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Nonholonomic Motion Planning for a Free-Falling Cat Using Quasi-Newton Method

Xin-Sheng Ge, Wei-Jia Zhao, Yan-Zhu Liu

The motion planning problem of a free-falling cat is investigated. Nonholonomicity arises in a free-falling cat subject to nonintegrable velocity constraints or nonintegrable conservation laws. When the total angular momentum is zero, the rotational motion of the cat subjects to nonholonomic constraints. The equation of dynamics of a free-falling cat is obtained by using the model of two symmetric rigid bodies. The control of system can be converted to the motion planning problem for a driftless system. Based on the input parameterization, the continuous optimal control problem is transformed into the discrete one. The quasi-Newton method of motion planning for nonholonomic multibody system is proposed. The effectiveness of the numerical algorithm is demonstrated by numerical simulation.

1 Introduction

It is well known that a cat, when released from an upside down configuration starting from rest, is able to land on her feet. At the end of 19th century, people began to try to explain this interesting phenomenon. Guyou and Marey (Liu, 1982) first explained from classical mechanics that the angular momentum of a falling cat is conserved. McDonald (1955) also represents this problem from a point of view of physiology. He believed a cat firstly contracts its front feet, then protracts the front feet while rotating its front body. Meanwhile, its rear body also experiences a rotation. According to the conservation law of angular momentum, the rotation angle of the front body is larger than that of the rear body in the opposite direction. This theory satisfied the principle of mechanics, however, in free-falling cat experiments, we hardly find any obvious protract-contract motion of the cat's feet. Лойцянский (1954) present another explanation that the rapid rotation of the cat's tail makes its body turn over in the opposite direction. But his conjecture can not stand either. Experiments show that a cat without tail can also finish the rotating motion. Kane and Scher (1969) proposed the dynamical explanation of the phenomenon that a free-falling cat usually lands on its feet. They assumed the cat's turning motion with its waist as the top point using the model of two symmetric rigid bodies. Based on this model, a set of governing equation was established and the general characteristic of the turning motion was obtained. Further numerical analysis showed that this model matches the experiment result very well. For the more general condition of two unsymmetrical rigid body's turning motion, a set of dynamics equation was set up by Yanzhu Liu (1982).

Recently, with the development of manned spacecrafts and exploratory researches of human turning motion under zero-gravity conditions, the research on a free-falling cat becomes a significant topic. Due to the non-integrable angle velocity, the first integration of the equation of cat's rotation is an equation with nonholonomic constrains, and it is a special nonlinear system. In this equation, the dimension of generalized coordinates is larger than that of the control input. Brockett (1993) first finished a systematic research on the optimal control problem of driftless nonholonmic system. Using control objective functions to construct Lagrangian functions, they reached conclusions under optimal input of sinusoidal function and elliptical function respectively. Murray and Sastry (1993) extended Brockett's conclusion to the control of nonholonomic chain system under sinusoidal input . A similar motion planning method was also given by Reyhanoglu and Mukherjee (1994), which used Stokes theorem and Taylor series expansion to analyze the dynamic model of nonholonomic system. For motion planning problems of nonholonomic control systems, various numerical methods were achieved by some researches. Fernandes et al (1995) formulated the nonholonomic motion planning problem as an optimal control problem, and developed a simple algorithm for a coupled rigid body system using ideas from Ritz approximation theory. Yih and Ro (1996, 1997) proposed the algorithms of near optimal motion planning using multipoint shooting and quasi Newton method for nonholonomic systems. Duleba and Sasiadek (2003) discuss a modification of the Newton algorithm applied to nonholonomic motion planning with energy optimization. The Lyapunov control method for solving motion planning was proposed by Tsuchiya et al. (2002). In this method, the control input is obtained by multiplying the gradient vector of the Lyapunov function by a tensor. Ge et al (2007, 2006) studied an optimal algorithm to find feasible trajectories for motion planning of a free-falling cat. Based on the Ritz approximation theory in functional analysis, they approximated a solution of an infinite-dimensional optimization problem by a family of finite-dimensional Fourier basis function expansion.

