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Deformation of an Elastic Spherical Shell under the Pressure of Viscous Incompressible Fluid

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The deformation of an elastic spherical shell under the pressure of viscous incompressible fluid is considered. Analytical formulas for calculating the components of normal and tangential deflections of the shell middle surface are obtained. A new mathematical model of an elastic spherical shell is offered on the basis of introduction of the Finite Element Method calculations. The comparison of the asymptotic and numerical results is performed.

1 Literature Review

The problem of flow of an absolutely rigid sphere by the viscous incompressible fluid has been solved by Stokes in 1851. He used the approximate method of neglecting inertial terms and exterior forces in the basic equations of motion. In the books by Kochin et al. (1965) and Landau et al. (1987) containing the classical solution of such problem, the surface strains are not considered, and the fluid motion is carried out at low Reynolds numbers.

In reviewing the literature, it can be also seen that some works deal with the problem of a solid sphere moving in stratified fluid (Greenslade (2000)), or with the problem of a spinning sphere that translates in the shear flow (Ben Salem et al. (1998)).

In Mullin et al. (2005) there are results of investigation of a novel dynamical system in which one, two or three solid spheres are free to move in a horizontal rotating cylinder which is completely filled with a highly viscous fluid. Prevailing part of the published papers concerning thin-shell structures (see, for example, Fung et al. (2000)) is devoted to investigation of the simplest case of strain of a spherical shell under the action of uniform external pressure.

Also, at present (see, for example, Kaplunov et al. (1992)) there are refined formulations of hydroelasticity problems that take into account, in particular, the influence of the fluid on the tangential motion of a shell caused by the Poisson effect. This makes it also possible to define the no-fluid-loss condition more exactly.

In our article the problem of deformation of a thin elastic spherical shell is considered under non-uniform external pressure from outside of a stream of the viscous incompressible fluid. A new mathematical model of an elastic spherical shell is offered on the basis of Finite Element Method calculations. We consider the boundary conditions that may occur in problems of penetrating micro-bodies into the blood vessels. In this analysis we consider general factors such as the shell deflections magnitude in the normal and tangential directions. Analytical formulas for calculating the components of normal and tangential deflections of the shell middle surface are obtained. A comparison of the asymptotic and numerical results is performed.

2 Introduction

We consider a viscous fluid flow about a sphere of radius a, which is at rest and centred at the origin, at low Reynolds numbers. In the case under consideration we take R

$$R = \frac{\rho U a}{\widetilde{\mu}}$$

as the Reynolds number, where ρ is the fluid density, $\widetilde{\mu}$ is the coefficient of dynamic viscosity. The fluid flow has the velocity U that is constant in magnitude and direction at infinity. The direction of flow and the sphere are

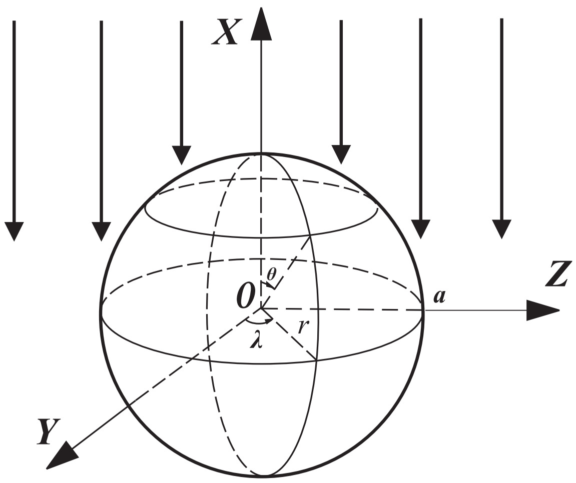

Figure 1: Spherical shell in the fluid flow.

shown in Fig. 1. The vector form of the Navier-Stokes equations, describing a motion of viscous fluid, is

