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A Mechanism-Based Approach to Life Prediction for a Nickel-Base Alloy subjected to Cyclic and Creep-Fatigue

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A large number of damage parameters have been proposed to estimate cyclic fatigue life predominantly at ambient temperatures. However, especially in aerospace and automotive industry fatigue models with a wide temperature application range are required. Here, the regimes of high temperature creep-fatigue and non-isothermal thermo-mechanical fatigue are of particular interest.
Within the present work a new mechanism-based life prediction approach is proposed for a nickel-base superalloy. The ability of the model to describe fatigue at low, intermediate, and also at high temperatures is investigated. Isothermal, as well as non-isothermal loading conditions are considered. The enhanced model formulation is based on the micro crack growth parameter Z_d introduced by Heitmann et al. (1984). The model incorporates a threshold concept and corrections for mean stress and creep effects. In addition the detrimental effects caused by oxygen-induced embrittlement of the near tip region are accounted for by a parabolic oxidation approach. Test data from literature is used to compare the proposed model to several other fatigue models. Basis for all life prediction approaches under investigation is the stress-strain response of the material obtained by finite element analysis. Therefore, an inelastic constitutive model is applied. The fatigue model accuracy is evaluated on a statistical basis through an evaluation of the variance in the ratio of predicted life to actual life. This is done for the entire test database as well as for subsets, like isothermal, thermo-mechanical, and dwell tests only.

1 Introduction

In the past many authors, e.g. Heitmann et al. (1984) and Riedel (1987) have suggested that a close relation between classic fracture mechanics and cyclic fatigue exists. Several fatigue models based on micro crack growth have been proposed of which the models of Dowling (1977), Miller et al. (1993), Nissley (1995), and Schmitt et al. (2002) are prominent examples. The simplest way to explain this connection is to estimate fatigue life based on linear elastic fracture mechanics (LEFM). Assuming that the power law introduced by Paris and Erdogan (1963) holds for fatigue crack growth (FCG), the ΔK-controlled growth of semi-circular surface cracks within an infinite body can be modeled by (1). The number of cycles to initiation N_i can now be obtained by integrating the cyclic crack growth rate da/dN from an initial crack length a_0 up to a critical length a_f. Therefore, a certain crack initiation criterion a_f (e.g. 0.4 mm) has to be defined. The initial crack length a_0 can be estimated based on the knowledge about the size of critical microstructures like inclusions, voids, critical phase boundaries between precipitates and matrix, or carbides. The resulting equation (2) is mathematically identical to common two-parameter live curve approaches correlating $\Delta\sigma$ and N_i. If the assumptions for (1) and (2) were correct, the exponent in such an empirical fatigue model can be expected to take the same absolute value like the Paris exponent n.

$$\frac{da}{dN} = A(\Delta K)^n = A\left(Y\Delta\sigma\sqrt{\pi a}\right)^n \tag{1}$$

$$N_i = \int_{a_0}^{a_f}\left(\frac{da}{dN}\right)^{-1} da = \frac{1}{AY^n\pi^{n/2}}\Delta\sigma^{-n}\int_{a_0}^{a_f} a^{-n/2}da \overset{\forall\, n\neq 2}{=} \frac{2\left(a_f^{(2-n)/2} - a_0^{(2-n)/2}\right)}{(2-n)AY^n\pi^{n/2}}\Delta\sigma^{-n} \tag{2}$$

Fundamental understanding of the fatigue crack growth mechanism has been obtained when Laird (1967), Pelloux (1969), and Neumann (1974) investigated crack tips with duplex slip system dislocation configurations in single crystals. The same mechanism can be expected to hold for transgranular crack growth in polycrystalline materials as well. A simple geometric model (3) explaining this type of cyclic crack growth by blunting and re-sharpening of the crack tip has been proposed in Pelloux (1969). This idealized process has also been called the alternating slip or alternating shear model.

$$\frac{da}{dN} = \beta \cdot \Delta CTOD \tag{3}$$

The important finding here was that *da/dN* is proportional to the cyclic crack tip opening displacement $\Delta CTOD$ (also called *COD*). The proportionality factor β is the so called irreversibility factor. β quantifies the amount of crack advance remaining after crack extension during loading and subsequent partial sintering of the crack tip region during unloading and compression. As long as new crack faces were not fully covered with oxides, healing could take place during unloading. This idea is supported by the observation that for inert environments often significantly decelerated crack growth rates *da/dN* can be measured. Pelloux (1969) estimated β to be approximately 1/2. According to Neumann (1974) β can be estimated based on the directions of the actual slip system by

$$\beta = \frac{\mathrm{cotan}(\theta)}{1 + 1/c} \tag{4}$$

where θ is the angle at which the crack tip slip bands cross the crack face. $c \geq 2$ is a material parameter describing the coarseness of the reversed slip mechanism. For $\theta = \pm 45°$, as it was originally assumed by Pelloux (1969), β becomes $\leq$ 1/3. Measurements of fatigue crack length by replica technique reported in Schweizer et al. (2007) revealed that the geometrical model maps the actual mechanisms fairly well.

