

Dissipative Heating and Thermal Fatigue Life Prediction for Structures Containing Piezoactive Layers

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A coupled dynamic problem of thermo-electro-mechanics for a layered beam is formulated based on the Kirchhoff–Love hypotheses. In the case of harmonic loading, a simplified formulation is given using the single frequency approximation and the concept of complex moduli. As an example, the problem of forced vibration of a three-layer sandwich beam (aluminium alloy core covered with piezoelectric layers) with hinged ends is solved in order to investigate the accuracy and applicability of the approximate monoharmonic approach and to study the self-heating caused by the electromechanical losses in piezoactive layers and the mechanical losses in electrically passive metal layer. The thermal fatigue life was estimated assuming that the structure fails if the temperature exceeds the Curie point for piezoceramics.

1 Introduction

The forced-vibration analysis of structures occupies a significant place in the dynamics of deformable systems. An accurate prediction of the dynamic response is a serious challenge, since the material of a structure may become plastic under intensive loading and/or exhibit viscous properties. Variable viscoelastoplastic behaviour should be studied when designing metal dampers for the vibrations of building structures under wind and seismic loads, devices for suppressing vibrations of pipelines, test specimens in low-cycle fatigue tests, etc.

The stringent requirements to modern complex devices have recently compelled many researchers to pay attention to the modelling and control of the vibration of flexible structures (Rao and Sunar, 1994; Liu et al., 1999; Vidoli and dell’Isola, 2001; Han and Lee, 1998; Bao et al., 1998; Fernandes and Pouget, 2002; Zhuk and Senchenkov, 2004; Sridharan and Kim, 2009; Zhuk and Guz, 2009). The rapid development of the modern technology necessitates changing over from the traditional methods of vibration control to new ones that allow implementing more complex and highly effective operating modes and observing numerous life and reliability criteria. As a result, the modelling of the vibration of flexible structures and their elements and study of the possibilities for controlling them have gained a new impetus and are aimed at the development of systems with improved or qualitatively new characteristics. The ultimate goal of such studies is to create a new generation of active adaptive materials capable of responding to an external load according to predefined criteria or a program and having the functions of self-checking, self-diagnostics, and self-restoration (Tani et al., 1998).

Using the electromechanical model (Zhuk and Senchenkov, 2004; Zhuk et al., 2011a,b) as the starting point, the simplified monoharmonic approach is developed in this paper here in order to describe the coupled thermo-electro-mechanical dynamic behaviour of thin wall structures consisting of elastic piezoactive and inelastic metal layers under cyclic electric or mechanical loading. As an example, a three-layer sandwich beam (aluminium alloy core covered with piezoelectric layers) with hinged ends is studied. The beam undergoes harmonic mechanical loading by the moments applied to its ends or by the harmonic voltages applied to the electrodes covering the piezoactive layers.

The developed formalism of thermo-electro-mechanical problem is used for estimating the self-heating caused by the electromechanical losses in piezoactive layers and the mechanical losses in electrically passive metal layer. For this purpose, the heating temperature evolution with time is computed. The thermal fatigue life is estimated assuming that the structure fails if the temperature exceeds the Curie point for piezoceramics. Using this criterion, the safe dissipative heating levels under harmonic loading are determined. The developed method is also successfully used for describing the thermal state under active damping regimes for other thin wall structures with piezoelectric layers for the case of small to moderate inelastic strains in the metal layer.

2 Two Approaches to Modelling Mechanical Response of Inelastic Solids under Cyclic Loading

Modelling the mechanical behaviour of inelastic materials is one of the major challenges to be accomplished in evaluating the durability or function characteristics of deformed solids. There are currently two approaches to solve such problems. One employs the constitutive equations valid for arbitrary or, at least, rather wide classes of loading histories (Chan et al., 1984; Chaboche, 1993; Sands et al., 2010). The other approach is based on simplified models for specific deformation history (Krempf, 2000; Zhuk and Senchenkov, 2004; Senchenkov et al., 2004).

Within the framework of unified flow theories used in the first approach, the time-dependent inelastic behaviour of crystalline solids is represented as part of a combined elastic-inelastic formulation that may not rely on a yield criterion or loading and unloading conditions. The same equations are applicable for all circumstances such as straining at prescribed rates, creep under constant stress, and stress relaxation under constant strain. An objective of a unified constitutive theory is that it should be suitable for certain classes of materials over a wide range of strain rates and temperatures. To be useful for engineers, the equations have to be reasonably simple, possess a firm physical basis and be consistent with the principles and constraints of mechanics and thermodynamics (Chan et al., 1984; Chaboche, 1993; Sands et al., 2010). The second approach was developed especially for one of the most important cases of periodic processes – for cyclic (particularly harmonic) loading. The idea of simplification can be clearly demonstrated by an example of linearly viscoelastic materials. In the case of harmonic deformation, all basic types of constitutive equations – integral (hereditary) or differential – have an algebraic form written in terms of complex stress and strain amplitudes. The relationship between these quantities is expressed in terms of complex-value moduli (Christensen, 1971).

The present paper is devoted to the comparison of the results obtained within the framework of the complete and simplified problem statements for the particular case of forced vibration of thin wall members containing piezoelectric layers. Both problem statements are briefly described below.

