

Transversal Shear Effect in Moderately Thick Shells from Materials with Characteristics Dependent on the Kind of Stress State under Creep-Damage Conditions: Theoretical Framework

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Dedicated to Prof. Josef Betten on the occasion of his 70th birthday

The refined theory of creep deformation and creep damage in moderately thick shells of revolution which accounts for transversal shears and additionally for nonlinear distribution across its thickness of the components of the strain tensor as well as of the angles of rotation of the triad of vectors defined the position of the arbitrary point of a shell is discussed. A constitutive model for describing the creep deformation and directional nature of damage under creep conditions in initially isotropic materials with characteristics dependent on the kind of the stress state has been used. The governing equations of the moderately thick shell theory under discussion are introduced, and the initial/boundary-value problem in the frame of the physical nonlinearity and geometrical linearity has been formulated. The numerical tool developed for analysis of creep deformation and creep damage in moderately thick shells of revolution using the proposed theory is discussed.

1 Introduction

Analysis of creep deformation and creep damage in moderately thick shells is of interest in many applications, because shells of this class are broadly used in nuclear, chemical, aircraft and space facilities at high temperatures, and under severe operational and accidental conditions. In this way, constitutive modeling of the actual material behavior including time-dependent irreversible deformation process (creep) as well as time-dependent microstructural changes (creep damage) which induce some material deterioration due to dislocations, impurity atoms and voids in the initial stage, microscopic cavities in the following, and microcracks in the final stage of the creep process, all of them, at the grain boundaries with some preferential orientation, is of great importance. Obviously, satisfactory prediction of the creep deformation and accurate estimation of the creep failure time for the moderately thick shells may be possible only in the case of the realistic description of the creep and creep damage features.

One of the creep and creep damage features of the initially isotropic polycrystalline materials is their different behavior under tensile and compressive loading types (Altenbach et al., 1995; Betten, 2002; Zolochovskij, 1988; Zolochovsky, 1982). Thus, there are two different creep curves obtained from uniaxial tests in tension and compression at the same temperature, and for one and the same absolute value of constant stress. Furthermore, the creep damage development under uniaxial tension and uniaxial compression is essentially different depending on the sign of the stress. In other words, there is the directional nature of the material deterioration under creep conditions. For example, the nucleation, growth and coalescence of microscopic cavities and wedge microcracks in polycrystalline materials under uniaxial tension occur along grain boundaries which are perpendicular to the axis of tensile loading. On the other hand, the appearance, growth and coalescence of cavities and microcracks under uniaxial compression take place at the grain boundary faces located parallel to the axis of compressive loading. Constitutive models based on different creep and creep damage responses in tension and compression have been discussed in Altenbach et al. (1995), Betten (2002) and Kletschkowski et al. (2004).

In fact, the effect of the kind of the stress state on the creep deformation and creep damage development is more complicated phenomenon for many initially isotropic polycrystalline materials, and it can not be identified using only material characteristics under tensile and compressive loading types (Altenbach et al., 1995). For example, the growth of the specific dissipation energy ϕ with time t up to creep rupture in an aluminum alloy AK4-1T at the temperature of $T=473$ K (Rubanov, 1989) under uniaxial tension, uniaxial compression, and pure shear realized under pure torsion conditions is shown in Figure 1 by circles. Here

$$\phi = \int_0^t \sigma_{kl} \dot{p}_{kl} d\tau \quad (1)$$

where σ_{kl} is the Cauchy stress tensor, p_{kl} is the infinitesimal creep strain tensor, and the dot above the symbol denotes a derivative with respect to time. It is interesting to note that the specific dissipation energy given by equation (1) and, therefore, the level of the creep damage in the aluminum alloy AK4-1T are largest under pure torsion, and they cannot be predicted using mentioned above constitutive models which are coupled with the experimental data under tension and compression (Altenbach et al., 1995). Thus, it is necessary to take into account the effect of shear stress under pure torsion conditions on the creep deformation and creep damage development in initially isotropic materials under discussion as an independent phenomenon. In this regard, a number of constitutive models coupled with three series of the basic creep experiments (uniaxial tension, uniaxial compression and pure torsion) have been developed (Altenbach et al., 1995; Betten et al., 1998; Kawai, 2002; Mahnken, 2003; Zolocheskij, 1988; Zolochesky, 1982; Zolochesky et al., 2007) in order to describe the effect of the kind of the stress state.