Dowling (1977) was among the first authors proposing a life prediction parameter to describe fatigue initiation in smooth specimens based on elastic plastic fracture mechanics (EPFM). For the Ramberg-Osgood type material law (5) he proposed the strain energy density parameter (6). This allowed the correlation of low temperature cyclic fatigue tests in line with crack propagation data obtained for compact tension (CT) and center cracked (CC) specimens. Similar to the geometrically derived relationship (3) his tests suggest that the micro crack growth parameter (6) can be correlated with *da/dN* through a simple power law similar to (1).

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'} \tag{5}$$

$$\frac{J_{Dowling}}{a} = \frac{4 \cdot 1.1215^2}{\pi} \frac{\Delta\sigma^2}{E} + \frac{1.1215^2 \cdot 8}{\pi(1+n')} f_S(n') \Delta\sigma \Delta\varepsilon_p \tag{6}$$

In case of a simple linear elastic material the Dowling parameter reduces to the energy release rate *G,* which equals $\Delta K^2/E$ under plain stress conditions. If we further assume that the alternating slip mechanism (3) holds and that $\Delta CTOD$ is proportional to the J-integral estimate in (6), the Dowling approach can be considered as a special case of (1) for a Paris exponent $n = 2$ (see Figure 1).

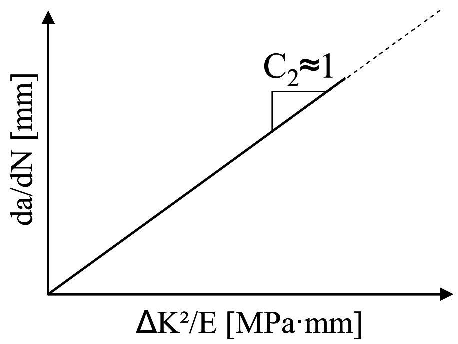

Figure 1. Crack propagation curve based on Z_d for a lin. elastic material is similar to Paris law

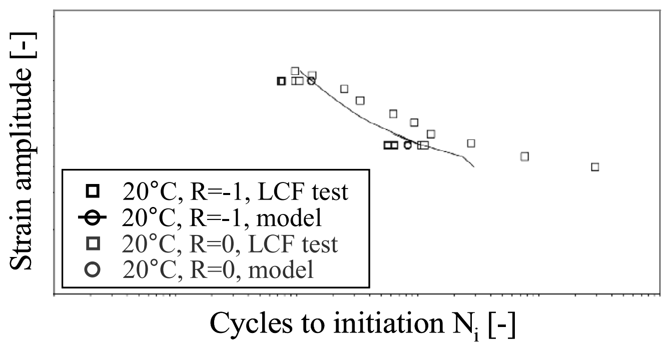

Figure 2. Comparison of the life curve resulting from Z_d model and fatigue tests at room temperature (see section 2)

Other authors, like Wüthrich (1982), confirmed the idea of an elastic-plastic energy criterion and found that the J-integral introduced by Rice and Rosengren (1968) and Hutchinson (1968) can be extended under certain assumptions to the cyclic J-integral, which has also been called Z integral in Wüthrich (1982).

At room temperature the Z_d parameter (7) originally proposed by Heitmann et al. (1984) is widely accepted and is often used to predict initiation life of specimens and components subjected predominately to low-cycle fatigue (LCF) conditions at ambient temperatures (c.p. Figure 2). Based on the idea of crack closure introduced by Elber (1971), Heitmann et al. (1984) incorporated an empirical expression for the effective stress σ_{eff} (8), accounting primarily for the effects of plasticity- and roughness-induced closure. Furthermore $Z_d \cdot a$ equals Z and can therefore be correlated with da/dN (9) when alternating slip mechanism (3) is assumed. Fatigue life to initiation (10) can now be obtained by integrating (9).

$$Z_d = \frac{Z}{a} = \frac{1.1215^2 \cdot 4\left(1-\nu^2\right)}{\pi}\frac{\Delta\sigma^2{}_{eff}}{E} + \frac{1.1215^2 \cdot 6}{\pi\sqrt{1+3n'}}\Delta\sigma\Delta\varepsilon_p \tag{7}$$

$$\Delta\sigma_{eff} = 3.74\Delta\sigma\left[3 - R_\sigma\right]^{-1.72} \tag{8}$$

$$\frac{da}{dN} = \beta \cdot \Delta CTOD = \beta \cdot \frac{d_{n'} Z_d \cdot a}{\sigma_{cy}} \tag{9}$$

$$N_i = \frac{\ln\left(a_f/a_0\right)}{\beta d_{n'}}\left[\frac{Z_d}{\sigma_{cy}}\right]^{-1} \tag{10}$$

In case of a linear elastic material the equation for Z_d reduces to the solution of a plain strain semi-circular surface crack where $Z_{el} = (1-\nu^2)\Delta K^2/E$. The plastic portion of Z_d was derived from the solution for a penny-shaped crack in an infinite body of power-law hardening material in He and Hutchinson (1981). Surface correction is done by applying 1.1215 (difference between penny-shaped crack and semi-circular crack according to LEFM). The function $d_{n'}$ describing the dependency of $\Delta CTOD$ and Z_d can be found in Shih (1981). This solution can be simplified to a polynomial function $d_{n'} = f_S(n')$. Finally N_i only depends on the term $ln(a_f/a_0)/(\beta d_{n'})$ in (10), which can often be treated as a single fitting parameter.