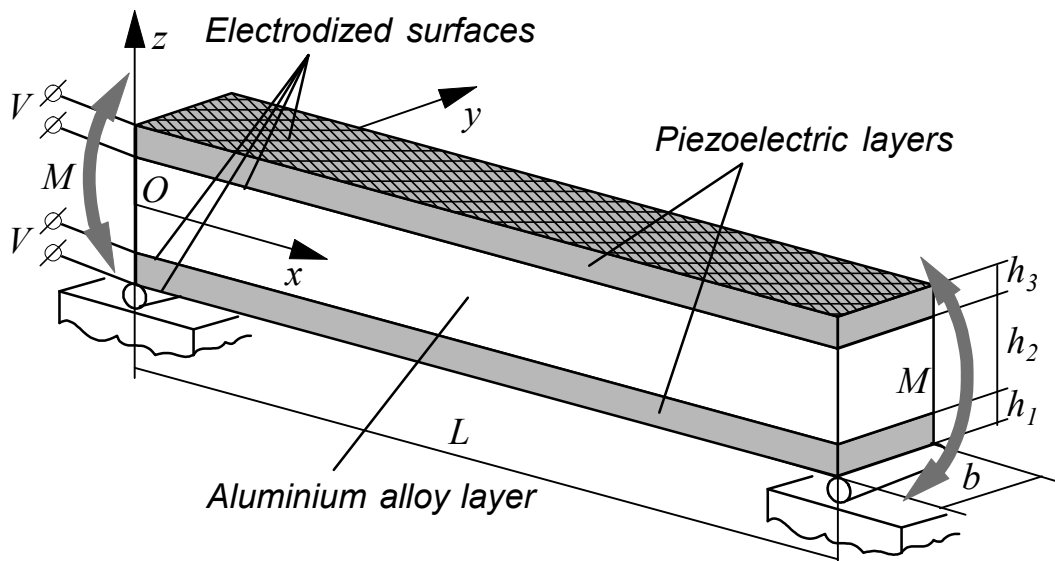


Figure 1: Beam geometries and the loading scheme

Let us briefly specify the complete statement of the electro-mechanical problem of the forced vibration and dissipative heating of layered beams composed of electrically passive and piezoelectric layers of constant thickness. The layers are in perfect contact with each other. Each s^{th} piezoelectric layer is covered with infinitely thin electrodes to which an electric potential difference $\varphi_z^{(s)}$ can be applied. The beam is referred to the rectangular Cartesian coordinate system $Oxyz$ as shown in Figure 1 for the particular case of hinged ("roller supported") three-layer sandwich beam consisting of a middle aluminium alloy layer and the outer PZT layers.

The beam length is L and its total thickness is $h = \sum h_s$, where h_s is the thickness of s^{th} layer. A surface equidistant to the surfaces of the layers is chosen to be the reference one. The layers are numbered in the z -direction. The coordinate of the upper surface of the s^{th} layer is denoted by z_s . The PZT layers are polarised in the positive or negative z -direction only. At this point, the material of the piezoactive layer is considered to be elastic transversely isotropic or viscoelastic one. The material of the electrically passive layers can demonstrate the inelastic response.

In deriving the equations of state, we assume that the total strain ε_{ij} includes elastic (or, in general case, viscoelastic), ε_{ij}^e , and inelastic, ε_{ij}^p , strain components. The constitutive equations for linear piezoelectric materials without pyroeffect can be written in the following form (Yang and Batra, 1995):

$$\sigma_{ij} = c_{ijkl}^D \varepsilon_{kl} - h_{mij} D_m, \quad E_n = \beta_{nm}^S D_m - h_{nkl} \varepsilon_{kl}, \quad i, j, k, l, m, n = 1, 2, 3 \leftrightarrow x, y, z, \quad (1)$$

where σ_{ij} and ε_{ij} are, respectively, the stress and strain tensors; E_i and D_i are, respectively, the electric field and electric displacement; c_{ijkl}^D is the isothermal elastic tensor at constant electric displacement; h_{ijk} are the piezoelectric constants; and β_{ij}^S is the permittivity matrix at constant strain. These equations were derived in the electrostatic approximation, i.e., the equations of electrostatics (Zhuk and Guz, 2009; Yang and Batra, 1995) are assumed to be true. The Bodner-Partom model was used to simulate the inelastic response of the electrically passive material (aluminium alloy), see (Chan et al. 1984). The detailed formalism can be found in (Zhuk and Guz, 2009).

In the case of long-term inelastic deformation, the complexity of the solution obtained within the framework of complete problem statement is due to the necessity of storing a large body of data and performing extensive computations to allow for the deformation history. To overcome these difficulties, in the practically important case of harmonic loading, an approximate monoharmonic model of thermo-electro-mechanical processes can be used. The simplified model is based on the concept of complex moduli, which are determined by a modified technique of equivalent linearization (Zhuk and Senchenkov, 2004; Senchenkov et al., 2004). In terms of these moduli, the initial problem is reduced to a scleronomous system of equations for complex amplitudes of mechanical and electrical variables – displacements, stresses, total and inelastic strains, voltage and electric current. Equations of the simplest version of the model for monophasic loading were developed in (Senchenkov et al., 2004).

3 Thermal Aspects

The single frequency approximation can be particularly useful for the investigation of the problems where the amplitudes of the main field variables play a major role, when the peculiarities of the hysteresis loop and specific behavior over the vibration cycle are not relevant (Senchenkov et al., 2004). Fatigue life prediction and estimation of the dissipative heating levels are among the useful results (Zhuk and Senchenkov, 2004; Senchenkov et al., 2004).