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = \mathbf{F} - \frac{1}{\rho}\nabla p + \widetilde{\nu}\nabla^2\mathbf{v}, \quad \text{div}\,\mathbf{v} = 0, \tag{1}$$

where $\mathbf{v}$ is the velocity vector or fluid, $\mathbf{F}$ is the vector of body force, ∇p is the stress vector of surface force, $\widetilde{\nu} = \frac{\widetilde{\mu}}{\rho}$ is the kinematic-viscosity coefficient. If the Reynolds number R is sufficiently small, i. e. for a given fluid we have either a quite low flow velocity, or a small radius of the sphere, then when integrating equation (1) one can neglect the inertial forces as compared to the viscous forces and stress forces. Neither are the body forces $\mathbf{F}$ taken into consideration. Then a slow motion of viscous flow is defined by the following equations

$$\nabla p = \widetilde{\mu}\nabla^2\mathbf{v}, \quad \text{div}\,\mathbf{v} = 0, \tag{2}$$

Eqt. (2) should be complemented with the boundary conditions

$$v_x = v_y = v_z = 0 \quad \text{for} \quad r = a, \quad \text{where} \quad r = \sqrt{x^2 + y^2 + z^2}. \tag{3}$$

Except conditions (3), the conditions at infinity

$$v_x \to U, \quad v_y \to 0, \quad v_z \to 0 \quad \text{for} \quad r \to \infty \tag{4}$$

should be satisfied. Apart from the Cartesian coordinates $(x,\ y,\ z)$ we introduce the spherical coordinates $(r,\ \theta,\ \lambda)$ as follows

$$x = r\cos\theta, \quad y = r\sin\theta\cos\lambda, \quad z = r\sin\theta\sin\lambda.$$

Because of the symmetry of motion with respect to the axis OX, from which the angle θ is measured, we have

$$v_r = v_r(r,\ \theta), \quad v_\theta = v_\theta(r,\ \theta), \quad v_\lambda = 0, \quad p = p(r,\ \theta).$$

In this case the equations of motion (2) appear as

$$\begin{gathered}
\frac{\partial p}{\partial r} = \widetilde{\mu}\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{2}{r}\frac{\partial v_r}{\partial r} + \frac{\text{ctan}\,\theta}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta} - \frac{2v_r}{r^2} - \frac{2\,\text{ctan}\,\theta}{r^2}v_\theta\right), \\
\frac{1}{r}\frac{\partial p}{\partial\theta} = \widetilde{\mu}\left(\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{2}{r}\frac{\partial v_\theta}{\partial r} + \frac{\text{ctan}\,\theta}{r^2}\frac{\partial v_\theta}{\partial\theta} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta}{r^2\sin^2\theta}\right), \\
\frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{2v_r}{r} + \frac{v_\theta\,\text{ctan}\,\theta}{r} = 0.
\end{gathered} \tag{5}$$

Boundary conditions (3) will change to the following

$$v_r(a,\ \theta) = 0, \quad v_\theta(a,\ \theta) = 0, \tag{6}$$

and conditions at infinity (4), as seen from Fig. 2, take the form

$$v_r \to -U\cos\theta, \quad v_\theta \to U\sin\theta \quad \text{for} \quad r \to \infty. \tag{7}$$

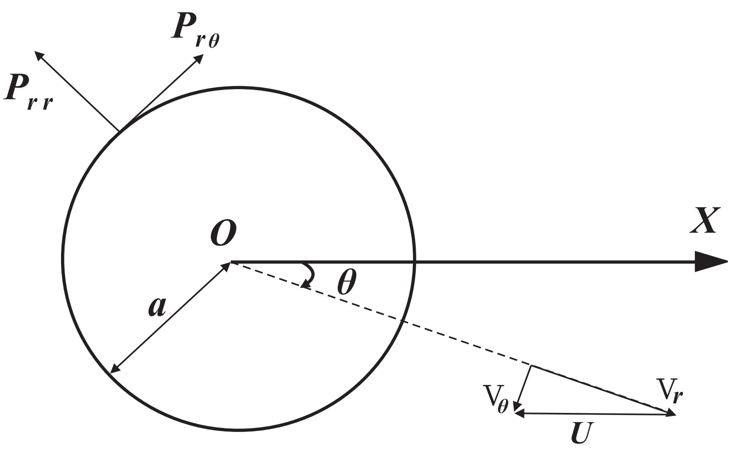

Figure 2: Vector directions of external forces p_{rr}, $p_{r\theta}$ and projections of fluid velocity v_r, v_θ.