Aim of the present work is to find a mechanism based approach which is able to describe isothermal cyclic fatigue as well as thermo-mechanical fatigue (TMF) in a unified manner based on the hysteresis data obtained by finite element analysis (FEA) of specimens. It is expected that using physically motivated fatigue models can decrease the number of model parameters.

Nomenclature

A	Paris coefficient	$f_S(n')$	Solution function for plastic J-integral
A_{cr}	Coefficient in creep damage equation	$J_{Dowling}$	Estimate for the cyclic J-integral
a, a_0, a_f	Actual, initial, final crack length	K'	Ramberg-Osgood coefficient
α	Norton creep proportionality factor	K_m	Oxidation correction for inelastic strains
α_m	Parabolic oxidation coefficient	$K_{p,eff}$	Effective oxidation constant
B	Norton creep coefficient	k'	Frequency modification exponent
$B_{0.1}$, B_{50}	0.1 and 50 percent quantile of the log-normal distribution	m	Norton creep exponent
		m_W	Walker exponent
b_{ox}	Parameter describing the strain rate sensitivity of oxidation damage D_{ox}	N_a / N_p	Ratio actual life to predicted life
		N_i	Number of cycles to initiation
β	Irreversibility factor of alternating slip	n_T	Number of interpolation temperatures
β_{ox}	Exponent on time in oxidation law	n	Paris exponent
C^*	Creep analogue to plastic J-integral	n'	Ramberg-Osgood exponent
C_1, C_2	Elastic life curve parameters	υ	Poison ratio
C_3, C_4	Inelastic life curve parameters	υ^*_0	Intrinsic activation volume
C_5	Coefficient in frequency modified term	p	Threshold correction parameter
C_{ox}	Coefficient in oxide growth equation	Φ_{ox}	Phase factor for oxidation damage
C_{th}	Proportionality factor of oxidation-induced crack closure approach	Q	Nominal activation energy for time-dependent damage contributions
c	Coarseness parameter of alternating slip	Q_0	Intrinsic activation energy

D_0	Diffusion coefficient	Q_{cr}	Activation energy for creep
$D_{CF},\ \widetilde{D}_{CF}$	Riedel's creep-fatigue damage parameter	Q_{ox}	Activation energy for oxidation
D_{fat}, D_{cr}, D_{ox}	Damage portions due to fatigue, creep,	R	Universal gas constant
	and oxide-induced degradation	R_σ	Stress ratio
da/dN	Cyclic crack growth rate	σ	Stress
$d_{n'}$	Correlation between Z and $\Delta CTOD$	σ_{amp}	Stress amplitude ($=0.5\Delta\sigma$)
$\Delta CTOD$	Cyclic crack tip opening displacement	σ_{cy}	Cyclic 0.2% yield strength
$\Delta\varepsilon_p+\Delta\varepsilon_{cr}=\Delta\varepsilon_{in}$	Plastic, creep, and inelastic strain range	σ_{UTS}	Ultimate tensile strength
$\Delta\varepsilon_{mech}$	Mechanical strain range	$\dot{\sigma}$	Stress rate
ΔK	Cyclic stress intensity factor range	T	Temperature in Kelvin
ΔK_{th}	Threshold stress intensity factor range	t	Time
$\Delta\sigma$	Stress range	t_{cyc}	Cycle time
$\Delta\sigma_{eff}=\sigma_{max}-\sigma_{op}$	Effective stress range due to closure	t_f	Creep rupture life (unit of time)
E	Young's modulus	t_T	Cumulated time during rising branch of
ε	Strain		hysteresis loop including hold time
$\dot{\varepsilon}_{cr}$	Mechanical strain rate	θ	Neumann's angle for alternating slip
$\dot{\varepsilon}_{mech}$	Creep strain rate	ξ_{ox}, ξ_{cr}	Parameters of phasing functions
$\dot{\varepsilon}_s$	Secondary creep rate	Y	Crack geometry correction factor
$\dot{\varepsilon}_{th}$	Thermal strain rate	Z	Cyclic J-integral
f_{cyc}	Test frequency ($=1/t_{cyc}$)	Z_d	Heitmann parameter

2 Experimental Database

For the fine grained wrought Ni-base alloy IN718 cyclic fatigue tests from Socie et al. (1985), Kim et al. (1988), Halford and McGaw (1995), and Nelson et al. (1992) were used to validate the capabilities of the fatigue models under consideration. Due to the large amount of publications related particularly to this alloy and its mechanical, chemical, and micro-structural properties we can be quite sure that most effects observed in this material are well known and the respective modeling approaches have been reviewed by numerous authors. In total 88 fatigue tests have been compiled to the overall database used for this paper. Details on each test are given in the appendix of Vöse et al. (2011). Chemical compositions and heat treatments of the specimens were also presented there and are similar.

