c_{55}(\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_1}{\partial x \partial z}) - F_{31}\frac{\partial^2 \phi}{\partial x^2} - \lambda_{31}\frac{\partial^2 \psi}{\partial x^2} + c_{31}\frac{\partial^2 u_1}{\partial x \partial z}$$

$$+c_{33}\frac{\partial^2 u_3}{\partial z^2} - F_{33}\frac{\partial^2 \phi}{\partial z^2} - \lambda_{33}\frac{\partial^2 \psi}{\partial z^2} - a_3\frac{\partial T}{\partial z} = \rho\frac{\partial^2 u_3}{\partial t^2}, \tag{12}$$

$$\lambda_{11}\frac{\partial^2 u_1}{\partial x^2} + \lambda_{31}(2\frac{\partial^2 u_1}{\partial x \partial z} + \frac{\partial^2 u_3}{\partial x^2}) + \lambda_{33}\frac{\partial^2 u_3}{\partial z^2} + \alpha_1\frac{\partial^2 \phi}{\partial x^2}$$

$$+\alpha_3\frac{\partial^2 \phi}{\partial z^2} + \gamma_1\frac{\partial^2 \psi}{\partial x^2} + \gamma_3\frac{\partial^2 \psi}{\partial z^2} - p_1\frac{\partial T}{\partial x} - p_3\frac{\partial T}{\partial z} = 0, \tag{13}$$

$$F_{11}\frac{\partial^2 u_1}{\partial x^2} + F_{31}(2\frac{\partial^2 u_1}{\partial x \partial z} + \frac{\partial^2 u_3}{\partial x^2}) + F_{33}\frac{\partial^2 u_3}{\partial z^2} + A_1\frac{\partial^2 \phi}{\partial x^2}$$

$$+A_3\frac{\partial^2 \phi}{\partial z^2} + \alpha_1\frac{\partial^2 \psi}{\partial x^2} + \alpha_3\frac{\partial^2 \psi}{\partial z^2} - m_1\frac{\partial T}{\partial x} - m_3\frac{\partial T}{\partial z} = 0, \tag{14}$$

$$T_0[a_1\frac{\partial^2 u_1}{\partial x \partial t} + a_3\frac{\partial^2 u_3}{\partial z \partial t} - m_1\frac{\partial^2 \phi}{\partial x \partial t} - m_3\frac{\partial^2 \phi}{\partial z \partial t} - p_1\frac{\partial^2 \psi}{\partial x \partial t} - p_3\frac{\partial^2 \psi}{\partial z \partial t}$$

$$+c\frac{\partial T}{\partial t}] = \frac{K_1}{1 + \tau_0\frac{\partial}{\partial t}}\frac{\partial^2 T}{\partial x^2} + \frac{K_3}{1 + \tau_0\frac{\partial}{\partial t}}\frac{\partial^2 T}{\partial z^2}, \tag{15}$$

where
$c_{11} = c_{1111}, \; c_{33} = c_{3333}, \; c_{55} = c_{3113} = c_{1331}, \; c_{31} = c_{3311} = c_{1133},$
$\lambda_{11} = \lambda_{111}, \; \lambda_{31} = \lambda_{311}, \; \lambda_{33} = \lambda_{333}, \; \gamma_1 = \gamma_{11}, \; \gamma_3 = \gamma_{33},$
$F_{11} = F_{111}, \; F_{33} = F_{333}, \; a_1 = a_{11}, \; a_3 = a_{33}, \; \alpha_1 = \alpha_{11}, \; \alpha_3 = \alpha_{33},$
$A_1 = A_{11}, \; A_3 = A_{33}, \; K_1 = K_{11}, \; K_3 = K_{33}.$

4 Plane Wave Propagation

It is assumed that the *x-z* plane has an infinite extension. Now, we seek the two-dimensional plane wave solutions of

Table 1: Physical constants of $LiNbO_3$

$\rho(Kgm^{-3})$	$c_{11}(Nm^{-2})$	$c_{33}(Nm^{-2})$	$c_{55}(Nm^{-2})$	$c_{31}(Nm^{-2})$
4.647×10^3	2.03×10^{11}	2.424×10^{11}	0.595×10^{11}	0.752×10^{11}
$\lambda_{11}(Cm^{-2})$	$\lambda_{33}(Cm^{-2})$	$\lambda_{31}(Cm^{-2})$	γ_1	γ_3
1.13	1.33	0.23	85.2	28.7
$F_{11}(Kg)$	$F_{33}(Kg)$	$F_{31}(Kg)$	$A_1(NA^{-2})$	$A_3(NA^{-2})$
0.2×10^{-2}	0.15×10^{-2}	0.1×10^{-2}	0.005	0.004
$K_1(Wm^{-1}K^{-1})$	$K_3(Wm^{-1}K^{-1})$	$c(Jkg^{-1}K^{-1})$	$a_1(K^{-1})$	$a_3(K^{-1})$
4	4.2	0.633	13.3×10^{-6}	10.3×10^{-6}
$\alpha_1(Cm^{-1}A^{-1})$	$\alpha_3(Cm^{-1}A^{-1})$	$p_1(NC^{-1}K^{-1})$		
0.02	0.03	0.133		
$p_3(NC^{-1}K^{-1})$	$m_1(Nm^{-1}A^{-1}K^{-1})$	$m_3(Nm^{-1}A^{-1}K^{-1})$		
0.103	0.006	0.004		