In this paper, the motion of a free-falling cat is formulated through a double rigid body model which can represent the front and rear half of its body. The motion equation of a free-falling cat is established based on multibody dynamics and conservation of angular momentum. When the total angular momentum is zero, the attitude motion of a free-falling cat subjects to nonholonomic constraints. The control of a free-falling cat can be converted to the motion planning problem without drift. To find a globally convergence strategy, the unit step functions are introduced to form the control inputs, and a quasi-Newton method is designed to solve the nonholonomic motion planning problem. Finally, the algorithm is tested through simulation, and the simulation results indicate that the algorithm is an effective approach to deal with a free-falling cat.

2 Kinematics in Mixture Theories

To simplify the free-falling motion model, the body of a cat is taken as two symmetric rigid bodies B_1 and B_2 which are joined at *O*. Assume the rigid bodies are torsion free. Only bending exists when the cat bends its spine. The coordinate systems $O\text{-}X_iY_iZ_i\,(i=1,\ 2)$ are prescribed as follows: OX_i is centroid axis of the rigid bodies pointing from *O* to the head of the cat ($i=1$) or the tail of the cat ($i=2$), OZ_i points to the abdomen of the cat.

The coordinate system $O\text{-}X_2Y_2Z_2$ is obtained by first rotating about axis OX_1 through angle ψ to obtain $O\text{-}X_1^*Y_1^*Z_1^*$, then rotating about axis OY_1^* through angle ϑ to obtain $O\text{-}X_1^\#Y_1^\#Z_1^\#$, and finally rotating about axis $OX_1^\#$ through angle φ to obtain $O\text{-}X_2Y_2Z_2$. After getting $O\text{-}X_2Y_2Z_2$, we construct a new coordinate system $O\text{-}X^*Y^*Z^*$, in which OX^* and OZ^* are along the bisector of $\angle X_1^*OX_1^\#$ and $\angle Z_1^*OZ_1^\#$ separately, and OY^* is coincident to OY_1^* and $OY_1^\#$. The angle γ, which equals to $\vartheta/2$, is the angularity between the front half (or rear half) spine and OX^*. X^*OZ^* is the spine-curving plane. ψ denotes the position of the plane in the cat's body.

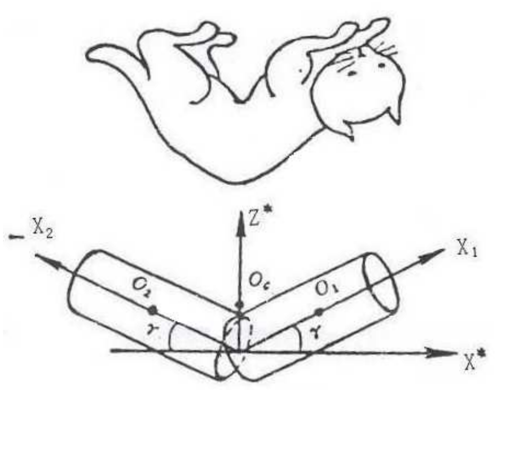

Figure 1: A free-falling cat model

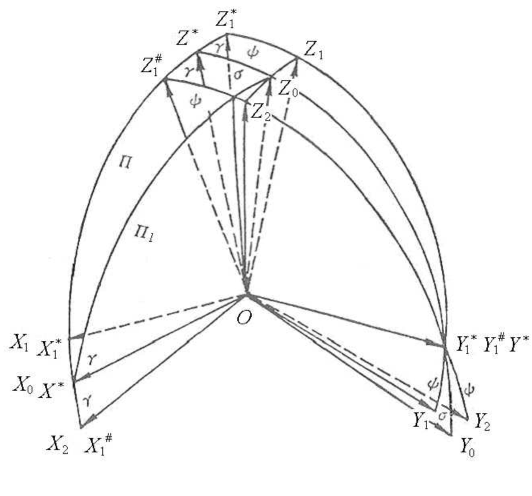

Figure 2 : Attitude angle transform

The angular velocity ω' of B_2 with respect to B_1 is obtained by projection on the $O\text{-}X^*Y^*Z^*$ coordinate system as

