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On Approximate Analytical Solutions of Nonlinear Thermal Emission Problems

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In this paper, a hybrid asymptotic method on the basis of a double asymptotic expansion for the investigation of the nonlinear inhomogeneous systems is described. This approach can be used for the solution of heat transfer problems that are reduced to the solution of the second order nonlinear differential equation with variable coefficients.

1 Introduction

Most of the mathematical models of real processes have a number of essential singularities that do not allow for an exact analytical solution. For the solution of such problems one has to apply straight numerical or approximate analytical methods. Among approximate analytical methods, asymptotic perturbation methods with a small parameter that naturally occur in the equations or are introduced artificially, are important (Samoilenko V. and Samoilenko Yu., 2007; Starun and Shkil, 2002).

As it is described in the papers (Gristchak and Dmitrieva, 1999; Geer and Andersen, 1990), one of effective asymptotic approaches is a hybrid method, the idea of which is in the creation of any asymptotic expansion (perturbation method, WKB method, etc.) and Galerkin's orthogonality principle. The application of the hybrid asymptotic-numerical method on the basis of the double asymptotic expansion together with the perturbation method in the nonlinear equations is one of the new trends in heat emission investigation.

2 Description of the Method

The main idea of the proposed hybrid approach for the solution of nonlinear problems consists of the following.

The nonlinear second order differential equation with variable coefficients

$$\varepsilon^2 U''(r) + a(r,\varepsilon)U'(r) - \beta b(r,\varepsilon)U^4(r) = 0, \qquad U(0) = 1,\ U'(1) = 0, \tag{1}$$

is analyzed, where ε, β are small parameters, $a(r,\varepsilon)$, $b(r,\varepsilon)$ are some continuously differentiable functions and $\frac{a^2(r)}{4\varepsilon^2} + \frac{a'(r)}{2} \neq 0$ for all $r \in [0;1]$.

To obtain the solution of the equation (1) we will use the method of double asymptotic expansion according to which, using Poincaré's (small parameter) method at the first stage, the function U is assumed as an expansion by the degrees of parameter β (external asymptotic):

$$U(r) = U_0(r) + \beta U_1(r) + \beta^2 U_2(r) + \dots \tag{2}$$

By equating coefficients at equal degrees of parameter β, as a result of the external expansion we obtain a system of linear differential equations for finding the unknown functions $U_0(r), U_1(r), \dots$:

$$\text{for the coefficients at } \beta^0 : \quad \varepsilon^2 U_0'' + a(r)U_0' = 0, \tag{3}$$

$$\text{for the coefficients at } \beta^1 : \quad \varepsilon^2 U_1'' + a(r)U_1' = b(r)U_0^4. \tag{4}$$

We will consider two terms in the expansion (2). Equations (3)–(4) could have been solved exactly, however in the exact solution of formula (2) the truncation of the series takes place and the hybrid solution gives an integral characteristic. As a result of Bernoulli's method, equation (3) takes a form

$$\varepsilon^2 Z'' - g(r)Z = 0, \tag{5}$$

where

$$g(r) = \frac{a^2(r)}{4\varepsilon^2} + \frac{a'(r)}{2}, \quad Y(r) = \exp\left(-\frac{1}{2\varepsilon^2}\int_0^r a(x)\,dx\right), \quad U_0(r) = Z(r)\cdot Y(r).$$

The homogeneous linear differential equation (5) has a small parameter ε^2, therefore we look for its general solution by the hybrid WKB-Galerkin method, according to which the solution of equation (5) is described by the analytical expression (internal asymptotic)

$$Z^H(r,\varepsilon) = \exp\left(\int_0^r (\delta_0\varphi_0 + \ldots)\,dx\right). \tag{6}$$