3 Constitutive Model Predictions

A proper constitutive model is essential to determine the correct stress-strain response governing the fatigue process. Especially during long time dwell and TMF loading maximum tensile stresses can change significantly. Not taking this into account will limit the performance of the fatigue model. A constitutive model able to describe rate-dependent as well as rate independent stress-strain behavior for IN718 has been published in Becker and Hackenberg (2010). The respective parameters have been fit for a number of strain-controlled isothermal constitutive tests. These tests include cycles with different load levels, hold times, strain rates, and several R-ratios. Cyclic TMF tests were later on used to validate the approach for non-isothermal conditions. An example of the stress history for one of the constitutive tests used for model regression is given in Figure 3.

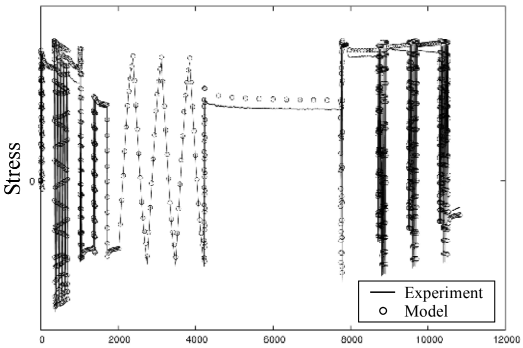

Figure 3. Stress history diagram for an isothermal constitutive tests involving complex load history

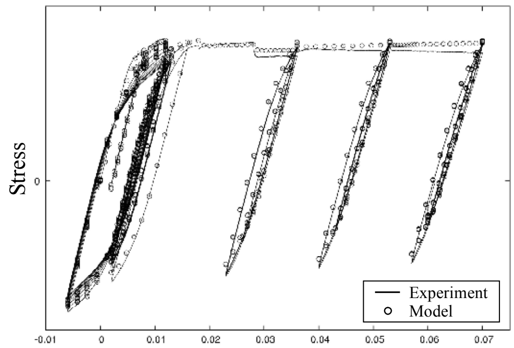

Figure 4. Stress-strain diagram resulting from complex load history

Based on the good description of the stress-strain behavior (see Figure 4) obtained for the set of proprietary tests, midlife hysteresis loops for each test within the database described in section 2 were determined by FEA. The simulation has been performed on a single 20 node brick element loaded with the appropriate stress- or strain boundary conditions respectively. A comparison of measured and predicted stress amplitudes in Vöse et al. (2011) has shown an overall good agreement between model and experiments.

4 Existing Fatigue Models

Numerous fatigue models have been proposed for application to isothermal- and TMF loading conditions. Within the current work a modified version of the model proposed in Neu and Sehitoglu (1989), described in Vöse et al. (2011), and an empirical model published recently in Vöse et al. (2011) shall be considered. Both will be assessed and compared to the predictive capabilities of the model described in section 5 of this paper. In this section only a short description including the current model equations of the two approaches will be provided. For more details the reader is referred to the original publications.

The Sehitoglu model is a MULTI MECHANISM FATIGUE MODEL in which great care has been taken in mapping the actual damage mechanism under various isothermal and TMF loading conditions. Several damage mechanisms (cyclic fatigue, creep, and oxidation) relevant under TMF conditions have been identified and fatigue life is modeled by linear superposition of those damage contributions (11). The description of Cyclic Fatigue is done by a conventional life curve approach (12) in which a Walker-like mean stress correction, described in Walker (1970) and Vöse et al. (2011), has been added. The creep damage formulation (13) follows the fundamental ideas of Monkman and Grant (1956), who estimated creep rupture life t_f based on the inverse of cumulated secondary creep strain during one cycle. An empirical TMF-phase factor has been added to account for increased creep damage accumulation during in-phase (IP) TMF cycling. Based on oxide measurements the model as proposed in Neu and Sehitoglu (1989) incorporates a sophisticated approach describing cyclic growth and cracking of oxides at the specimen surfaces (14-16). To fit all model parameters according to the proposals made in Neu and Sehitoglu (1989) and Sehitoglu and Boismier (1990) special tests like cyclic tests in vacuum, out-of-phase (OP) TMF tests, creep rupture tests and static measurements of the oxide thickness are needed to determine the model parameters properly. Since some of the required experiments are often not available one alternative is to use a large number of tests to calibrate the parameters by applying an optimization algorithm.

