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Derivation of Theory of Thermoviscoelasticity by Means of Two-component Cosserat Continuum

E. A. Ivanova

We consider a mechanical model of a two-component medium whose first component is the classical continuum and the other component is the continuum having only rotational degrees of freedom. We show that the proposed model can be used for description of thermal and dissipative phenomena. Interpretation of the temperature, entropy and other thermodynamic quantities given in accordance with the proposed model is no more than the mechanical analogy. However, use of these mechanical counterparts allows one to obtain the well-known equations describing the thermal and diffusion processes within the framework of the model. The mathematical description of the proposed mechanical model includes as special cases not only the classical formulation of coupled problem of thermoelasticity but also the formulation of the coupled problem of thermoelasticity with the hyperbolic type heat conduction equation. In the context of the proposed mechanical model, an original interpretation of the volume (acoustic) viscosity and the shear viscosity is offered.

1 Introduction

There exist different macroscopic and microscopic models of internal damping like Ziman (1960); Truesdell (1965); Eringen (1980); Christensen (1971); Kondepudi and Prigogine (1998). There exists an extensive literature on the construction of diverse theories of viscoelasticity and thermoviscoelasticity. The models describing the properties of real materials have been proposed. A great number of the specific problems has been solved. The general approaches and some models are included in the textbooks on continuum mechanics like Truesdell (1965); Christensen (1971) and in the books meant for design engineers, e. g., Rabotnov (1988). However not all of the theoretical problems concerned with dissipative processes in a continuous medium are solved. Now the problem of the nature of internal damping in materials remains unclear. The point of view that internal damping is connected with thermal effects is widespread — see, for example, Ziman (1960). We are sure that the internal damping and heat conduction should be considered as a result of interaction of atoms with the infinite surrounding medium which can be called the *thermal ether*. We propose the mechanical model of the thermal ether which is the continuum of particles interacting by the elastic moments. We consider some model problems of the elastic interaction of thermal ether with the particle imbedded in it. As a result we show that the influence of the thermal ether on the particle can be modelled by the damping moment proportional to the angular momentum of the particle. Using of the damping moment in the model of a two-component medium allows us to describe the internal damping and the heat conduction mechanism.

2 Linear Theory of Continuum of One-Rotor Gyrostats

Now we construct the linear theory of the material medium consisting of one-rotor gyrostats. The one-rotor gyrostat is a particle which consists of the carrier body and the rotor (see Figure 1). The rotor can rotate independently of rotation of the carrier body, but it can not translate relative to the carrier body.

Let vector $\mathbf{r}$ determine the position of some point of space. We introduce the following notations: $\rho(\mathbf{r},t)$ is the mass density of the material medium at a given point of space; $\mathbf{v}(\mathbf{r},t)$ is the velocity field; $\mathbf{u}(\mathbf{r},t)$ is the displacement field; $\tilde{\mathbf{P}}(\mathbf{r},t)$, $\tilde{\boldsymbol{\omega}}(\mathbf{r},t)$ are the fields of the rotation tensors and the angular velocity vectors of the carrier bodies; $\mathbf{P}(\mathbf{r},t)$ and $\boldsymbol{\omega}(\mathbf{r},t)$ are fields of the rotation tensors and the angular velocity vectors of the rotors. We assume that in the reference configuration the tensors $\tilde{\mathbf{P}}(\mathbf{r},t)$ and $\mathbf{P}(\mathbf{r},t)$ are equal to the unit tensor. Therefore, upon the linearization near the reference position they take the form

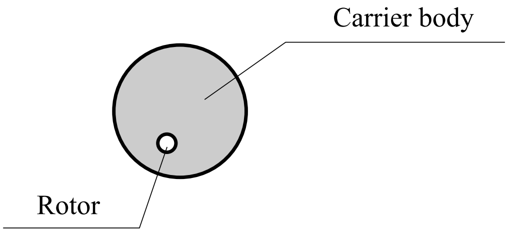

Figure 1. One-rotor gyrostat

$$\tilde{\mathbf{P}}(\mathbf{r},t) = \mathbf{E} + \boldsymbol{\varphi}(\mathbf{r},t) \times \mathbf{E}, \qquad \mathbf{P}(\mathbf{r},t) = \mathbf{E} + \boldsymbol{\theta}(\mathbf{r},t) \times \mathbf{E}, \tag{1}$$

where $\mathbf{E}$ is the unit tensor; $\boldsymbol{\varphi}(\mathbf{r},t)$, $\boldsymbol{\theta}(\mathbf{r},t)$ are the rotation vector fields of carrier bodies and rotors, respectively. Kinematic relations in the linear approximation are

$$\mathbf{v} = \frac{d\mathbf{u}}{dt}, \qquad \tilde{\boldsymbol{\omega}} = \frac{d\boldsymbol{\varphi}}{dt}, \qquad \boldsymbol{\omega} = \frac{d\boldsymbol{\theta}}{dt}. \tag{2}$$

The carrier bodies of the gyrostats are the classical particles. The rotors of the gyrostats represent body-points whose tensors of inertia are the spherical parts of tensors. The kinetic energy of such body-point takes the form

$$K = m_* \left(\frac{1}{2}\mathbf{v}\cdot\mathbf{v} + B\,\mathbf{v}\cdot\boldsymbol{\omega} + \frac{1}{2}J\,\boldsymbol{\omega}\cdot\boldsymbol{\omega} \right). \tag{3}$$

Here m_* is the mass of the body-point, B and J are the moments of inertia of the body-point. The model of the body-point (3) was proposed by P. A. Zhilin, see e. g. Zhilin (2003, 2006). A substantiation of the model can be found in Ivanova (2011). The linear momentum and angular momentum of the body-point (3) are

$$\mathbf{K}_1 = m_*\,(\mathbf{v} + B\,\boldsymbol{\omega}), \qquad \mathbf{K}_2 = m_*\,(B\,\mathbf{v} + J\,\boldsymbol{\omega}). \tag{4}$$

The equations of balance of linear momentum for the gyrostats and of angular momentum for the carrier bodies of gyrostats take the form

$$\nabla\cdot\boldsymbol{\tau} + \rho_*\mathbf{f} = \rho_*\frac{d}{dt}\big(\mathbf{v} + B\boldsymbol{\omega}\big), \qquad \nabla\cdot\boldsymbol{\mu} + \boldsymbol{\tau}_\times + \rho_*\mathbf{m} = \rho_*\frac{d}{dt}\big(\mathbf{I}_0\cdot\tilde{\boldsymbol{\omega}}\big). \tag{5}$$

Here $\boldsymbol{\tau}$ is the stress tensor, $\boldsymbol{\mu}$ is the moment tensor modelling the interaction of the carrier bodies of gyrostats, $(\ )_\times$ denotes the vector invariant of a tensor, $\mathbf{f}$ is the mass density of external forces, $\mathbf{m}$ is the mass density of external moments acting on the carrier bodies of gyrostats, ρ_* is the mass density of the material in the reference configuration, $\mathbf{I}_0$ is the mass density of the inertia tensors of carrier bodies in the reference configuration. The equation of balance of angular momentum for the rotors of gyrostats takes the form

$$\nabla\cdot\mathbf{T} + \rho_*\mathbf{L} = \rho_*\frac{d}{dt}\big(B\mathbf{v} + J\boldsymbol{\omega}\big), \tag{6}$$

where $\mathbf{T}$ is the moment tensor modelling the interaction of the rotors of gyrostats, $\mathbf{L}$ is the mass density of the external moments acting on the rotors, B and J are the mass density of the moment of inertia of the rotors. The moment of inertia B is the coefficient of the production of linear and angular velocities in the expression for the kinetic energy of rotor (3). The moment of inertia J is the coefficient of the squared angular velocity in this expression.