equations (11) to (15) in the $x - z$ plane assuming that the independent variables can be described by the following harmonic functions as

$$\{u_1, \, u_3 \, T, \, \phi, \, \psi\} = \{\bar{u}_1, \, \bar{u}_3 \, \bar{T}, \, \bar{\phi}, \, \bar{\psi}\}e^{\iota k(\sin\theta x + \cos\theta z - vt)}, \tag{16}$$

where θ is angle of propagation, k is wavenumber, v is wave speed, $(\sin\theta, \cos\theta)$ is the projection of the wave normal onto $x - z$ plane. and $\bar{u}_1, \, \bar{u}_3 \, \bar{T}, \, \bar{\phi}, \, \bar{\psi}$ are constants. Using equation (16) in equations (11) to (15), we obtain a homogeneous system of five equations in $\bar{u}_1, \, \bar{u}_3 \, \bar{T}, \, \bar{\phi}, \, \bar{\psi}$. The system has non-trivial solution if the determinant of the coefficients of $\bar{u}_1, \, \bar{u}_3 \, \bar{T}, \, \bar{\phi}, \, \bar{\psi}$ vanishes, i.e.,

$$A\zeta^3 + B\zeta^2 + C\zeta + D = 0, \tag{17}$$

where $\zeta = \rho v^2$ and the the expressions for A, B, C and D are given in Appendix.
The dispersion equation (17) is a cubic equation in v^2 with complex coefficients. The three roots $v_i{}^2$, $(i = 1, 2, 3)$ of equation (17) correspond to quasi-P (qP), quasi-T (qT) and quasi-SV (qSV) waves, respectively. Further, $v_i = Re(v_i) + iIm(v_i)$ holds so that $Re(v_i)$ represent the wave speeds of plane waves.

If we neglect magnetic and thermal fields, the equation (17) reduces to

$$D_4\zeta^2 - [D_4(D_1 + D_2) + D_3{}^2 + L_2{}^2]\zeta$$
$$+[D_1D_2D_4 + D_1D_3{}^2 - D_4L_1{}^2 - 2D_3L_1L_2 + D_2L_2{}^2] = 0, \tag{18}$$

which gives the speeds of quasi-P and quasi-SV waves in a transversely isotropic electro-elastic media.

If we neglect electric, magnetic and thermal fields, the equation (17) reduces to

$$\zeta^2 - (D_1 + D_2)\zeta + (D_1D_2 - L_1{}^2) = 0, \tag{19}$$

which gives the speeds of quasi-P and quasi-SV waves in a transversely isotropic elastic media.

5 Numerical Results and Discussion

Following Weis and Gaylord (1985), the relevant physical constants of $LiNbO_3$ at $T_0 = 298K$ are considered as in Table 1.
  Using a Fortran code of the Cardan method, the equation (17) is solved numerically for wave speeds of qP, qT and qSV waves. The wave speeds of qP, qT and qSV are shown graphically against the frequency (ω), thermal relaxation time (τ_0), electric coupling coefficient (λ_{11}), magnetic coupling coefficient (F_{11}) and angle of propagation (θ) in Figures 1 to 7.

5.1 Effect of Frequency

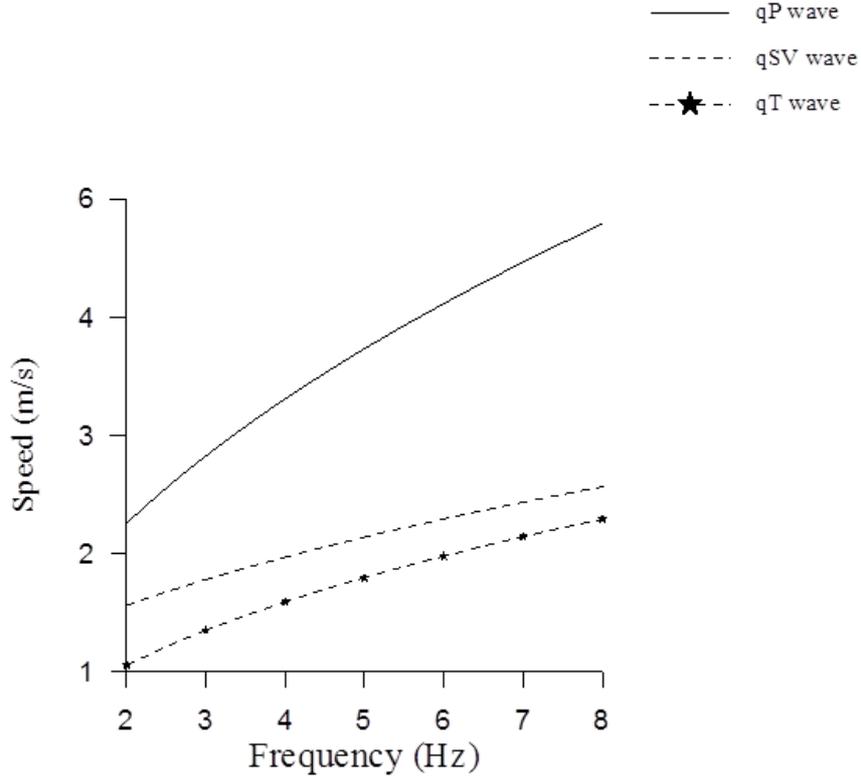The wave speeds of qP, qT and qSV are plotted against the frequency ($2Hz \le \omega \le 8Hz$) in Fig. 1, when the