Let us consider the coupled thermo-electromechanical problem for the tree-layer beam shown in Figure 1 when the mismatch between the heat expansion coefficients of the constituents is not taken into account. To incorporate the thermal effects under harmonic loading, one should start with the energy balance equation which yields the general form of heat conduction equation. Introducing temperature increase as $\theta = T - T_0$ and averaging the heat conduction equation over the period of vibration as well as neglecting the contribution of thermoelastic terms to the heating (Senchenkov et al., 2004), one can obtain the heat conduction equation in the form

$$c_V \dot{\theta} = k(\theta_{,xx} + \theta_{,zz}) - 2\theta \alpha_y / b + D'(x, z), \quad (2)$$

where T and T_0 are current and initial temperatures, c_V is the specific heat capacity at constant volume, k is the thermal conductivity coefficient, α_y is the heat transfer coefficient from the beam faces perpendicular to the Oy axis, D' is the density of the dissipation power due to electromechanical losses, b is the width of the beam. The perfect thermal contacts at the material interfaces are assumed, i.e. the temperatures on and the thermal fluxes through the interfaces are equal. Let us emphasise that θ and D' are used to denote the averaged temperature and dissipation respectively.

The dissipative function for the piezoactive layer can be written as follows (Karnaukhov and Kirichok, 1986)

$$D'(x, z) = \frac{\omega}{2} \text{Im} \left[\tilde{\sigma}_{xx} \tilde{\varepsilon}_{xx}^* + \tilde{E}_z \tilde{D}_z^* \right], \quad (3)$$

where the complex conjugate quantities are marked with an asterisk. Taking account of the amplitude constitutive relations, the final form of the expression for the averaged dissipation function for the piezoactive layer contains three terms that determine mechanical, dielectric and piezoelectric losses. In the case of the electrically passive metallic layer, only the term which specifies the mechanical losses appears.

It is assumed that convective heat exchange with the environment happens on the each beam face. Corresponding boundary conditions as well as the initial conditions for temperature are to be added to complete the problem statement.

Let us emphasize here that the heat conduction problem is solved as a two-dimensional problem in the plane Oxy . It allows the heat generation and distribution along longitudinal and thickness dimensions of the beam to be investigated.

4 Material Properties

In order to solve the model problem of forced vibration under harmonic mechanical or electrical loading of a hinged beam containing piezoelectric layers, we need to specify both the actual and the cyclic properties of the constituents. The following beam lay-up is considered: the middle layer is made of aluminium alloy and the outer layers are made of piezoactive material. The piezoceramics PZT is chosen as a material of the piezoactive layers,

It was mentioned above that within the framework of the complete problem statement, the response of the PZT to the harmonic loading is considered to be the elastic one. The density of piezoceramics is equal to 7300 kg/m^3 . The relevant moduli for transversely isotropic material are taken as follows

$$\begin{aligned} c_{11}^D &= 1.12 \cdot 10^5 \text{ MPa}, \quad c_{12}^D = 0.64 \cdot 10^5 \text{ MPa}, \quad c_{13}^D = 0.5 \cdot 10^5 \text{ MPa}, \quad c_{33}^D = 1.25 \cdot 10^5 \text{ MPa}, \\ h_{31} &= -5.0 \cdot 10^8 \text{ V/m}, \quad h_{33} = 1.8 \cdot 10^9 \text{ MPa}, \quad \beta_{33}^S = 1.33 \cdot 10^8 \text{ m/F}. \end{aligned}$$

The response of the aluminium alloy within the framework of the complete problem statement is described by a set of the physical properties and the Bodner-Partom model parameters, which are chosen as

$$\begin{aligned} \rho &= 2692.65 \text{ kg/m}^3, \quad E = 0.816 \cdot 10^5 \text{ MPa}, \quad \nu = 0.34, \quad D_0 = 10^4 \text{ c}^{-1}, \quad n = 2.06, \\ m_1 &= 0.182 \text{ MPa}^{-1}, \quad m_2 = 3.7 \text{ MPa}^{-1}, \quad K_0 = 323.6 \text{ MPa}, \quad K_1 = 647.4 \text{ MPa}, \quad D_1 = 80 \text{ MPa}. \end{aligned}$$

Here density, Young's modulus and Poisson's ratio are denoted by ρ , E and ν respectively. For complete explanation of the model parameters D_0 , n , m_1 , m_2 , K_0 , K_1 and D_1 see, for example, (Chan et al., 1984).

Making use of the data listed above and the complete problem statement derived in Zhuk and Guz, 2009, it is possible to compute both the transient and the steady state response of the beam to harmonic loading.

The cyclic properties of material should be determined either experimentally or obtained numerically (within the framework of chosen material model) as a response to the harmonic loading. The complex moduli for aluminium alloy are taken from (Senchenkov et al., 2004) where the detailed procedure for calculating them is also described. Dependencies of the storage modulus, G' , the loss modulus, G'' , and the real and imaginary parts of the plasticity coefficients κ_p' and κ_p'' on the stress intensity, e_i , are also shown by Senchenkov et al. (2004). Let us emphasise that for the simplified model adopted here, the volumetric modulus K_V is a real quantity. The value of this modulus for the aluminium alloy equals $8.5 \cdot 10^4 \text{ MPa}$.