The constitutive equations are

$$\begin{aligned}
\boldsymbol{\tau}^T &= \boldsymbol{\tau}_0^T + {}^4\mathbf{C}_1\cdot\cdot\boldsymbol{\varepsilon} + {}^4\mathbf{C}_2\cdot\cdot\boldsymbol{\kappa} + {}^4\mathbf{C}_4\cdot\cdot\boldsymbol{\vartheta},\\
\boldsymbol{\mu}^T &= \boldsymbol{\mu}_0^T + \boldsymbol{\varepsilon}\cdot\cdot{}^4\mathbf{C}_2 + {}^4\mathbf{C}_3\cdot\cdot\boldsymbol{\kappa} + {}^4\mathbf{C}_5\cdot\cdot\boldsymbol{\vartheta},\\
\mathbf{T}^T &= \mathbf{T}_0^T + \boldsymbol{\varepsilon}\cdot\cdot{}^4\mathbf{C}_4 + \boldsymbol{\kappa}\cdot\cdot{}^4\mathbf{C}_5 + {}^4\mathbf{C}_6\cdot\cdot\boldsymbol{\vartheta}.
\end{aligned} \tag{7}$$

Here $\boldsymbol{\tau}_0$, $\boldsymbol{\mu}_0$, $\mathbf{T}_0$ are the initial stresses, ${}^4\mathbf{C}_k$ are the stiffness tensors, $\boldsymbol{\varepsilon}$, $\boldsymbol{\kappa}$, $\boldsymbol{\vartheta}$ are the strain tensors. The strain tensors are determined by the formulas

$$\boldsymbol{\varepsilon} = \nabla \mathbf{u} + \mathbf{E} \times \boldsymbol{\varphi}, \qquad \boldsymbol{\kappa} = \nabla \boldsymbol{\varphi}, \qquad \boldsymbol{\vartheta} = \nabla \boldsymbol{\theta}. \tag{8}$$

The structure of the stiffness tensors and the concrete values of the stiffness coefficients are determined by the physical properties of the medium. As evident from Eq. (7), in the general case all stress tensors depend on the all strain tensors.

Thus, the basic equations describing the dynamics of the elastic continuum of one-rotor gyrostats are presented above. The detailed derivation of these equations can be found in Ivanova (2010).

3 Continuum of One-Rotor Gyrostats and a Model of Thermoviscoelastic Medium

We consider the material continuum (see Figure 2) that consists of one-rotor gyrostats. Free space between the gyrostats is filled up by body-points the structure of which coincides with the structure of rotors belonging to the gyroststs. The body-points in the space between the gyrostats are the elementary particles of a continuum which will be called the thermal ether in what follows. We consider only the gyrostats continuum. The body-points continuum (thermal ether) positioned in space between gyrostats is an external factor with respect to continuum under study. That is why we will model the influence of the thermal ether on the gyrostats by an external moment in the equation of the rotors motion.

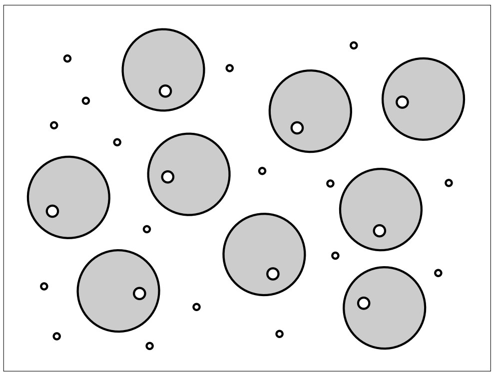

Figure 2. Elementary volume of continuum of one-rotor gyrostats deep in the thermal ether

The interaction of carrier bodies of the gyrostats is charged with the mechanical processes. The interaction of the rotors models the thermal processes. The spherical part of the moment tensor characterizing the interaction of the rotors is the analogue of the temperature. The corresponding angular deformation is the analogue of the entropy. The interference of the carrier bodies and the rotors provides the interplay of the mechanical and the thermal processes. The particles of the thermal ether realize the mechanism of thermal conductivity and viscosity.

4 Basic Hypotheses and Reductive Model of Continuum of One-Rotor Gyrostats

Let us consider a special case of the linear theory of one-rotor gyrostats continuum. We start with a formulation of three hypotheses.

Hypothesis 1. Vector $\mathbf{L}$ (the mass density of external actions on the rotors of gyrostats) is a sum of the moment $\mathbf{L}_h$ characterizing external actions of all sorts and the moment of linear viscous damping

$$\mathbf{L}_f = -\beta(B\mathbf{v} + J\boldsymbol{\omega}). \tag{9}$$

The moment (9) characterizes the influence of the thermal ether. Structure of the moment is chosen in accordance with the results of solving two model problems. One of them is presented in Ivanova (2011), and the second will be considered in what follows.

Hypothesis 2. The moment interaction between the carrier bodies of gyrostats is supposed to be characterized by the antisymmetric tensor; there is no influence of the external moment upon the carrier bodies of gyrostats; and the inertia tensors of the carrier bodies can be neglected

$$\boldsymbol{\mu} = -\boldsymbol{\mu}_v \times \mathbf{E}, \qquad \mathbf{m} = 0, \qquad \mathbf{I}_0 = 0. \tag{10}$$

Hypothesis 3. The moment stress tensor $\mathbf{T}$ characterizing the interactions between rotors is the sum of the spherical part of tensor and the antisymmetric tensor

$$\mathbf{T} = T\mathbf{E} - \mathbf{M} \times \mathbf{E}. \tag{11}$$

Now we construct the model of continuum that is based on the hypotheses stated above. In view of assumptions (10) the motion of carrier bodies of gyrostats is described by the equations

$$\nabla \cdot \boldsymbol{\tau} + \rho_* \mathbf{f} = \rho_* \frac{d}{dt}\big(\mathbf{v} + B\boldsymbol{\omega}\big), \qquad \nabla \times \boldsymbol{\mu}_v = \boldsymbol{\tau}_\times. \tag{12}$$

Representing $\boldsymbol{\tau}$ as a sum of the symmetric and antisymmetric tensors

$$\boldsymbol{\tau} = \boldsymbol{\tau}^s - \mathbf{q} \times \mathbf{E}, \qquad \mathbf{q} = \frac{1}{2}\boldsymbol{\tau}_\times, \tag{13}$$

we rewrite Eq. (12) in the form

$$\nabla \cdot \boldsymbol{\tau}^s - \nabla \times \mathbf{q} + \rho_* \mathbf{f} = \rho_* \frac{d}{dt}\big(\mathbf{v} + B\boldsymbol{\omega}\big), \qquad \nabla \times \boldsymbol{\mu}_v = 2\mathbf{q}. \tag{14}$$

In view of assumptions (9), (11) the equation of motion of the rotors takes the form

$$\nabla T - \nabla \times \mathbf{M} - \beta\rho_*(B\mathbf{v} + J\boldsymbol{\omega}) + \rho_* \mathbf{L}_h = \rho_* \frac{d}{dt}\big(B\mathbf{v} + J\boldsymbol{\omega}\big). \tag{15}$$