$$\omega' = (\dot{\psi}+\dot{\varphi})\cos\gamma\,\boldsymbol{i}^* + 2\dot{\gamma}\,\boldsymbol{j}^* + (\dot{\psi}-\dot{\varphi})\sin\gamma\,\boldsymbol{k}^* \tag{1}$$

According to the torsion free condition, the velocity component along axis OX^* must be zero, and then we obtain the relationship $\dot{\varphi} = -\dot{\psi}$. Since the initial condition is also torsion free, we get $\varphi = -\psi$ by employing integration. Considering the relation between angles φ and ψ , the equation (1) can be simplify as

$$\omega' = 2(\dot{\gamma}\boldsymbol{j}^* + \dot{\psi}\sin\gamma\boldsymbol{k}^*) . \tag{2}$$

After bending the spine, the center of mass of the cat O_c locates on axis OZ^* . If we move the origin from O to O_c , the axis $O_c X_0$ keeps a steady horizontal direction during the observation of a free-falling cat. A new coordinate system O_c-XYZ is built, in which $O_c X$ is coincident to $O_c X_0$, the axis $O_c Z$ goes upward and vertically to the ground. During the process of free falling, the inertia forces in O_c-XYZ are balanced with gravity. When we consider rotating about the center of mass and coordinates O_c-XYZ can be taking as the inertial reference frame. Set vertical plane XO_cZ as Π_0 , and let ϕ be the clockwise angle from plane Π_0 to Π_1. The purpose of rotational motion of the cat is to make its abdomen from facing upward to downward, namely, the angle ϕ from 0 to π .

The angular velocities of $B_i\,(i=1,\ 2)$ with respect to O-$X^*Y^*Z^*$ reference frame are

$$\begin{aligned} \omega_1^* &= -\dot{\psi}\cos\gamma\boldsymbol{i}^* - \dot{\gamma}\boldsymbol{j}^* - \dot{\psi}\sin\gamma\boldsymbol{k}^* \\ \omega_2^* &= \omega_1^* + \omega' = -\dot{\psi}\cos\gamma\boldsymbol{i}^* + \dot{\gamma}\boldsymbol{j}^* + \dot{\psi}\sin\gamma\boldsymbol{k}^* \end{aligned} \tag{3}$$

Letting A、 B、 C、 m and a be the central inertia moments, mass and distance between centroid and O of $B_i\,(i=1,\ 2)$ respectively. The moment of momentum $\boldsymbol{H}_i$ of B_i with respect to O_c could be computed. The vector $\boldsymbol{H}_i$ can be decomposed into components with respect to in the O - $X_iY_iZ_i\,(i=1,\ 2)$ coordinate systems; we have (Liu, 1982)

$$\begin{Bmatrix} H_{1x} \\ H_{1y} \\ H_{1z} \end{Bmatrix} = \begin{Bmatrix} H_{2x} \\ -H_{2y} \\ -H_{2z} \end{Bmatrix} = \begin{bmatrix} A_c & -F_c & -E_c \\ -F_c & B_c & -D_c \\ -E_c & -D_c & C_c \end{bmatrix} \begin{Bmatrix} p \\ q \\ r \end{Bmatrix} \tag{4}$$