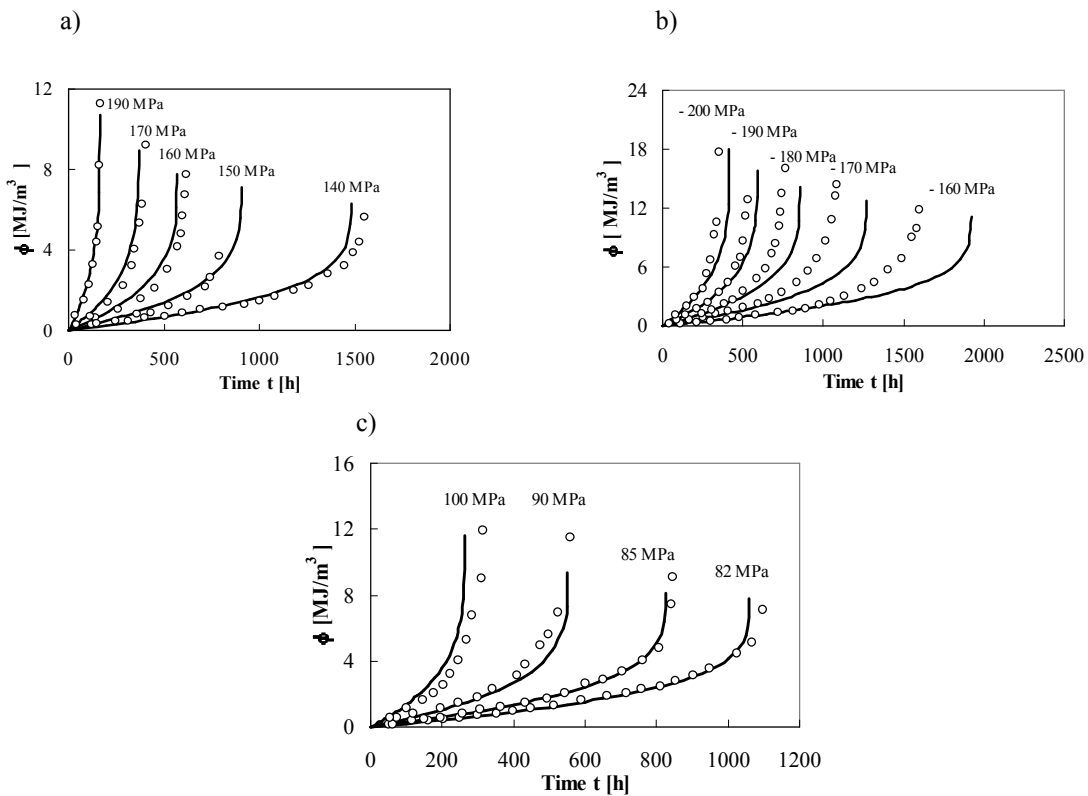


Figure 1: Specific Dissipation Energy vs. Time: a) Uniaxial Tension; b) Uniaxial Compression; c) Pure Torsion

In the past, various authors have analyzed the creep deformation and creep damage development in thin and moderately thick shells (or plates) using the classical theories based on the Kirchoff-Love (or Kirchoff 's) hypotheses, and the refined (shear deformation) theories, as a rule, by Timoshenko or Reissner, respectively. However, most of them assumed no effect of the kind of the stress state on the creep behaviour and creep damage growth for the materials of shells and plates. For the first time, the analysis of creep deformation for thin shells based on the Kirchoff-Love assumptions taking into account different behavior of materials in tension and compression has been performed by Zolochesky (1982), and subsequently by Betten and Borrmann (1987). In the following, the effect of the kind of the stress state on the creep behavior and creep damage development for thin shells composed of the aluminum alloy AK4-1T discussed above has been investigated by Altenbach and Zolochesky (1991), and subsequently by Zolochesky et al. (2007) using the Kirchoff-Love hypotheses. Creep-damage analysis of moderately thick shells and plates based on the refined shear deformation theories and constitutive model of Leckie-Hayhurst has been performed by Altenbach and Naumenko (2002), Altenbach et al. (2004), Bodnar and Chrzanowski (1994), Ganczarski and Skrzypek (2004), Sichov (1998), however, the realistic material behavior similar to the creep of the aluminum alloy AK4-1T (Figure 1) was not considered. Thus, to the best of the authors' knowledge, up to now no investigations exist of the effect of the kind of the stress state on the

creep deformation and creep damage development in moderately thick shells made from materials, such as the aluminum alloy AK4-1T. The aim of this paper is to formulate the theoretical framework for such investigations.

2 Constitutive Theory

A creep theory for initially isotropic polycrystalline materials at small strains and under multiaxial loading which will be discussed in this section was recently proposed by Betten et al. (1998) and Zolochovsky et al. (2007). The principal eigenvalues of the stress tensor are assumed to be distinct. Therein, the creep damage is assumed to be related to dislocation creep and microstructural changes at the grain boundary faces located perpendicular to the direction of the maximum principal stress. Introduction into consideration of the scalar damage variable in a form of the specific dissipation energy ϕ and putting into the expression for the equivalent stress σ_e in a creep potential an eigenvector $\mathbf{m} = \{m_k\}_{k=1}^3$ associated with the maximum principal stress σ_{\max} give the possibility to reproduce the effect of the kind of the stress state on the creep deformation and creep damage development in initially isotropic materials as well as to describe the directional nature of the material deterioration under creep conditions. In this way, a satisfactory agreement has been obtained between the results generated from the proposed creep theory and the experimental data under proportional loading with two-dimensional stress state for isothermal processes of various polycrystalline materials (Betten et al., 1998). The constitutive equation and creep damage evolution equation in the case of the Norton-type power relation have the following structure (Zolochovsky et al., 2007):

$$\dot{p}_{kl} = \sigma_e^m \left(1 - \frac{\phi}{\phi_*}\right)^{-q} \left(\frac{AI_1 \delta_{kl} + B\sigma_{kl}}{\sigma_2} + \alpha C m_k m_l\right), \quad \dot{\phi} = \sigma_e^{m+1} \left(1 - \frac{\phi}{\phi_*}\right)^{-q} \quad (2)$$

with

$$\sigma_e = \sigma_2 + \alpha \sigma_1, \quad \sigma_1 = C \sigma_{\max}, \quad \sigma_{\max} = \sigma_{kl} m_k m_l, \quad \sigma_2 = \sqrt{AI_1^2 + BI_2}, \quad I_1 = \sigma_{kl} \delta_{kl}, \quad I_2 = \sigma_{kl} \sigma_{kl}, \quad \phi \in [0, \phi_*]$$