Following (Schwartz, 2002; Sabata et al., 2007), the cyclic behaviour of PZT is considered to be a viscoelastic one with relatively small loss moduli. Complex moduli for this material are assumed to be independent of the frequency, temperature and strain intensity. Real parts of the relevant complex moduli are chosen to be equal to

the correspondent actual moduli for the complete problem statement given in the previous subsection. The loss tangents for transversely isotropic material are (Pugachev, 1984)

$$\tan \delta_{c_{11}} = c_{11}^{D''}/c_{11}^{D'} = 0.0128, \quad \tan \delta_{c_{12}} = c_{12}^{D''}/c_{12}^{D'} = 0.0144, \quad \tan \delta_{c_{13}} = c_{13}^{D''}/c_{13}^{D'} = 0.02, \quad \tan \delta_{c_{33}} = c_{33}^{D''}/c_{33}^{D'} = 0.015,$$

$$\tan \delta_{h_{31}} = h_{31}^{S''}/h_{31}^{S'} = -0.0125, \quad \tan \delta_{h_{33}} = h_{33}^{S''}/h_{33}^{S'} = 0.0142, \quad \tan \delta_{\beta_{33}} = \beta_{33}^{S''}/\beta_{33}^{S'} = -0.0215$$

Here the storage moduli are denoted by prime and the loss moduli are denoted by the double prime.

For temperatures below the Curie point for piezoceramics, which is the region of our main interest, the complex moduli for an aluminium alloy exhibit a weak dependence on the temperature and, therefore, can be considered as temperature independent quantities for the purposes of the work presented here. Manufacturers provide different data for the Curie point, T_C , ranging from 150°C to 350°C depending on the PZT type and composition. In the present investigation, it is chosen to be equal to 200°C, and therefore, in terms of the temperature increase $\theta_C = T_C - T_0 = 180^\circ\text{C}$. The following parameters were used to solve the heat conduction problem:

$$\alpha_V = 100 \text{ W/m}^2\cdot\text{K}, \quad T_0 = 20 \text{ }^\circ\text{C}; \quad c_V = 2.48 \cdot 10^6 \text{ J/m}^3\cdot\text{K} \text{ and } k = 90 \text{ W/m}\cdot\text{K} \text{ for the aluminium alloy; and}$$

$$c_V = 3.58 \cdot 10^6 \text{ J/m}^3\cdot\text{K} \text{ and } k = 1.25 \text{ W/m}\cdot\text{K} \text{ for the PZT piezoceramics.}$$

5 Results and Discussion

The model problem of vibration and dissipative heating of a sandwich beam under harmonic mechanical or electric loading is discussed in this section. The vibrations in the vicinity of the first resonance frequency occur in the first symmetric mode. In all cases, the upper PZT piezoceramic layer is polarized in the positive direction of z -axis, and the lower layer is polarised in the opposite direction. Two types of the beam support are considered here: a simply supported beam and a hinged ("roller supported") one.

5.1 Limits of the Applicability of the Simplified Model

The nature of the approximations introduced within the framework of the simplified monoharmonic approach imposes the obvious limitations on the model use. All of them are considered here for the isothermal case. The version of the monoharmonic approach adopted here does not deal with the mean strain and stress. To exclude them from the consideration and prevent formation of the hollow arch instead of the beam due to the inelastic deformation, a gradual linear growth of the load amplitude (both, mechanical and electrical) should be provided at the beginning for several cycles – the so called “adaptation period”. After that, the amplitude of loading remains constant. This initial adaptation period lasted for fifty cycles in our study. The next fifty cycles are used as a stabilization period to allow for the steady-state vibration of the beam to be reached. Then, the amplitudes of the investigated quantities are computed.

Analysis of the typical transient response of the beam to harmonic mechanical as well as electric excitation in the presence of both physical and geometrical nonlinearities demonstrated that after the first 50 cycles of vibration, when the load amplitude reaches the steady state, the deflection amplitude shows a small decrease caused by the hardening of the aluminium layers adjacent to the piezoceramics. After the completion of the transient adaptation period, the cycles of vibration become symmetric about a horizontal line. The total strain, the inelastic strain and the stress as well as the electric current taken from the PZT layer electrodes tend to form the stable loops after the short transition period. The cycle of stress response is symmetric during the steady-state vibration. The elastic limit of the material is reached with increasing load. This time point corresponds to the change in the envelope angle. In the presence of geometrical nonlinearity alone, the history of inelastic strain is not symmetric about zero because of the deformation of the beam axis. If, however, only physical nonlinearity is present, the history of inelastic deformation is symmetric. In the sensor mode, the electric current from one of the piezoelectric layers is also characterized by a cycle weakly asymmetric about zero. The reason is that the charge induced by the inverse piezoelectric effect is determined by the deformation of the piezoelectric layer, which is not symmetric about zero due to the geometrical nonlinearity.

In general, the asymmetry of the deformation cycles can be attributed to the geometrical nonlinearity, particularly associated with the deformation of the beam axis. Since the axis undergoes tensile deformation, the strain in the stretched layers increases and the strain in the compressed layers decreases within each half-cycle. The situation

is similar for the electric current. If the geometrical nonlinearity is disregarded, all the curves will be symmetric and the strain of the beam axis will be zero.

Let us emphasize here that the two types of loading (mechanical and electrical) are equivalent for the particular problem under consideration, in the sense that they cause the same deflections and produce the same levels of the stress, the total strain and the inelastic strain. Therefore, only one set of figures for mechanical loading can be shown.

As expected, the closer the frequency is to the resonance, the smaller the influence of the higher harmonics is on the response of the system. In other words, the monoharmonic approximation becomes more accurate if the system vibrates in the resonance regime, because the amplitude of vibration with the excitation frequency is amplified at the resonance.