Figure 1: Variations of the speeds of the qP, the qT and the qSV waves against the frequency (ω) when the propagation angle is $\theta = 45^o$ and the thermal relaxation time is $\tau_0 = 0.005$ s
.

angle of propagation is $\theta = 45^o$ and the thermal relaxation time is $\tau_0 = 0.005s$. For $\omega = 2Hz$, the speeds of qP, qT and qSV waves are $2.482 \times 10^4 ms^{-1}, 1.159 \times 10^4 ms^{-1}$ and $1.717 \times 10^4 ms^{-1}$, respectively. The speeds of qP, qT and qSV increase with the increase in the frequency and attain values $5.272 \times 10^4 ms^{-1}, 2.522 \times 10^4 ms^{-1}$ and $2.824 \times 10^4 ms^{-1}$, respectively for $\omega = 8Hz$.

5.2 Effect of Thermal Relaxation

The wave speeds of qP, qT and qSV are plotted against the thermal relaxation time ($0 \leq \tau_0 \leq 0.5s$) in Fig. 2, when the angle of propagation is $\theta = 45^o$ and the frequency is $\omega = 5Hz$. For $\tau_0 = 0$, the speeds of qP, qT and qSV waves are $4.113 \times 10^4 m.s^{-1}, 2.024 \times 10^4 ms^{-1}$ and $2.340 \times 10^4 ms^{-1}$, respectively. The speeds of qP, qT and qSV decrease with the increase in relaxation time and finally attain values $1.606 \times 10^4 ms^{-1}, 0.278 \times 10^4 ms^{-1}$ and $0.375 \times 10^4 ms^{-1}$, respectively for $\tau_0 = 0.5s$.

5.3 Effect of Electric Coupling

The wave speeds of qP, qT and qSV are plotted against the electric coupling coefficient ($0 \leq \lambda_{11} \leq 2Cm^{-2}$) in Fig. 3, when the angle of propagation is $\theta = 45^o$, the frequency is $\omega = 5Hz$ and the thermal relaxation time is $\tau_0 = 0.005s$. For $\lambda_{11} = 0$, the speeds of qP, qT and qSV waves are $4.1085 \times 10^4 ms^{-1}, 1.9775 \times 10^4 ms^{-1}$ and $2.3547 \times 10^4 ms^{-1}$, respectively. The speeds of qP and qT waves increase very slightly with the increase in the electric coupling coefficient (λ_{11}), whereas the speed of qSV wave decreases very slightly.