In these notations ϕ_* is a critical value of ϕ that corresponds to creep rupture time, I_1 and I_2 are the first and the second invariants of the stress tensor, α is a weight coefficient, and δ_{kl} is the Kronecker delta. Material parameters m , q , A , B and C can be found (Zolochovsky et al., 2007) using the experimental results, such as,

$$\dot{p}_{11} = K_+ \sigma_{11}^m \left(1 - \frac{\phi}{\phi_*}\right)^{-q}, \quad \phi = \sigma_{11} p_{11} \text{ (in tension); } \dot{p}_{11} = -K_- |\sigma_{11}|^m \left(1 - \frac{\phi}{\phi_*}\right)^{-q}, \quad \phi = \sigma_{11} p_{11} \text{ (in compression) and}$$

$$2\dot{p}_{12} = K_0 \sigma_{12}^m \left(1 - \frac{\phi}{\phi_*}\right)^{-q}, \quad \phi = 2\sigma_{12} p_{12} \text{ (under pure torsion).}$$

Approximation of the creep curves up to creep rupture for the aluminum alloy AK4-1T given by equation (2) with the critical value $\phi^* = \frac{1}{2}(3I_2 - I_1^2)(a - bI_1)$

and with such values of the material constants as $K_+ = 55.0 \text{ GPa}^{-m} \text{ h}^{-1}$, $K_- = 22.5 \text{ GPa}^{-m} \text{ h}^{-1}$, $K_0 = 1.14 \cdot 10^4 \text{ GPa}^{-m} \text{ h}^{-1}$, $m = 8$, $q = 3$, $a = 0.4 \text{ GPa}^{-1}$, $b = 0.4 \text{ GPa}^{-2}$ is shown in Figure 1 by solid line.

Taking into account the natural scatter of experimental data, particularly marked in compression, the results of approximation of the creep curves and the experimental data are in the satisfactory agreement. The material

parameters in equations (2) can be found as $\alpha C = K_+^{\frac{1}{m+1}} - K_-^{\frac{1}{m+1}}$, $\sqrt{2B} = K_0^{\frac{1}{m+1}} - \alpha C$, $A = K_-^{\frac{2}{m+1}} - B$ and have the

numerical values $\alpha C = 0.148 \text{ GPa}^{\frac{m}{m+1}} \text{ h}^{\frac{1}{m+1}}$, $B = 3.58 \text{ GPa}^{\frac{2m}{m+1}} \text{ h}^{\frac{2}{m+1}}$, $A = -1.58 \text{ GPa}^{\frac{2m}{m+1}} \text{ h}^{\frac{2}{m+1}}$. Note that the

application of the constitutive theory under discussion is the creep deformation and creep damage development in moderately thick shells. The reader, who is interested to the constitutive modeling under three-dimensional stress state, is referred to Altenbach et al. (1995), Ganczarski and Skrzypek (2001), and Zolochovskij (1988).

3 Basic Equations of the Shell Theory

A moderately thick shell of revolution (Figure 2) made from the material with the creep and creep damage characteristics dependent on the kind of the stress state is considered with reference to a cartesian coordinate

system x, y, z with a triad of orthonormal vectors $\vec{e}_x, \vec{e}_y, \vec{e}_z$ obeying the condition $\vec{e}_k \cdot \vec{e}_m = \delta_{km}$ ($k, m = x, y, z$), where z is the coordinate directed along the axis of revolution, and coordinates x, y are in the plane perpendicular to the direction z . The position of the arbitrary point M of a shell may be determined in the local curvilinear coordinate system s, φ, ζ with a triad of orthonormal (before deformation) vectors $\vec{e}_s, \vec{e}_\varphi, \vec{e}_\zeta$ obeying the condition $\vec{e}_k \cdot \vec{e}_m = \delta_{km}$ ($k, m = s, \varphi, \zeta$), where s ($s_0 \leq s \leq s_n$) is the length of the coordinate meridian arc referenced from the end $s = s_0$, φ is the circumferential coordinate, ζ ($\zeta_0 \leq \zeta \leq \zeta_N$) is the distance of the point M from the coordinate surface referenced in the direction of the outer normal, coordinates ζ_0 and ζ_N correspond to the inner and outer surfaces of a shell. In our approach, the coordinate meridian surface does not coincide with the middle surface of a shell. Assume that the moderately thick shell is initially unstressed and undeformed at a temperature T_0 , and it is then subjected to an axisymmetrically and statically applied thermal and force loading leading to the meridional, transversal shear and torsional deformations. In a cartesian coordinate system x, y, z the parametric equations of the coordinate surface have the form

$$x = r(s) \cos \varphi, \quad y = r(s) \sin \varphi, \quad z = z(s) \quad (3)$$

where r is the distance from a point of this surface to the axis of revolution. The combination of equations $\varphi = 0$ and (3) leads to the equations of the coordinate meridian in the plane zOx

$$z = z(s), \quad x = r(s) \quad (4)$$

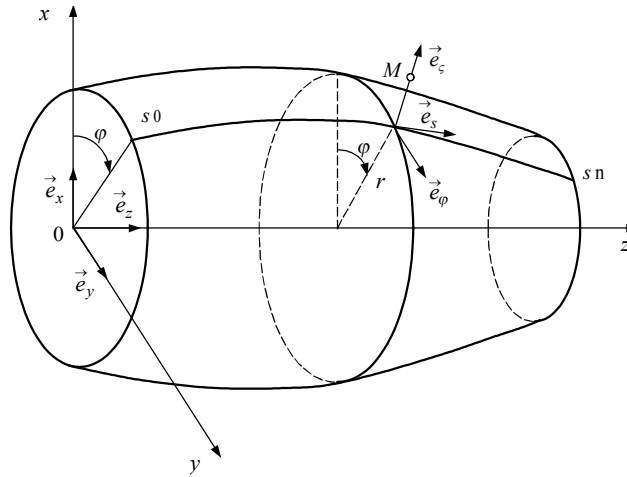


Figure 2: The Meridian of a Shell of Revolution

In the curvilinear coordinate system s, φ, ζ the Lamé's parameters can be defined as

$$H_s = 1 + \zeta k_s = a_s, \quad H_\varphi = r(1 + \zeta k_\varphi) = r a_\varphi, \quad H_\zeta = 1 \quad (5)$$

where k_s, k_φ are the principal curvatures of the coordinate surface given by

$$k_s = \theta', \quad k_\varphi = \frac{\sin \theta}{r} \quad (6)$$

and $(\pi - \theta)$ is the angle between the normal to the coordinate surface and the axis z . Here and in the following derivations we use the abbreviation $(\dots)' = \frac{d(\dots)}{ds}$.