$$1/N_i = D_{fat} + D_{cr} + D_{ox} \tag{11}$$

$$\Delta\varepsilon_{mech}(1-R_\sigma)^{m_W-1}2^{1-m_W} = C_1\left(1/D_{fat}\right)^{C_2} + C_3\left(1/D_{fat}\right)^{C_4} \tag{12}$$

$$D_{cr} = \frac{1}{t_{cyc}}\int_0^{t_{cyc}} \exp\left[-\frac{1}{2}\left(\frac{\dot{\varepsilon}_{th}/\dot{\varepsilon}_{mech}-1}{\xi_{cr}}\right)^2\right]\cdot A_{cr}\dot{\varepsilon}_s dt \tag{13}$$

$$D_{ox} = \left[\frac{C_{ox}}{\Phi_{ox}K_{p.eff}}\right]^{-1/\beta_{ox}} \frac{2\left(\Delta\varepsilon_{mech}\right)^{2/\beta_{ox}+1}}{\left(\dot{\varepsilon}_{mech}\right)^{1-b_{ox}/\beta_{ox}}} \tag{14}$$

$$K_{p.eff} = \frac{1}{t_{cyc}} \int_0^{t_{cyc}} D_0 \exp(-Q_{ox}/RT) dt \tag{15}$$

$$\Phi_{ox} = \frac{1}{t_{cyc}} \int_0^{t_{cyc}} \exp\left[-\frac{1}{2} \left(\frac{\dot{\varepsilon}_{th}/\dot{\varepsilon}_{mech} + 1}{\xi_{ox}} \right)^2 \right] dt \tag{16}$$

In addition an EMPIRICAL FATIGUE MODEL is considered here. This shall answer the question whether a simple life curve approach is able to describe fatigue at several temperatures and temperature ranges properly for design purposes. Similar to the original frequency modification model presented in Coffin (1976) the model described in (17) does not differentiate between distinct damage mechanisms. It is basically an empirical approach in which the life curves for various LCF temperatures are modeled by considering a thermally activated fatigue mechanism. In this context it is assumed that stress concentrations within the grains lead to localized damage causing fatigue. A combination of high temperatures and high cyclic stresses can accelerate this process.

$$\Delta\varepsilon_{mech} \left(\frac{1-R_\sigma}{2} \right)^{m_W - 1} \left(1 + C_5 \int \exp\left[\frac{-1}{RT} \left(Q_0 - \overset{*}{\upsilon_0} \sigma_{amp} \left(1 - \frac{\sigma_{amp}}{2\sigma_{UTS}} \right) \right) \right] dt \right)^{k'} = C_1 (N_i)^{C_2} + C_3 (N_i)^{C_4} \tag{17}$$

As proposed in Vöse et al. (2011) the temperature dependency of fatigue tests can be described by an Arrhenius term, whereas an additional dependency on stress amplitude, similar to the ideas of Warren and Wei (2008), has to be added in order to describe the influence of mechanical cycling on the intrinsic activation energy of the thermally activated fatigue phenomenon.

5 Micro Crack Growth Model

In Heitmann et al. (1984) and Dowling (1977) it has been observed that the predicted crack growth rate for small cyclic load levels is overestimated by the Z_d and Dowling parameter approach (7-9). This has been interpreted as a small deviation of the model which only leads to slightly conservative life estimates. Since these publications were concerned primarily with low-life LCF tests this seems reasonable. However, if many tests within the range of small amplitudes are available, this model weakness has to be addressed. A quite simple way to do that is to add a proper threshold formulation as they are known from LEFM. Here an approach proposed in Newman (1981) is applied (18) to the elastic portion of Z_d.

$$Z'_d = \frac{1.1215^2 \cdot 4\left(1-\nu^2\right)}{\pi} \frac{\Delta\sigma^2{}_{eff}}{E} \left[1 - \left(\frac{\Delta K_{th} \sqrt{\pi / a}}{1.1215 \cdot 2\Delta\sigma_{eff}} \right)^p \right]^2 + \frac{1.1215^2 \cdot 6}{\pi \sqrt{1+3n'}} \Delta\sigma\Delta\varepsilon_p \tag{18}$$

The formulation contains the temperature independent threshold stress intensity factor ΔK_{th} which also models the R-dependency of the closure phenomenon described in Suresh (1998).

At elevated temperatures creep effects start to increase and the assumption of a simple Ramberg-Osgood type of material (5) is no longer justified. In the following a generalized form of the material behavior (5) including a Norton creep formulation is assumed (19).

$$\dot{\varepsilon}_{mech} = \frac{\dot{\sigma}}{E} + \frac{1}{n'K'^{1/n'}} (\Delta\sigma)^{1/n'-1} \dot{\sigma} + B\sigma^m \tag{19}$$

$$\dot{\varepsilon}_{mech} = \frac{\dot{\sigma}}{E} + \frac{1}{n'K'^{1/n'}} (\Delta\sigma)^{1/n'-1} \dot{\sigma} + \alpha \exp\left(\frac{-Q_{cr}}{RT} \right) \sigma^m \tag{20}$$

Assuming an explicit temperature dependency by including an Arrhenius term in (19) leads to the more general form (20) which is expected to be valid also under TMF conditions. A similar model for secondary creep rate in IN718 including an additional backstress has also been investigated in Han and Chaturvendi (1987). Now several simple cases of the material law (19) shall be investigated.