We take only one term in the expansion (6) and apply the phase integral method to find the unknown value of function φ_0 and Galerkin's orthogonality criterion to find the value of parameter δ_0. A detailed solution of the equation (1) by the WKB-method and also the calculation of the function φ_0 is described in the paper (Gristchak and Pogrebitskaya, 2009). So,

$$Z^H(r) = c_{1,2}\exp\left(\int_0^r \delta_{0_{1,2}} g^{\frac{1}{2}}(x)\,dx\right),$$

where

$$\delta_{0_{1,2}} = \frac{g(0)-g(1)}{4\int_0^1 \sqrt{g^3(x)}\,dx} \pm \sqrt{\frac{1}{\varepsilon^2} + \left(\frac{g(1)-g(0)}{4\int_0^1 \sqrt{g^3(x)}\,dx}\right)^2}.$$

Then

$$U_0(r) = c_{1,2}\exp\left(\int_0^r (\delta_{0_{1,2}} g^{\frac{1}{2}}(x) - \frac{1}{2\varepsilon^2}a(x))\,dx\right)$$

is a general solution of the equation (3).

Using the notation

$$G_1(r) = \exp\left(\int_0^r \delta_{0_1} g^{\frac{1}{2}}(x)\,dx\right), \quad G_2(r) = \exp\left(\int_0^r \delta_{0_2} g^{\frac{1}{2}}(x)\,dx\right),$$

$$e(r) = \exp\left(\frac{1}{2\varepsilon^2}\int_0^r a(x)\,dx\right),$$

the solution can be written in the form

$$U_0(r) = c_1\frac{G_1(r)}{e(r)} + c_2\frac{G_2(r)}{e(r)}.$$

Using the variation of arbitrary constants method for solving the equation (4), function $U_1(r)$ can be written as:

$$U_1(r) = k_1(r)\frac{G_1(r)}{e(r)} + k_2(r)\frac{G_2(r)}{e(r)}.$$

We also form the system of equations for calculating the unknown functions $k_1(r)$ and $k_2(r)$

$$k_1'(r)\frac{G_1(r)}{e(r)} + k_2'(r)\frac{G_2(r)}{e(r)} = 0,$$

$$k_1'(r)\frac{G_1(r)}{e(r)}\left(\delta_{0_1}g^{\frac{1}{2}}(r)-\frac{a(r)}{2\varepsilon^2}\right)+k_2'(r)\frac{G_2(r)}{e(r)}\left(\delta_{0_2}g^{\frac{1}{2}}(r)-\frac{a(r)}{2\varepsilon^2}\right)=\frac{b(r)U_0^4(r)}{\varepsilon^2}. \tag{7}$$

The solution of the system (7) is

$$k_1(r)=-\frac{1}{\varepsilon^2(\delta_{0_2}-\delta_{0_1})}\left(\int\limits_0^r\frac{e(x)b(x)U_0^4(x)}{g^{\frac{1}{2}}(x)G_1(x)}\,dx+t_1\right),$$

$$k_2(r)=\frac{1}{\varepsilon^2(\delta_{0_2}-\delta_{0_1})}\left(\int\limits_0^r\frac{e(x)b(x)U_0^4(x)}{g^{\frac{1}{2}}(x)G_2(x)}\,dx+t_2\right).$$

Then the solution of the equation (4) takes a form

$$U_1(r)=-\frac{G_1(r)}{\varepsilon^2(\delta_{0_2}-\delta_{0_1})e(r)}\left(\int\limits_0^r\frac{e(x)b(x)U_0^4(x)}{g^{\frac{1}{2}}(x)G_1(x)}\,dx+t_1\right)+$$

$$+\frac{G_2(r)}{\varepsilon^2(\delta_{0_2}-\delta_{0_1})e(r)}\left(\int\limits_0^r\frac{e(x)b(x)U_0^4(x)}{g^{\frac{1}{2}}(x)G_2(x)}\,dx+t_2\right).$$