Let us consider the basic equations for the shell under discussion in the frame of the geometrically linear formulation. At present, there exist a number of the refined shell models taking into account the transversal shear deformation. The reader, who is interested in details to these refined theories, is referred to Tovstik P.E. and Tovstik T.P. (2007). In our approach, the displacements of the arbitrary point M of a shell are determined as follows (Grigorenko et al., 1987, Mindlin, 1951):

$$u_s = u + \zeta \psi_s, \quad u_\varphi = v + \zeta \psi_\varphi, \quad u_\zeta = w \quad (7)$$

where u, v are the displacements of a point of the coordinate surface in the directions s and φ ; w is the deflection of the coordinate surface; ψ_s and ψ_φ are the total angles of rotation of a rectilinear element, initially perpendicular to the coordinate surface before deformation. The components $\varepsilon_{ss}, \varepsilon_{\varphi\varphi}, \varepsilon_{s\varphi}, \varepsilon_{s\zeta}$ of the strain tensor as well as the angles of rotation θ_s, θ_φ of the triad $\vec{e}_s, \vec{e}_\varphi, \vec{e}_\zeta$ due to its declination after deformation are connected with corresponding parameters of the coordinate surface by the following relations (Grigorenko et al., 1987; Novozhilov, 1958):

$$\varepsilon_{ss} = \frac{\varepsilon_s + \zeta \kappa_s}{a_s} (s, \varphi), \quad 2\varepsilon_{s\varphi} = \frac{\omega_s}{a_s} + \frac{\omega_\varphi}{a_\varphi} + \zeta \frac{\tau_s}{a_s} + \zeta \frac{\tau_\varphi}{a_\varphi}, \quad 2\varepsilon_{s\zeta} = \frac{\gamma_s}{a_s} (s, \varphi), \quad \theta_s = \psi_s - \frac{\gamma_s}{a_s} (s, \varphi) \quad (8)$$

where

$$\begin{aligned} \varepsilon_s = u' + k_s w, \quad \varepsilon_\varphi = \rho u + k_\varphi w, \quad \kappa_s = \psi_s', \quad \kappa_\varphi = \rho \psi_s, \quad \omega_s = v', \quad \omega_\varphi = -\rho v, \quad \tau_s = \psi_\varphi' \\ \tau_\varphi = -\rho \psi_\varphi, \quad \vartheta_s = k_s u - w', \quad \vartheta_\varphi = k_\varphi v, \quad \gamma_s = \psi_s - \vartheta_s, \quad \gamma_\varphi = \psi_\varphi - \vartheta_\varphi, \quad \rho = \frac{\cos \theta}{r} \end{aligned} \quad (9)$$

Here $\varepsilon_s, \varepsilon_\varphi$ and κ_s, κ_φ are the strains of the coordinate surface and the parameters of its change in curvature in the directions s and φ ; $\omega_s, \omega_\varphi, \tau_s, \tau_\varphi$ are parameters that specify the change of the angle between axes s and φ ; $\vartheta_s, \vartheta_\varphi$ are the angles of rotation of the normal to the coordinate surface; γ_s and γ_φ are the angles of rotation due to transversal shears, and the symbol (s, φ) implies that the new equation follows from the equation under consideration by the cyclic substitution of the subscripts s and φ .

Note that the normal stress $\sigma_{\zeta\zeta}$ is negligibly small compared to the other stresses, and therefore it vanishes in the present paper, so that $\sigma_{\zeta\zeta} = 0$. Let now introduce the forces and the moments in the coordinate surface of a shell under discussion as the integral characteristics of the components of the stress tensor over the shell thickness in the following form (Grigorenko et al., 1987; Novozhilov, 1958):

$$\begin{aligned} N_s = F(\sigma_{ss} a_\varphi) (s, \varphi), \quad N_{s\varphi} = F(\sigma_{s\varphi} a_\varphi) (s, \varphi), \quad Q_s = F(\sigma_{s\zeta} a_\varphi) (s, \varphi) \\ M_s = F(\sigma_{ss} a_\varphi \zeta) (s, \varphi), \quad M_{s\varphi} = F(\sigma_{s\varphi} a_\varphi \zeta) (s, \varphi) \end{aligned} \quad (10)$$

where $N_s, N_{s\varphi}$ and Q_s are the membrane force, shear force and transversal force acting in the cross section $s = \text{const}$ (Figure 3); M_s and $M_{s\varphi}$ are bending moment and twisting moment in the same cross section; $N_\varphi, N_{\varphi s}, Q_\varphi, M_\varphi$ and $M_{\varphi s}$ are the analogous forces and moments in the cross section $\varphi = \text{const}$. Here and in the following integrals we use the abbreviation $F(\dots) = \int_{\zeta_0}^{\zeta_N} (\dots) d\zeta$.