5.4 Effect of Magnetic Coupling

The wave speeds of qP, qT and qSV are plotted against the magnetic coupling coefficient ($0 \leq F_{11} \leq 1 \times 10^{-2}Kg$) in Fig. 4, when the angle of propagation $\theta = 45^o$, the frequency $\omega = 5Hz$ and the thermal relaxation time $\tau_0 = 0.005s$. For $F_{11} = 0$, the speeds of qP, qT and qSV waves are $4.186 \times 10^4 ms^{-1}, 2.056 \times 10^4 ms^{-1}$ and $2.277 \times 10^4 ms^{-1}$, respectively. The speeds of qP and qT waves decrease with the increase in magnetic coupling coefficient (F_{11}), whereas the speed of qSV wave increases.
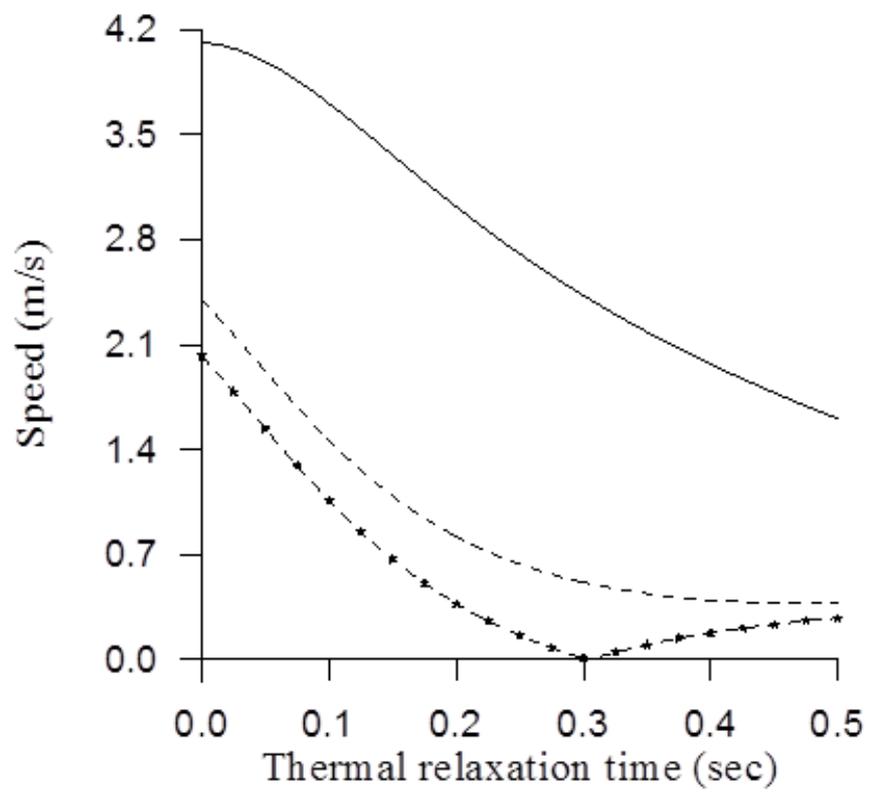
Figure 2: Variations of the speeds of the qP, the qT and the qSV waves against the thermal relaxation time τ_0, when the frequency is $\omega = 5Hz$ and the propagation angle is $\theta = 45^o$
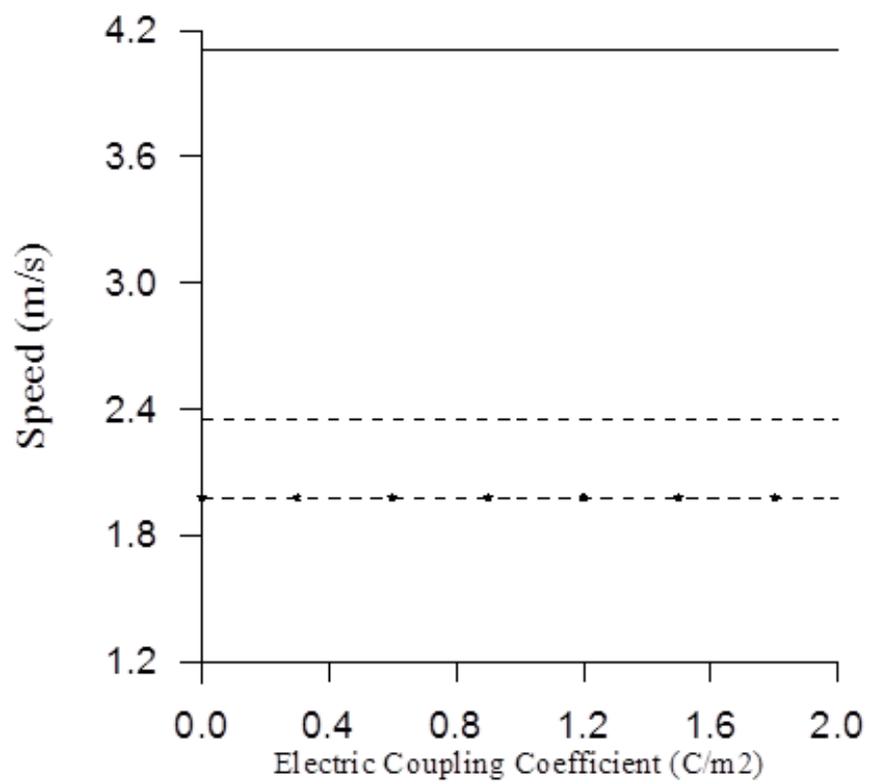
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Figure 3: Variations of the speeds of the qP, the qT and the qSV waves against the electrical coupling coefficient (λ_{11}), when the thermal relaxation time is $\tau_0 = 0.005s$, the frequency is $\omega = 5Hz$ and the propagation angle is $\theta = 45^o$
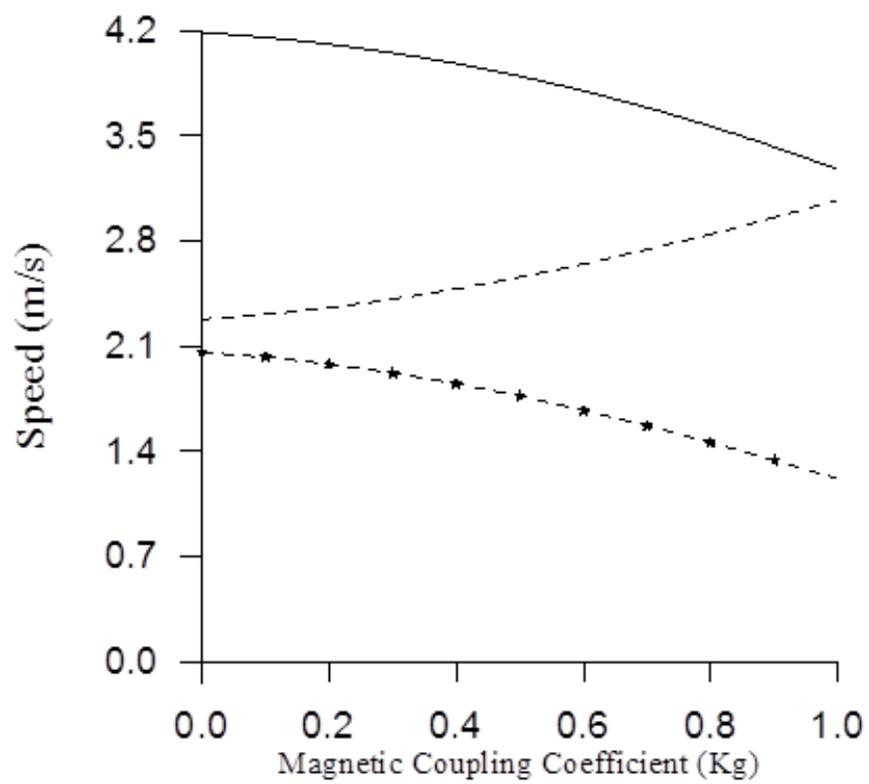
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Figure 4: Variations of the speeds of the qP, the qT and the qSV waves against the magnetic coupling coefficient (F_{11}), when the thermal relaxation time is $\tau_0 = 0.005s$, the frequency is $\omega = 5Hz$ and the angle of propagation is $\theta = 45^o$
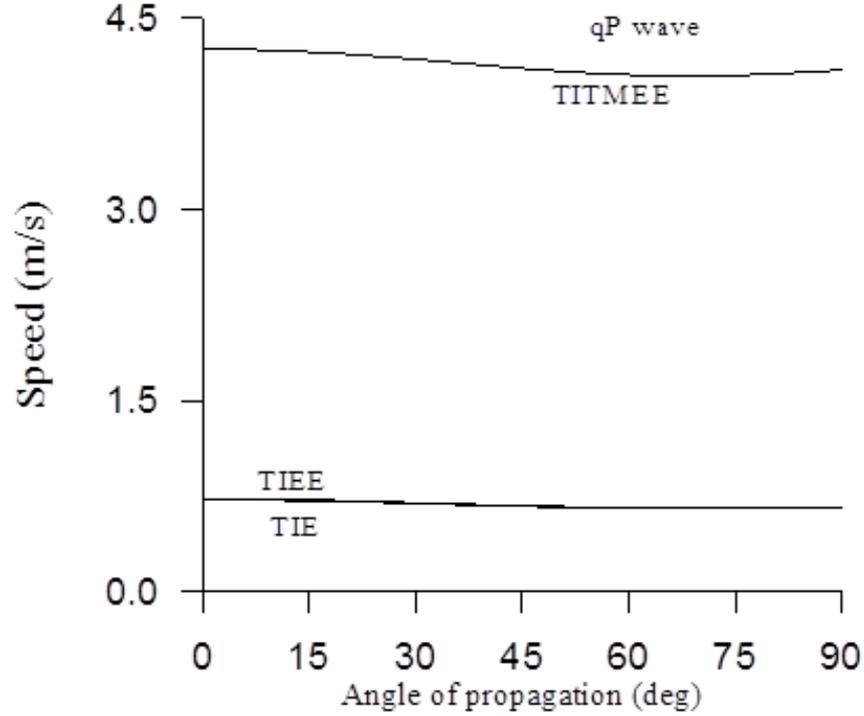
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Figure 5: Variations of speeds of qP wave against angle of propagation (θ), when thermal relaxation time $\tau_0 = 0.005s$ and frequency $\omega = 5Hz$