Solutions of problem (5) — (7) are obtained in papers Kochin et al. (1965), Landau, Lifshits (1987), and are given by

$$v_r(r,\ \theta) = -\,U\cos\theta\left[1 - \frac{3}{2}\frac{a}{r} + \frac{1}{2}\frac{a^3}{r^3}\right],$$
$$v_\theta(r,\ \theta) = U\sin\theta\left[1 - \frac{3}{4}\frac{a}{r} - \frac{1}{4}\frac{a^3}{r^3}\right], \tag{8}$$
$$p(r,\ \theta) = -\,\frac{3}{2}\widetilde{\mu}\frac{Ua}{r^2}\cos\theta.$$

Taking into account (8) one can calculate the components of the stress tensor. In the case under study, stresses acting at the sphere surface from the approach flow direction are as follows

$$p_{r\theta} = -\frac{3}{2}\frac{\widetilde{\mu}U}{a}\sin\theta, \quad p_{rr} = \frac{3}{2}\frac{\widetilde{\mu}U}{a}\cos\theta. \tag{9}$$

3 Basic Equations

We summarize some information from Vintson (1991), Bauer et al. (1993) on the theory of elastic shells based on the Kirchoff-Love hypothesis. Let the shell displacement along the normal to its middle surface be small as compared to its thickness. In this connection the shell equations are linearized. Then, the equilibrium of a thin elastic spherical shell under external pressure $\mathbf{p} = (p_1, p_3)$ is described by the dimensionless equations

$$\frac{dT_1}{d\theta} + \mathrm{ctan}\,\theta\,(T_1 - T_2) + N_1 = p_1, \tag{10}$$
$$\frac{dN_1}{d\theta} - (T_1 + T_2) + N_1\,\mathrm{ctan}\,\theta = p_3, \tag{11}$$
$$\frac{dM_1}{d\theta} + \mathrm{ctan}\,\theta\,(M_1 - M_2) = N_1, \tag{12}$$

the elasticity relations

$$M_1 = \mu^4\left(\frac{d\gamma_1}{d\theta} + \nu\,\mathrm{ctan}\,\theta\gamma_1\right), \quad M_2 = \mu^4\left(\nu\frac{d\gamma_1}{d\theta} + \mathrm{ctan}\,\theta\gamma_1\right), \tag{13}$$
$$T_1 = \varepsilon_1 + \nu\varepsilon_2, \quad T_2 = \varepsilon_2 + \nu\varepsilon_1, \tag{14}$$
$$\varepsilon_1 = \frac{du}{d\theta} + w, \quad \varepsilon_2 = u\,\mathrm{ctan}\,\theta + w \tag{15}$$

and the deformation-displacement equations

$$\gamma_1 = -\frac{dw}{d\theta} + u. \tag{16}$$

In these equations u and w are the components of the displacement vector, u is directed along the line tangent to the shell generatrix and w is the component of normal displacement; T_1, T_2, N_1, M_1, M_2 are the dimensionless stress-resultants and stress-couples, p_1 and p_3 are the tangential and normal components of external forces, ε_1, ε_2 are the relative elongations, $\mu = h^2/12$ is a small parameter, h is the shell thickness, γ_1 is the angle of rotation. Tangential and normal components of external forces p_1 and p_3 are obtained in papers Kochin et al. (1965), Landau et al. (1987), and are given by

$$p_1 = K\sin\theta, \quad p_3 = -K\cos\theta, \quad K = -\frac{3}{2}\frac{\widetilde{\mu}U(1-\nu^2)}{Eha}, \tag{17}$$

where $\widetilde{\mu}$ is the dynamic viscosity coefficient, U is the velocity of fluid flow, ν is Poisson's ratio, E is Young's modulus.

Between the dimensionless and the dimension variables there is the obvious connection

$$\{T_i, N_1\} = \frac{1-\nu^2}{Eha}\{\widehat{T}_i, \widehat{N}_1\}, \quad M_i = \frac{1-\nu^2}{Eha^2}\widehat{M}_i, \quad \{h, u, w\} = \frac{1}{a}\{\widehat{h}, \widehat{u}, \widehat{w}\},\ i = 1, 2.$$

Consider the problem-solving method on the interval $0 \le \theta \le \frac{\pi}{2}$. Further, the superscript (k), $k = 1, 2$, denotes variables corresponding to the first $(0 \le \theta \le \frac{\pi}{2})$ and second $(\frac{\pi}{2} \le \theta \le \pi)$ parts of the shell meridian, respectively. Later, we shall omit the superscript in formulae which are valid for both parts. The expressions for $(k = 2)$ are obtained from $(k = 1)$ by using the even and uneven properties of all functions. Naturally we shall assume that all of the unknown functions are continuous and do not have exceptions in the poles.