The effect of the higher harmonics can be more pronounced if the excitation frequency differs significantly from the resonance. In order to investigate this aspect of the beam behaviour, the forced vibrations were considered at the different frequencies of loading. Due to the type of nonlinearity dealt with (i.e. elastic-viscoplastic response of the electrically passive material), the additional factor influencing the system response is the level of loading. The higher loading leads to a more complex behaviour of the beam as a result of the interplay between physical nonlinearity, geometrical nonlinearity and electro-mechanical coupling. It was shown that for a hinged "roller supported" beam shown in Figure 1, under the moderate loading levels and the deflections up to 1.5 of the beam thickness, the influence of geometrical nonlinearity is rather small, in contrast to the problem studied in (Zhuk and Guz, 2009), where the membrane force played a significant role.

In general, it was found that the simplified approach provides a reliable and accurate estimation of amplitudes of the main mechanical parameters and the electric current taken from piezoelectric layers, if the influence of the geometrical nonlinearity is small. This is true not only in the immediate vicinity of the first resonance, but also for the region of quasi-static response.

5.2 Amplitude-Frequency Characteristics

There are two main points in understanding the nature and estimating the level of the dissipative heating under intensive harmonic loading: amplitude-frequency characteristics and distribution of the dissipation function. The amplitude-frequency characteristics are very useful for investigation of the applicability of the approximate monoharmonic approach to the solution of the class of problems under consideration and for estimation of its accuracy and the limits of applicability. They provide the qualitative and quantitative analysis of cyclic response of the inelastic material and give the general idea about the dangerous frequency domain. The set of amplitude-frequency characteristics for different parameters of electromechanical state is obtained within the frameworks of both, the complete and the approximate problem statements. Since the considered mechanical and electrical loading produces the identical response of the beam, only the set of plots for mechanical loading will be shown below.

The amplitude-frequency characteristics of the forced vibration under mechanical excitation of a hinged beam with $L = 2.0$ m, $h = 0.6 \cdot 10^{-1}$ m, $h_1 = h_3 = 0.2 \cdot 10^{-2}$ m ($h_2 = 0.56 \cdot 10^{-1}$ m), $b = 0.3 \cdot 10^{-1}$ m are presented in Figure 2 for six different values of the controlled parameter (amplitude of loading bending moment) $M_0 = 0.2 \cdot 10^{-3}$, $0.2 \cdot 10^{-1}$, 0.2 , 0.5 , 1.0 and 2.0 kN·m. The amplitudes of the stress and the inelastic strain at the most stressed point of the metal are given in Figures 2(a) and 2(b). The results obtained within the framework of the approximate approach are shown by the solid lines. The results of direct solution of the problem using the complete model are indicated by the squares. All calculations are performed for $M_0 = 1.0$ kN·m. Computing the transient response is extremely time and resource consuming, especially in the vicinity of the resonance.

All dependences presented in Figure 2 demonstrate nonlinearity of the soft type. The quantities have finite amplitudes in the resonance domain. Inelastic strains are observed over a limited region of frequencies. This region has a tendency to grow if the controlled parameters M_0 or V_0 increase. The curves for stress amplitude under a significant load causing inelastic deformation - see the lines for $M_0 = 0.2$, 0.5 , 1.0 and 2.0 kN·m in Figure 2(a) - have a plateau-like segment expanding as the controlled parameter increases. This peculiarity is due to the weak hardening of cyclic stress-strain curve for aluminium alloy. Figure 2(b) allows one to determine the frequency region where the inelastic deformation occurs. The high temperature release is expected in it. In general, the results obtained within the frameworks of the complete and the approximate monoharmonic problem statements agree well with each other.

The analysis of Figure 2 shows that the simplified model works well if the geometrical nonlinearity is not taken into account. For other beam geometries and boundary conditions, the squares of rotation angles (which may appear due to the comparatively large deflections) and the membrane force (in the case of the pin hinged ends, for example) may have to be accounted for to appropriately describe the geometrically non-linear behaviour. This requires the harmonics higher than the first one to be taken into consideration.