The energy balance equation for the elastic continuum of one-rotor gyrostats is written as

$$\frac{d(\rho_* U_m)}{dt} = \boldsymbol{\tau}^T \cdot\cdot \frac{d\boldsymbol{\varepsilon}}{dt} + \boldsymbol{\mu}^T \cdot\cdot \frac{d\boldsymbol{\kappa}}{dt} + \mathbf{T}^T \cdot\cdot \frac{d\boldsymbol{\vartheta}}{dt} \tag{16}$$

where U_m is the internal energy density per unit mass; the strain tensors $\boldsymbol{\varepsilon}$, $\boldsymbol{\kappa}$, $\boldsymbol{\vartheta}$ are determined by formulae (8). In view of Eq. (13) the first term in Eq. (16) can be reduced as follows

$$\boldsymbol{\tau}^T \cdot\cdot \frac{d\boldsymbol{\varepsilon}}{dt} = \boldsymbol{\tau}^s \cdot\cdot \frac{d\boldsymbol{\varepsilon}}{dt} + (\mathbf{q} \times \mathbf{E}) \cdot\cdot \frac{d\boldsymbol{\varepsilon}}{dt} = \boldsymbol{\tau}^s \cdot\cdot \frac{d\boldsymbol{\varepsilon}^s}{dt} + \mathbf{q} \cdot \frac{d\boldsymbol{\gamma}}{dt}, \tag{17}$$

where the following notations are used

$$\boldsymbol{\varepsilon}^s = \frac{1}{2}\Big(\nabla\mathbf{u} + \nabla\mathbf{u}^T\Big), \qquad \boldsymbol{\gamma} = \nabla \times \mathbf{u} - 2\boldsymbol{\varphi}. \tag{18}$$

Let us note that the trace of $\boldsymbol{\varepsilon}$ is equal to the trace of $\boldsymbol{\varepsilon}^s$, and therefore we will use the notation $\varepsilon = \operatorname{tr}\boldsymbol{\varepsilon} = \operatorname{tr}\boldsymbol{\varepsilon}^s$. In view of assumption (10) the second term in Eq. (16) takes the form

$$\boldsymbol{\mu}^T \cdot\cdot \frac{d\boldsymbol{\kappa}}{dt} = (\boldsymbol{\mu}_v \times \mathbf{E}) \cdot\cdot \frac{d\boldsymbol{\kappa}}{dt} = \boldsymbol{\mu}_v \cdot \frac{d\boldsymbol{\kappa}_\times}{dt}, \qquad \boldsymbol{\kappa}_\times = \nabla \times \boldsymbol{\varphi}. \tag{19}$$

We suppose that the strain vector $\boldsymbol{\kappa}_\times$, on which the moment vector $\boldsymbol{\mu}_v$ works, is equal to zero

$$\nabla \times \boldsymbol{\varphi} = 0. \tag{20}$$

However, the moment vector $\boldsymbol{\mu}_v$ has a finite value. It is possible if the corresponding stiffness tends to infinity. Indeed, in the linear theory $\boldsymbol{\mu}_v$ and $\nabla \times \boldsymbol{\varphi}$ are related by the constitutive equation $\boldsymbol{\mu}_v = C\,\nabla \times \boldsymbol{\varphi}$, where C is the stiffness. If $\nabla \times \boldsymbol{\varphi} = 0$ then $\boldsymbol{\mu}_v = 0$ in the case of a finite value of C and only when $C \to \infty$ vector $\boldsymbol{\mu}_v$ has a finite value. In the last case the constitutive equation becomes indeterminate, and vector $\boldsymbol{\mu}_v$ is found as a result of solution of Eq. (14). In view of assumption (11) the last in Eq. (16) can be reduced as follows:

$$\mathbf{T}^T \cdot\cdot \frac{d\boldsymbol{\vartheta}}{dt} = T\mathbf{E} \cdot\cdot \frac{d\boldsymbol{\vartheta}}{dt} + (\mathbf{M} \times \mathbf{E}) \cdot\cdot \frac{d\boldsymbol{\vartheta}}{dt} = T\, \frac{d(\mathrm{tr}\, \boldsymbol{\vartheta})}{dt} + \mathbf{M} \cdot \frac{d\boldsymbol{\vartheta}_\times}{dt}. \tag{21}$$

Using the results of transformations (17), (19), (21) and taking into account assumption (20) we write down the energy balance equation (16) in the form

$$\frac{d(\rho_* U_m)}{dt} = \boldsymbol{\tau}^s \cdot\cdot \frac{d\boldsymbol{\varepsilon}^s}{dt} + \mathbf{q} \cdot \frac{d\boldsymbol{\gamma}}{dt} + T\, \frac{d\vartheta}{dt} + \mathbf{M} \cdot \frac{d\boldsymbol{\psi}}{dt}\,, \tag{22}$$

where the following notations are used

$$\vartheta = \mathrm{tr}\, \boldsymbol{\vartheta}, \qquad \boldsymbol{\psi} = \boldsymbol{\vartheta}_\times, \qquad \boldsymbol{\vartheta} = \nabla \boldsymbol{\theta}. \tag{23}$$

Taking into account elasticity of the medium we obtain the Cauchy–Green relations

$$\boldsymbol{\tau}^s = \frac{\partial(\rho_* U_m)}{\partial \boldsymbol{\varepsilon}^s}, \qquad \mathbf{q} = \frac{\partial(\rho_* U_m)}{\partial \boldsymbol{\gamma}}, \qquad T = \frac{\partial(\rho_* U_m)}{\partial \vartheta}, \qquad \mathbf{M} = \frac{\partial(\rho_* U_m)}{\partial \boldsymbol{\psi}}. \tag{24}$$

According to the energy balance equation (22) the energy density is the function of four independent variables: $\boldsymbol{\varepsilon}^s$, $\boldsymbol{\gamma}$, ϑ $\boldsymbol{\psi}$. Let us define the energy density as

$$\begin{aligned} &\rho_* U_m = \boldsymbol{\tau}_0 \cdot\cdot \boldsymbol{\varepsilon}^s + \mathbf{q}_0 \cdot \boldsymbol{\gamma} + T_* \vartheta + \mathbf{M}_* \cdot \boldsymbol{\psi} + G\, \mathrm{dev}\, \boldsymbol{\varepsilon}^s \cdot\cdot \mathrm{dev}\, \boldsymbol{\varepsilon}^s + \\ &+ \frac{1}{2} K_{ad}\, \varepsilon^2 + \Upsilon\, \varepsilon\, \vartheta + \frac{1}{2} K \vartheta^2 + \frac{1}{2}\, A\, \boldsymbol{\gamma} \cdot \boldsymbol{\gamma} + D\, \boldsymbol{\gamma} \cdot \boldsymbol{\psi} + \frac{1}{2}\, \Gamma\, \boldsymbol{\psi} \cdot \boldsymbol{\psi}\,. \end{aligned} \tag{25}$$

Here $\boldsymbol{\tau}_0$, $\mathbf{q}_0$, T_*, $\mathbf{M}_*$ are the initial stresses, K_{ad} is the adiabatic modulus of compression (the adiabatic bulk modulus), G is the shear modulus, Υ, K, A, D Γ are constants whose physical sense will be discussed further. The notation “dev” is used for the deviator part of tensor. Substituting Eq. (25) into the Cauchy–Green relations (24) we obtain

$$\begin{aligned} &\boldsymbol{\tau}^s = \boldsymbol{\tau}_0 + K_{ad}\, \varepsilon \mathbf{E} + 2G\, \mathrm{dev}\, \boldsymbol{\varepsilon} + \Upsilon \vartheta \mathbf{E}, \qquad \mathbf{q} = \mathbf{q}_0 + A\, \boldsymbol{\gamma} + D\, \boldsymbol{\psi}, \\ &T = T_* + \Upsilon\, \mathrm{tr}\, \boldsymbol{\varepsilon} + K\vartheta, \qquad \mathbf{M} = \mathbf{M}_* + D\, \boldsymbol{\gamma} + \Gamma\, \boldsymbol{\psi}\,. \end{aligned} \tag{26}$$

So, the reduced model of continuum of one-rotor gyrostats is described by Eqs. (2), (14), (15), (18), (20), (23), (26).