Having found functions $U_0(r)$, $U_1(r)$ and substituting them to the series (2), we obtain the approximate analytical solution in the form

$$U^H(r)=c_1\frac{G_1(r)}{e(r)}+c_2\frac{G_2(r)}{e(r)}+$$

$$+\beta\left(-\frac{G_1(r)}{\varepsilon^2(\delta_{0_2}-\delta_{0_1})e(r)}\left(\int\limits_0^r\frac{e(x)b(x)U_0^4(x)}{g^{\frac{1}{2}}(x)G_1(x)}\,dx+t_1\right)+\right.$$

$$\left.+\frac{G_2(r)}{\varepsilon^2(\delta_{0_2}-\delta_{0_1})e(r)}\left(\int\limits_0^r\frac{e(x)b(x)U_0^4(x)}{g^{\frac{1}{2}}(x)G_2(x)}\,dx+t_2\right)\right). \tag{8}$$

Note that by taking only two terms in the expansion (2) we obtain the approximate analytical solution (8) for the nonlinear homogeneous equation (1).

We separately transform the function $U_0^4(r)$:

$$U_0^4(r)=\frac{1}{e^4(r)}\left(c_1^4G_1^4(r)+4c_1^3c_2G_1^3(r)G_2(r)+6c_1^2c_2^2G_1^2(r)G_2^2(r)+4c_1c_2^3G_1(r)G_2^3(r)+c_2^4G_2^4(r)\right). \tag{9}$$

Substituting expression (9) in formula (8) we obtain

$$U^H(r)=c_1\frac{G_1(r)}{e(r)}+c_2\frac{G_2(r)}{e(r)}+$$

$$+\beta(-\frac{G_1(r)}{\varepsilon^2(\delta_{0_2}-\delta_{0_1})e(r)}\{\int\limits_0^r\frac{b(x)}{e^3(x)g^{\frac{1}{2}}(x)}\left(c_1^4G_1^3+4c_1^3c_2G_1^2G_2+\right.$$

$$\left.+6c_1^2c_2^2G_1G_2^2+4c_1c_2^3G_2^3+c_2^4G_1^{-1}G_2^4\right)\,dx+t_1\}+$$

$$+\frac{G_2(r)}{\varepsilon^2(\delta_{0_2}-\delta_{0_1})e(r)}\{\int\limits_0^r\frac{b(x)}{e^3(x)g^{\frac{1}{2}}(x)}\left(c_1^4G_1^4G_2^{-1}+4c_1^3c_2G_1^3+\right.$$

$$\left.+6c_1^2c_2^2G_1^2G_2+4c_1c_2^3G_1G_2^2+c_2^4G_2^3\right)\,dx+t_2\}).$$

We introduce the notations

$$f(x) = \frac{b(x)}{e^3(x) g^{\frac{1}{2}}(x)} \qquad h(x) = \int\limits_0^x g^{\frac{1}{2}}(\tau)\, d\tau,$$

$$I_1(r) = \int\limits_0^r f(x) \exp(3\delta_{0_1} h(x))\, dx, \qquad I_4(r) = \int\limits_0^r f(x) \exp(3\delta_{0_2} h(x))\, dx,$$

$$I_2(r) = \int\limits_0^r f(x) \exp((2\delta_{0_1} + \delta_{0_2}) h(x))\, dx, \qquad I_5(r) = \int\limits_0^r f(x) \exp((4\delta_{0_2} - \delta_{0_1}) h(x))\, dx,$$

$$I_3(r) = \int\limits_0^r f(x) \exp((\delta_{0_1} + 2\delta_{0_2}) h(x))\, dx, \qquad I_6(r) = \int\limits_0^r f(x) \exp((4\delta_{0_1} - \delta_{0_2}) h(x))\, dx. \tag{10}$$

