TECHNISCHE MECHANIK, Band 28, Heft 3-4, (2008), 194 – 203
Manuskripteingang: 15.Oktober 2007

The Influence of Elastic Base on Stability of Rotor-Oil Film Bearing Systems

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The oil film bearings (or journal bearings) are usually used for large rotating machinery systems. The oil film bearings contribute to dynamic characteristics of the systems by influencing the natural frequencies from hydrodynamic properties of oil in the bearing cage. The stability of the systems is changeable by elastic base. This is subject of the paper, and the vibration measurement of the foundation on an elastic base could be used for assessement of the technical conditions of the oil-film bearing and rotor-bearing systems.

1 Introduction

In Nguyen Cao Menh (2004) the stability of rotor - oil film bearing systems is investigated with neglect of base elasticity and the vibration of the system is measured by proximity probe at journal of shaft. If the outside of bearings hardly connected with foundation is placed on elastic base in the vertical direction and the vibration can be measured at the foundation (that means on the non-rotating part of the system) then the modelling of the system has one degree of freedom more than before. For estimating the effect of different parameters of the system on its technical condition such as an unstable oil whirl at a sub-harmonic frequency, one of the qualities, which provide many informations for it, is the vibration of the rotor shaft and the vibration of the foundation. The elasticity of the base has influence on stability of the system.

On the basis of analysing the vibrations in a mathematical modelling and a vibration measurement it is possible to make remarks on the technical conditions and to propose solutions for reduction of unnormal phenomena in the system.

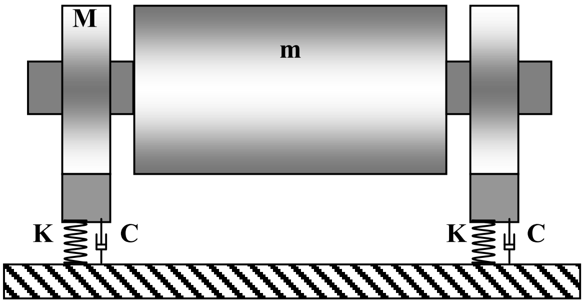
2 The Modelling of the System

The modelling of the system is described in the Figure 1 and Figure 2. For $K=\infty$ the system has been investigated in Nguyen Cao Menh (2004), and the motion equations are in the form

$$\begin{aligned} m\ddot{x} + 2(D_{11}\dot{x} + D_{12}\dot{y} + K_{11}x + K_{12}y) &= m e \omega^2 \cos\omega t \\ m\ddot{y} + 2(D_{21}\dot{x} + D_{22}\dot{y} + K_{21}x + K_{22}y) &= m e \omega^2 \sin\omega t \end{aligned} \tag{1}$$

where m is the mass of the rotor, e – the eccentricity of the rotor, ω – the angular frequency of the rotor, $\omega=2\pi f$, and f is the rotor revolutions per second, D_{ij} and K_{ij} (i,j = 1,2) are the damping and stiffness coefficients respectively of the oil-film bearing (Figure 2). These coefficients can be calculated from the parameters of the oil-film bearing by a hydrodynamic theory.

On the Figure 1, if K and C are finite then the system has one degree of freedom more than before, and the motion equations are as follows:



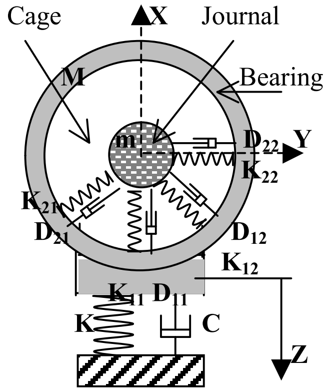


Figure 1. Rotor- journal bearing system on elastic base

Figure 2. Modelling of the system in Figure 1

$$
\begin{aligned}
& m\ddot{x} + 2[D_{11}(\dot{x} - \dot{z}) + D_{12}\dot{y} + K_{11}(x - z) + K_{12}y] = m e \omega^2 \cos\omega t \\
& m\ddot{y} + 2[D_{21}(\dot{x} - \dot{z}) + D_{22}\dot{y} + K_{21}(x - z) + K_{22}y] = m e \omega^2 \sin\omega t \\
& M\ddot{z} + 2Kz + 2C\dot{z} - 2[D_{11}(\dot{x} - \dot{z}) + D_{21}(\dot{x} - \dot{z}) + K_{11}(x - z) + K_{21}(x - z)] = 0
\end{aligned}
\tag{2}
$$