5 Model of the “Thermal Ether”

We consider a continuum (see Figure 3) consisting of the body-points (3), (4). Let us assume the following.

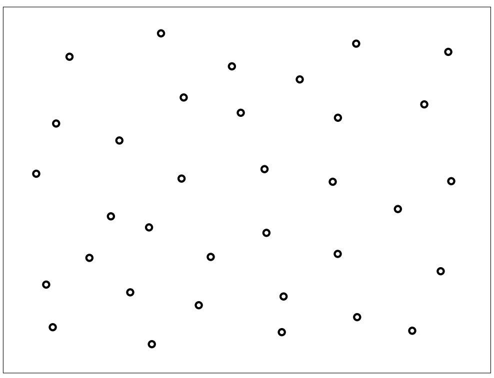

Figure 3. Elementary volume of continuum consisting of body-points

Hypothesis 1. The external forces and the force interactions between the particles of the medium are zero:

$$\mathbf{f} \equiv 0, \qquad \boldsymbol{\tau} \equiv 0. \tag{27}$$

Hypothesis 2. The moment stress tensor $\mathbf{T}$ is an isotropic tensor:

$$\mathbf{T} = T\mathbf{E}. \tag{28}$$

Hypothesis 3. The external moments and the initial moment stresses are absent:

$$\mathbf{L} \equiv 0, \qquad \mathbf{T}_0 \equiv 0. \tag{29}$$

We will call the model of elastic continuum satisfying the hypotheses (27)–(29) the thermal ether. A body of finite size in the medium dissipates energy into the medium due to the moment interactions.

In view of Eqs. (27)–(29) the equations of motion of the medium under consideration take the form

$$\tilde{\rho}\frac{d}{dt}\big(\mathbf{v} + \hat{B}\boldsymbol{\omega}\big) = 0, \qquad \nabla T = \tilde{\rho}\frac{d}{dt}\big(\hat{B}\mathbf{v} + \hat{J}\boldsymbol{\omega}\big) \tag{30}$$

where $\tilde{\rho}$ is the mass density; $\hat{B}$, $\hat{J}$ are the moments of inertia. The linear approximations of the kinematic relations are

$$\mathbf{v} = \frac{d\mathbf{u}}{dt}, \qquad \boldsymbol{\omega} = \frac{d\boldsymbol{\theta}}{dt}. \tag{31}$$

Here $\mathbf{u}(\mathbf{r}, t)$, $\boldsymbol{\theta}(\mathbf{r}, t)$ are the displacement vector field and the rotation vector field correspondingly. Using the notation

$$\vartheta = \operatorname{tr} \boldsymbol{\vartheta} \equiv \nabla \cdot \boldsymbol{\theta} \tag{32}$$

and taking into account Eqs. (27), (28) the energy balance equation can be rewritten in the form

$$\frac{d}{dt}(\tilde{\rho}U_m) = T\,\frac{d\vartheta}{dt}. \tag{33}$$

The elasticity assumption allows us to obtain the Cauchy–Green relations

$$T = \frac{\partial(\tilde{\rho}U_m)}{\partial\vartheta}. \tag{34}$$

Let us specify the internal energy in the simplest form

$$\tilde{\rho}U_m = \frac{1}{2}\tilde{k}\,\vartheta^2 \tag{35}$$

where $\tilde{k}$ is the coefficient of stiffness. Then the constitutive equation takes the form

$$T = \tilde{k}\,\vartheta. \tag{36}$$

It follows from Eqs. (30), (31), (32), (36) that the thermal ether is described by the wave equation

$$\Delta\vartheta - \frac{\tilde{\rho}(\hat{J} - \hat{B}^2)}{\tilde{k}}\,\frac{d^2\vartheta}{dt^2} = 0. \tag{37}$$

6 Spherical Source in the "Thermal Ether"

The simplest model illustrating the process of dissipation of the body-point energy into the thermal ether has been considered in Ivanova (2011). Namely, the model problem of the interaction of a body-point with one-dimensional semi-infinite continuum of body-points is solved. The problem of the interaction of a body-point with the thermal ether in the case of spherical symmetry (see Figure 4) is more complicated but a more appropriate model of the process of dissipation. Now we consider the spherical source of radius r_0 consisting of the body-points (3), (4).

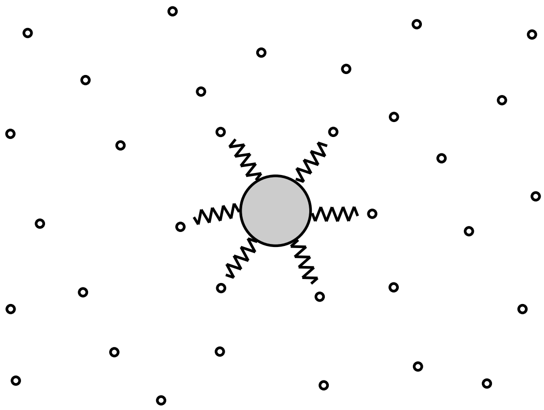

Figure 4. Interaction of the spherical source with the thermal ether

We suppose that the source can pulsate, and the change of its radius is characterized by the variable $\xi(t)$. At the same time the body-points of the spherical source rotate about its radius. The angles of rotation of all body-points are assumed to be the same and they are characterized by the variable $\psi(t)$. Thus, the kinematics of the spherical source is described by the displacement vector and by the rotational vector

$$\boldsymbol{\xi} = \xi(t)\,\mathbf{e}_r \qquad \boldsymbol{\psi} = \psi(t)\,\mathbf{e}_r \tag{38}$$

where $\mathbf{e}_r$ is the unit vector of the spherical coordinate system. The inertia properties of the spherical source are characterized by the mass m evenly distributed on the source surface and the moments of inertia B, J. The spherical source interacts with the thermal ether by means of an elastic connection. The elastic connection constitutes the system of the identical springs working in torsion. Each of them connects the body-point of the spherical source with the body-point of the thermal ether (see Figure 3). The stiffness of the connection per unit area of spherical source is characterized by the stiffness k_*/r_0 where coefficient r_0^{-1} is introduced in order to the dimension of stiffness k_* be the same as the dimension of stiffness of the thermal ether. In the problem with the spherical symmetry kinematics of the thermal ether is described by the variable $\theta_r(r,t)$ which is the angle of rotation of the medium about the unit vector $\mathbf{e}_r$. Eq. (32) takes the form

$$\vartheta = \frac{\partial \theta_r}{\partial r} + \frac{2}{r}\,\theta_r \tag{39}$$

and the equation of motion of the thermal ether (37) takes the form

$$\frac{\partial^2 (r\vartheta)}{\partial r^2} - \frac{1}{c^2}(r\vartheta)^{\cdot\cdot} = 0, \qquad c^2 = \frac{\tilde{k}}{\tilde{\rho}(\hat{J} - \hat{B}^2)}. \tag{40}$$

The boundary conditions for the thermal ether are posed on the border of the source

$$\tilde{k}\,\vartheta\Big|_{r=r_0} = -\frac{k_*}{r_0}\big(\psi - \theta_r|_{r=r_0}\big). \tag{41}$$