where

$$\begin{aligned} &A_c = A + ma^2\cos^2\gamma\,\sin^2\gamma,\quad B_c = B + ma^2\cos^2\gamma(\cos^2\gamma + \sin^2\gamma\,\cos^2\psi) \\ &C_c = C + ma^2\cos^2\gamma(\cos^2\gamma + \sin^2\gamma\,\sin^2\psi),\quad D_c = ma^2\cos^2\gamma\,\sin^2\gamma\,\cos\psi\,\sin\psi \\ &E_c = ma^2\cos^3\gamma\,\sin\gamma\,\cos\psi,\quad F_c = ma^2\cos^3\gamma\,\sin\gamma\,\sin\psi \end{aligned} \tag{5}$$

and

$$\begin{aligned} p &= [(\dot{\psi}\,\cos\gamma + \dot{\gamma}\,\sin\gamma\,\cos\psi\,\sin\psi)/(1-\sin^2\gamma\cos^2\psi) - \dot{\phi}]\cos\gamma - \dot{\psi} \\ q &= [(\dot{\psi}\,\cos\gamma + \dot{\gamma}\,\sin\gamma\,\cos\psi\,\sin\psi)/(1-\sin^2\gamma\cos^2\psi) - \dot{\phi}]\sin\gamma\,\sin\psi - \dot{\gamma}\,\cos\psi \\ r &= [\dot{\phi} - (\dot{\psi}\,\cos\gamma + \dot{\gamma}\,\sin\gamma\,\cos\psi\,\sin\psi)/(1-\sin^2\gamma\cos^2\psi)]\sin\gamma\,\cos\psi - \dot{\gamma}\,\sin\psi \end{aligned} \tag{6}$$

Adding $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, the sum is the total moment of momentum of the cat respect to O_c . After transformation to the O - $X^*Y^*Z^*$ coordinate system, the component of the sum along axis OX^* is

$$\begin{aligned} \boldsymbol{H} = -2\{&[A\cos^2\gamma + (B\sin^2\psi + C\cos^2\psi)\sin^2\gamma][\dot{\phi} - (\dot{\psi}\,\cos\gamma + \dot{\gamma}\,\sin\gamma\,\cos\psi\,\sin\psi)/(1-\sin^2\gamma\cos^2\psi)] \\ &+ A\dot{\psi}\,\cos\gamma + (B-C)\dot{\gamma}\,\sin\gamma\,\cos\psi\,\sin\psi\}\,\boldsymbol{i}^* \end{aligned} \tag{7}$$

During the process of falling of cat, the moment with respect to centroid is zero. Since the angular momentum $\boldsymbol{H}$ is conservative, the assumption of invariance of direction of axis O_cX^* or O_cX is proved to be true. Considering $\boldsymbol{H} \equiv 0$, we can obtain the motion equation from equation (7) given by

$$\dot{\phi} = \frac{\{\dot{\psi}\,\cos\gamma\,\sin\gamma\,[\lambda + (1-\varepsilon)\cos^2\psi] + \dot{\gamma}\,\cos\psi\,\sin\psi(1-\varepsilon + \lambda\,\sin^2\gamma)\}\sin\gamma}{(1-\sin^2\gamma\,\cos^2\psi)[1 + (\lambda - \varepsilon\,\cos^2\psi)\sin^2\gamma]} \tag{8}$$

where $\lambda = (B-A)/A$, $\varepsilon = (B-C)/A$ are parameters associated with the mass of cat. Equation (8) is the nonholonomic attitude motion equation of free-falling cat.

3 The quasi-Newton method for nonholonomic motion planning

The motion planning is to find control input to steer a nonholonomic system from an initial configuration to final configuration along a feasible trajectory in time *T*. We can formulate the motion planning problem as a nonlinear optimal control problem using a performance functional. Without loss of generality we assume that a free-falling cat has been formulated in the form

$$\dot{\boldsymbol{x}} = \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}, \qquad \boldsymbol{x}_0, \boldsymbol{x}_f \in \boldsymbol{R}^3 \tag{9}$$