First small scale creep, i.e. creep displacement fields are dominant only in the vicinity of the crack tip, shall be discussed. Under these conditions Riedel and Rice (1980) have argued that ΔK is the appropriate loading parameter for very short time scales, whereas the creep counterpart C^* to the J-integral is the correct loading parameter after a certain critical time has been spent. Based on these findings an estimate for *da/dN* by alternating slip (21) can be given by assuming a transient behavior in between.

$$\left.\frac{da}{dN}\right|_{el.-viscous} = \frac{\beta}{2\sigma_{cy}} \frac{1.25\cdot 4\left(1-\nu^2\right)}{\pi} \frac{\Delta\sigma_{eff}^2}{E}\left[1+(m+1)Bt_T K'^m\right]^{1/m} a \tag{21}$$

In the second case severe plastic deformations in the entire specimen consequently cause large deformations around the crack tip. During monotonically increasing loads the J-integral is the correct loading parameter to describe these large scale deformation fields. Assuming Masing behavior the cyclic extension Z_p (22) based on the rising branch of the hysteresis is able to describe *ΔCTOD* properly.

$$\left.\frac{da}{dN}\right|_{plastic} = \frac{\beta d_{n'} Z_p}{\sigma_{cy}} = \frac{\beta d_{n'}}{\sigma_{cy}} \frac{2.4}{\sqrt{1+3n'}} \Delta\sigma\Delta\varepsilon_p a \tag{22}$$

In the third case, due to the analogy between non-linear elastic power-law hardening material and power-law creeping material, the solution for a semi-circular surface crack in (22) can be transformed immediately to the case of a large scale creeping material (23).

$$\left.\frac{da}{dN}\right|_{creep} = \frac{\beta d_m}{\sigma_{cy}} \frac{2.4}{\sqrt{1+3/m}} \sigma\dot{\varepsilon}_{cr}\cdot t_T^{(m+1)/m} a \tag{23}$$

In Riedel (1987) it is argued that the latter two equations (22) and (23) can be combined to only a single term incorporating large scale plastic hardening as well as large scale creep behavior (24). He further assumed that linear superposition of small- and large-scale deformation fields at the crack tip can be applied to introduce the so called creep-fatigue damage parameter D_{CF} (25). This new parameter is introduced similar to Z_d and reduces to (7) at ambient temperatures where creep effects are negligible.

$$\left.\frac{da}{dN}\right|_{inelastic} = \frac{\beta d_{n'}}{\sigma_{cy}} \frac{2.4}{\sqrt{1+3n'}} \Delta\sigma\Delta\varepsilon_{in}\left[1+\frac{\Delta\varepsilon_{cr}}{\Delta\varepsilon_p}\right]^{1/m} a \tag{24}$$

$$D_{CF} = \frac{1}{\sigma_{cy}}\left\{\frac{1.25\cdot 4\left(1-\nu^2\right)}{\pi}\frac{\Delta\sigma_{eff}^2}{E}\left[1+(m+1)Bt_T K'^m\right]^{1/m} + \frac{2.4}{\sqrt{1+3n'}}\Delta\sigma\Delta\varepsilon_{in}\left[1+\frac{\Delta\varepsilon_{cr}}{\Delta\varepsilon_p}\right]^{1/m}\right\} \tag{25}$$

$$\begin{aligned}\widetilde{D}_{CF} = \frac{1}{\sigma_{cy}} & \left\{1.45\frac{\Delta\sigma_{eff}^2}{E}\left[1+(m+1)\alpha K'^m \int \exp\left(\frac{-Q_{cr}}{RT}\right)dt\right]^{1/m}\left[1-\left(\frac{\Delta K_{th}\sqrt{\pi/a}}{1.1215\cdot 2\Delta\sigma_{eff}}\right)^p\right]^2\right. \\ & \left. + \frac{2.4}{\sqrt{1+3n'}}\Delta\sigma\Delta\varepsilon_{in}\left[1+\alpha\int\left(\frac{K'\sigma}{\Delta\sigma}\right)^m \exp\left(\frac{-Q_{cr}}{RT}\right)dt\right]^{1/m}\right\}\end{aligned} \tag{26}$$

In case of non-isothermal loading conditions we propose that by using the material behavior from (20) the modified $\widetilde{D}_{CF}$ parameter can be formulated as described in (26). A similar model has already been published in Schmitt et al. (2002) where the similarity of both creep corrections has been used to introduce an empirical correction function *F* which describes the time-dependent creep effects on fatigue life.

From Maier et al. (2000) it is known that beside the creep effects, already accounted for when applying D_{CF}, environmental effects like oxidation can be crucial when predicting fatigue life. The effects of oxidation on FCG in IN718 have been investigated by numerous authors (e.g. Reuchet and Rémy (1983), Hoffelner (1987), Pineau (1988), Soboyejo and Deffeyes (1991), Krupp et al. (2004)) and their results, suggesting that there is a strong dependency, cannot be neglected. Two effects have major influence on crack growth and shall therefore be incorporated into the final modeling approach presented here. Several authors have reported for a number of polycrystalline alloys that crack propagation at high temperatures was accelerated, when oxides were found by fractographic investigations. Two mechanisms are supposed to cause this acceleration effect. Either cracks grow by oxygen-induced embrittlement of grain boundaries or by the formation of a brittle subsurface layer. The second effect has been reported for LCF tests as well as for crack propagation tests predominately at high temperatures. It has been found in Suresh et al. (1981) that FCG can be decelerated or even stopped, if the thickness of the oxide layer, formed at the freshly exposed faces around the crack tip, is in the order of ΔCTOD. The situation is similar to that of a single tensile overload. After inserting a sufficient amount of oxide material into the region, the crack tip field experiences higher compressive stresses and is decelerated. In case of repeated oxide growth at the crack tip and all over the crack faces it seems reasonable that crack arrest may be caused by this phenomenon. Since analytical models for oxide-induced closure are primarily based on LEFM, e.g. Chen (2002), the following simple approach has been chosen to account for both oxidation effects (27).