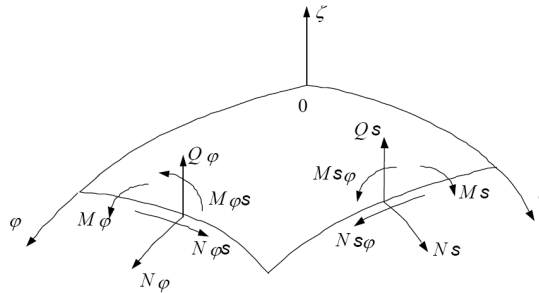


Figure 3: Forces and Moments in a Shell

The equations of equilibrium (Grigorenko et al., 1987; Novozhilov, 1958) are given by

$$\begin{aligned} (rN_s)' - r\rho N_\varphi + rk_s Q_s + rq_s &= 0, & (rN_{s\varphi})' + r\rho N_{\varphi s} + rk_\varphi Q_\varphi + rq_\varphi &= 0 \\ (rQ_s)' - rk_s N_s - rk_\varphi N_\varphi + rq_s &= 0, & (rM_s)' - r\rho M_\varphi - rQ_s + rm_s &= 0, & (rM_{s\varphi})' + r\rho M_{\varphi s} - rQ_\varphi + rm_\varphi &= 0 \end{aligned} \quad (11)$$

where q_s, q_φ, q_ζ are the distributed loads referred to the coordinate surface, and m_s, m_φ are the distributed moments caused by these applied loads.

The physical equations of the moderately thick shell theory will be formulated here by the assumptions of the initial isotropy for the shell material, no effect of the damage on the elastic deformation of the shell, and the effect of the kind of the stress state in the shell on the creep deformation and creep damage development. Additionally, the total strains in a shell are assumed to be composed of an elastic part, thermal part and a part due to creep, and using the generalized Hooke's law we obtain

$$\begin{aligned} \sigma_{ss} &= B_{11}\varepsilon_{ss} + B_{12}\varepsilon_{\varphi\varphi} - \sigma_{ss}^\circ, & \sigma_{\varphi\varphi} &= B_{12}\varepsilon_{ss} + B_{22}\varepsilon_{\varphi\varphi} - \sigma_{\varphi\varphi}^\circ \\ \sigma_{s\zeta} &= 2B_{44}\varepsilon_{s\zeta} - \sigma_{s\zeta}^\circ, & \sigma_{\varphi\zeta} &= 2B_{55}\varepsilon_{\varphi\zeta} - \sigma_{\varphi\zeta}^\circ, & \sigma_{s\varphi} &= 2B_{66}\varepsilon_{s\varphi} - \sigma_{s\varphi}^\circ \end{aligned} \quad (12)$$

where the nonzero components of the symmetrical matrix $[B] = (B_{ij})$ ($i, j = 1, 2, \dots, 6$), and the additional terms related to the creep and thermal expansion are expressed as

$$\begin{aligned} B_{11} = B_{22} &= \frac{2G}{1-\nu}, & B_{12} &= \nu B_{11}, & B_{44} = B_{55} = B_{66} &= G, & \sigma_{ss}^\circ &= B_{11}(p_{11} + \alpha_T \Delta T) + B_{12}(p_{22} + \alpha_T \Delta T) \\ \sigma_{\varphi\varphi}^\circ &= B_{12}(p_{11} + \alpha_T \Delta T) + B_{22}(p_{22} + \alpha_T \Delta T), & \sigma_{s\zeta}^\circ &= 2B_{44}p_{13}, & \sigma_{\varphi\zeta}^\circ &= 2B_{55}p_{23}, & \sigma_{s\varphi}^\circ &= 2B_{66}p_{12} \end{aligned} \quad (13)$$

Here G, ν and α_T are the shear modulus, Poisson's ratio and the coefficient of linear thermal expansion, respectively, $\Delta T = T - T_0$. The creep strains in equation (13) are defined by equation (2) describing the effect of the kind of the stress state in the shell on the creep deformation and creep damage growth in the shell material. The combination of equations (8), (12) and (10) with integration over the shell thickness leads to the physical equations of the shell theory under discussion as follows:

$$\begin{pmatrix} \vec{X}_s \\ \vec{X}_\varphi \end{pmatrix} = \begin{bmatrix} [C] & [K] \\ [K]^T & [D] \end{bmatrix} \begin{pmatrix} \vec{\varepsilon}_s \\ \vec{\varepsilon}_\varphi \end{pmatrix} - \begin{pmatrix} \vec{X}_s^\circ \\ \vec{X}_\varphi^\circ \end{pmatrix}, \quad \vec{Q} = [L]\vec{\gamma} - \vec{Q}^\circ \quad (14)$$

where

$$\begin{aligned} \vec{X}_s &= \{N_s, N_{s\varphi}, M_s, M_{s\varphi}\}^T (s, \varphi), & \vec{\varepsilon}_s &= \{\varepsilon_s, \omega_s, \kappa_s, \tau_s\}^T (s, \varphi), & \vec{Q} &= \{Q_s, Q_\varphi\}^T, & \vec{\gamma} &= \{\gamma_s, \gamma_\varphi\}^T \\ \vec{X}_s^\circ &= \{N_s^\circ, N_{s\varphi}^\circ, M_s^\circ, M_{s\varphi}^\circ\}^T = F \left(a_\varphi \{ \sigma_{ss}^\circ, \sigma_{s\varphi}^\circ, \zeta \sigma_{ss}^\circ, \zeta \sigma_{s\varphi}^\circ \}^T \right) (s, \varphi), & \vec{Q}^\circ &= F \left(\{ a_\varphi \sigma_{s\zeta}^\circ, a_s \sigma_{\varphi\zeta}^\circ \}^T \right) \end{aligned} \quad (15)$$