$$\boldsymbol{G}(\boldsymbol{x}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \dfrac{\cos\gamma \sin^2\gamma\,[\lambda+(1-\varepsilon)\cos^2\psi]}{(1-\sin^2\gamma\cos^2\psi)[1+(\lambda-\varepsilon\cos^2\psi)\sin^2\gamma]} & \dfrac{\cos\psi\sin\psi\sin\gamma\,(1-\varepsilon+\lambda\sin^2\gamma)}{(1-\sin^2\gamma\cos^2\psi)[1+(\lambda-\varepsilon\cos^2\psi)\sin^2\gamma]} \end{bmatrix}$$

where $\boldsymbol{x} = (\psi, \ \gamma, \ \phi)^{\mathrm{T}} \in \boldsymbol{R}^3$ is the configuration variable, $u \in \boldsymbol{R}^2$ is the control input noted as $u_1 = \dot{\psi}$ and $u_1 = \dot{\gamma}$, $\boldsymbol{G}(\boldsymbol{x}) \in \boldsymbol{R}^{3\times 2}$ is a regular 2-dimensional distribution and the cost function to be minimized is

$$J(\boldsymbol{u}) = \int_{\boldsymbol{0}}^{\mathrm{T}} <\boldsymbol{u}, \boldsymbol{u}> \mathrm{d}t \tag{10}$$

We assume that the system is controllable (Fernandes et al. 1995) and thus there exists a solution $u^* \in L_2([0, T])$ for the problem. Here, $L_2([0, T])$ denotes the Hilbert space of measurable vector valued functions of the form $u(t) = [u_1(t), \ u_2(t)]^{\mathrm{T}}$, $t \in [0, T]$. By Ritz's approximation theory, the problem of nonholonomic motion planning is equivalent to an infinite dimensional optimization problem.

One can use the calculus of variations to find the necessary optimality conditions for the nonlinear optimal control problem. Since the system is nonlinear, the optimal control depends on the solution of the nonlinear multi-point boundary value problem (Yih 1996, Murray 1994). Numerical schemes such as multi-point shooting method and quasi-linearization method can be used to solve the boundary value problem. However, one needs to find an initial solution close to the optimum point in order to ensure the convergence of these numerical algorithms.

Also, since only necessary conditions are satisfied, the minimum point is not guaranteed to be the solution of the optimization problem. For complicated underactuated systems such as the rigid spacecraft with two wheels, it is a very difficult task to test the optimality conditions.

The problems mentioned above can be overcome by using control parameterization method. Using the parameterization of the control variables, one can transform the infinite dimensional optimization problem to a finite dimensional one. In this paper, each control input is approximated by a piecewise constant control,which makes it easier to implement the algorithm. The control inputs can be expressed as follows

$$\boldsymbol{u}(t) = \sum_{i=0}^{\boldsymbol{N}-1} h_{j,i+1}[1(t-t_i) - 1(t-t_{i+1})] \tag{11}$$

where $1(t)$ is the unit step function, $h_{j,i+1}$ is the control input in $[t_i, \ t_{i+1}]$, $j = 1, 2, \cdots, m$. Then, one can recast the optimal motion planning as an optimal parameter searching problem. The advantages of this indirect approach are: First, we do not need a very close starting guess to the optimal solution because a globally convergent method can be utilized to find the minimum solution for the motion planning problem. Second, the parameterization of the control variables makes the algorithm easier to implement.

By using penalty functions to describe the cost function, the cost function can be approximated by

$$J(h, \xi) = \sum_{j=1}^{m} \sum_{i=0}^{\boldsymbol{N}-1} (h_{j,i})^2 + \xi \left\| \boldsymbol{x}(T) - \boldsymbol{x}_{\boldsymbol{f}} \right\|^2 \tag{12}$$

where parameter ξ is the penalty factor, $\boldsymbol{x}(T)$ is the solution of Equation (9) at time $t = T$ with initial condition x_0.