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5.5 Effect of Angle of Propagation

The wave speed of qP wave in a Transversely Isotropic Thermo-Magneto-Electro-Elastic ($TITMEE$) medium is plotted against the angle of propagation ($0^o \leq \theta \leq 90^o$) in Fig. 5, when the frequency is $\omega = 5Hz$ and the thermal relaxation time is $\tau_0 = 0.005s$. For $\theta = 0^o$, the speed of qP waves is $4.262 \times 10^4 ms^{-1}$. The speeds of the qP wave first decreases with the increase in the angle of propagation and attain its minimum value $4.050 \times 10^4 ms^{-1}$ at $\theta = 68^o$. Thereafter, the speed of qP wave wave increases till $\theta = 90^o$. This variation is compared with those for Transversely Isotropic Electro-Elastic ($TIEE$) and Transversely Isotropic Elastic (TIE) cases to observe the effect of electric, magnetic and thermal coupling on the speed of qP wave at each angle of propgation. The speed of the qP wave is found larger at decimal places in TIEE as compared to TIE medium. However in figure 5, the variations for TIE and TIEE have been shown by same curve. The wave speed of the qT wave in $TITMEE$ medium is plotted against the angle of propagation ($0^o \leq \theta \leq 90^o$) in Fig. 6, when the frequency is $\omega = 5Hz$ and the thermal relaxation time is $\tau_0 = 0.005s$. For $\theta = 0^o$, the speed of the qT wave is $2.098 \times 10^4 ms^{-1}$. The speed of the qT waves first decreases with the increase in angle of propagation and attain its minimum value $1.935 \times 10^4 ms^{-1}$ at $\theta = 68^o$. Thereafter, the speed of this wave increases till $\theta = 90^o$. This wave does not appear in TIE and TIEE cases. The wave speed of the qSV wave in $TITMEE$ is plotted against the angle of propagation ($0^o \leq \theta \leq 90^o$) in Fig. 7, when frequency $\omega = 5Hz$ and thermal relaxation time $\tau_0 = 0.005s$. For $\theta = 0^o$, the speed of qSV wave is $2.316 \times 10^4 ms^{-1}$. The speed of the qSV wave first increases with the increase in angle of propagation and attain its maximum value $2.364 \times 10^4 m.s^{-1}$ at $\theta = 58^o$ and then decreases till grazing incidence. This variation is compared with those for Transversely Isotropic Electro-Elastic ($TIEE$) and Transversely Isotropic Elastic (TIE) cases to observe the effect of electric, magnetic and thermal coupling on speed of qSV wave at each angle of incidence. The speed of qP wave is found larger at decimal places in TIEE as compared to TIE medium. However in figure 7, the variations for TIE and TIEE have been shown by same curve.