We suppose that the shell equator ($\theta = \pi/2$) is clamped so that the boundary conditions for system (10) — (16) can be written as

$$u^{(1)}(\pi/2) = u^{(2)}(\pi/2) = 0, \tag{18}$$

$$w^{(1)}(\pi/2) = w^{(2)}(\pi/2) = 0, \tag{19}$$

$$M_1^{(1)}(\pi/2) = M_1^{(2)}(\pi/2) = 0, \quad \gamma_1^{(1)}(\pi/2) = \gamma_1^{(2)}(\pi/2), \tag{20}$$

$$u^{(1)}(0) = u^{(2)}(\pi) = 0, \tag{21}$$

$$\gamma^{(1)}(0) = \gamma^{(2)}(\pi) = 0, \tag{22}$$

$$N_1^{(1)}(0) = N_1^{(2)}(\pi) = 0. \tag{23}$$

Thus, the mathematical formulation of the problem is the system of equations and conditions (10) — (16), (18) — (23). In the zeroth-order approximation (by substituting $\mu = 0$ into (13)) we obtain momentless system (10) — (11) from which one can find the tangential forces $T_1(\theta)$ and $T_2(\theta)$

$$T_1(\theta) = \frac{1}{\sin^2\theta}(-K\cos\theta + C_1), \tag{24}$$

$$T_2(\theta) = -T_1 - p_3 = \frac{1}{\sin^2\theta}(-K\cos\theta + C_1) + K\cos\theta. \tag{25}$$

Here C_1 is an integration constant. The elimination of the function w from system (15), taking into accout (14), gives a first-order differential equation with respect to function u

$$\frac{du}{d\theta} - \mathrm{ctan}\,\theta\, u = \frac{1}{1-\nu}\left(\frac{2}{\sin^2\theta}(-K\cos\theta + C_1) - K\cos\theta\right). \tag{26}$$

The solution of equation (26) has the form

$$u^{(k)}(\theta) = \frac{\sin\theta}{1-\nu}\left[K\,\mathrm{ctan}^2\theta - K\ln\sin\theta + C_1^{(k)}\left[\ln\left[\tan\frac{\theta}{2}\right] - \frac{\cos\theta}{\sin^2\theta}\right] + C_2^{(k)}\right]. \tag{27}$$

The unknown constants $C_1^{(k)}$ and $C_2^{(k)}$ can be found from boundary conditions (18), (21). The tangential component of shell displacement is

$$u^{(1)}(\theta) = \frac{K\sin\theta}{1-\nu}\left[\frac{-\cos\theta}{1+\cos\theta} - \ln\left[1+\cos\theta\right]\right], \quad 0 \le \theta \le \frac{\pi}{2}. \tag{28}$$

The normal component of the shell displacement w taking into account (14), (15), (24), (25), (28) is

$$w^{(1)}(\theta) = \frac{K}{1-\nu^2}\left[1+\nu-\cos\theta\left[2+\nu+(1+\nu)\ln\left[1+\cos\theta\right]\right]\right], \quad 0\le\theta\le\frac{\pi}{2}. \tag{29}$$

It should be noted, that expression (29) for $w^{(k)}$ does not satisfy boundary conditions (19), because the next term of the approximation is of the same order as the main term $w^{(k)}$. Therefore, we consider the influence of the boundary disturbance near the clamped equator for $\theta = \pi/2$.