Then finally the hybrid asymptotic solution of the equation (1) takes a form

$$U^H(r) = c_1 \frac{G_1(r)}{e(r)} + c_2 \frac{G_2(r)}{e(r)} +$$

$$+\beta \left(-\frac{G_1(r)}{\varepsilon^2(\delta_{0_2} - \delta_{0_1}) e(r)} \left(c_1^4 I_1(r) + 4c_1^3 c_2 I_2(r) + 6c_1^2 c_2^2 I_3(r) + 4c_1 c_2^3 I_4(r) + c_2^4 I_5(r) + t_1 \right) + \right.$$

$$\left. + \frac{G_2(r)}{\varepsilon^2(\delta_{0_2} - \delta_{0_1}) e(r)} \left(c_1^4 I_6(r) + 4c_1^3 c_2 I_1(r) + 6c_1^2 c_2^2 I_2(r) + 4c_1 c_2^3 I_3(r) + c_2^4 I_4(r) + t_2 \right) \right). \tag{11}$$

The integrals (10), which are quadrature free in general, are included to the solution (11). So to find their approximate values the method of their analytical estimate can be used.

To estimate integrals $I_j(r)$, $j = 1 \ldots 6$, applying the method of integration by parts, we get

$$\int\limits_0^r f(x) \exp(kh(x))\, dx = \frac{1}{k} \left[\frac{f(r)}{h'(r)} \exp(kh(r)) - \frac{f(0)}{h'(0)} \exp(kh(0)) \right] +$$

$$+ \frac{1}{k^2} \left[\left(\frac{f(0)}{h'(0)} \right)' \frac{\exp(kh(0))}{h'(0)} - \left(\frac{f(r)}{h'(r)} \right)' \frac{\exp(kh(r))}{h'(r)} \right] +$$

$$+ \frac{1}{k^3} \left[\left(\left(\frac{f(r)}{h'(r)} \right)' \frac{1}{h'(r)} \right)' \frac{\exp(kh(r))}{h'(r)} - \left(\left(\frac{f(0)}{h'(0)} \right)' \frac{1}{h'(0)} \right)' \frac{\exp(kh(0))}{h'(0)} \right] + O\left(\frac{1}{k^4} \right).$$

where $k = 3\delta_{0_1}, 2\delta_{0_1} + \delta_{0_2}, \delta_{0_1} + 2\delta_{0_2}, 3\delta_{0_2}, 4\delta_{0_2} - \delta_{0_1}, 4\delta_{0_1} - \delta_{0_2}$.

An analytical estimation of the integrals is described in detail in the paper (Gristchak and Kabak,1999). Arbitrary constants c_1, c_2 and t_1, t_2 are found from initial conditions $U_0'(1) = 0$, $U_0(0) = 1$ and $U_1'(1) = 0$, $U_1(0) = 0$ correspondingly.

3 Application of the Method

This approach has been investigated in the paper (Pogrebitskaya, 2008) and used for the applied problem of mathematical physics that describes the process of the diffusion of heat in a sphere.

We apply this method to the concrete problem of thermal emission. We consider a problem of thermal emission of ring plates of a trapezoidal intersection radiator that is described by the following nonlinear second order differential equation with variable coefficients in the dimensionless form

$$\mathrm{tg}\alpha U''(r) + \mathrm{tg}\alpha \left(\frac{1}{r+\rho} - \frac{1}{(1-r) + \theta \mathrm{tg}^{-1}\alpha} \right) U'(r) - \frac{\beta U^4(r)}{(1-r) + \theta \mathrm{tg}^{-1}\alpha} = 0, \tag{12}$$

$$U(0) = 1, \quad U'(1) = 0,$$

where k is the coefficient of the heat conduction, R is the flow radius, R_B is radius of the base of the edge, R_T is radius of the vertex of the edge, $r = (R - R_B)/(R_T - R_B)$, T is the temperature, T_B is the temperature of the base of the edge, $U = T/T_B$, Z_B is the thickness of the edge at the base, Z_T is the thickness of the edge at the top, α is the angle of the edge narration, e is the blackness degree, σ is the Stefan-Boltzmann constant,

$$\beta = (R_T - R_B)e\sigma T_B^3/k\cos\alpha, \ \theta = Z_T/(R_T - R_B), \ \rho = R_B/(R_T - R_B).$$