$$\frac{da}{dN} = \left\langle \beta d_{n'}\left(\widetilde{D}_{CF} \cdot a/\sigma_{cy}\right) - C_{th}\overline{\alpha_m}\sqrt{t_T} \right\rangle + \overline{\alpha_m}/2\sqrt{t_{cyc}/N_i} \tag{27}$$

$$\overline{\alpha}_m = \sqrt{\frac{1}{t}\int_0^t \alpha_m dt} \tag{28}$$

$$\alpha_m = \sqrt{D_0 \exp\left(-Q_{ox}/RT\right)}\left[1 + K_m \Delta\varepsilon_{in}\right] \tag{29}$$

The acceleration in crack growth rate *da/dN* due to oxidation is modeled following the additive superposition of several growth rate contributions as proposed in Wei and Landes (1969), Miller et al. (1993), Dai et al. (1995), and Maier et al. (2000). Simple summation of several crack growth rate contributions can be justified by assuming that at any given time there might be a critical crack in the specimen growing by the fastest mechanism, i.e. oxidation assisted or by blunting and re-sharpening.

Following the proposals of Reuchet and Rémy (1983) oxidation is modeled by a parabolic oxide growth equation. An empirical dependency on the cyclic inelastic strain range has been added to account for the observation that oxide formation largely depends on the amount accumulated dislocation density. The same oxide growth equation describing crack acceleration is used also to describe the assumed formation of oxide wedges within the crack faces, which decelerates *da/dN* primarily at small crack lengths a. This is modeled by subtracting a proportional part of the oxide layer formed between the opened crack faces from the cyclic crack growth increment caused by alternating slip. The Macauley bracket in (27) ensures that this can only decelerate or stop the blunting mechanism. Whereas the oxide-assisted crack growth contribution is expected to dominate growth of small arrested surface cracks until the crack finally starts to grow by alternating slip. The form of (27) makes it necessary that fatigue life N_i based on this model has to be determined by numerical integration.

Equation (29) does not account for the specific oxidation kinetics of IN718, reported in England and Virkar (1999) and Wang and Chen (2006), where several different oxides (e.g. Cr_2O_3 and CrO_3) can form. However in England and Vikar (1999) it has been found that a parabolic law represents the overall oxidation behavior of IN718 reasonably well. Estimates of the apparent activation energy for oxidation can be taken from Krupp et al. (2004), England and Vikar (1999), or from Greene and Finfrock (2001), setting a range within which the actual model parameter Q_{ox} must be found.

6 Fatigue Model Predictions

After fitting the parameters for all three fatigue models to the entire tests database, the intrinsic scatter within the life predictions shall be investigated. A common diagram visualizing the actual scatter for each model is the plot predicted life N_p versus actual life N_a (see Figures 5 to 7). In addition the 0.1% and 50% quantiles (also called $B_{0.1}$ and B_{50}) according to an assumed log-normal distribution of the quantity N_a / N_p will be presented here. The ratio $B_{50}/B_{0.1}$ can be understood as a measure for the combined scatter originating from model imperfections and experimental scatter. A small value of $B_{50}/B_{0.1}$ is desired, since this quantity is also a measure for the mean safety factor which has to be applied to the average model predictions in design applications. A safety factor of $1/B_{0.1}$ allows component design with only 0.1% probability that failure occurs prior to the N_i predicted. On the other hand B_{50} shows, if the predictions for a certain set of data tends to be predicted conservatively or not.