Actually, restricting the control input at the initial and final time leads to another constraint, which introduces a new penalty factor ζ. .Once N, ξ and ζ are chosen, the cost function given by (10) is a function of h which can be expressed as

$$J(h) = <h, h> + \xi \left\| f(h) - \boldsymbol{x}_f \right\|^2 + \zeta(h_{t=0}^{\mathrm{T}} h_{t=0} + h_{t=\mathrm{T}}^{\mathrm{T}} h_{t=\mathrm{T}}) \tag{13}$$

where $h_{i=0} = [h_{1,1} \quad h_{2,1}]^{\mathrm{T}}$, $h_{i=\mathrm{T}} = [h_{1,N} \quad h_{2,N}]^{\mathrm{T}}$. Therefore the problem now is to find h such that the cost function in (10) is minimized.

A robust and globally convergent quasi-Newton method is used to find the solution for the optimal motion planning problem. Quasi-Newton methods use an iterative process to approximate the inverse Hessian matrix so that no calculation for the second derivatives is needed for carrying out the search of the optimal parameters. Using line search, the quasi-Newton method also combines a globally convergence strategy with a fast local convergence rate of Newton's method.

Define h^* to be the minimum of the cost functional $J(h)$. Let h_k be the current parameter and Δh_k be the difference between h^* and h_k as follows

$$h^* = h_k + \Delta h_k \tag{14}$$

where

$$\Delta h_k = -(\nabla_h^2 J(h_k))^{-1} \nabla_h J(h_k) \tag{15}$$

The above update algorithm is known as Newton's method which is quadraticly convergent in the neighborhood of the minimum. However, there are problems associated with Newton's method. First, the computation of the inverse of the Hessian matrix is a very difficult task. Here we use the BFGS (Broyden-Felether-Goldfarb-Shanno) algorithm to update the inverse of a Hessian matrix. BFGS method has been known to have a global convergence. Using the BFGS algorithm to approximate the inverse of the Hessian matrix, $(\nabla_h^2 J(h_k)^{-1})$ can be replaced by B_k which is given by (Joshi, 2004)

$$B_{k+1} = B_k + (1 + \frac{\gamma_k^{\mathrm{T}} B_k \gamma_k}{\delta_k^{\mathrm{T}} \gamma_k})(\frac{\delta_k \delta_k^{\mathrm{T}}}{\delta_k^{\mathrm{T}} \gamma_k}) - (\frac{\delta_k \gamma_k^{\mathrm{T}} B_k + B_k \gamma_k \delta_k^{\mathrm{T}}}{\delta_k^{\mathrm{T}} \gamma_k}) \tag{16}$$

where

$$\begin{aligned} \delta_k &= h_{k+1} - h_k \\ \gamma_k &= \nabla J(h_{k+1}) - \nabla J(h_k) \end{aligned} \tag{17}$$

We can use a line search to censure the global convergence of the quasi-Newton method. The following algorithm can be used to update vector h such that a minimization of J will be reached

$$h_{k+1} = h_k + \lambda_k p_k \tag{18}$$

where

$$p_k = -B_k \nabla J(h_k) \tag{19}$$

To minimize $J(h_k + \lambda_k p_k)$ with respect to λ by the line search, Define

$$\hat{J}(\lambda) = J(h_k + \lambda_k p_k) \tag{20}$$

With the following conditions

$$\hat{J}(0) = J(h_k),\ \hat{J}'(0) = \nabla J(h_k)^{\mathrm{T}} p_k,\ \hat{J}(1) = J(h_k + p_k) \tag{21}$$

To guarantee that $\{ h_k \}$ converge to a minimize of the cost $J(h)$ and to avoid very small decreases in $J(h)$ relative to the lengths of the steps, we can write the step-acceptance criteria as follows (Yih, 1997)

$$\hat{J}(1) \le \hat{J}(0) + \xi \hat{J}'(0), \quad \xi \in (0, 1) \tag{22}$$

If $\hat{J}(1)$ dose not satisfy Equation (22), we can approximate $\hat{J}(\lambda)$ by the following quadratic model which satisfies conditions (21)

$$\hat{J}(\lambda) = [\hat{J}(1) - \hat{J}(0) - \hat{J}'(0)]\lambda^2 + \hat{J}'(0)\lambda + \hat{J}(0) \tag{23}$$