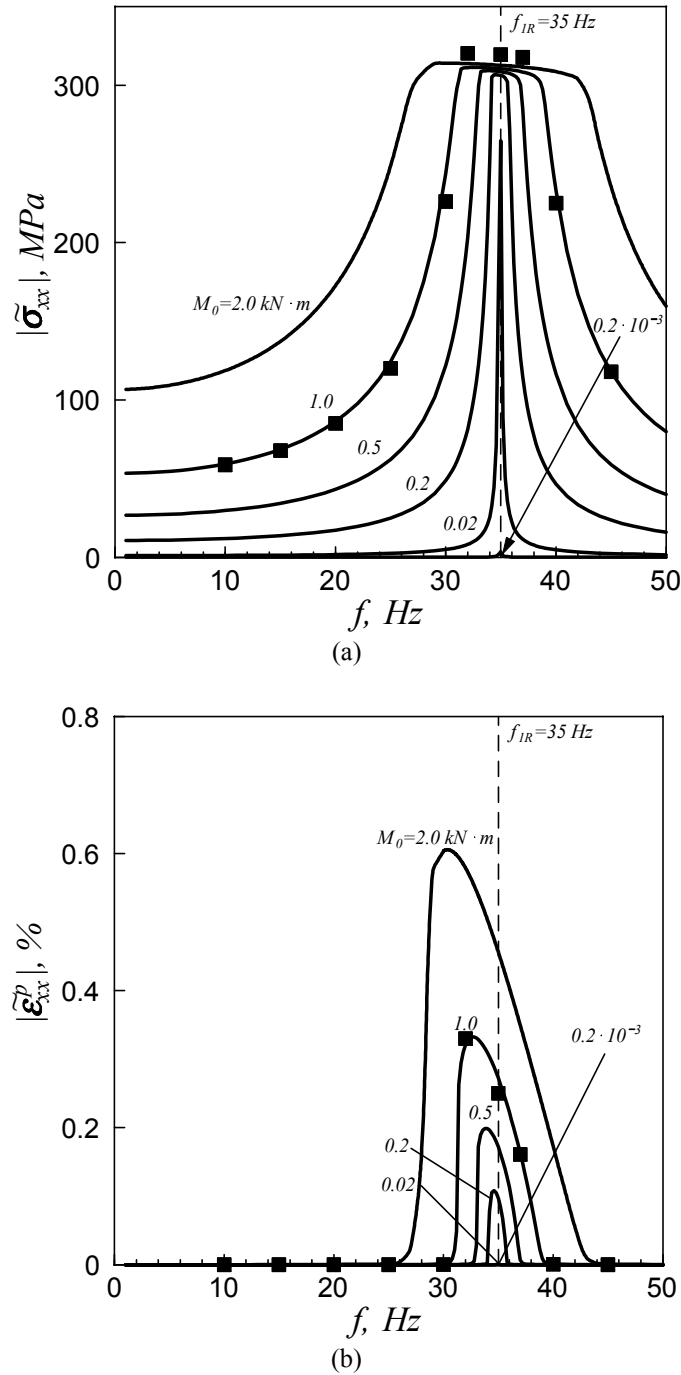
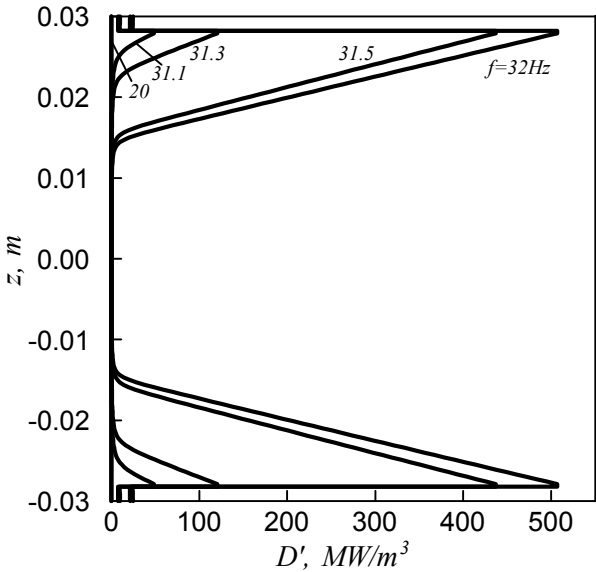
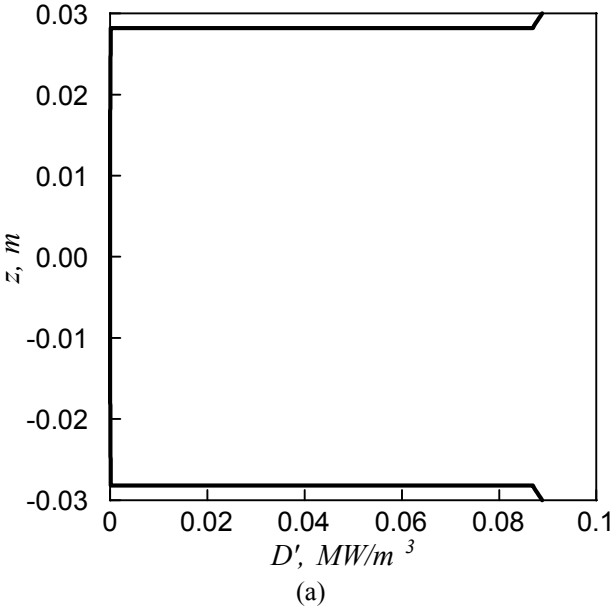


Figure 2: Amplitude-frequency characteristics of the forced vibration for a three layer beam excited mechanically or electrically: (a) amplitude of stress; (b) amplitude of inelastic strain plotted against frequency at the point $(L/2, -h_2/2)$

5.3 Dissipative Heating and Thermal Fatigue Life Prediction

The dissipative heating is essentially defined by the distribution of dissipative function over the body. In turn, it is ruled by the spatial distributions of the amplitudes of the main field electro-mechanical characteristics. The peak values of them are located in the central cross-section of the beam and can be attributed to the dynamic effect. It worth mentioning that inelastic deformation occurs in the regions adjacent to the piezoelectric layers. This fact is decisive for the dissipative function shaping and, therefore, the heating zones formation.

Examples of central cross-section distributions of the dissipative function for $f = 10$ Hz and for the set of frequencies in the resonance vicinity are shown in Figures 3(a) and 3(b) respectively. The numbers in Figure 3(b) display the frequency values. Far away from the resonance region, the metal layer deforms elastically and does not contribute to the heating over the vibration period. In this case, the energy losses (mechanical, piezoelectric and dielectric) occur in the piezoelectric layers only, see Figure 3(a). The situation changes dramatically if even a small inelastic strain appears in the metal layer, Figure 3(b). Then the mechanical losses in the metal dominate. The maximal losses occur at the central cross-section in the regions adjacent to the piezoceramics. Presence of inelastic strain generates significant loss of mechanical energy that, being dissipated as heat, leads to an appreciable temperature raise. For a large number of cycles, this can cause an overheating of the piezoceramics and deterioration of its piezoelectric properties.



(b)

Figure 3: Distributions of the dissipative function along the beam central cross-section for (a) loading frequency $f=10$ Hz and (b) for the set of frequencies in the resonance vicinity.