The equations of motion of the elementary part of the spherical source are

$$\frac{m}{4\pi r_0^2}\big(B\ddot{\xi} + J\ddot{\psi}\big) = -\frac{k_*}{r_0}\big(\psi - \theta_r|_{r=r_0}\big), \qquad \frac{m}{4\pi r_0^2}\big(\ddot{\xi} + B\ddot{\psi}\big) = f. \tag{42}$$

Here f is an external force per unit area of the spherical source. Now we formulate the initial conditions. Suppose that the thermal ether is at rest at an initial time, i. e. the displacements and angles of rotation as well as the linear and angular velocities of particles of the continuum are equal to zero at the initial time. Let us assume the following initial conditions for the spherical source

$$\xi(0) = \xi_0, \qquad \psi(0) = \psi_0, \qquad \dot{\xi}(0) = v_0, \qquad \dot{\psi}(0) = \omega_0. \tag{43}$$

We will look for the solution of the Eq. (40) in the form given by d'Alembert and Euler:

$$\vartheta(r,t) = \frac{1}{r} f(r-ct) + \frac{1}{r} g(r+ct). \tag{44}$$

Since there are no perturbations at infinity we can assert that $g(r+ct)=0$. In view of zero initial conditions for the thermal ether we see that the function $f(s-ct)$ is not equal to zero only on the negative semiaxis. Hence

$$\vartheta(r,t) = \begin{cases} 0, & r > ct, \\ \dfrac{1}{r} f(r-ct), & r < ct. \end{cases} \tag{45}$$

It is easy to show that

$$\frac{\partial (r\vartheta)}{\partial r} = -\frac{1}{c} (r\vartheta)^{\cdot}. \tag{46}$$

By using Eqs. (39), (40), (46) we can show that

$$\vartheta = -\left(\frac{r}{c}\dot{\vartheta} + \frac{r}{c^2}\ddot{\theta}_r\right). \tag{47}$$

In view of Eq. (47) the boundary condition for the thermal ether (41) takes the form

$$-\frac{\tilde{k}\, r_0}{c} \left(\dot{\vartheta} + \frac{1}{c}\ddot{\theta}_r\right)\bigg|_{r=r_0} = -\frac{k_*}{r_0}\big(\psi - \theta_r|_{r=r_0}\big). \tag{48}$$

Let us express the difference $(\psi - \theta_r|_{r=r_0})$ from Eq. (48) and put it in the first equation of (42). As a result we obtain

$$\frac{m}{4\pi r_0^2}\big(B\ddot{\xi} + J\ddot{\psi}\big) = -\frac{\tilde{k}\, r_0}{c} \left(\dot{\vartheta} + \frac{1}{c_r}\ddot{\theta}_r\right)\bigg|_{r=r_0}. \tag{49}$$

We integrate Eq. (49) taking into account the initial conditions. We obtain

$$\frac{m}{4\pi r_0^2}\big(B\dot{\xi} + J\dot{\psi}\big) = -\frac{\tilde{k}\, r_0}{c} \left(\vartheta + \frac{1}{c}\dot{\theta}_r\right)\bigg|_{r=r_0} + \frac{m}{4\pi r_0^2}\big(Bv_0 + J\omega_0\big). \tag{50}$$

Let us express $\vartheta|_{r=r_0}$ from Eq. (50) and substitute it in the boundary condition for the thermal ether (41). As a result we obtain

$$-\frac{\tilde{k}}{c}\, \dot{\theta}_r\Big|_{r=r_0} - \frac{mc}{4\pi r_0^3}\big(B\dot{\xi} + J\dot{\psi}\big) + \frac{mc}{4\pi r_0^3}\big(Bv_0 + J\omega_0\big) = -\frac{k_*}{r_0}\big(\psi - \theta_r|_{r=r_0}\big). \tag{51}$$

We express the difference $(\psi - \theta_r|_{r=r_0})$ from Eq. (51) and put it in the first equation of (42)

$$\frac{m}{4\pi r_0^2}\big(B\ddot{\xi} + J\ddot{\psi}\big) = -\frac{\tilde{k}}{c}\, \dot{\theta}_r\Big|_{r=r_0} - \frac{mc_r}{4\pi r_0^3}\big(B\dot{\xi} + J\dot{\psi}\big) + \frac{mc_r}{4\pi r_0^3}\big(Bv_0 + J\omega_0\big). \tag{52}$$

Let us integrate Eq. (52) taking into account the initial conditions

$$\frac{m}{4\pi r_0^2}\big(B\dot{\xi} + J\dot{\psi}\big) = -\frac{\tilde{k}}{c}\, \theta_r|_{r=r_0} + \frac{mc}{4\pi r_0^3}\Big[-\big(B\xi + J\psi\big) + \big(Bv_0 + J\omega_0\big)t + B\xi_0 + J\psi_0\Big]. \tag{53}$$

Then we express $\theta_r|_{r=r_0}$ from Eq. (53) and put it in the first equation of (42). As a result we obtain the following system of equations:

$$m\big(B\ddot{\xi} + J\ddot{\psi}\big) + m\beta\big(B\dot{\xi} + J\dot{\psi}\big) + \frac{mk_*}{r_0^2\tilde{\rho}(\hat{J} - \hat{B}^2)}\big(B\xi + J\psi\big) + 4\pi r_0 k_* \psi = \\ = \frac{mk_*}{r_0^2\tilde{\rho}(\hat{J} - \hat{B}^2)}\Big[(Bv_0 + J\omega_0)t + B\xi_0 + J\psi_0\Big], \qquad m\big(\ddot{\xi} + B\ddot{\psi}\big) = 4\pi r_0^2 f \tag{54}$$

where coefficient β is determined by the formula

$$\beta = \frac{ck_*}{r_0\tilde{k}} = \frac{k_*/r_0}{\sqrt{\tilde{k}\tilde{\rho}(\hat{J} - \hat{B}^2)}}\,. \tag{55}$$

A comparison of Eq. (54) with the equations obtained in Ivanova (2011) for the case of the interaction of a body-point with the one-dimension continuum shows that although these equations somewhat differ from each other, they have one important similarity. Both of them have the dissipative terms proportional to the angular momentum and the same dependence of the coefficient of viscous damping β on the parameters of the model.

7 Hyperbolic Type Thermoelasticity

Now we consider the one-rotor gyrostat continuum which is described by Eqs. (2), (14), (15), (18), (20), (23), (26). The quantity T characterizing the spherical part of tensor of the moment interaction of the rotors is supposed to have the sense of temperature, and the quantity ϑ characterizing the spherical part of the corresponding strain tensor plays role of the volume density of entropy. Dimensions of the temperature and the entropy introduced in the framework of the proposed model are different from dimensions of those in classical thermodynamics. This problem can be solved by introduction of a normalization factor a

$$T = aT_a, \qquad \vartheta = \frac{1}{a}\,\vartheta_a. \tag{56}$$

Here T_a is the absolute temperature measured by a thermometer; ϑ_a is volume density of the absolute entropy. Let us introduce the similar relations for the remaining variables

$$\boldsymbol{\theta} = \frac{1}{a}\,\boldsymbol{\theta}_a, \qquad \boldsymbol{\omega} = \frac{1}{a}\,\boldsymbol{\omega}_a, \qquad \mathbf{M} = a\mathbf{M}_a, \qquad \boldsymbol{\psi} = \frac{1}{a}\,\boldsymbol{\psi}_a, \qquad \mathbf{L}_h = a\mathbf{L}_h^a. \tag{57}$$