6 Conclusion

The governing equations of a generalized thermo-magneto-electro-elastic media are derived in the x-z plane. The plane wave solution of the governing equations indicates the existence of three quasi plane waves, namely, the qP, the qT and the qSV waves, respectively. The mechanical and thermal fields in the medium are mainly responsible for the existence of the three quasi-plane waves. The electric and magnetic fields in the medium just modify the speeds of these plane waves. The wave speeds of these plane waves are computed numerically for $LiNbO_3$. From
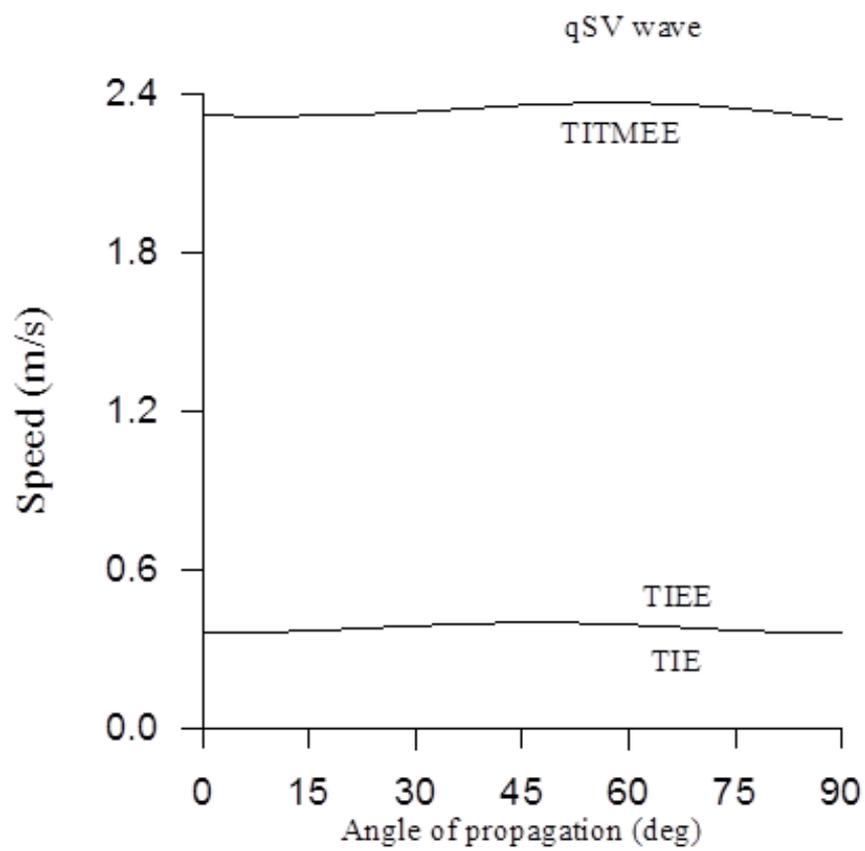
Figure 6: Variations of the speeds of the qT wave against the angle of propagation (θ), when the thermal relaxation time is $\tau_0 = 0.005s$ and the frequency is $\omega = 5Hz$
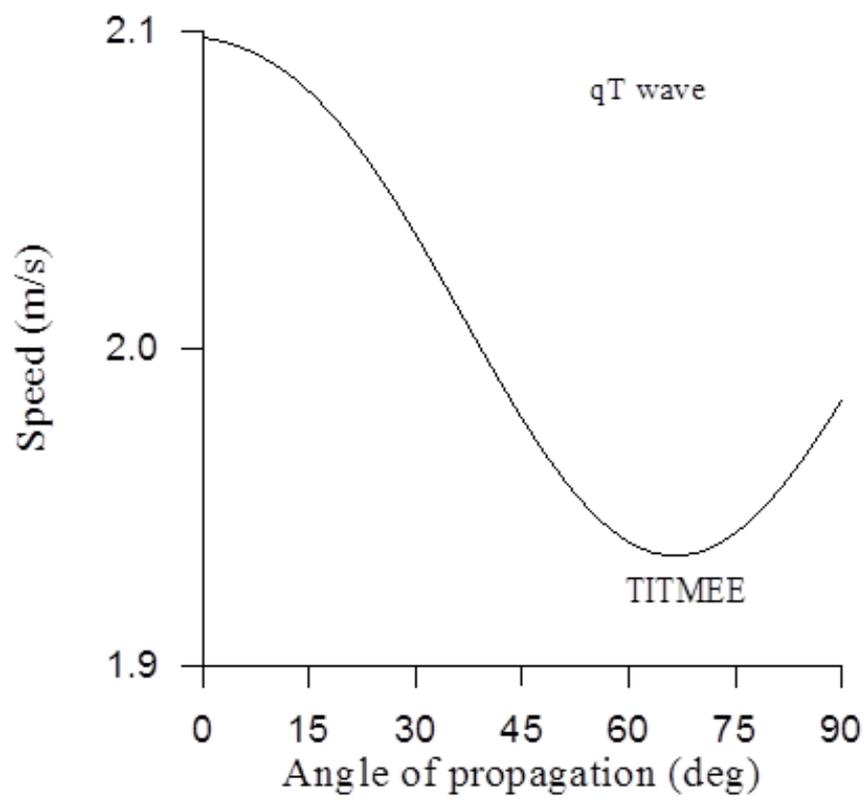
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Figure 7: Variations of the speeds of the qSV wave against the angle of propagation (θ), when the thermal relaxation time is $\tau_0 = 0.005s$ and the frequency is $\omega = 5Hz$