By introducing the notations

$$
h = \frac{m}{M},\ k = \frac{2K}{M},\ d_1 = \frac{2C}{M},\ d_{ij} = \frac{2D_{ij}}{m},\ \ k_{ij} = \frac{2K_{ij}}{m}
\tag{3}
$$

the equations (2) become

$$
\begin{aligned}
& \ddot{x} + d_{11}(\dot{x} - \dot{z}) + d_{12}\dot{y} + k_{11}(x - z) + k_{12}y = e\cos\omega t \\
& \ddot{y} + d_{21}(\dot{x} - \dot{z}) + d_{22}\dot{y} + k_{21}(x - z) + k_{22}y = e\sin\omega t \\
& \ddot{z} + d_1\dot{z} + kz - h(d_{11} + d_{21})(\dot{x} - \dot{z}) - h(k_{11} + k_{21})(x - z) = 0
\end{aligned}
\tag{4}
$$

These equations can be written in the form

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix}
=
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-k_{11} & -d_{11} & -k_{12} & -d_{21} & k_{11} & d_{11} \\
0 & 0 & 0 & 1 & 0 & 0 \\
-k_{12} & -d_{12} & -k_{22} & -d_{22} & k_{21} & d_{21} \\
0 & 0 & 0 & 0 & 0 & 1 \\
h(k_{11} + k_{21}) & h(d_{11} + d_{21}) & 0 & 0 & -k - h(k_{11} + k_{21}) & -d - h(d_{11} + d_{21})
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}
+
\begin{bmatrix} 0 \\ e\cos\omega t \\ 0 \\ e\sin\omega t \\ 0 \\ 0 \end{bmatrix}
\tag{5}
$$

where we denote

$$
x_1 = x;\quad x_2 = \dot{x};\quad x_3 = y;\quad x_4 = \dot{y};\quad x_5 = z;\quad x_6 = \dot{z}
\tag{6}
$$

Before solving the equation (5) we have to assess the coefficients d_{ij} and k_{ij} .

3 Calculation of damping and stiffness coefficients in the oil film bearing

The system consists of a rotor and two oil-film bearings. When the rotor turns, the journal is rotating in the bearing cage with a full incompressible fluid that causes pressure from the fluid (oil film) on the surfaces of the journal and the bearing. Some parameters are denoted in the Figure 3. On the basis of a hydrodynamic theory of short circular oil-film bearings and the assumption that the oil-film thickness is the same as the bearing clearance, see Kramer (1993), the Reynolds Equation takes the form

$$\frac{\partial^2 p}{\partial z^2} = \frac{6\eta}{h^3(\varphi,t)}[e_j(\omega - 2\dot{\gamma})\sin(\varphi-\gamma) - 2\dot{e}_j\cos(\varphi-\gamma)] \tag{7}$$

where p is pressure of the fluid in the bearing cage, η is viscosity coefficient, and the other parameters are denoted in the Figure 3.

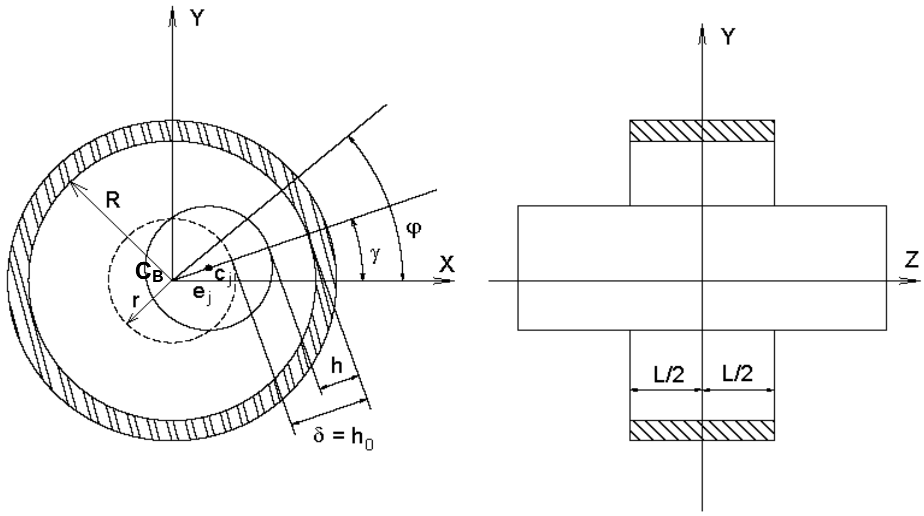


Figure 3. Some parameters of oil film bearing in rotation

After solving (7), it is possible to calculate the forces acting from fluid in the bearings to the surfaces of the bearings and the journals, and then to compute the coefficients D_{ij} and K_{ij} . On the basis of the results in Kramer (1993) and Nguyen Cao Menh (2004, 2006) and with denotations (3), we have the following formulae for calculating the coefficients k_{ij} and d_{ij}:

$$\begin{aligned}
k_{11} &= (g/\delta)[2\pi^2 + (16-\pi^2)\varepsilon^2]A(\varepsilon) \\
k_{12} &= \frac{g\pi}{4\delta}\frac{\pi^2 - 2\pi^2\varepsilon^2 - (16-\pi^2)\varepsilon^4}{\varepsilon(1-\varepsilon^2)^{1/2}}A(\varepsilon) \\
k_{21} &= -\frac{g\pi}{4\delta}\frac{\pi^2 + (32+\pi^2)\varepsilon^2 + (32-2\pi^2)\varepsilon^4}{\varepsilon(1-\varepsilon^2)^{1/2}}A(\varepsilon) \\
k_{22} &= \frac{g}{\delta}\frac{\pi^2 + (32+\pi^2)\varepsilon^2 + (32-2\pi^2)\varepsilon^4}{1-\varepsilon^2}A(\varepsilon)
\end{aligned} \tag{8}$$

$$\begin{aligned}
d_{11} &= \frac{g\pi}{2\delta\omega}\frac{(1-\varepsilon^2)^{1/2}}{\varepsilon}[\pi^2 + (2\pi^2-16)\varepsilon^2]A(\varepsilon) \\
d_{12} &= d_{21} = -\frac{g}{\delta\omega}[2\pi^2 + (4\pi^2 - 32)\varepsilon^2]A(\varepsilon)
\end{aligned} \tag{9}$$

$$d_{22} = \frac{g\pi}{2\delta\omega} \frac{\pi^2 + (48 - 2\pi^2)\varepsilon^2 + \pi^2\varepsilon^4}{\varepsilon(1-\varepsilon^2)^{1/2}} A(\varepsilon)$$

and

$$A(\varepsilon) = \frac{4}{[\pi^2 + (16 - \pi^2)\varepsilon^2]^{3/2}} \tag{10}$$

where $\varepsilon = e_j/\delta$, and e_j is the distance between the center of shaft journal C_j and the center of the bearing C_B in the rotor rotation, δ is the clearance between journal and bearing, $\delta = R\text{-}r$ (Figure 3). The Value ε has the following relation with rotating speed of the rotor shaft, Kramer (1993):

$$f = \frac{C(1-\varepsilon^2)^2}{\varepsilon\sqrt{1-\varepsilon^2 + (\frac{4}{\pi}\varepsilon)^2}} \tag{11}$$

where $C = \frac{2\delta^2 Mg}{\pi^2 \eta L^3 D}$, $D = 2R$. (12)

The coefficients h, k and d_1 will be given in the rotor-bearing system.

4 Results of Modelling and Calculation

Let us consider the system, in which there is a rotor with mass m supported by two oil-film bearings (Figure 1) and the bearings are connected with machine body with mass M placed on elastic base, that has the following parameters:

- Mass of the rotor m = 10,000 kg.
- Mass of machine body M = 20,000 kg, then $h = m/M = 0.5$
- Diameter of the bearings $D = 2R = 0.4$ m
- Length of the oil-film bearings $L = 0.2$ m
- Diameter of the journal $2r = 0.3994$ m
- Clearance $\delta = R\text{-}\ r = 0.0003$ m
- Dynamic viscosity coefficient $\eta = 0.03$ Ns/m^2.
- Mass eccentricity $e = 0.01$m
- Elastic stiffness of the base is given in the form $k = 2K/M = 6000$ (rad/s)^2
- Damping coefficient of the base is $d_1 = 2D/M = 0.5$ (1/s)

In this case, the coefficients in equation system (8), (9) dependent on the rotation speed f of the rotor via ε from (11). If $f = 0$ then $\varepsilon = 1$, the bigger f is the smaller gets ε. When f increases with time, ε is changed in the interval (1,0). The relation between f and ε is given in Figure 4.