Here the superscript 'T' denotes the transposition operation, the components of the matrixes $[C], [K], [D], [L]$ are the stiffness characteristics of the shell, so that $[C] = (C_{ij})$ ($i, j = 1, 2, \dots, 4$) ($C \leftrightarrow K \leftrightarrow D$), $[L] = (L_{pq})$ ($p, q = 1, 2$), the symbol ($C \leftrightarrow K \leftrightarrow D$) means that the formal replacement of C by K and D is permissible, and vectors $\vec{X}_s^\circ, \vec{X}_\varphi^\circ, \vec{Q}^\circ$ are due to the creep and thermal expansion. Note that the matrixes $[C], [D], [L]$ are the symmetrical ones, so that $[C] = [C]^T$ ($C \leftrightarrow D \leftrightarrow L$). The nonzero components of the matrixes $[C], [K], [D], [L]$ can be defined as

$$C_{ij} = F\left(c_{ij} \frac{a_\varphi}{a_s}\right), \quad K_{ij} = F(k_{ij}), \quad D_{ij} = F\left(d_{ij} \frac{a_s}{a_\varphi}\right) \quad (16)$$

$$L_{11} = F\left(B_{44} \frac{a_\varphi}{a_s}\right), \quad L_{22} = F\left(B_{55} \frac{a_s}{a_\varphi}\right)$$

$$c_{p+2,q} = c_{p,q+2} = \zeta c_{pq}, \quad c_{p+2,q+2} = \zeta^2 c_{pq} \quad (p, q = 1, 2) \quad (c \leftrightarrow k \leftrightarrow d)$$

$$c_{11} = B_{11}, \quad c_{22} = k_{22} = d_{22} = B_{66}, \quad k_{11} = B_{12}, \quad d_{11} = B_{22}$$

Thus, the kinematic equations (9), static equations (11) and physical ones (14) form a complete system of the governing equations describing the creep deformation and creep damage development in the moderately thick shells composed from materials with creep and damage characteristics dependent on the kind of the stress state.

4 Initial/ Boundary-value Problem

In the case under consideration all unknowns in the coordinate surface of a moderately thick shell depend on the time and additionally only on the meridional coordinate s . In this way, it is necessary to find the displacements u , v and w , total angles of rotation of a rectilinear element ψ_s and ψ_φ , angles of rotation of the normal to the coordinate surface ϑ_s and ϑ_φ , angles of rotation due to transversal shears γ_s and γ_φ , forces N_s , N_φ , $N_{s\varphi}$, $N_{\varphi s}$, Q_s and Q_φ , moments M_s , M_φ , $M_{s\varphi}$ and $M_{\varphi s}$, components of the deformation of the coordinate surface ε_s , ε_φ , κ_s , κ_φ , ω_s , ω_φ , τ_s and τ_φ . Thus, it is necessary to find the 27 unknowns in the coordinate surface of a moderately thick shell. For this purpose, there are twelve kinematic equations (9), five static equations (11) and ten physical equations (14), thus, in general, 27 basic equations. Here, we keep in the mind that physical equations (14) are highly nonlinear in nature due to the creep dependence of the components of vectors $\vec{X}_s^\partial, \vec{X}_\varphi^\partial, \vec{Q}^\partial$.

The problem to investigate the creep deformation and creep damage growth in a moderately thick shell will be now formulated as the initial/ boundary-value problem. In this regard, a vector of resolving functions is introduced as

$$\vec{Y} = \{y_1, \dots, y_{10}\}^T = \left\{ \vec{N}, \vec{u} \right\}^T \quad (17)$$

$$\vec{N} = \{N_1, \dots, N_5\}^T = \{rN_s, rN_{s\varphi}, rM_s, rM_{s\varphi}, rQ_s\}^T, \quad \vec{u} = \{u_1, \dots, u_5\}^T = \{u, v, \psi_s, \psi_\varphi, w\}^T$$

Governing equations (9), (11) and (14) of the shell theory under discussion can then be transformed to the following system of nonlinear differential equations presented in vector form

$$\vec{Y}' = [P]\vec{Y} + \vec{f} \quad (18)$$

Here the matrix $[P]$ has a block structure, so that $[P] = \begin{bmatrix} [p] & [m] \\ [n] & [s] \end{bmatrix}$, $[p] = (p_{ij}) \quad (i, j = 1, 2, \dots, 5)$

$(p \leftrightarrow s \leftrightarrow m \leftrightarrow n)$, and the relations between matrixes $[p]$, $[s]$, $[m]$, $[n]$ are given as $[s] = -[p]^T$, $[m] = [m]^T \quad (m \leftrightarrow n)$. The matrixes $[p]$, $[m]$, $[n]$ have the following nonzero components:

$$p_{ij} = (-1)^{i+1} \mu_{ji} \rho \quad (i, j = 1, 2, \dots, 4), \quad p_{15} = -k_s, \quad p_{25} = -l_{12} k_\varphi$$

$$p_{35} = 1, \quad p_{45} = l_{12}, \quad p_{51} = \mu_{11} k_\varphi + k_s, \quad p_{5j} = \mu_{j1} k_\varphi \quad (j = 2, 3, 4)$$

$$m_{1j} = (-1)^{1+j} \Lambda_{1j} \rho \cos \theta \quad (j = 2, 3, 4), \quad m_{23} = -\Lambda_{23} \rho \cos \theta$$

$$m_{24} = \Lambda_{24} \rho \cos \theta - l_{22} \sin \theta, \quad m_{34} = -\Lambda_{34} \rho \cos \theta$$

$$m_{ii} = \Lambda_{ii} \rho \cos \theta \quad (i = 1, 3), \quad m_{22} = \Lambda_{22} \rho \cos \theta + l_{22} k_\varphi \sin \theta$$