4 Boundary Effect of the Problem

We introduce a vector $\mathbf{Y}$ which containes the unknown functions

$$\mathbf{Y} = (u,\ w,\ T_1,\ \gamma_1,\ M_1,\ N_1)^T,$$

then system (10) — (16) can be presented in vector form (30):

$$A(\theta,\mu)\frac{\partial\mathbf{Y}(\theta)}{\partial\theta} + B(\theta,\mu)\mathbf{Y}(\theta) = \mathbf{P}, \tag{30}$$

where $\mathbf{P} = (p_1,\ p_3,\ 0,\ 0,\ 0,\ 0)^T$; and the matrices $A(\theta,\mu)$ and $B(\theta,\mu)$ take the forms

$$A(\theta,\mu) = \begin{pmatrix} \operatorname{ctan}\theta(1-\nu) & 0 & 1 & 0 & 0 & 0 \\ (1+\nu) & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & \operatorname{ctan}\theta\mu^4(1-\nu) & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu^4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$B(\theta,\mu) = \begin{pmatrix} b_{11} & 0 & 0 & 0 & 0 & 1 \\ b_{21} & 2(1+\nu) & 0 & 0 & 0 & b_{26} \\ 0 & 0 & 0 & b_{34} & 0 & -1 \\ b_{41} & (1+\nu) & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{54} & -1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Some elements of the matrix $B(\theta,\mu)$ can be written as

$$\begin{array}{lll} b_{11} = -\operatorname{ctan}^2\theta(1-\nu), & b_{21} = (1+\nu)\operatorname{ctan}\theta, & b_{26} = -\operatorname{ctan}\theta, \\ b_{34} = -\operatorname{ctan}^2\theta\mu^4(1-\nu), & b_{41} = \nu\operatorname{ctan}\theta, & b_{54} = \nu\mu^4\operatorname{ctan}\theta\nu. \end{array}$$

According to the procedure proposed by Goldenveizer (see, for example, Goldenveizer (1961)), the asymptotic solution of the boundary value problem (30), (18) — (23), can be expressed as

$$\mathbf{Y} = \mathbf{Y}_a + \mathbf{Y}_b. \tag{31}$$

The vector $\mathbf{Y}_a$ describes the main stress-strain state and is also a solution of a momentless system obtained from (30) when $\mu = 0$. In the problem we have $\mathbf{Y}_a = \left(u_a^{(k)},\ w_a^{(k)},\ T_{1a}^{(k)},\ \gamma_{1a}^{(k)},\ 0,\ 0\right)^T$, where $u_a^{(k)}$ and $w_a^{(k)}$ are obtained from (28), (29), $T_{1a}^{(k)}$ and $\gamma_{1a}^{(k)}$ are obtained from (14), (15) è (16).
The vector $\mathbf{Y}_b$ describes a boundary effect near the clamped equator ($\theta = \pi/2$). Following the boundary effect method, one still requires that

$$\mathbf{Y}_b = \widetilde{\mathbf{Y}}(\theta)\exp\left(\frac{1}{\mu}\int_0^\theta q\,dt\right), \tag{32}$$

where $\widetilde{\mathbf{Y}} = (\widetilde{u},\ \widetilde{w},\ \widetilde{T}_1,\ \widetilde{\gamma}_1,\ \widetilde{M}_1,\ \widetilde{N}_1)^T$. Substituting (31) into (30), one arrives at system (33) for the definition of the vector $\widetilde{\mathbf{Y}}$

$$A(\theta,\mu)\frac{\partial\widetilde{\mathbf{Y}}(\theta)}{\partial\theta} + \left(B(\theta,\mu) + \frac{q}{\mu}A(\theta,\mu)\right)\widetilde{\mathbf{Y}}(\theta) = 0. \tag{33}$$

The vector $\widetilde{\mathbf{Y}}(\theta)$ can be defined as

$$\widetilde{\mathbf{Y}}(\theta) = D\cdot\widehat{\mathbf{Y}}(\theta), \tag{34}$$

where the diagonal matrix D is given by

$$D = \begin{pmatrix} \mu^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu^0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu^1 \end{pmatrix}.$$

Neglecting high degrees of the small parameter μ, i. e. the values $A(\theta,\mu)\frac{\partial\widetilde{\mathbf{Y}}(\theta)}{\partial\theta}$ in system (33) can be dropped, and taking into account (34), one has the system of homogeneous linear equations

$$\widehat{T}_1 + \operatorname{ctan}\theta(1-\nu)\widehat{u} = 0, \qquad \widehat{w}(1+\nu) + \widehat{u}q = 0, \tag{35}$$

$$\widehat{N}_1 + \widehat{u}(1-\nu) = 0, \qquad \widehat{M}_1 - q\widehat{\gamma}_1 = 0, \tag{36}$$

$$\widehat{N}_1 - q\widehat{M}_1 = 0, \qquad \widehat{\gamma}_1 + q\widehat{w} = 0. \tag{37}$$