By setting $\hat{J}'(\hat{\lambda}) = 0$, we obtain

$$\hat{\lambda} = \frac{-\hat{J}'(0)}{2(\hat{J}(1) - \hat{J}(0) - \hat{J}'(0))} \tag{24}$$

It is easy to verify that $\hat{J}''(\hat{\lambda}) > 0$, thus $\hat{\lambda}$ minimizes $\hat{J}(\lambda)$. Now we can replace λ_k in Equation (18) by $\hat{\lambda}$ to update vector h. Hence, the quasi-Newton iteration procedure is described as follows:

Step 1: Setting up initial and final configurations x_0, $x_f \in R^3$ and $G(q) \in R^{3\times 2}$.

Step 2: Assign initial parameters: ξ、 ζ、 h_0、 δ、 B_0 and the control error r_e.

Step 3: Solve the differential equations given by (9), and compute $J(h_0)$ using (13).

Step 4: Compute $\nabla J(h_0)$ using (15), substituting h_0 and $\nabla J(h_0)$ into equation (19) to compute p_0 , and substituting $\nabla J(h_0)$, h_0 and p_0 into equations (21)~(22) and (24) to compute λ_0 .

Step 5: Compute h_k (k=1) using (18), and check the condition $\|\Delta h\| < r_e$. If the condition is not satisfied, compute B_k (k=1) using (16), and repeat Step 2, otherwise exit.

4 Numerical simulation

Assume that during the process of a cat's free falling, only its spine bends, there is no rotation between the front and rear body. The cat bends its spine forward to all the directions in turn and keeps angle γ constant. When the front body of the cat moves a whole circle, the whole body of the cat turns radian π in the reverse direction, i.e. when angle ψ changes from 0 to 2π , the angle ϕ changes from 0 to 2π . From the experiment data, $\lambda \approx 3$, $|\varepsilon| \ll 1$. In the simulation experiment, $\lambda = 3, \varepsilon = 0.01$, $N = 20$, $\xi = 120$ diag[30 7.8 2.5] , $\zeta = 194.85$, $e = 10^{-6}$, the time interval of falling is t =1s. The prescribed time space in simulation computation is 0.05s. We denote the initial position and the end position of the free-falling cat as

$$x_0 = (0 \quad \pi/6 \quad 0)^{\mathrm{T}} \quad , \qquad x_f = (2\pi \quad \pi/6 \quad \pi)^{\mathrm{T}}$$

The simulation results are shown in Figure 4~7, where Figure 4 shows plots of the optimal control inputs for the middle joint of the double rigid body. Figure 5-7 shows the attitude optimal trajectory of the cat during its falling. The two ends of the curves are separately the initial point and the landing point. We can see from Figure 4 that the control input curve is not as smooth as that in some other papers such as (Ge, 2006) when we choose the stepwise shape function (11), which means that smooth conditions of it have to be considered if smooth control input is required. From Figure 5 and Figure 7, it is obvious the cat experiences a steady rotation. There's no detour behavior in the turndown motion. From Figure 6, we can see the bending angle has a small amplitude variation. These simulation results are very inosculate to the experiment record.

