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A Model for the Threshold of the Rolling Transition

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A dynamic transition has been identified recently by numerical simulations in the case of a granular flow along a rotating boundary (Rioual and Le Quiniou (2011)). We showed that this transition appears above a certain critical microscopic friction coefficient particle/boundary μ^* *from which the particles roll without any sliding. We propose here a model at the scale of the microstructure which aimes to be close to the physical process considered and predicts also a threshold for the apparition of this dynamic transition. We discuss the discrepancy on a quantitative level in terms of friction properties at the contact.*

1 Introduction

The hydrodynamics of dense granular flows is of interest in several areas of nature and engineering. It still presents unexpected issues in these different contexts. As the energy in granular flows is dissipated mainly at the scale of the contacts between naturally polydisperse grains by surface friction, attrition and eventually comminution (Davies and McSaveney (2009)), an understanding of the physical mechanisms involved at the scale of the micro structure would be of high relevance (see for instance de Gennes (1996)). Bridging the gap between the scale of the contact (scale of the dissipation) and the scale of the continuum is still an open challenge in the dynamics of granular matter.
On a more practical point of view, a local study of the flow may at the same time represent an important complementary input for hydrodynamic theories which have been commonly used to model granular flows as in depth averaged approaches and in which microstructural studies have not always been considered.

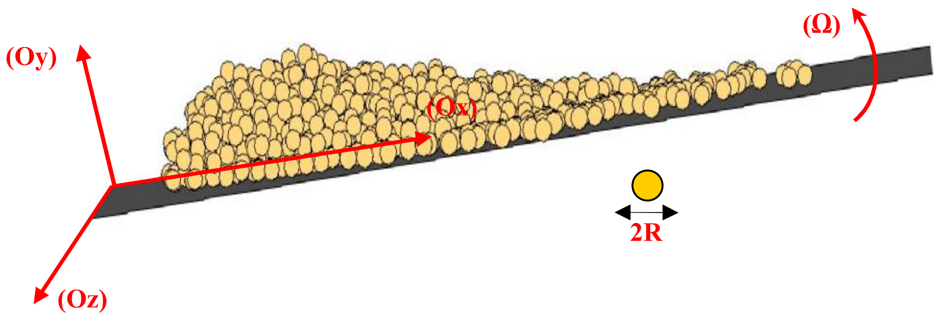

Figure 1: Sketch of a granular packing flowing along a vertical boundary rotating (direction of the flow (Ox); axis of rotation (Oz)) at a velocity Ω

We treat here the specific case of a granular packing flowing along a vertical rotating boundary (See Figure 1). The packing flows under the effect of the rotating boundary driven by the inertial forces (Coriolis and Centrifugal force). This situation is here directly inspired from a practical situation encountered in environmental engineering (spreading of a dry granular material).
At the scale of the entire packing, we observed by numerical simulations a localisation of the deformation at the basis of the flow (involving typically the first layers of grains at the basis of the flow). This may be reminiscent of a shear banding phenomenon observed in other contexts in granular physics (Shall and Van Hecke (2010); Koval-Junior (2008)). This confirms that the stress field in the flow is not invariant in the direction perpendicular to the boundary and may have to be carefully considered for modelling the bulk properties of this kind of flow. This phenomenon is not described in the course of this short article where we will be more concerned by the motion of the first layer of particles at the contact with the boundary.

2 Friction of the Granular Flow along the Boundary

We used recently experiments and numerical simulations by the discrete element method (D.E.M.) to study the friction properties of a granular material flowing along a vertical rotating wall (Rioual and Le Quiniou (2011)). In the case of a single particle under flow, we showed the relevance of defining two friction coefficients specifically related to an impact process μ_{impact} for a particle in collisional interaction with a boundary (as in Foerster and Louge (1994)) and an enduring contact process $\mu_{contact}$ for a particle sliding continuously along the same boundary (Le Quiniou and Rioual (2009)).
In the case of a collective flow of particles along the same rotating boundary, we calculated an effective friction coefficient μ_{eff} (Eq.(1)) defined as the ratio between the sum of the tangential forces to the sum of the normal forces on all the contacts between the particles and the boundary inside a specific characteristic box (typical size of the sample equal to 10 bead diameters).

$$\mu_{eff} = \sum T / \sum N \tag{1}$$

We observe on Figure 2 that two regimes appear in the evolution of the effective friction coefficient as a function of the microscopical friction coefficient. (i) a first regime corresponding to solid body friction where the effective friction coefficient is equal to the microscopic friction coefficient. (ii) a second regime above a critical microscopic friction coefficient μ^* at the basis of the flow from which the effective friction coefficient saturates. Numerical simulations showed furthermore that this critical value corresponds to a dynamic transition towards a rolling regime where the particles roll without any sliding at the boundary. In this rolling phase, the saturated value of the effective friction coefficient depends only on the tangential elastic component of the contact force law as shown recently (Rioual and Le Quiniou (2011)).
We would like here to propose a model for the prediction of the critical friction coefficient μ^*.