$$m_{44} = \Lambda_{44} \rho \cos \theta + l_{22} r, \quad m_{i5} = (-1)^{i+1} \Lambda_{1i} k_\varphi \cos \theta \quad (i = 1, 2, \dots, 4) \quad (19)$$

$$m_{55} = \Lambda_{11} k_\varphi \sin \theta, \quad n_{ij} = \frac{\lambda_{ij}}{r} \quad (i, j = 1, 2, \dots, 4), \quad n_{55} = \frac{\lambda_{11}}{r}$$

$$[\lambda] = (\lambda_{ij}) \quad (i, j = 1, 2, \dots, 4) \quad (\lambda \leftrightarrow \mu \leftrightarrow \Lambda)$$

$$[\lambda] = [C]^{-1}, \quad [\mu] = [\lambda][K]$$

$$[\Lambda] = [D] - [K]^T [\lambda][K] = [D] - [\mu]^T [K] = [D] - [K]^T [\mu]$$

$$l_{11} = \frac{1}{L_{11}}, \quad l_{12} = L_{12} l_{11}, \quad l_{22} = L_{22} - l_{12} L_{12}$$

Note that the matrixes $[\lambda]$ and $[\Lambda]$ are symmetrical ones. The free-term vector \vec{f} that includes the surface loads as well as the effects of creep and thermal expansion has the following components:

$$\begin{aligned} f_1 &= \eta_1 \cos \theta - r q_s, & f_2 &= -\chi_2 \sin \theta - \eta_2 \cos \theta - r q_\varphi, & f_3 &= \eta_3 \cos \theta - r m_s, \\ f_4 &= r \chi_2 - \eta_4 \cos \theta - r m_\varphi, & f_5 &= \eta_1 \sin \theta - r q_\zeta, & f_{5+i} &= \xi_i \quad (i = 1, 2, \dots, 4), & f_{10} &= \chi_1 \end{aligned} \quad (20)$$

where

$$\chi_1 = l_{11} Q_s^\partial, \quad \chi_2 = l_{12} Q_s^\partial - Q_\varphi^\partial, \quad \vec{\xi} = [\lambda] \vec{X}_s^\partial, \quad \vec{\eta} = [\mu]^T \vec{X}_s^\partial - \vec{X}_\varphi^\partial \quad (21)$$

The system of the nonlinear differential equations (18) must be complemented by the boundary conditions at the ends of the moderately thick shell

$$[G] \vec{Y} = \vec{g} \quad (22)$$

where $[G]$ and \vec{g} are the specific rectangular matrix and vector of the boundary conditions used, so that $[G] = (G_{ij}) \quad (i = 1, 2, \dots, 5; j = 1, 2, \dots, 10)$, $\vec{g} = \{g_1, \dots, g_5\}^T$.

Thus, the analysis of the effect of the kind of the stress state on the meridional, transversal shear and torsional deformations under creep-damage conditions in the moderately thick shells of revolution subjected to an axisymmetrically and statically applied thermal and force loading reduces to a nonlinear one-dimensional boundary-value problem given by equations (18) and (22). Due to the time dependence of the components of the vector \vec{f} related to the creep, the boundary-value problem under discussion should be considered simultaneously with the initial-value problem (with respect to time) for the ordinary differential equations (2) with the natural initial conditions $p_{11} = p_{22} = p_{13} = p_{23} = p_{12} = \phi = 0$ at $t=0$. Thus, the direct integration of the initial-value problem for equations (2) by one of the numerical methods with explicit or implicit schemes involves reducing the nonlinear boundary-value problem of creep to the solution of a sequence of linear boundary value problems with known components of the vector \vec{f} related to the creep. The fourth-order Runge-Kutta-Merson's method with automatic selection of time step sizes will be used in order to solve the initial-value problem for equations (2). This time integration algorithm has been applied for the first time in the study of creep of structures, to the authors' best knowledge, by Zolochovsky (1982) for the numerical analysis of creep deformation in thin shells. It has been later used in creep and creep damage analyses by many authors, for example, by Hayhurst et al., 1984; Sichov, 1998, and Ling et al., 2000. Each linearized boundary-value problem will be solved by the discrete orthogonal shooting method of Godunov given in Grigolyuk and Shalashilin (1991). After finding the basic unknowns in the coordinate surface of a moderately thick shell the strains $\varepsilon_{ss}, \varepsilon_{\varphi\varphi}, \varepsilon_{s\zeta}, \varepsilon_{\varphi\zeta}, \varepsilon_{s\varphi}$ will be calculated from equations (8) while the stresses $\sigma_{ss}, \sigma_{\varphi\varphi}, \sigma_{\varphi\zeta}, \sigma_{s\varphi}$ will be found from equations (12). Since the shear stress $\sigma_{s\zeta}$ found from equations (12) does not satisfy the boundary conditions at the shell surfaces it will be determined as a result of integration directly from the equilibrium equation taken in terms of the stresses together with the

corresponding boundary conditions. The strain ε_{cc} can be found as a sum of an elastic part defined by the generalized Hooke's law, thermal part and a part due to creep.

5 Conclusions

The effect of the kind of the stress state on the creep behavior and creep damage of the initially isotropic polycrystalline materials under two-dimensional stress state can be identified using three series of the basic experiments up to creep rupture (uniaxial tension, uniaxial compression and pure torsion). A constitutive model for describing the creep and directional nature of damage in initially isotropic materials with characteristics dependent on the kind of the stress state has been applied to the modeling of the meridional, transversal shear and torsional deformations under creep-damage conditions in the moderately thick shells of revolution. The refined shear deformation theory of a shell has been considered, and the approach of establishing the basic equations for the moderately thick shells under creep and creep damage conditions has been introduced. To solve the formulated initial/boundary-value problem, the fourth-order Runge-Kutta-Merson's method of time integration with the combination of the discrete orthogonal shooting method of Godunov is used. Some applications of the proposed refined shell theory will be considered in a forthcoming paper.