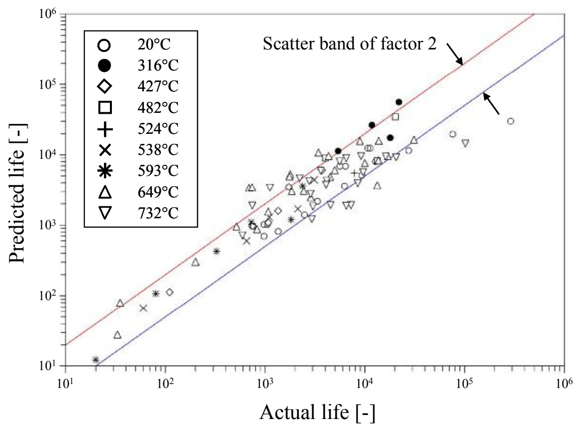

Figure 5. Predicted life vs. Actual life for Multi-mechanism fatigue model

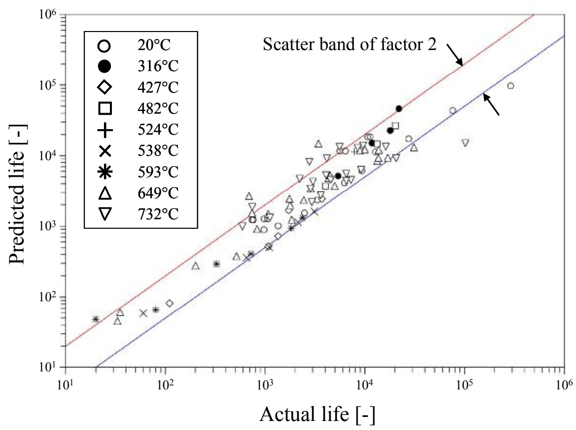

Figure 6. Predicted life vs. Actual life for empirical fatigue model

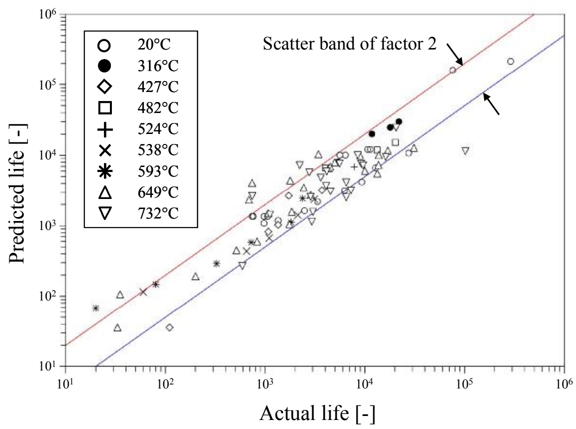

Figure 7. Predicted life vs. Actual life for micro crack growth model

As we can see in Figures 5 to 7 the scatter bands observed for all three approaches is are very similar. For the data set considered the model accuracy does not largely depend on the type of model (empirical, multi-mechanism, or micro crack growth model).

Table 1. Statistical evaluation of several subsets within the test database

Subset	N tests	B_{50}	$B_{50}/B_{0.1}$
20°C LCF	18	0.943	5.49
316°C LCF	4	0.476	1.52
427°C LCF	5	1.080	4.76
538°C LCF	5	1.120	4.17
593°C LCF	6	0.824	6.88
649°C LCF	18	1.064	7.07
732°C LCF	13 + 1	1.541	11.16
Slow LCF (t_{cyc}≥60s)	20	1.270	6.57
Slow LCF incl. dwell	5	0.875	18.35
All LCF	72	1.096	6.21
0° (in-phase)	7	0.579	10.97
180°/ ± 90°	9	0.755	2.85

Since we are interested in a more detailed assessment of the new model, for a comparison the B_{50} and $B_{50}/B_{0.1}$ values given in Table 1 shall be considered now. As we can see LCF life predictions for almost all temperature ranges except 732°C show little scatter. This also holds for slow LCF tests without hold times as well as for OP and diamond shape TMF conditions. For all tests except the four tests at 316°C B_{50} lies between 0.5 and 2, indicating that the model predicts these subsets in the mean case within a factor of two or even better.
Larger deviations can be observed for slow LCF tests including dwell time and for IP-TMF conditions. In both cases B_{50} is below unity and non-conservative estimates have to be expected for these loading conditions.

7 Discussion

In total the micro crack growth model described in section 5 includes 10 temperature independent parameters. But since the critical crack length a_f is usually defined to 0.4 mm only 9 independent parameters have to be determined during regression. A comparison with the two existing models, in which one or even two temperature dependent parameters are required, is presented in Table 2. If the number of discrete temperatures n_T for which temperature-dependent parameters are defined is five, one can easily see that the newly proposed model incorporates less parameters.

Table 2. Comparison of the $B_{50}/B_{0.1}$ scatter bands for the three fatigue models under investigation

Model	Frequency mod.	Sehitoglu	Mechanism based
Performance ($B_{50}/B_{0.1)}$	5.82	8.84	7.42
Number of parameters	$7+2n_T$	$9+n_T$	9

Fatigue is modeled through a unified process of micro crack evolution. The integration of several damage contributions is expected to be much more physical than the application of a Miner-like approach within the Sehitoglu model.
Despite their lack in physics phenomenological approaches like the frequency modification model are still valuable in the industry since they can provide an improved fit quality at the expense of a larger number of parameters and in general a large test database.

8 Conclusions

A relatively large database of 88 LCF and TMF Tests from 4 different literature sources was considered for different modeling approaches, namely a new mechanism-based, an established multi-mechanism, and an empirical approach. The stress-strain response at midlife, used in the fatigue models considered, was simulated by applying the constitutive model of Becker and Hackenberg (2010).

A new unified mechanism based LCF-TMF model was presented. For a wide range in life ($10^2 \ldots 10^6$ cycles) the predictions were found to be comparable to existing fatigue models considered here. Even though the phenomenological frequency modification model gives the best scatter fit for the actual data, the proposed mechanism-based model incorporates most physics and has the least number of parameters. An additional strength is that estimates for the model parameters can be based on the theory of small fatigue crack growth. For the given database the Sehitoglu model is showing the largest scatter when fitted according to the optimization strategy.

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