Hence, the determinant of system (35) — (37) is $\Delta = q^4 + \left(1-\nu^2\right) = 0$, thus

$$q = \frac{\pm 1 \pm i}{\sqrt{2}}\left(1-\nu^2\right)^{1/4}, \quad \text{where } i \text{ is the imaginary unit.} \tag{38}$$

Solving system (35) — (37) one can find the eigenvector $\widehat{\mathbf{Y}}(\theta)$. Then $\widetilde{\mathbf{Y}}\,(\theta)$ can be derived as

$$\widetilde{\mathbf{Y}}\,(\theta) = \left(-\mu,\ -\frac{q}{1+\nu},\ \operatorname{ctan}\theta(1-\nu)\mu,\ \frac{q^2}{1+\nu}\,\mu^{-1},\ \frac{q^3}{1+\nu}\,\mu^2,\ \frac{q^4}{1+\nu}\,\mu\right)^T. \tag{39}$$

Thus, w_b (the second component of the vector Y_b) is defined as

$$w_b = -\frac{q}{1+\nu}\cdot\exp\left(\frac{1}{\mu}\int_0^\theta q\,dt\right). \tag{40}$$

Taking into account (38) expression (40) takes the form

$$w_b = -\frac{1}{1+\nu}\sum_{j=1}^{4} G_j\cdot q_j\cdot\exp\left(\frac{1}{\mu}\int_0^\theta q_j\,dt\right). \tag{41}$$

The numerical constants G_j can be derived by substituting $w = w_a + w_b$ into boundary conditions (19). The substituting procedure and the definition of the constants G_j have been made in the software Mathematica 5.1 code. Then one obtains

$$w_b(\theta) = -\frac{K}{1-\nu}\exp\left(\frac{\left(1-\nu^2\right)^{1/4}\left(\theta-\frac{\pi}{2}\right)}{\sqrt{2}\mu}\right)\cos\left(\frac{\left(1-\nu^2\right)^{1/4}\left(\theta-\frac{\pi}{2}\right)}{\sqrt{2}\mu}\right). \tag{42}$$

5 Research Results: Comparison with Numerical Calculations

The presented numerical calculations were performed in ANSYS 13 for a spherical shell of radius $a = 0.2$ m and thickness $h = 0.002$ m with the following material properties: $E = 2.07\cdot 10^{11}$ N/m^2 (Young's modulus), $\nu = 0.3$ (Poisson's ratio). The velocity of fluid flow is $U = 0.01$ m/sec, $\widetilde{\mu} = 1$ Pa ·sec is the dynamic viscosity coefficient.

Figure 3 shows the meridian-section of the spherical shell. The continous line shows the deformed shape, the dashed one shows the undeformed shape of the shell.

The graphs of functions $u\,(\theta)$ and $w\,(\theta)$ are presented in Fig. 4 and Fig. 5, respectively. The continous line shows the approximate solution obtained by asymptotic formulas (28), (29), (31), (42). The dotted line shows the numerical results obtained by using the Finite Element Method in ANSYS 13. As one can see, the shell deflections computed by FEM (Bathe (1984)) amplify the asymptotic results.

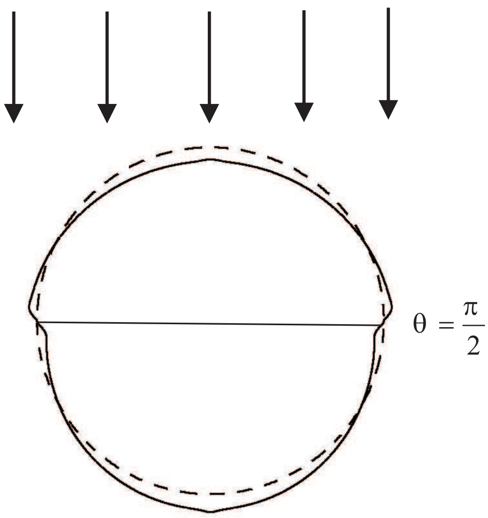

Figure 3: The shell shape under external pressure.