Since the new force characteristics are multiplied by a and the new kinematic characteristics are divided by a, after substituting Eqs. (56), (57) into the energy balance equation (22) the normalization factor a is cancelled and Eq. (22) takes the form

$$\frac{d(\rho_* U_m)}{dt} = \boldsymbol{\tau}^s \cdot\cdot \frac{d\boldsymbol{\varepsilon}^s}{dt} + \mathbf{q}\cdot\frac{d\boldsymbol{\gamma}}{dt} + T_a\frac{d\vartheta_a}{dt} + \mathbf{M}_a\cdot\frac{d\boldsymbol{\psi}_a}{dt}\,. \tag{58}$$

By introducing new parameters

$$B_a = \frac{B}{a}, \qquad J_a = \frac{J}{a^2}, \qquad \Upsilon_a = \frac{\Upsilon}{a}, \qquad K_a = \frac{K}{a^2}, \qquad D_a = \frac{D}{a}, \qquad \Gamma_a = \frac{\Gamma}{a^2}, \tag{59}$$

the normalization factor a can be eliminated from all equations. Indeed, after substituting Eqs. (2), (56), (57), (59) into Eqs. (14), (15) we obtain

$$\nabla\cdot\boldsymbol{\tau}^s - \nabla\times\mathbf{q} + \rho_*\mathbf{f} = \rho_*\frac{d^2}{dt^2}\big(\mathbf{u} + B_a\boldsymbol{\theta}_a\big), \qquad \nabla\times\boldsymbol{\mu}_v = 2\mathbf{q}. \tag{60}$$

$$\nabla T_a - \nabla\times\mathbf{M}_a - \beta\rho_*\frac{d}{dt}(B_a\mathbf{u} + J_a\boldsymbol{\theta}_a) + \rho_*\mathbf{L}_h^a = \rho_*\frac{d^2}{dt^2}\big(B_a\mathbf{u} + J_a\boldsymbol{\theta}_a\big). \tag{61}$$

It is well known that applying the linear theory is admissible in certain range of temperatures and entropy densities. Therefore we introduce deviations of the quantities introduced above from their reference values T_a^*, $\mathbf{M}_a^*$ (which are not zero) and ϑ_a^*, $\boldsymbol{\psi}_a^*$ (which can be considered to be zero notice that without loss of generality):

$$T_a = T_a^* + \tilde{T}_a, \qquad \mathbf{M}_a = \mathbf{M}_a^* + \tilde{\mathbf{M}}_a, \qquad \vartheta_a = \vartheta_a^* + \tilde{\vartheta}_a, \qquad \boldsymbol{\psi}_a = \boldsymbol{\psi}_a^* + \tilde{\boldsymbol{\psi}}_a. \tag{62}$$

Taking into account Eqs. (56), (57), (59), (62) and neglecting the initial stresses $\boldsymbol{\tau}_0$, $\mathbf{q}_0$ we reduce the constitutive equations (26) to the form

$$\begin{aligned} &\boldsymbol{\tau}^s = K_{ad}\,\varepsilon\mathbf{E} + 2G\,\mathrm{dev}\,\boldsymbol{\varepsilon} + \Upsilon_a\tilde{\vartheta}_a\mathbf{E}, \qquad \mathbf{q} = A\,\boldsymbol{\gamma} + D_a\,\tilde{\boldsymbol{\psi}}_a, \\ &\tilde{T}_a = \Upsilon_a\,\mathrm{tr}\,\boldsymbol{\varepsilon} + K_a\tilde{\vartheta}_a, \qquad \tilde{\mathbf{M}}_a = D_a\,\boldsymbol{\gamma} + \Gamma_a\,\tilde{\boldsymbol{\psi}}_a\,. \end{aligned} \tag{63}$$

It is easy to see that the normalization factor a is absent both in the equations of motion (60), (61) and in the constitutive equations (63). That is due to the special choice of relations (59) between new and old parameters.

Now we consider a special case when the parameters A, D_a, Γ_a, B_a are equal to zero, and the remaining parameters are calculated by

$$\beta J_a = \frac{T_a^*}{\rho_*\lambda}, \qquad K_a = \frac{T_a^*}{\rho_* c_v}, \qquad \Upsilon_a = -\frac{\alpha K_{iz} T_a^*}{\rho_* c_v}\,, \tag{64}$$

where c_v is the specific heat at constant volume, λ is the heat-conduction coefficient, K_{iz} is the isothermal modulus of compression (the isothermal bulk modulus), α is the volume coefficient of thermal expansion,

$$K_{ad} = K_{iz}\frac{c_p}{c_v}, \qquad c_p - c_v = \frac{\alpha^2 K_{iz} T_a^*}{\rho_*} \qquad \Rightarrow \qquad K_{ad} = K_{iz} + \frac{\alpha^2 K_{iz}^2 T_a^*}{\rho_* c_v} \tag{65}$$

where c_p is the specific heat at constant pressure. In Ivanova (2010) it is shown that in the special case under consideration Eqs. (18), (20), (60), (61), (63) can be reduced to the well known equations of the coupled problem of thermoelasticity including the hyperbolic type heat conduction equation.

8 Volume and Shear Viscosities

Now we abandon the assumption that the parameters A, D_a, Γ_a, B_a are equal to zero. In what follows an isentropic process is considered, i.e. the volume density of entropy is assumed to be constant

$$\vartheta_a = \vartheta_a^* = \mathrm{const} \qquad \Rightarrow \qquad \tilde{\vartheta}_a = 0 \qquad \Rightarrow \qquad \tilde{T}_a = \Upsilon_a\,\varepsilon. \tag{66}$$

In this case choosing the parameter B_a as

$$\beta B_a = -\frac{\alpha K_{iz} T_a^*}{\rho_* c_v \eta_v} \tag{67}$$

we can show that the following equation is the consequence of Eqs. (2), (14), (15), (18), (20), (23), (26):

$$\eta_v\,\Delta\varepsilon - \rho_*\,\frac{d\varepsilon}{dt} - \beta^{-1}\rho_*\,\frac{d^2\varepsilon}{dt^2} = \rho_*\Psi_v, \qquad \Psi_v = \frac{\alpha\,c_v\eta_v}{c_p - c_v}\,\nabla\cdot\mathbf{L}_h^a. \tag{68}$$

It is easy to see that Eq. (68) agrees with the self-diffusion equation. The only difference is that the former contains the inertial term. Therefore the parameter η_v has the sense of the volume (acoustic) viscosity. Since η_v is the coefficient in the equation describing the isentropic process it will be called the isentropic volume viscosity.

Now we pass on to discussion of the shear viscosity. Let us choose elastic modulus A as

$$A = \frac{\lambda\,(\eta_q - \eta_s)(K_{ad} - K_{iz})}{c_v\,\eta_v^2}\,. \tag{69}$$

The parameters D_a and Γ_a we represent in the form

$$D_a = -\frac{\alpha K_{iz} T_a^* \eta_s}{\rho_* c_v \eta_v}\,, \qquad \Gamma_a = \frac{(\eta_q - \eta_s)\,T_a^*}{\lambda\,\rho_*}\,. \tag{70}$$

The physical sense of the constants η_s and η_q will be discussed further.