To study the latter effect, the heating time histories for the beam are computed. The maximum heating is observed at the points $x = 1$ m; $z = \pm 0.28 \cdot 10^{-1}$ m which is the most stressed points of the metal layer where the maximal inelastic strain occurs. The temperature time histories at the point for different frequencies are shown in Figure 4. The numbers indicate the frequency values. The maximum heating is observed in the resonance region. Comparison with Figure 2 shows that the greater the deflection and, as a result, the inelastic strain the shorter the time period is needed to reach the Curie point. Far away from the resonance region, the temperature increase is not high enough to cause the dangerous temperature levels. Under these conditions, the vibrations can continue for a very long time period without compromising the piezoelectric properties of the ceramics. The distribution of the temperature through the thickness of the beam is almost uniform at the advanced stages of the process. At the beginning, it corresponds to the dissipative function shown in Figure 3. Then the temperature is approximately constant through the beam thickness, but still having a local maxima at the points $x = 1$ m; $z = \pm 0.28 \cdot 10^{-1}$ m. It happens due to the high thermal conductivity of the core aluminium alloy. Nevertheless, due to dynamic effects and pure heat conductivity of piezoceramics, the effect of the heating localization in the central part of the beam is easily observed. That is the region where the temperature reaches the Curie level first and depolarisation begins.

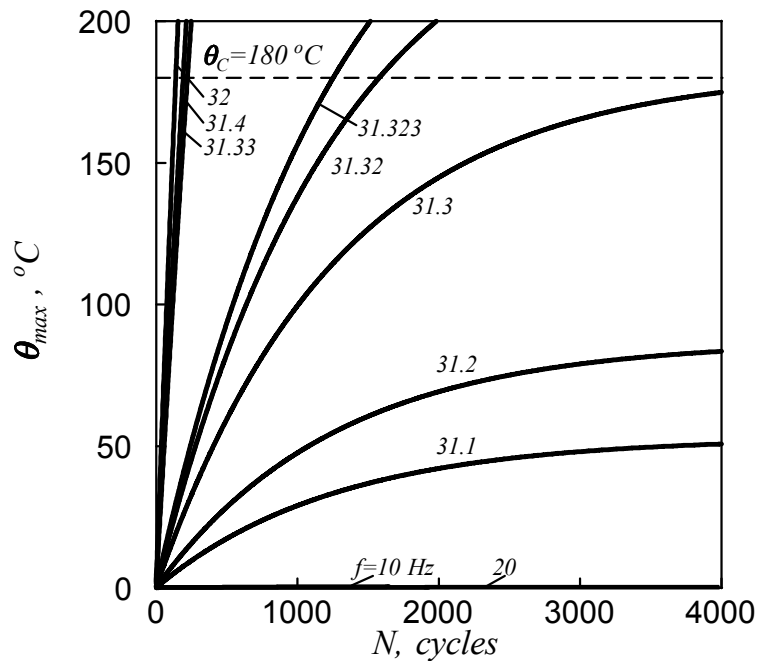


Figure 4: Time-temperature histories at the point $x = 1$ m; $z = 0.28 \cdot 10^{-1}$ m for different frequencies.

To avoid dangerous overheating, safe vibration regimes must be found. For this purpose, the thermal fatigue life characteristic, N_θ , can be introduced. It corresponds to the classical fatigue life, but with the Curie temperature taken as the failure criterion. A typical curve $M_0 - N_\theta$ for frequency $f = 32$ Hz (the fastest heating regime) is shown in Figure 5. The line divides the plane into two regions: a region above the curve where thermal failure can occur at a sufficiently high number of cycles and a region below the curve where thermal failure will not occur regardless of the number of cycles. For each value of the controlled parameter, the safe operating regime can easily be established from the appropriate curve. Small values of thermal fatigue life belong to the near-the-resonance region and match well with the domain where the amplitudes of inelastic strain become noticeable; see

Figure 2(b). Taking account of the information presented in Figure 5, one can easily estimate the useful life expectancy of a structure for a specified loading level.

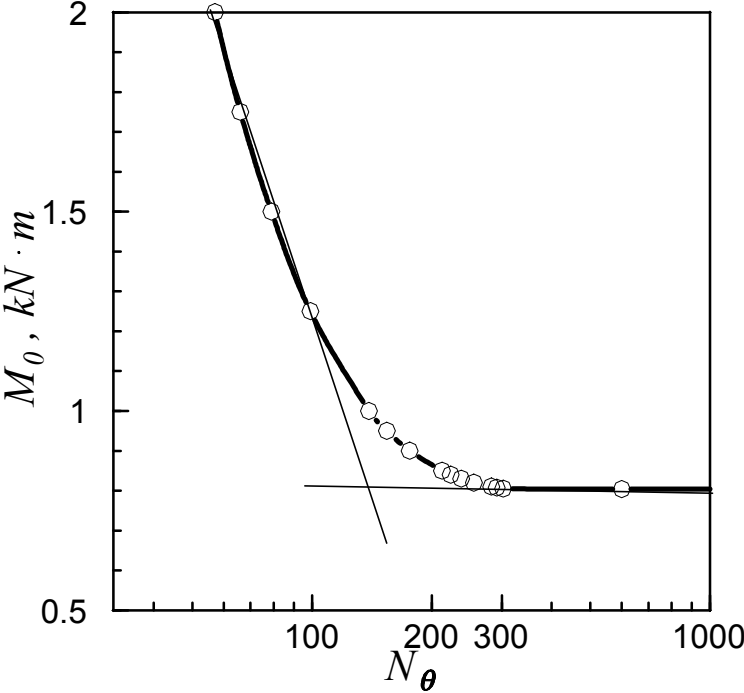


Figure 5: Typical thermal fatigue curve $M_0 - N_\theta$ for frequency $f = 32$ Hz.