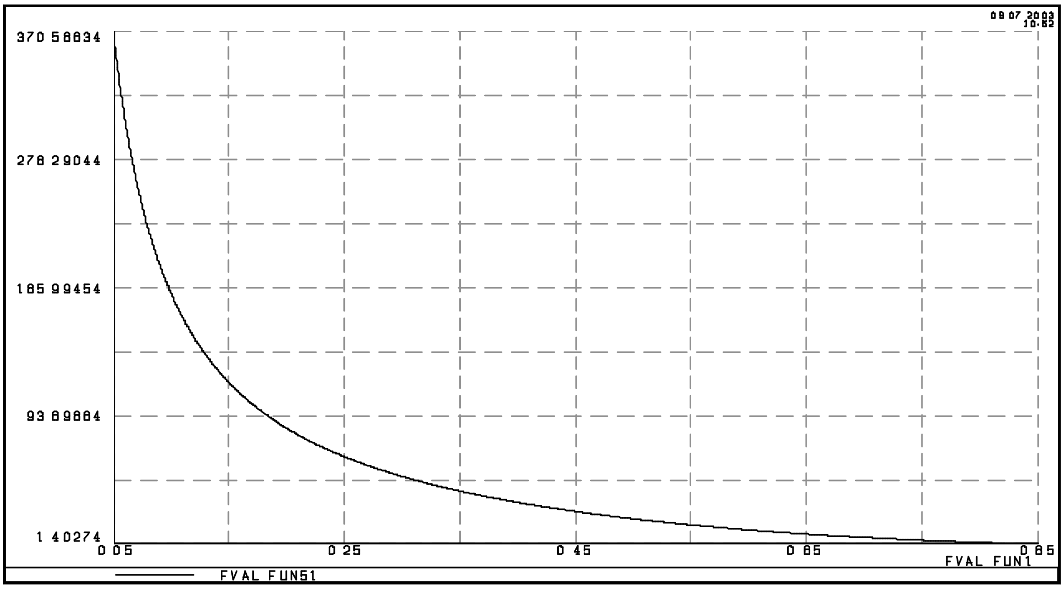


Figure 4. Graphics of function *f(ε)*

If the rotation speed *f* is given then *ε* can be found from Figure 4. For example, we introduce some values of rotation speed and according values of *ε*, which will be used later.

f	50.0634	60.0185	70.1072	71.0001	72.0410
ε	0.3000	0.2633	0.2337	0.2313	0.2287

By integrating the equation system (5) with *f* and *ε* given one is able to obtain the solutions or vibrations of the system. The vibration diagrams show that the system motions are either in stable or unstable conditions. By spectral analysis of vibrations it can be seen at what domains of the frequencies there is a phenomenon of oil whirl, and at what frequencies there is an unstable condition. Notice that on the basis of the obtained results in Nguyen Cao Menh (2004, 2006), the oil whirl phenomenon for oil film bearing could happen at approximately half of the rotation frequency. The system is unstable if the oil whirl phenomenon is large, that means in the vibration spectrum graphics the peak at the oil whirl frequency is bigger than the peak at the rotation frequency f. In the following there are some cases in detail.

1) Case 1: *Rotation speed f = 60.0185 (rev/s).*

On the basis of Figure 4, $\varepsilon = 0.2633$ and the vibration diagram of the system is described in Figure 5 for a time interval (0,20). It shows that the vibration is stable in this case. The vibration spectra for x_1 , x_3 , x_5 are presented in the Figure 6 for time interval (15,20) and show that there exists only one spectral peak at $f = 60$ Hz., that means there is no oil whirl phenomenon.

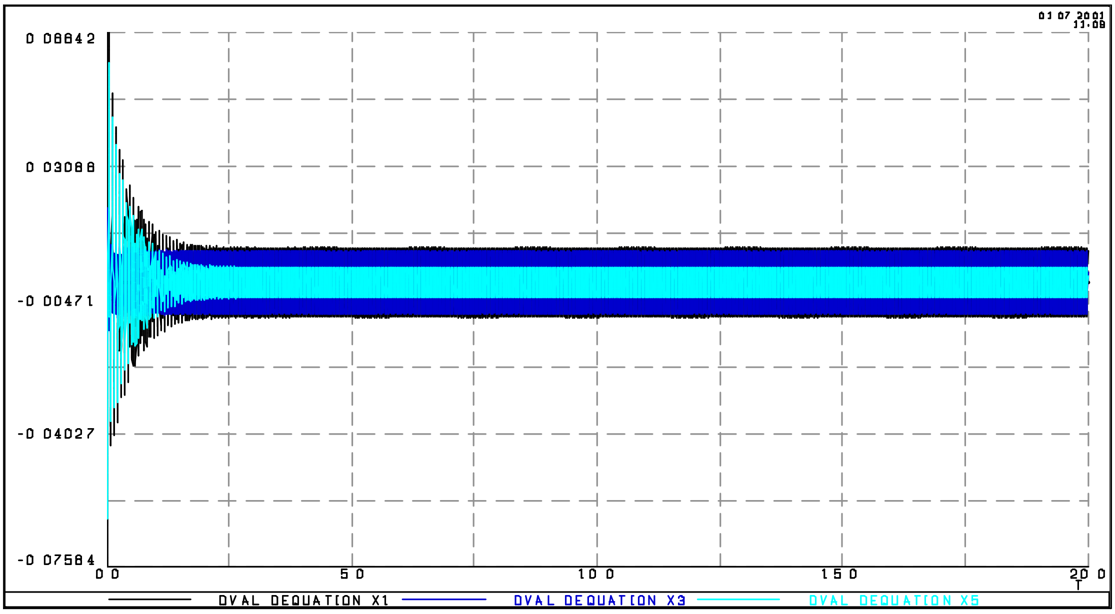


Figure 5. Vibration x_1 ,x_3 ,x_5 in time interval (0, 20)