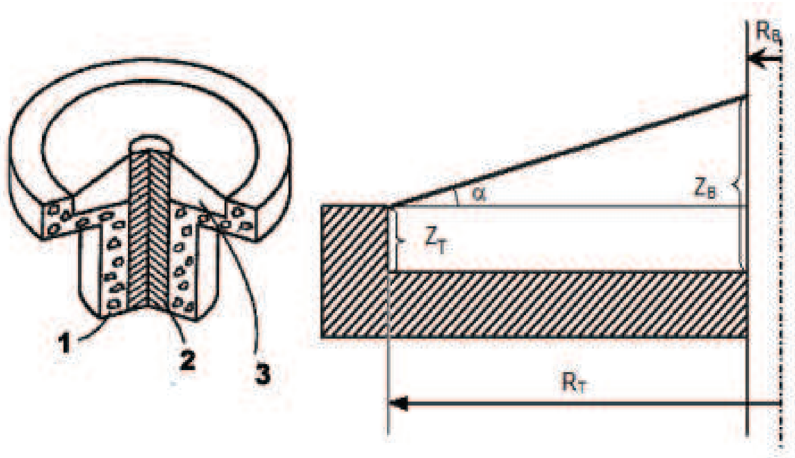

Figure 1. Scheme for annular fins of trapezoidal profile 1 is the insolation; 2 is the heat conducting substructure of the edge; 3 is the ring shaped edge of the trapezoidal profile

The scheme for annular fins of trapezoidal profile is shown in the Figure 1. While solving the problem we suppose the following assumptions that led to the equation (12):

1. The heat transmission by heat conduction is going in the radial direction.

2. The heat flow and temperature distribution in the edge do not depend on time.

3. There are no heat sources and flows in the edge.

4. The temperature at the base of the edge is constant.

5. The edge is made from a homogeneous material the properties of which do not depend on the temperature.

6. The edge surface does not absorb any radial energy from the outside.

7. There is no heat transmission from the vertex of the edge.

8. The edge is a grey body.

Here α, θ and ρ are three parameters that characterize the geometry of the edge ($\text{tg}\alpha = \varepsilon^2$). Value $\alpha = 0$ corresponds to the rectangular profile. This case will be considered separately. Value $\theta = 0$ corresponds to the triangle profile. Inequality to zero of α and θ points to a trapezoidal edge profile. Straight edges with each of these three profiles can be assumed if $\rho \geq 10$.

To obtain the solution of the equation (12) the hybrid asymptotic method as described above is used. We determine the results of a comparison of the approximates that were received by the usage of the hybrid approach and the phase integrals method with numerous results that were received by the use of the fourth order Runge-Kutta method for the concrete parameters of the system under consideration. Calculations have been made in the environment of the software "Mathcad".

The values of the parameters that characterize the geometry of the edge are taken from the paper (Keleer and Holdrege, 1970): $\alpha = 6^o$, $\theta = 0,1$ and $\rho = 0,5$, then $\varepsilon = \sqrt{\text{tg}\alpha} = 0,324$.

We trace the influence of the small parameter β on the correlation of the results. So in Fig. 2, 3, 4 it is seen that with the rising of β the divergence of the results increases. The hybrid approach is a rather satisfactory approximation of the numerical solution for all values of the parameter of external expansion. The results that are received by the usage of the phase integral method show a smaller precision when $\beta = 0,05$, and they become unacceptable at all when $\beta = 0,07$ and $\beta = 0,1$.