Analogous to the isentropic process, suppose that vector $\boldsymbol{\psi}_a$ is a constant:

$$\boldsymbol{\psi}_a = \boldsymbol{\psi}_a^* = \mathrm{const} \qquad \Rightarrow \qquad \tilde{\boldsymbol{\psi}}_a = 0 \qquad \Rightarrow \qquad \tilde{\mathbf{M}}_a = D_a\,\boldsymbol{\gamma}. \tag{71}$$

We can show that from Eqs. (2), (14), (15), (18), (20), (23), (26) and the supposition (71) follows the equation with respect to the unknown $\nabla\times\mathbf{v}$. Neglecting the terms characterizing the external effects and the terms containing the second time derivatives we obtain an approximate form of the equation. By using the notations (69), (70) the equation takes the form

$$\eta_s\,\Delta\nabla\times\mathbf{v} = \rho_*\frac{d}{dt}\nabla\times\mathbf{v}. \tag{72}$$

It is easy to see that Eq. (72) is equivalent to the equation of vortex motion of a viscous fluid. Hence the parameter η_s has the sense of the shear viscosity. Since Eq. (72) describes the isentropic process the coefficient η_s will be called the isentropic shear viscosity.

Let us consider the process where the antisymmetric part of the stress tensor is equal to zero

$$\mathbf{q} = 0 \qquad \Rightarrow \qquad \tilde{\boldsymbol{\psi}}_a = -\frac{A}{D_a}\boldsymbol{\gamma} \qquad \Rightarrow \qquad \tilde{\mathbf{M}}_a = \left(D_a - \frac{A\Gamma_a}{D_a}\right)\boldsymbol{\gamma}. \tag{73}$$

Taking into account the condition (73) we reduce Eqs. (2), (14), (15), (18), (20), (23), (26) to the equation with respect to $\nabla\times\mathbf{v}$. Let us neglect the terms characterizing the external effects and the terms contain the second time derivatives. As a result we obtain

$$\eta_q\,\Delta\nabla\times\mathbf{v} = \rho_*\frac{d}{dt}\nabla\times\mathbf{v} + \rho_*\boldsymbol{\Psi}_q, \qquad \boldsymbol{\Psi}_q = \frac{2AJ_a}{D_aB_a - AJ_a}\,\frac{d^2\boldsymbol{\varphi}}{dt^2}\,. \tag{74}$$

Vector $\boldsymbol{\Psi}_q$ in Eq. (74) has the sense of the source term. The parameter η_q represents the shear viscosity the value of which, generally, differs from the value of the isentropic shear viscosity η_s.

9 Coupled Problem of Thermoviscoelasticity

Now we write down Eqs. (18), (20), (60), (61), (63) in view of expressions for the parameters of the model (64), (67), (69), (70):

$$\begin{aligned}
&\nabla\cdot\boldsymbol{\tau}^s - \nabla\times\mathbf{q} + \rho_*\mathbf{f} = \rho_*\frac{d^2\mathbf{u}}{dt^2} - \frac{\alpha K_{iz}T_a^*}{\beta c_v\eta_v}\,\frac{d^2\boldsymbol{\theta}_a}{dt^2}\,, \qquad \nabla\times\boldsymbol{\mu}_v = 2\mathbf{q}\,, \qquad \nabla\times\boldsymbol{\varphi} = 0,\\
&\nabla\tilde{T}_a - \nabla\times\tilde{\mathbf{M}}_a + \frac{\alpha K_{iz}T_a^*}{c_v\eta_v}\left(\frac{d\mathbf{u}}{dt} + \frac{1}{\beta}\frac{d^2\mathbf{u}}{dt^2}\right) - \frac{T_a^*}{\lambda}\left(\frac{d\boldsymbol{\theta}_a}{dt} + \frac{1}{\beta}\frac{d^2\boldsymbol{\theta}_a}{dt^2}\right) = -\rho_*\mathbf{L}_h^a,\\
&\boldsymbol{\tau}^s = \left[\left(K_{iz} - \frac{2}{3}\,G\right)\varepsilon - \alpha K_{iz}\,\tilde{T}_a\right]\mathbf{E} + 2G\,\boldsymbol{\varepsilon}^s, \qquad \boldsymbol{\varepsilon}^s = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right), \quad \varepsilon = \mathrm{tr}\,\boldsymbol{\varepsilon}^s,\\
&\mathbf{q} = \frac{\lambda\,(\eta_q-\eta_s)(K_{ad}-K_{iz})}{c_v\eta_v^2}\,\boldsymbol{\gamma} - \frac{\alpha K_{iz}T_a^*\,\eta_s}{\rho_*c_v\eta_v}\,\nabla\times\boldsymbol{\theta}_a\,, \qquad \boldsymbol{\gamma} = \nabla\times\mathbf{u} - 2\boldsymbol{\varphi}\,,\\
&\nabla\cdot\boldsymbol{\theta}_a = \frac{\rho_*c_v}{T_a^*}\,\tilde{T}_a + \alpha K_{iz}\,\varepsilon, \qquad \tilde{\mathbf{M}}_a = -\frac{\alpha K_{iz}T_a^*\eta_s}{\rho_*c_v\eta_v}\,\boldsymbol{\gamma} + \frac{(\eta_q-\eta_s)\,T_a^*}{\lambda\,\rho_*}\nabla\times\boldsymbol{\theta}_a\,.
\end{aligned} \tag{75}$$

Here the reference values of ϑ_a and $\boldsymbol{\psi}_a$ are considered to be equal to zero. Hence, $\tilde{\vartheta}_a = \nabla\cdot\boldsymbol{\theta}_a$ and $\tilde{\boldsymbol{\psi}}_a = \nabla\times\boldsymbol{\theta}_a$.

The first equation in (75) is the linear momentum balance equations for the gyrostats. This equation differs from the analogous equation for the classical Cosserat continuum by the last term on the right-hand side of the equation. The second equation is the reduced angular momentum balance equation for the carrier bodies of gyrostats, and the third one represents the kinematic restriction related to the rotation of the carrier bodies. The fourth equation in (75) is the angular momentum balance equation for the rotors of gyrostats. It has the thermodynamical sense. If we take the divergence of both sides of this equation and exclude $\nabla\cdot\boldsymbol{\theta}_a$ by using the tenth equation in (75) we obtain the heat conduction equation. The other equations in (75) are the constitutive equations and the expressions for the strain tensors. Let us discuss the fourth equation in (75) in more detail. The motion of the rotors of gyrostats cause the appearance of waves in the thermal ether. As a result the certain part of energy of the material particles is spent on the formation of these waves. We suppose that the internal damping mechanism and the heat conduction mechanism are provided due to the material medium energy dissipation into the thermal ether, and the third and fourth terms on the right-hand side of the fourth equation in (75) are account for this process. To be exact, the third term (containing $\mathbf{u}$) provides the internal damping mechanism and the fourth term (containing $\boldsymbol{\theta}_a$) provides the heat conduction mechanism.

Let us transform Eqs. (75) to the form corresponding to the classical continuum without microstructure. It is well-known that an arbitrary vector can be represented in terms of the scalar and vector Helmholtz potentials. We use this representation for dynamic term containing vector $\boldsymbol{\theta}_a$ in the first equation (75)

$$-\rho_* B_a \frac{d^2\boldsymbol{\theta}_a}{dt^2} = \nabla p - \nabla\times\mathbf{t}, \qquad \nabla\cdot\mathbf{t} = 0. \tag{76}$$