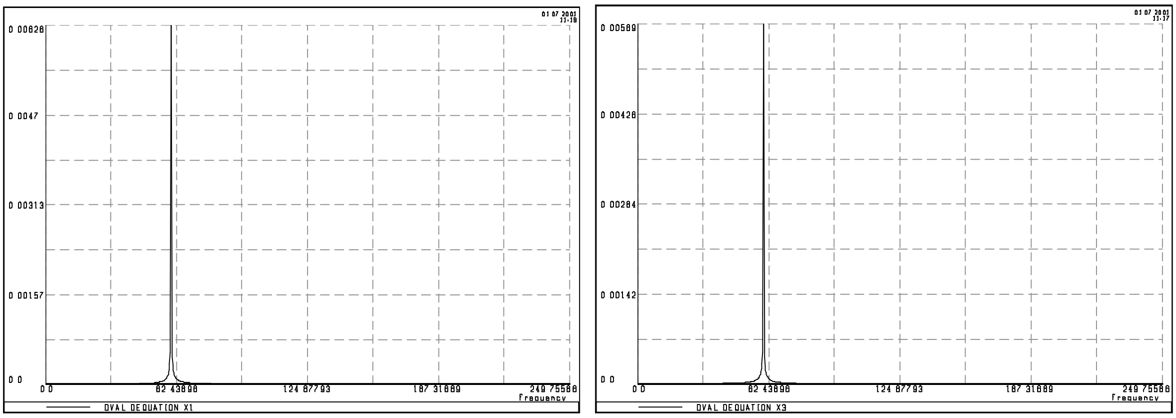


Figure 6. Vibration spectra of x_1 , x_3 and it is the same for x_5 in time interval (15, 20)

2) <u>Case 2:</u> *Rotation speed f = 70.1072(rev/s) then ε = 0.2337*

In this case, the vibration of the system is still stable (Figure 7), but it's spectra show that the oil whirl phenomena occurs in small intensity (Figure 8, 9 for x_1 and x_5)