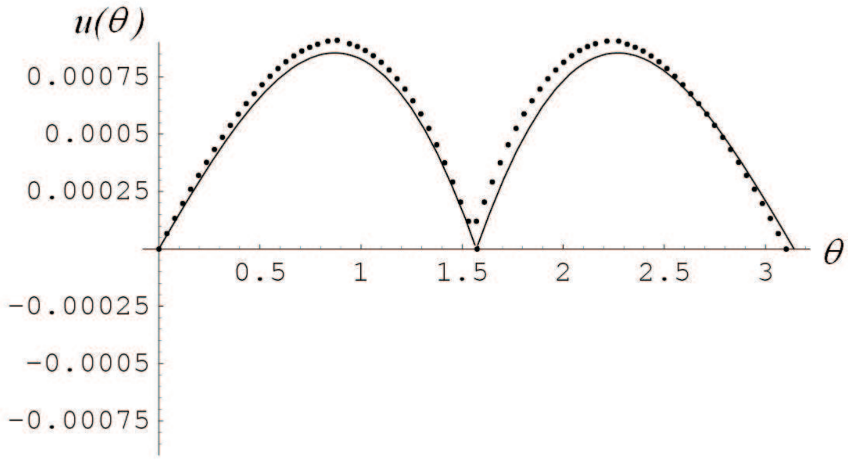

Figure 4: The graph of function $u(\theta)$.

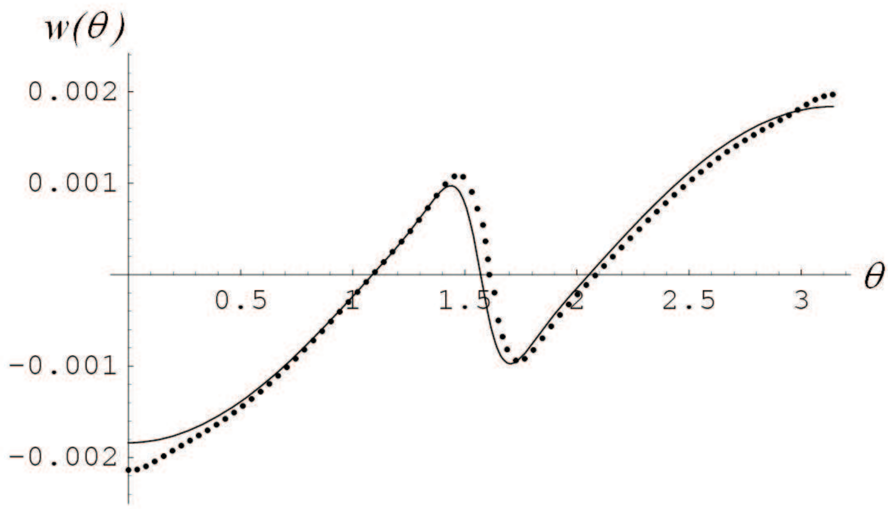

Figure 5: The graph of function $w(\theta)$.

Since the external normal to the shell is taken as the positive direction of w, it follows from the graph (Fig. 5) that the spherical shell is stretched. As follows from the graph, w is an uneven function ($w(\theta) = -w(-\theta)$) with respect to the line passing across the point $\theta = \pi/2$. In the graph (Fig. 4) on can see, the problem considered is symmetric with respect to the line passing across the point $\theta = \pi/2$ because u is an even function ($u(\theta) = u(-\theta)$). These function properties were used in the numerical integrating of problem (10) — (16). For example, for the even functions in the problem we have $f(\theta) = -f(\pi - \theta)$, for uneven functions $f(\theta) = f(\pi - \theta)$. Thus, after introducing a new variable $\xi = \pi - \theta$ we get the system of equations identical to (10) — (16), in the variable ξ. Therefore, we have considered the problem on the interval $0 \leq \theta \leq \frac{\pi}{2}$ and used the even and uneven properties of all functions mentioned to obtain a full solution.

6 Conclusion

As numerous calculations show, such an approximate solution obtained by using expressions (28), (29), (31), (42) describes sufficiently accurately the shell deflections.

In conclusion we note that the simple approximation asymptotic formulas for the the shell deflections are derived. A comparison of asymptotic and FEM results shows the reliability of the formulae presented. Besides, the advantage of the asymptotic formulas is their relative simplicity and effective applications compared with the finite element method codes.

Acknowledgment

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