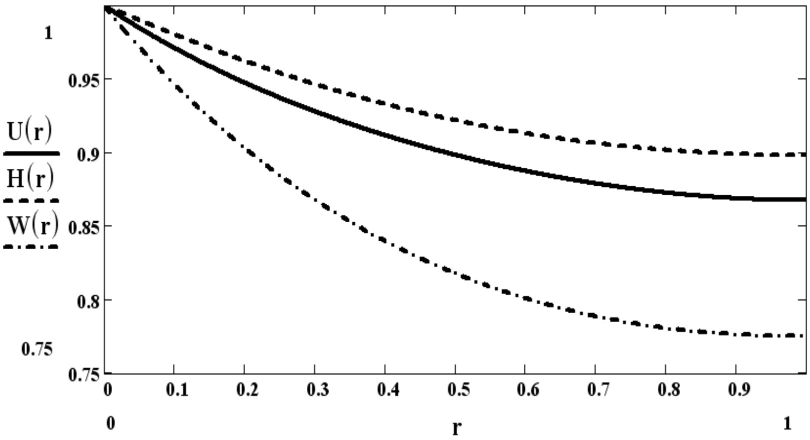

Figure 2. Comparison of the asymptotic solutions of Eq. (12) with the numerical solution for $\beta = 0,05$, where $U(r)$ is the numeral solution, $H(r)$ is the hybrid solution, $V(r)$ is the WKB solution

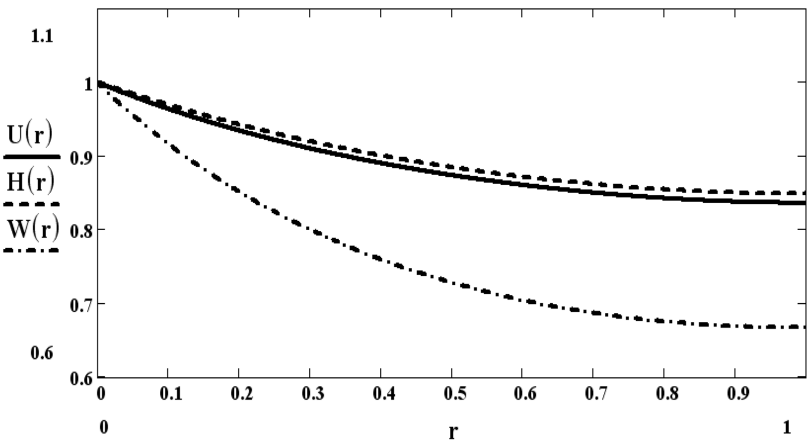

Figure 3. $U(r)$ is the numerical solution, $H(r)$ is the hybrid solution, $V(r)$ is the WKB solution

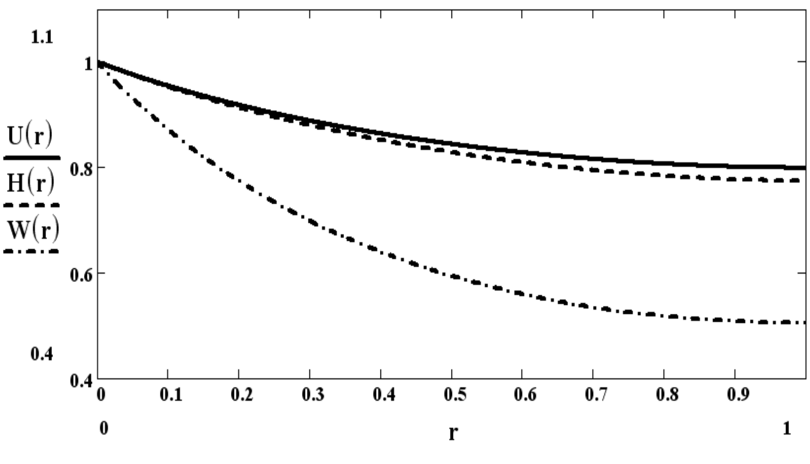

Figure 4. $U(r)$ is the numerical solution, $H(r)$ is the hybrid solution, $V(r)$ is the WKB solution

We put $\alpha = 51^o$, then $\varepsilon = 1,111$. As it is seen in Fig. 5, 6, 7, the hybrid approximation reiterate the character of the numerical solution.