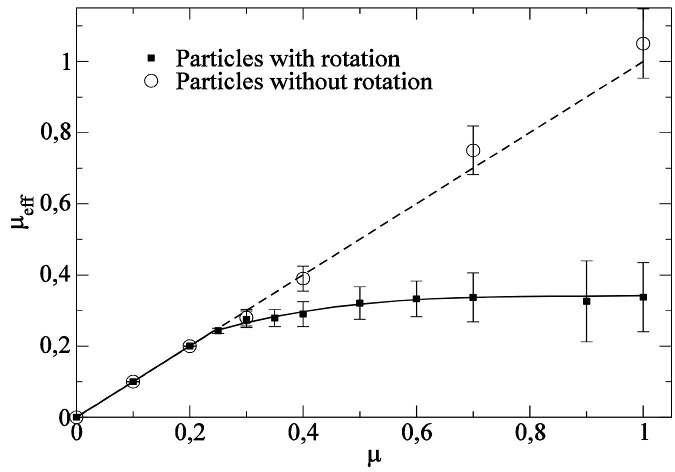

Figure 2: Mean effective friction at the boundary as a function of the friction coefficient particle/vane for particles with and without rotational degrees of freedom (friction particle/particle $\mu_p = 0{,}2$

3 Balancing Theory for the Rolling Transition

3.1 Rolling Transition for a Single Particle along the Boundary

We suppose a particle (mass m) moving in permanent contact along a boundary under a given force field.
- We consider here the case where the boundary is rotating at a velocity Ω.
We can place ourselves in the rotating frame. In such case, the applied force field is composed of the inertial forces (Centrifugal force $m\Omega^2 x$, Coriolis force $2m\Omega\, dx/dt$) as well as the friction force $T = \mu N$. The equations of motion for the particle can be written as:

$$\begin{cases} m\,d^2x/dt^2 = m\Omega^2 x - \mu N & (2) \\ N = 2m\Omega\, dx/dt & (3) \\ I\,d^2\theta/dt^2 = \mu NR & (4) \end{cases}$$

As the dynamics of the particle is essentially controlled at the scale of the contact with the boundary, a natural variable to characterize the dynamics of the particle at this scale is the sliding velocity of the flowing particle with respect to the static boundary: V

The sliding velocity is here defined as the velocity of the point at the lower extremity of the particle minus the velocity of the boundary at the same position x (which is 0):

$$V = \frac{dx}{dt} + \frac{Rd\theta}{dt}$$

- The other variable remains the position of the center of gravity of the moving particle:

x

Using eqs. (3) and (4), we deduce:

$$d\theta/dt = (5\mu\Omega/R)x + \alpha \qquad (5)$$

where α depends on the initial conditions. We can choose the origin of the axis such that $d\theta/dt = 0$ for $x = 0$.
Newton's law of mechanics for the moving particle can be written again as a dynamical equation with respect to these two new variable (x, V) as the following:

$$\frac{dV}{dt} = \left(1 - 15\mu^2\right)\Omega^2 x + 3\mu\Omega V \qquad (6)$$

Equ.6 is supposed to describe the temporal evolution of the sliding velocity of the particle in contact with the boundary. The first term on the right hand side is a forcing term which represents the competition the centrifugal force and the friction at the contact point.