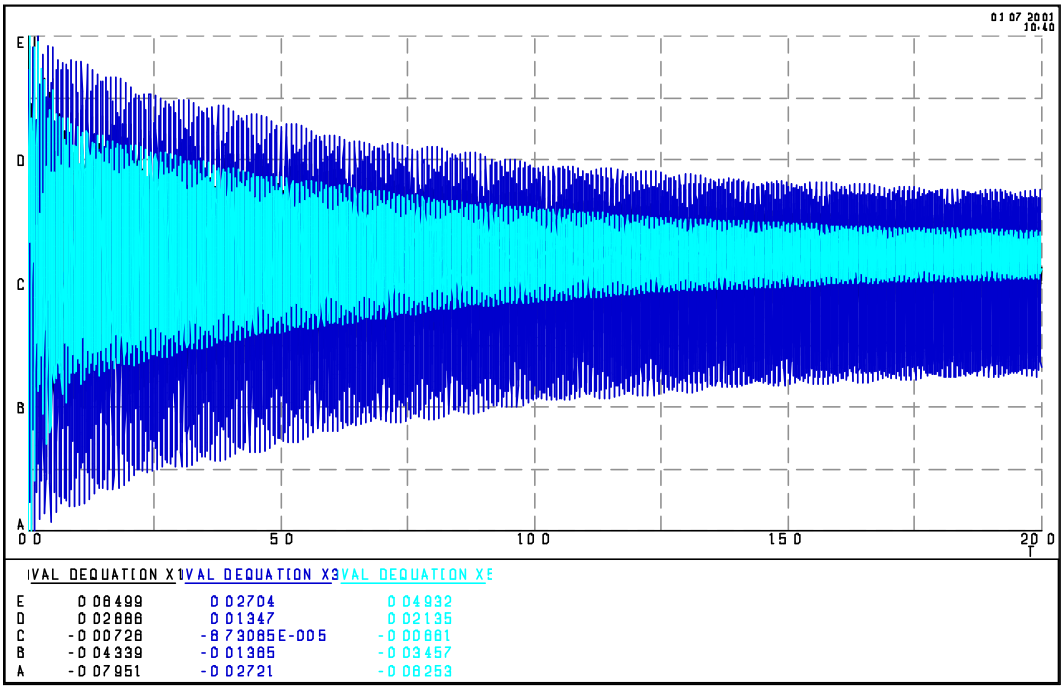


Figure 7. Vibration of the system $f = 70.1072$

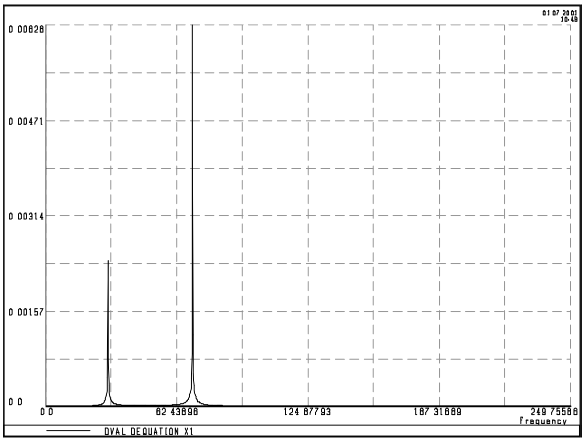


Figure 8. Vibration Spectrum of x_1 f=70.1072

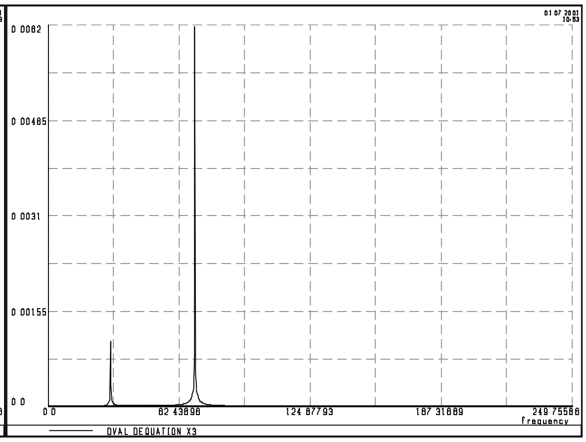


Figure 9. Vibration Spectrum of x_5 f = 70.1072

3) <u>Case3:</u> *Rotation speed f =71 (rev/s) then ε =0.2313*

In this case, the vibration is increased with time (Figure 10), that means the system is unstable, and it's spectra show that the peak at an oil whirl frequency 32 Hz is dominant.

In this case, for the stabilization of the system vibrations under stable conditions, we can propose some techniques.

a) <u>Decrease stiffness of elastic base (K)</u> : In this case the rotation speed f is not changed, but k is decreased from 6000 (rad /s)^2 to 2000 (rad /s)^2 , therefore the elastic base is decreased, the vibrations have the form of Figure 11 (transient process), and their spectra are given in Figure 12 for the vibration at the end interval of time (stationary process). It shows that the spectral peak at the oil whirl frequency is small, and the spectral peak at the rotating frequency is dominant, that means the system is in stable condition.
b) <u>Decrease clearance δ :</u> In this case the rotation speed is not changed, that means f = 71.Hz, but δ decreases from 0.0003 m to 0.00023 m, the vibrations and their spectra in a transient process have the form of Figure 13, and the spectrum of x_1 is given in Figure 14 for the vibration at the end interval of time, and the spectra

of x_3 , x_5 are of the same form. It shows that there is only one spectral peak at the rotation frequency, that means the system is in stable condition.

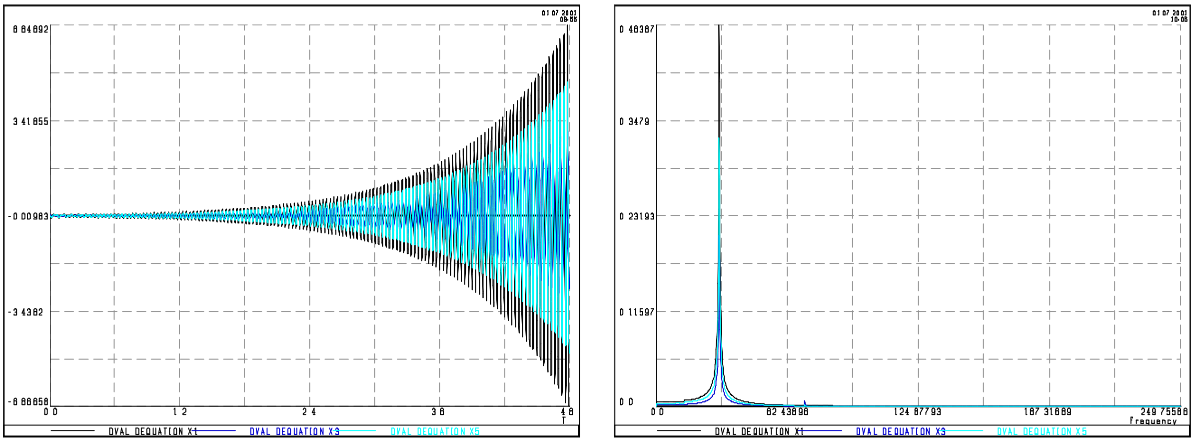


Figure 10. Vibration and its spectrum for f =71 Hz, One spectral peak at frequency 32 Hz (oil whirl frequency)