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References

- Altenbach, H.; Altenbach, J.; Naumenko, K.: Edge effects in moderately thick plates under creep-damage conditions. *Technische Mechanik*, 24, 3-4, (2004), 254-263.
- Altenbach, H.; Altenbach, J.; Zolochovsky, A.: *Erweiterte Deformationsmodelle und Versagenskriterien der Werkstoffmechanik*. Deutscher Verlag für Grundstoffindustrie, Stuttgart (1995).
- Altenbach, H.; Naumenko, K.: Shear correction factors in creep-damage analysis of beams, plates and shells. *JSME International Journal, Series A*, 45, 1, (2002), 77-83.
- Altenbach, H.; Zolochovsky, A.: Kriechen dünner Schalen aus anisotropen Werkstoffen mit unterschiedlichem Zug-Druck-Verhalten. *Forsch. Ingenieurwesen*, 57, 6, (1991), 172-179.
- Betten, J.: *Creep Mechanics*. Springer-Verlag, Berlin (2002).
- Betten, J.; Borrmann, M.: Stationäres Kriechverhalten innendruckbelasteter dünnwandiger Kreiszyinderschalen unter Berücksichtigung des orthotropen Werkstoffverhaltens und des CSD-Effects. *Forsch. Ingenieurwesen*, 53, 3, (1987), 75-82.
- Betten, J.; Sklepus, S.; Zolochovsky, A.: A creep damage model for initially isotropic materials with different properties in tension and compression. *Engng. Fracture Mech.*, 59, (1998), 623-641.
- Bodnar, A.; Chrzanowski, M.: Cracking of creeping plates in terms of continuum damage mechanics. *Mech. Teor. i Stos.*, 32, 1, (1994), 31-41.
- Ganczarski, A.; Skrzypek, J.: Anisotropic thermo-creep-damage in 3D thick plate vs Reissner's approach. In: R. Kienzler, H. Altenbach, I. Ott, eds., *Theories of Plates and Shells, EUROMECH Colloquium 444, Lecture Notes in Applied and Computational Mechanics*, vol.16, pages 39-44, Springer-Verlag, Berlin (2004).
- Ganczarski, A.; Skrzypek, J.: Application of the modified Murakami's anisotropic creep-damage model to 3D rotationally-symmetric problem. *Technische Mechanik* 21, (2001), 251-260.

- Grigolyuk, E.I.; Shalashilin, V.I.: *Problems of Nonlinear Deformation: The Computational Method Applied to Nonlinear Problems in Solid Mechanics*. Kluwer, Dordrecht (1991).
- Grigorenko, Ya. M. ; Vasilenko, A. T.; Golub, G. P.: *Statics of Anisotropic Shells with Finite Shear Rigidity*. Naukova Dumka, Kiev (1987).
- Hayhurst, D.R.; Dimmer, P.R.; Morrison, G.J.: Development of continuum damage in the creep rupture of notched bars. *Trans. R. Soc. London*, A 311, (1984), 103-129.
- Kawai, M.: Constitutive modeling of creep and damage behaviors of the non-Mises type for a class of polycrystalline metals. *Int. J. Damage Mech.*, 11, (2002), 223-246.
- Kletschkowski, T.; Schomburg, U.; Bertram, A.: An endochronic viscoplastic approach for materials with different behavior in tension and compression. *Mechanics of Time-dependent Materials*, 8, (2004), 119 – 135.
- Ling, L.; Tu, S.-T.; Gong, J.-M.: Application of Runge-Kutta-Merson algorithm for creep damage analysis. *Int. J. Press. Vessel Piping*, 77, (2000), 243-248.
- Mahnken, R.: Creep simulation of asymmetric effects by use of stress mode dependent weighting functions. *Int. J. Solids Structures*, 40, (2003), 6189-6209.
- Mindlin, R.D.: Influence of rotatory inertia and shear on flexural motions of isotropic elastic plates. *Trans. ASME. J. Applied Mech.*, 18, 1, (1951), 31-38.
- Novozhilov, V. V.: *Elasticity Theory*. Sudpromgiz, Leningrad (1958).
- Rubanov, V.V.: *Experimental Foundation of the Constitutive Equations of Creep for Materials with Different Behaviour in Tension and Compression*. Ph.D. thesis, Siberian Branch of the Russian Academy of Sciences, Institute of Hydrodynamics, Novosibirsk (1987).
- Sichov, A.: Eine verbesserte Schalentheorie unter Einbeziehung geometrischer Nichtlinearität sowie der Schubverzerrungen in Problemen der Kriechmechanik. *Bulletin of the National Technical University "Kharkov Polytechnic Institute"*, 17, (1998), 68-72.
- Tovstik P.E., Tovstik T.P.: On the 2D models of plates and shells including the transversal shear. *ZAMM*, 87, 2, (2007), 160-171.
- Zolochevskij, A. A.: Kriechen von Konstruktionselementen aus Materialien mit von der Belastung abhängigen Charakteristiken. *Technische Mechanik*, 9, (1988), 177–184.
- Zolochevsky, A. A. : *Creep of Thin Shells for Materials with Different Behavior in Tension and Compression*. Ph.D. thesis, National Academy of Sciences of Ukraine, Institute of Mechanical Engineering Problems, Kharkov (1982).
- Zolochevsky, A.; Galishin, A.; Sklepus, S.; Voyiadjis, G.Z.: Analysis of creep deformation and creep damage in thin-walled branched shells from materials with different behavior in tension and compression. *Int. J. Solids Structures*, 44, (2007), 5075–5100.

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