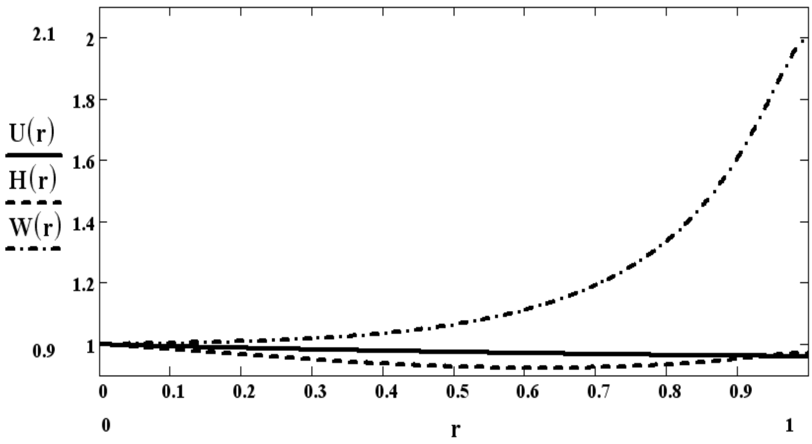

Figure 5. Comparison of the hybrid and WKB solutions of Eq. (12) with the numerical solution for $\beta = 0,05$, where $U(r)$ is the numerical solution, $H(r)$ is the hybrid solution, $V(r)$ is the WKB solution

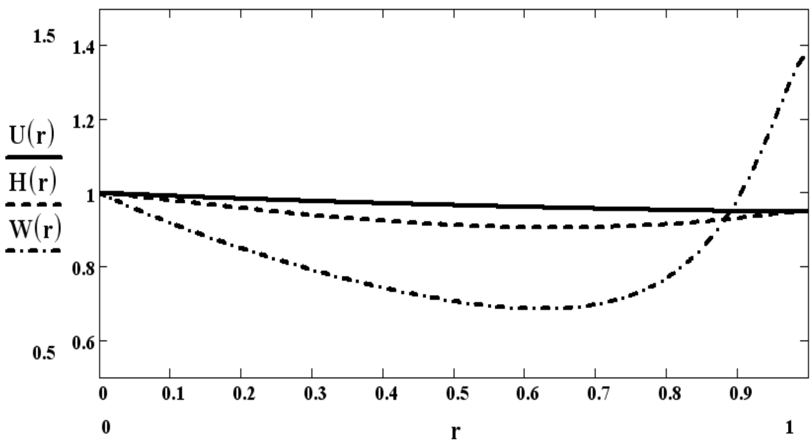

Figure 6. $U(r)$ is the numerical solution, $H(r)$ is the hybrid solution, $V(r)$ is the WKB solution $\beta = 0,07$

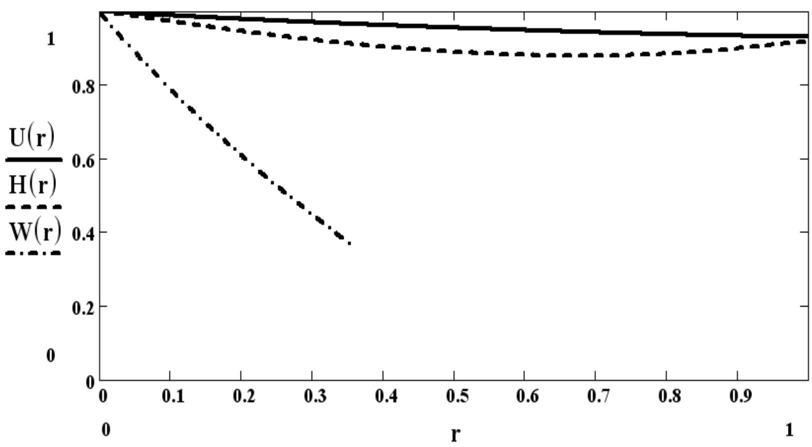

Figure 7. $U(r)$ is the numerical solution, $H(r)$ is the hybrid solution, $V(r)$ is the WKB solution $\beta = 0,1$

We put $\theta = 0,05$ and $\rho = 10$, then the obtained results also show the increase of the divergence between numerical and hybrid methods at the whole interval of change of the independent variable r. For this value ρ (even if the parameter of the internal expansion $\varepsilon < 1$) WKB gives unacceptable result. It is seen from data in Table 1.