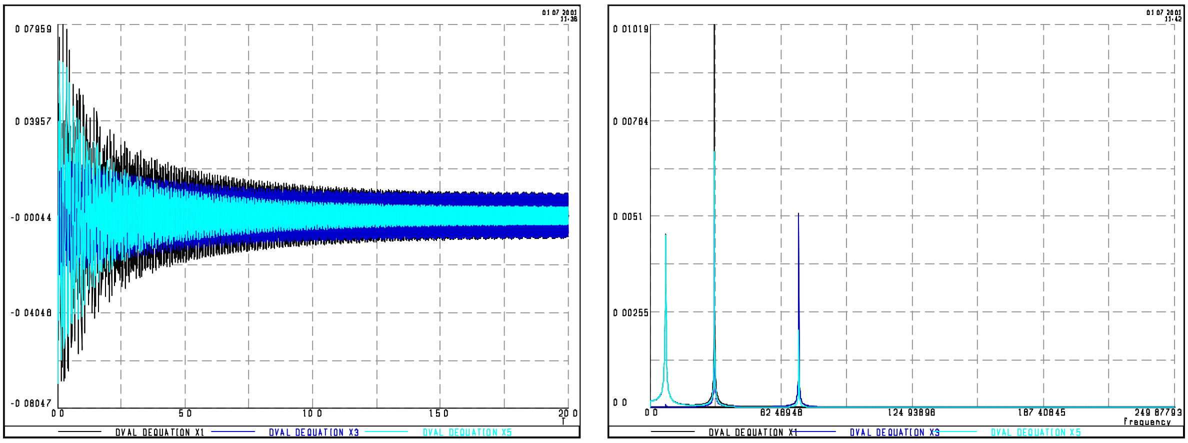


Figure 11. Vibration of the system for f = 71 Hz, but the system will be stable when the stiffness k decreases from 6000 $(rad/s)^2$ to 2000 $(rad/s)^2$

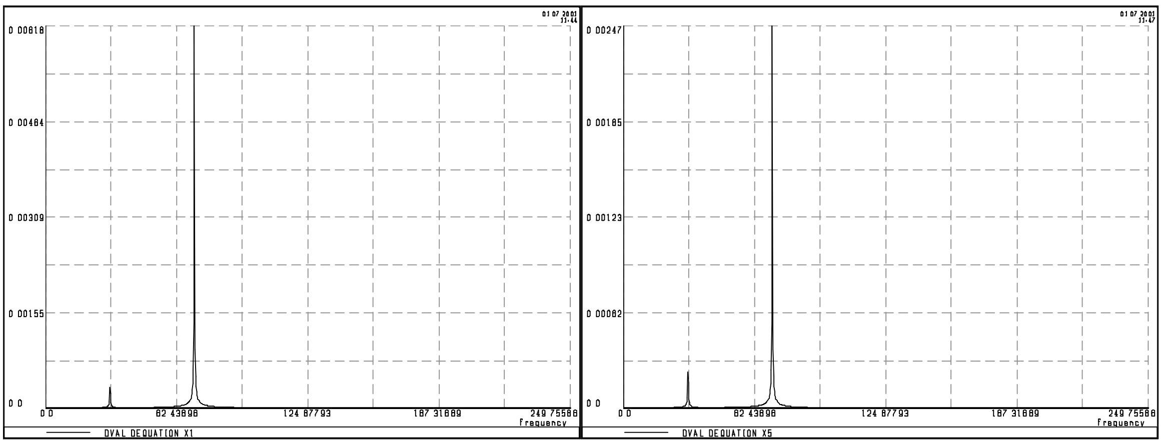


Figure 12. Vibration spectra of x_1 and x_5 for f = 71 Hz, but the system will be stable when the stiffness k decreases from 6000 $(rad/s)^2$ to 2000 $(rad/s)^2$