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numerical results and discussion, it is observed that the wave speed of plane waves are affected significantly by the change in the frequency, the thermal relaxation time and the electric and magnetic parameters. The wave speeds of these plane waves also depend on the angle of propagation. This paper may be useful for researchers and engineers who solve practical problems involving propagation of elastic waves in structural elements made of isotropic materials or laminated composites.

Appendix

The expressions for A, B, C and D are given as

$$A = \bar{c}(D_4 D_7 - D_5^2) + \bar{P}(PD_7 - MD_5) - \bar{M}(PD_5 - MD_4),$$

$$\begin{aligned} B = &\ D_8(D_4 D_7 - D_5^2) - \bar{c}[(D_1 + D_2)(D_4 D_7 - D_5^2) + 2L_2 L_3 D_5 - L_2^2 D_7 - L_3^2 D_4 \\ &- D_3^2 D_7 + 2D_3 D_5 D_6 - D_4 D_6^2] - \bar{P}[(D_1 + D_2)(PD_7 - MD_5) \\ &- a_3 \cos\theta(D_3 D_7 - D_5 D_6) - D_6(PD_6 - MD_3) - a_1 \sin\theta(L_2 D_7 - L_3 D_5) \\ &+ L_3(ML_2 - PL_3)] + \bar{M}[(D_1 + D_2)(PD_5 - MD_4) + a_3 \cos\theta(D_3 D_5 - D_4 D_6) \\ &- D_3(MD_3 - PD_6) + a_1 \sin\theta(L_2 D_5 - L_3 D_4) - L_2(ML_2 - PL_3)] \\ &+ \bar{a}_3 T_0 \cos\theta[a_3 \cos\theta(-D_4 D_7 + D_5^2) + D_3(PD_7 - MD_5) \\ &- D_6(PD_5 - MD_4)] + \bar{a}_1 T_0 \sin\theta[a_1 \sin\theta(-D_4 D_7 + D_5^2) \\ &+ L_2(PD_7 - MD_5) - L_3(PD_5 - MD_4)], \end{aligned}$$