Table 1. The results of the numerical and hybrid solutions for the parameter $\varepsilon < 1$.

r	$\rho = 10$ $\theta = 0,05$ $\alpha = \pi/30$								
	$\beta = 0,01$			$\beta = 0,05$			$\beta = 0,1$		
	U	U^H	U^{WKB}	U	U^H	U^{WKB}	U	U^H	U^{WKB}
0	1	1	1	1	1	1	1	1	1
0,1	0,994	0,993	1,027	0,977	0,969	1,036	0,963	0,938	1,047
0,2	0,988	0,987	1,055	0,956	0,939	1,076	0,93	0,879	1,102
0,3	0,983	0,981	1,086	0,937	0,911	1,122	0,901	0,824	1,168
0,4	0,978	0,975	1,118	0,92	0,885	1,175	0,874	0,772	1,45
0,5	0,973	0,971	1,153	0,904	0,862	1,234	0,851	0,725	1,336
0,6	0,969	0,967	1,189	0,89	0,84	1,301	0,831	0,682	1,44
0,7	0,966	0,964	1,227	0,879	0,822	1,373	0,814	0,645	1,556
0,8	0,963	0,961	1,263	0,869	0,808	1,449	0,801	0,616	1,681
0,9	0,961	0,96	1,294	0,863	0,798	1,517	0,792	0,595	1,795
1,0	0,96	0,96	1,309	0,861	0,794	1,551	0,789	0,587	1,855

Table 2. The results of the numerical and hybrid solutions for the parameter $\varepsilon > 1$.

r	$\rho = 10$ $\theta = 0,05$											
	$\alpha = \pi/4$						$\alpha = \pi/3$					
	$\beta = 0,01$		$\beta = 0,05$		$\beta = 0,1$		$\beta = 0,01$		$\beta = 0,05$		$\beta = 0,1$	
	U	U^H	U	U^H	U	U^H	U	U^H	U	U^H	U	U^H
0	1	1	1	1	1	1	1	1	1	1	1	1
0,1	0,999	0,993	0,995	0,989	0,992	0,985	0,999	0,992	0,997	0,99	0,995	0,988
0,2	0,998	0,986	0,991	0,979	0,983	0,971	0,999	0,984	0,994	0,981	0,99	0,976
0,3	0,997	0,979	0,987	0,97	0,975	0,958	0,998	0,977	0,992	0,972	0,984	0,965
0,4	0,996	0,973	0,982	0,961	0,968	0,945	0,998	0,97	0,989	0,963	0,979	0,954
0,5	0,995	0,968	0,978	0,953	0,96	0,933	0,997	0,964	0,987	0,955	0,975	0,944
0,6	0,994	0,963	0,974	0,945	0,953	0,922	0,997	0,958	0,984	0,947	0,97	0,934
0,7	0,994	0,959	0,97	0,938	0,946	0,912	0,996	0,953	0,981	0,941	0,965	0,926
0,8	0,993	0,956	0,967	0,933	0,94	0,903	0,996	0,949	0,979	0,935	0,961	0,918
0,9	0,992	0,955	0,963	0,929	0,934	0,897	0,995	0,946	0,977	0,931	0,957	0,912
1,0	0,992	0,956	0,962	0,929	0,93	0,895	0,995	0,946	0,975	0,931	0,954	0,911

As a result of the investigation, a closed analytical solution of the thermal emission problem with variable characteristics is obtained. Comparison of the results with the straight numerical method show that the given approximate analytical solution holds both for small and for large parameter values of the internal expansion. For example, if the phase integrals method is effective for the parameter $\beta \leq 0,05$ but for the hybrid WKB-Galerkin approach the range of values of the external expansion parameter increases, in practice, twice, namely up to $\beta = 0,1$.

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