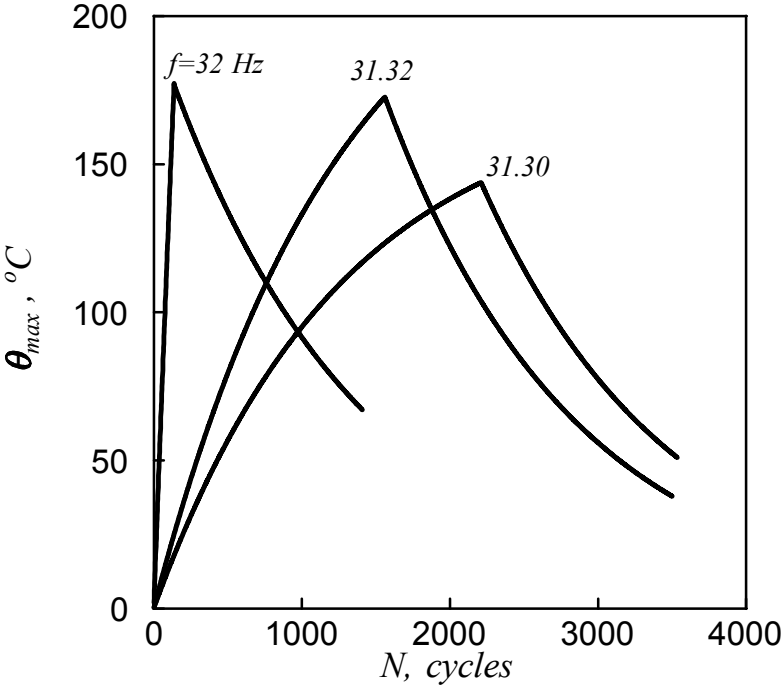


Figure 6: Time temperature histories at the maximum heating point $x = 1$ m; $z = 0.28 \cdot 10^{-1}$ m before and during active suppression of the vibration

To prevent overheating, active suppression of the vibration can be used. As it was mentioned above, in the cases of electric and mechanical excitation, the beam responds similarly to both types of loading. Hence, it is possible to damp mechanically excited vibrations by choosing the appropriate amplitude of voltage, V_0 , applied to the piezoelectric layers. The relationship between the controlled parameters of the electric (V_0) and mechanical (M_0) loading can be established by parametrizing the dependences of the deflection amplitudes vs M_0 and deflection amplitudes vs V_0 with respect to deflection for each frequency. As a result, the dependency V_0 vs M_0 is obtained. Despite of different levels of deflections and inelastic strains causing the strong nonlinearity, it appears the dependence is linear within the frequency range being considered and, therefore, can be specified by solving a linear problem.

Under suppression regime, dissipation is equal to zero in the electrically passive aluminium alloy layer if it deforms entirely elastically. The piezoelectric layers continue to provide heating, though, as a consequence of deformation induced internal friction and electrical activity. In this case, the term defining dielectric losses is the only nonzero one. The computations reveal that at the most dangerous points of PZT layers ($x = 1$ m; $z = \pm 0.3 \cdot 10^{-1}$ m) for $f = 32$ Hz and $M_0 = 1$ kN·m (maximum heating), the dielectric term contributes 15.6 % of the total dissipated energy, while the mechanical losses account for 78.3 % and the piezoelectric losses contribute a further 6.1 %. In turn, losses in the PZT layers are much smaller than those in the aluminium alloy layer, if vibrations are not suppressed; see Figure 3(b). Therefore, under the suppression regime, the power of the heat source decreases dramatically, and gradual cooling starts as a result of conduction inside the beam and heat convection to the outside. Several time temperature histories are shown in Figure 6 for frequencies 32, 31.32 and 31.30 Hz. In all cases, the immediate cooling begins, preventing the depolarisation of the piezoceramic layers. A stationary thermal state with nonzero temperature rise will be reached after a significant time interval.

6 Conclusions

When applied to layered thin wall structures containing piezoelectric active layers and undergoing harmonic electro-mechanical loading, the model based on the monoharmonic approximation and the concept of complex moduli is a powerful technique for determining the amplitudes of the main mechanical and electrical variables. This technique works very well for a systems consisting of elastic or viscoelastic piezoactive and inelastic electrically passive layers if the loading and boundary conditions allow for neglecting the influence of geometrical nonlinearity and hence ensure a symmetrical vibration cycle. If geometrical nonlinearity is significant, the second harmonic has to be taken account of.

Analysis of the data presented in the paper shows that the approximate model correctly estimates the amplitudes of the electric and mechanical parameters for small or moderate inelastic deformation of the passive layers. The nature of the approximate model does not allow considering large inelastic strains.

The proposed technique is useful for estimating the self-heating caused by electromechanical losses in the piezoactive layers and mechanical losses in the electrically passive metal layer. It is shown that the small temperature increases due to the dissipation of electromechanical energy over separate cycles of vibration can lead to a significant temperature rise for multi-cycle processes. Though the approximate approach is not designed for describing the transient mechanical response of structures, it is capable of simulating the heating temperature evolution with time. The problems of determining the safe dissipative heating levels under harmonic loading, predicting the thermal fatigue life, and modeling the thermal state under active damping regimes for thin wall layered structures containing piezoelectric layers can be successfully solved using this approach.

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