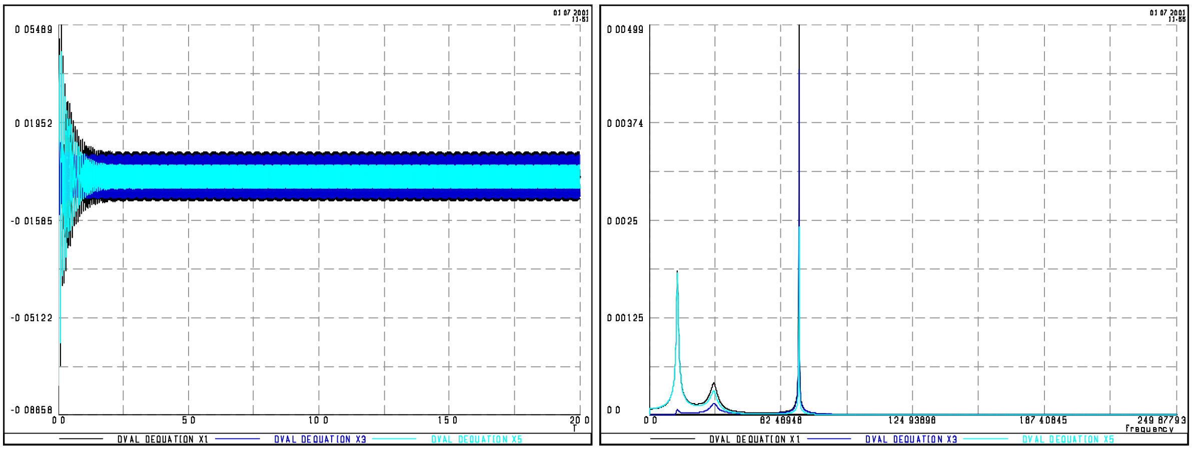


Figure 13. Vibrations and its spectra with $f = 71$. Hz, clearance $\delta = 0.00023$ m

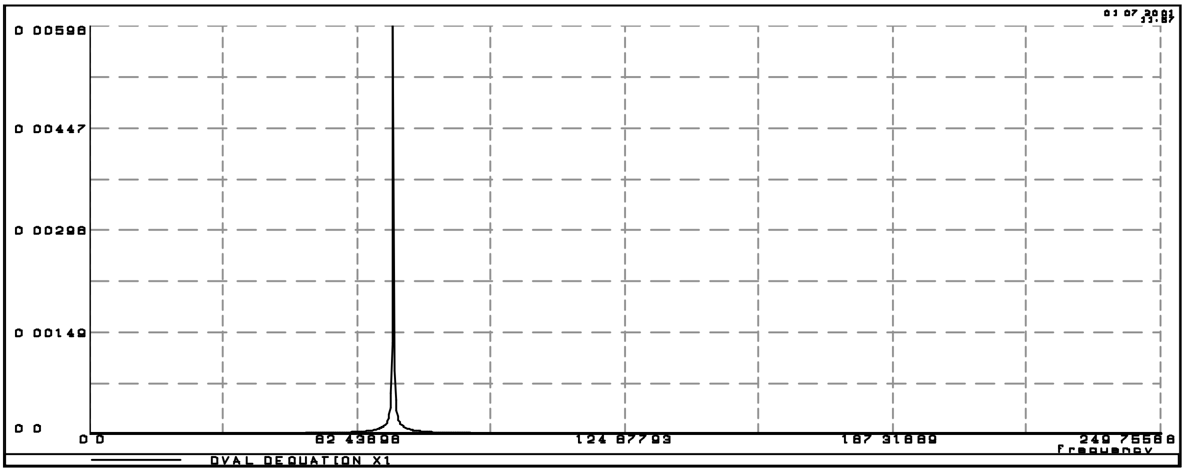


Figure 14. Spectrum of x_1 for the case $f = 71$.Hz, clearance $\delta = 0.00023$ m

5 Conclusion

In this paper, the rotor-oil film bearing system is investigated by simulation. The obtained results show that

- the vibrations of x_1 and x_5 (that means in directions x and z) have similar forms, therefore if the vibrations could be measured on the non-rotating part of the machine (for example, outside of bearings) then the processing of this measurement data can be used for the assessment of technical conditions of the rotor-journal bearing system (Figure 12).

- the rotor-bearing system is stable if in the diagram of the vibration spectra the peak at the rotation frequency is bigger than at the oil whirl frequency (about half of the rotation frequency, see Kramer (1993), Bently (1998)) (Figures 8 and 9).

- if the system is unstable by the oil whirl phenomenon, then the system could be stabilized by one of the following techniques: (i) decrease of rotation speed, (ii) decrease of clearance (iii) change stiffness of elastic base.

Acknowledgment: This publication is completed with financial support from Council for the Natural Science of Vietnam.

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