Here p is the scalar potential, $\mathbf{t}$ is the vector potential. By using notation (76) we rewrite Eqs. (75) in the form

$$\begin{aligned}
&\nabla\cdot\tilde{\boldsymbol{\tau}}^s - \nabla\times\tilde{\mathbf{q}} + \rho_*\mathbf{f} = \rho_*\frac{d^2\mathbf{u}}{dt^2}\,, \qquad \nabla\times\tilde{\boldsymbol{\mu}}_v = 2\tilde{\mathbf{q}}\,, \qquad \nabla\times\boldsymbol{\varphi} = 0,\\
&\tilde{\boldsymbol{\tau}}^s = \left[\Big(K_{iz} - \frac{2}{3}\,G\Big)\varepsilon - \alpha K_{iz}\,\tilde{T}_a + p\right]\mathbf{E} + 2\,G\,\boldsymbol{\varepsilon}^s, \qquad \boldsymbol{\varepsilon}^s = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right),\\
&\tilde{\mathbf{q}} = \frac{\lambda\,(\eta_q - \eta_s)(K_{ad} - K_{iz})}{c_v\eta_v^2}\,\boldsymbol{\gamma} - \frac{\alpha K_{iz}T_a^*\,\eta_s}{\rho_* c_v\eta_v}\;\tilde{\boldsymbol{\psi}}_a + \mathbf{t}\,, \qquad \boldsymbol{\gamma} = \nabla\times\mathbf{u} - 2\boldsymbol{\varphi}\,,\\
&\Delta p = \frac{\alpha K_{iz}}{\beta\eta_v}\left[\rho_*\frac{d^2\tilde{T}_a}{dt^2} + \frac{\alpha K_{iz}T_a^*}{c_v}\,\frac{d^2\varepsilon}{dt^2}\right], \qquad \Delta\mathbf{t} = \frac{\alpha K_{iz}T_a^*}{\beta c_v\eta_v}\,\frac{d^2\tilde{\boldsymbol{\psi}}_a}{dt^2}\,, \qquad \varepsilon = \operatorname{tr}\boldsymbol{\varepsilon}^s,\\
&\Delta\tilde{T}_a - \frac{\rho_* c_v}{\lambda}\left[\frac{d\tilde{T}_a}{dt} + \frac{1}{\beta}\frac{d^2\tilde{T}_a}{dt^2}\right] = \alpha K_{iz}T_a^*\left(\frac{1}{\lambda} - \frac{1}{c_v\eta_v}\right)\left[\frac{d\varepsilon}{dt} + \frac{1}{\beta}\frac{d^2\varepsilon}{dt^2}\right] - \rho_*\nabla\cdot\mathbf{L}_h^a,\\
&(\eta_q - \eta_s)\Delta\tilde{\boldsymbol{\psi}}_a - \rho_*\left(\frac{d\tilde{\boldsymbol{\psi}}_a}{dt} + \frac{1}{\beta}\frac{d^2\tilde{\boldsymbol{\psi}}_a}{dt^2}\right) =\\
&= \frac{\lambda\,\alpha K_{iz}}{c_v\eta_v}\left[\eta_s\,\Delta\nabla\times\mathbf{u} - \rho_*\left(\frac{d\nabla\times\mathbf{u}}{dt} + \frac{1}{\beta}\frac{d^2\nabla\times\mathbf{u}}{dt^2}\right)\right] - \frac{\lambda\,\rho_*^2}{T_a^*}\,\nabla\times\mathbf{L}_h^a.
\end{aligned} \tag{77}$$

The first equation in (77) is the dynamic equation of the Cosserat continuum consisting of the ordinary classical particles whose interaction is characterized by the symmetrical stress tensor $\tilde{\boldsymbol{\tau}}^s = \boldsymbol{\tau}^s + p\,\mathbf{E}$ and stress vector $\tilde{\mathbf{q}} = \mathbf{q} + \mathbf{t}$. Quantities p and $\mathbf{t}$ are the thermodynamic stresses. The constitutive equations for p and $\mathbf{t}$ are represented by the differential equations (the eighth and ninth equations in (77)). The eleventh equation in (77) is the heat conduction equation. The twelfth one is an auxiliary equation which is necessary to determine vector $\mathbf{t}$. Notice that the eleventh and twelfth equations follow from the angular momentum balance equation for the rotors of gyrostats (the fourth equation in (75)). It is easy to see that the thermodynamic stresses p and $\mathbf{t}$ vanish when $\eta_v \to \infty$. In that case the problem of thermoviscoelasticity turns into the hyperbolic type thermoelasticity problem.

10 Comparison with the Quantum Mechanical Approach

According to the quantum-mechanical ideas of Landau and Lifschitz (1989, 1979), in solid bodies the dependence of the acoustical absorption factor γ on the frequency ω is determined by the following formula (see Prokhorov (1992), p. 658)

$$\gamma = 1,1\,c_v\,T_a\,\Gamma_*^2\,\frac{\omega^2\tau}{c^3\rho_*(1+\omega^2\tau^2)}\,, \tag{78}$$

where Γ_* is the Grüneisen constant ($\Gamma_* = \alpha K_{ad}/c_v$), τ is the relaxation time scale (of order 10^{-11} sec). The diagram of dependence of the acoustical absorption factor on frequency (78) is represented in Figure 5. According to Eq. (78) in the range of relatively low frequencies γ is proportional to ω^2 (Akhiezer mechanism of absorption), at frequencies of $10^{10} - 10^{11}$ Hz the acoustical absorption factor γ is proportional to ω (Landau–Rumer mechanism of absorption), and at higher frequencies γ tends to a constant.

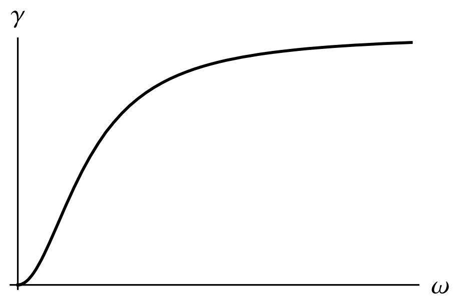

Figure 5. Dependence of the acoustical absorption factor on frequency

According to the classical theory of thermoelasticity the acoustical absorption factor γ is proportional to ω^2 at all frequencies. The dependence of γ on the frequency obtained by using of the hyperbolic type theory of thermoelasticity is in qualitative agreement with Eq. (78). Notice that the hyperbolic type theory of thermoelasticity contains one additional parameter compared with the classical theory of thermoelasticity. Neither the classical nor the hyperbolic type theory of thermoelasticity does not allow us to achieve quantitative agreement between theoretical and experimental values of γ. The proposed theory of thermoviscoelastisity, as well as the hyperbolic type theory of thermoelasticity, is in qualitative agreement with the dependence given by Eq. (78). Furthermore, the proposed theory contains the additional parameters which can be chosen so that the theoretical values of the acoustical absorption factor and the absorption factor of transverse waves will be in quantitative agreement with the their experimental values.

11 Conclusion

Thus, we construct the theory of thermoviscoelasticity (in frame of the continuum mechanics) which possesses the following properties. In the area of low frequencies it leads to the consequences similar to the consequences obtained from the classical theories. In the area of hypersonic frequencies the proposed theory leads to the consequences similar to the consequences obtained from the quantum-mechanical theories. Moreover the proposed theory allows us to give the mechanical interpretation of the mechanism of the thermal conductivity and the mechanism of the internal damping. In future we plan to carry out further development of the proposed theory. The first direction of the development is concerned with consideration of nonlinear effects in the context of the same mechanical model. This is necessary for describing the behavior of substance in the states near the phase transitions and heat-conduction processes under the circumstances of quickly varying and superhigh temperatures. The second direction is modification of the mechanical model by taking into account the additional degrees of freedom. This is necessary for introducing the chemical potential and a number of additional physical characteristics of the medium.

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Address: Prof. Elena A. Ivanova, Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences, Bolshoy pr. V.O., 61, 199178, St. Petersburg, Russia and Department of Theoretical Mechanics, St. Petersburg State Polytechnical University, Politechnicheskaja 29, 195251, St. Petersburg, Russia.
email: `elenaivanova239@post.ru`