$$\begin{aligned} C = &\ D_8[(D_1 + D_2)(D_4 D_7 - D_5^2) + L_2^2 D_7 - 2L_2 L_3 D_5 + L_3^2 D_4 + D_3^2 D_7 \\ &- 2D_3 D_5 D_6 + D_4 D_6^2] + \bar{c}[(D_1 D_2 - L_1^2)(D_4 D_7 - D_5^2) \\ &- 2L_1 L_2 D_3 D_7 + 2L_1 L_3 D_3 D_5 + 2L_1 L_2 D_5 D_6 - 2L_1 L_3 D_4 D_6 + L_2^2 D_2 D_7 \\ &- 2L_2 L_3 D_2 D_5 + L_2^2 D_6^2 - 2L_2 L_3 D_3 D_6 + L_3^2 D_2 D_4 \\ &+ L_3^2 D_3^2 + D_1 D_3^2 D_7 - 2D_1 D_3 D_5 D_6 + D_1 D_4 D_6^2] \\ &+ \bar{P}[(D_1 D_2)(PD_7 - MD_5) + a_3 \cos\theta D_1(D_3 D_7 - D_5 D_6) \\ &+ D_1 D_6(PD_6 - MD_3) - L_1^2(PD_7 - MD_5) \\ &- a_3 \cos\theta L_1(L_2 D_7 - L_3 D_5) + L_1 D_6(ML_2 - PL_3) - a_1 \sin\theta L_1(D_3 D_7 - D_5 D_6) \\ &+ a_1 \sin\theta D_2(L_2 D_7 - L_3 D_5) + a_1 \sin\theta D_6(L_2 D_6 - L_3 D_3) \\ &+ L_1 L_3(MD_3 - PD_6) - D_2 L_3(ML_2 - PL_3) - a_3 \cos\theta L_3(L_2 D_6 - L_3 D_3)] \\ &- \bar{M}[(D_1 D_2)(PD_5 - MD_4) + a_3 \cos\theta D_1(D_3 D_5 - D_4 D_6) \\ &- D_1 D_3(MD_3 - PD_6) - L_1^2(PD_5 - MD_4) - a_3 \cos\theta L_1(L_2 D_5 - L_3 D_4) \\ &+ L_1 D_3(ML_2 - PL_3) - a_1 \sin\theta L_1(D_3 D_5 - D_4 D_6) \\ &+ a_1 \sin\theta D_2(L_2 D_5 - L_3 D_4) + a_1 \sin\theta D_3(L_2 D_6 - L_3 D_3) \\ &+ L_1 L_2(MD_3 - PD_6) - D_2 L_2(ML_2 - PL_3) - a_3 \cos\theta L_2(L_2 D_6 - L_3 D_3)] \\ &+ \bar{a}_3 T_0 \cos\theta[a_3 \cos\theta D_1(D_4 D_7 - D_5^2) - D_1 D_3(PD_7 - MD_5) \\ &+ D_1 D_6(PD_5 - MD_4) - a_1 \sin\theta L_1(D_4 D_7 - D_5^2) \\ &- a_1 \sin\theta D_3(L_2 D_7 - D_5 L_3) + a_1 \sin\theta D_6(L_2 D_5 - L_3 D_4) + L_1 L_2)(PD_7 - MD_5) \\ &+ a_3 \cos\theta L_2(L_2 D_7 - L_3 D_3) - L_2 D_6(ML_2 - PL_3) \\ &- L_1 L_3(PD_5 - MD_4) - a_3 \cos\theta L_3(L_2 D_5 - L_3 D_4) + D_3 L_3(ML_2 - PL_3)] \\ &+ \bar{a}_1 T_0 \sin\theta[a_3 \cos\theta L_1(-D_4 D_7 + D_5^2) \\ &- L_1 D_3(PD_7 - MD_5) + L_1 D_6(PD_5 - MD_4) + a_1 \sin\theta D_2(D_4 D_7 - D_5^2) \\ &+ a_1 \sin\theta D_3(D_3 D_7 - D_5 D_6) - a_1 \sin\theta D_6(D_3 D_5 - D_4 D_6) \\ &- L_2 D_2(PD_7 - MD_5) - a_3 \cos\theta L_2(D_3 D_7 - D_5 D_6) \\ &+ L_2 D_6(MD_3 - PD_6) - L_3 D_2(PD_5 - MD_4) \\ &+ a_3 \cos\theta L_3(D_3 D_5 - D_4 D_6) - L_3 D_3(MD_3 - PD_6)], \end{aligned}$$

$$\begin{aligned} D = &-(D_8 D_1 D_2 + L_1^2)(D_4 D_7 - D_5^2) - 2L_1 L_2 D_3 D_7 + 2L_1 L_3 D_3 D_5 \\ &- 2L_1 L_3 D_4 D_6 + 2L_1 L_2 D_5 D_6 + L_2^2 D_2 D_7 - 2L_2 L_3 D_2 D_5 \\ &+ L_2^2 D_6^2 - 2L_2 L_3 D_3 D_6 + L_3^2 D_2 D_4 \\ &+ L_3^2 D_3^2 + D_1 D_3^2 D_7 - 2D_1 D_3 D_5 D_6 + D_1 D_4 D_6^2, \end{aligned}$$

and

$$L_1 = (c_{31} + c_{55})\sin\theta\cos\theta, \quad L_2 = 2\lambda_{31}\sin\theta\cos\theta + \lambda_{11}\sin^2\theta,$$
$$L_3 = 2F_{31}\sin\theta\cos\theta + F_{11}\sin^2\theta,$$
$$D_1 = c_{11}\sin^2\theta + c_{55}\cos^2\theta, \quad D_2 = c_{55}\sin^2\theta + c_{33}\cos^2\theta,$$
$$D_3 = \lambda_{31}\sin^2\theta + \lambda_{33}\cos^2\theta, \quad D_4 = \gamma_1\sin^2\theta + \gamma_3\cos^2\theta,$$
$$D_5 = \alpha_1\sin^2\theta + \alpha_3\cos^2\theta, \quad D_6 = F_{31}\sin^2\theta + F_{33}\cos^2\theta,$$
$$D_7 = A_1\sin^2\theta + A_3\cos^2\theta, \quad D_8\left(\tau_0 - \frac{\iota}{\omega}\right) = K_1\sin^2\theta + K_3\cos^2\theta,$$

$$P = p_1 \sin\theta + p_3 \cos\theta, \ M = m_1 \sin\theta + m_3 \cos\theta, \ \bar{P} = \frac{P}{\rho}, \ \bar{M} = \frac{M}{\rho},$$
$$\bar{a}_3 = \frac{a_3}{\rho}, \ \bar{a}_1 = \frac{a_1}{\rho}, \ \bar{c} = \frac{c}{\rho}.$$

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