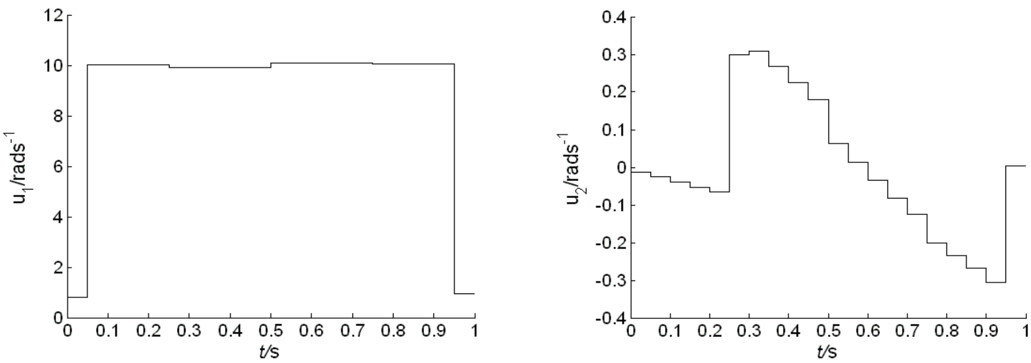

Figure 4: The optimal control input for free-falling cat

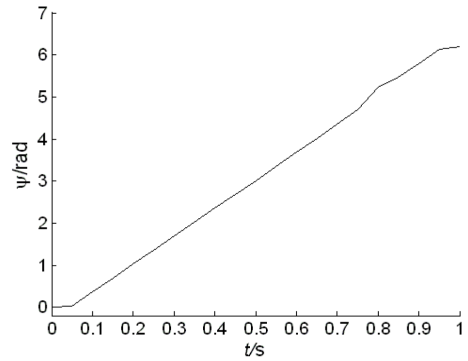

Figure 5: The optimal trajectory of angle ψ

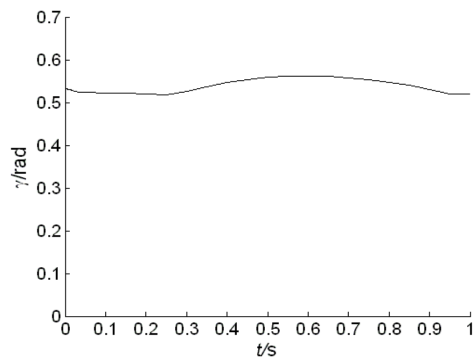

Figure 6: The optimal trajectory of angle γ

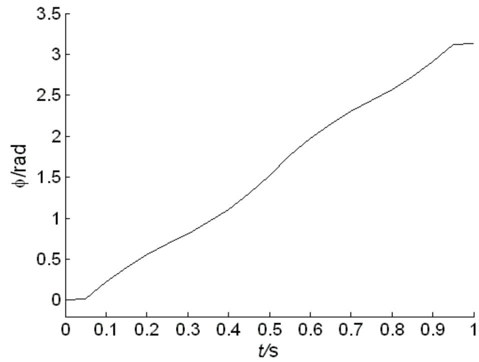

Figure 7: The optimal trajectory of angle ϕ

5 Conclusion

From the modeling of free-falling cat and numerical analysis, we get the following conclusion:

1) The nonlinear control problem of free-falling cat can be transformed to a nonholonmic motion planning problem of a driftless system.
2) The nonholonomic motion planning problem can be solved effectively by quasi-Newton method, which implements the attitude planning of free-falling cat and the optimal of control input. During the simulation computation, the quasi-Newton method shows fast convergence speed and good accuracy.
3) Using the stepwise shape function as the control input is a new attempt. It has some advantages such as convenient in use and high convergent speed. However, it brings about some difficulties in designing actuators in the engineering practices.

Acknowledgments

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Addresses: Prof. Xin-Sheng Ge, College of Mechanical & Electrical Engineering, Beijing Information Science & Technology University, Beijing, 100192, China. Prof. Wei-Jia Zhao, Science College of Qingdao University, Qingdao 266071, China. Prof. Yan-Zhu Liu, Department of Engineering Mechanics, Shanghai Jiaotong University, Shanghai, 200030, China.
email: gebim@vip.sina.com: zhweijia@vip.soho.com: liuyzhuc@online.sh.cn