We see as a consequence that the system defined by: ($V = 0; \frac{dV}{dt} = 0$) allows a solution if the following condition is fulfilled: $\mu_0 \geq 1/\sqrt{15}$ (0,25)

We can note also that this situation can occur in other contexts as for instance for the flow of a particle under gravity along an inclined boundary (inclination at an angle α). In this situation, the evolution equation for the sliding velocity at the contact can be expressed in the same way as:

$$\frac{dV}{dt} = g\sin\alpha - (7/2)\mu g\cos\alpha \qquad (7)$$

In this case, the rolling transition corresponding to the condition that the above equation cancels occurs above a critical friction coefficient equal to $(2/7)\tan\alpha$

3.2 Rolling Transition in the Case of a Dense Flow of Particles

We propose to take into account the influence of the neighbourhood of the bead in contact with the boundary in the force balance presented above (Fig.3). We propose also to model the phenomenon based on the two following hypothesis:

a- The existence of enduring contacts between the upper beads and the beads along the boundary. The impulsive transfer of momentum and associated fluctuations are not taken into account here. This hypothesis is based on the simulations of (Rioual and Le Quiniou (2011)).
b- A two dimensional scheme considering that the bead is placed at the basis of the flow and on average, the considered bead has two neighbours which contribute significantly to the spin because of the

traction/compression deformation rate imposed by the force field and equal friction forces apply in the transverse direction (Oz) on both sides of the bead.

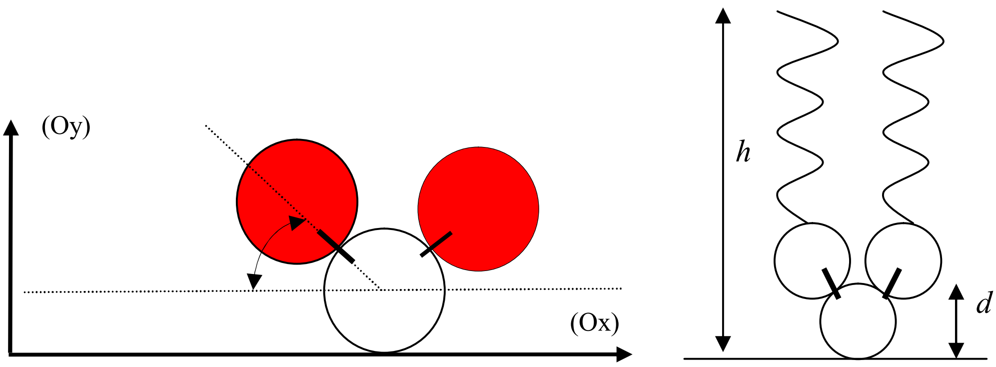

Figure 3: A scheme of the geometrical configuration along the boundary in a 2D representation

Considering that on average the contacts occur at an angular position between 0 and a maximal angular position θ_0, we get an additional purely frictional contribution in the force balance at the contact on both side of the bead equal to $-2\mu_p N' \langle \cos\theta \rangle$
N' corresponds to the normal force exerted by the upper layers of the packing applied at the contact $N' = \alpha N$.
We can choose here a value $\alpha = 1$ as the size of one bead is small compared to the typical height of the flow.
The averaging applies thus only on the angular position of the contact point.
We see then that the condition ($V = 0, \frac{dV}{dt} = 0$) is fulfilled if

$$\mu^* = 1/\sqrt{15} + 2\mu_p \sin(\theta_0) \tag{8}$$

θ_0 can here be related to the local compacity of the granular flow. We obtain thus a formula for the critical friction coefficient with respect to the micromechanical parameters of the problem.

4 Discussion

Experiments have been performed on the flow of a single particle along a rotating wall where a first bouncing phase was accounted for (Le Quiniou and Rioual (2009)). We show a discrepancy with the predicted value of the critical friction coefficient in the case of a single particle remaining in permanent contact along the boundary which is the value derived here and in the case of a dense flow, the matching with the results from numerical simulations (Fig.2) is doubtful for a reasonable range of parameters as θ_0. As pointed out recently ((Le Quiniou and Rioual (2009)), the choice of constant friction coefficients in the model is still questionable. This is problematic in this kind of situation and may limit any quantitative prediction at this stage.

5 Conclusion

We proposed in this article an analytical derivation of the threshold for the rolling transition which was studied in experiments (for a single particle) and in discrete element numerical simulations (for a dense granular flow) without any precise knowledge of the microstructural properties in the bulk of the flow.
We introduced for this purpose a new dynamical variable: the sliding velocity which is the relevant quantity for characterizing the dynamics at the scale of the contact between the particle and the boundary and we reformulated the dynamic equations for the particle with respect to this new variable. We were able to give a prediction for the threshold of the friction coefficient at the boundary using a mixed local/global approach i.e. based on a local description of the force balance in the granular flow. The ability to reproduce quantitatively the

values for the threshold friction coefficient is uncertain. We point out the difficulty to treat correctly the frictional interactions between particles in contact in the present model at this stage where rigid body mechanics may find here partially its limits.

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