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Modeling and Dynamic Behavior of Electrostatic Tranducers with Distributed Parameters

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The dynamics of an electrostatic transducer as a 2-field system with distributed parameters is studied, here in form of a 1-dimensional capacitor composed of a flexible strip with clamped ends and a rigid plate opposite including inductance and resistance effects. Both electrical and mechanical input signals are taken into consideration, and the influence on the electrical charge between the plates as well the relative displacement of the plates is discussed in detail. First, a formulation of the governing non-linear boundary value problem is presented. Next the simplified time-independent boundary value problem is derived when only a polarization DC voltage is applied. Then, the equations of motion describing the dynamic behavior of the transducer in the neighborhood of the steady state are generated. The RITZ *method for discretization is introduced to obtain simplified equation sets for approximate solutions. Starting with the roughest 1-term truncation for both displacement and charge to be interpreted as the classical lumped-parameter transducer, the discussion is finally generalized studying truncations of higher order.*

1 Introduction

Within MEMS applications, electrostatic beam-, plate- or membrane-shaped transducers are representatives with remarkable potential (see Batra et al. (2007), for instance). Recently, some research work on statics and dynamics of such structural systems with distributed parameters has been presented (see Abdel-Rahman et al. (2002); Batra et al. (2006); Najar et al. (2005); Younis and Nayfeh (2003), for instance) but in all cases, the electrically loaded mechanical system is analyzed and not the electro-mechanical system with a full interaction between mechanical and electrical field quantities.

On the other hand, electromechanical lumped-parameter transducers are classical two-domain non-linear systems where the dynamics for the use as actuator but also as sensor is well-understood (see, e. g., Crandall et al. (1968)).

Obviously, there is a gap to be bridged in the present contribution by analyzing such an interacting electromechanical system as a distributed 2-field system. To hold the analysis relatively straightforward, a 1-parametric capacitor composed of a flexible strip (arranged along the x-axis of a Cartesian reference base) and a rigid plate opposite is considered where inductance and resistance effects are included. In general, electrical as well mechanical input signals are acting, and their influence on both the electrical charge between the plates and the relative displacement of the plates is discussed in detail where all variables are space- and time-dependent.

The paper is arranged as follows. The description of the model is presented within the first section. The formulation of the governing non-linear boundary value problem is given in the second section applying HAMILTON's principle. For the discussion within the third section, 2-term solutions for the variables displacement and charge are composed. There are time-independent components as solutions of the stationary boundary value problem when only a polarization DC voltage is applied. Superimposed are small dynamic components due to small oscillating input signals describing the dynamic behavior of the transducer in the neighborhood of the steady state. Their computation is based on the evaluation of the corresponding time-dependent boundary value problem. Then, the RITZ method for discretization is applied to obtain simplified equation sets for approximate solutions. The roughest 1-term truncation can be interpreted as the classical lumped-parameter transducer to couple a 1-degree-of-freedom mechanical sub-system and a 1-degree-of-freedom electrical circuit. Higher-order truncations finally improve the statements about the steady state, its stability, and the dynamic behavior in the neighborhood of the equilibrium.

2 Modeling

Consider an electrostatic transducer system with distributed parameters shown in Fig. 1. The structural system is modeled as an electrically conducting strip in form of a BERNOULLI-EULER beam (density ρ, length ℓ, cross-

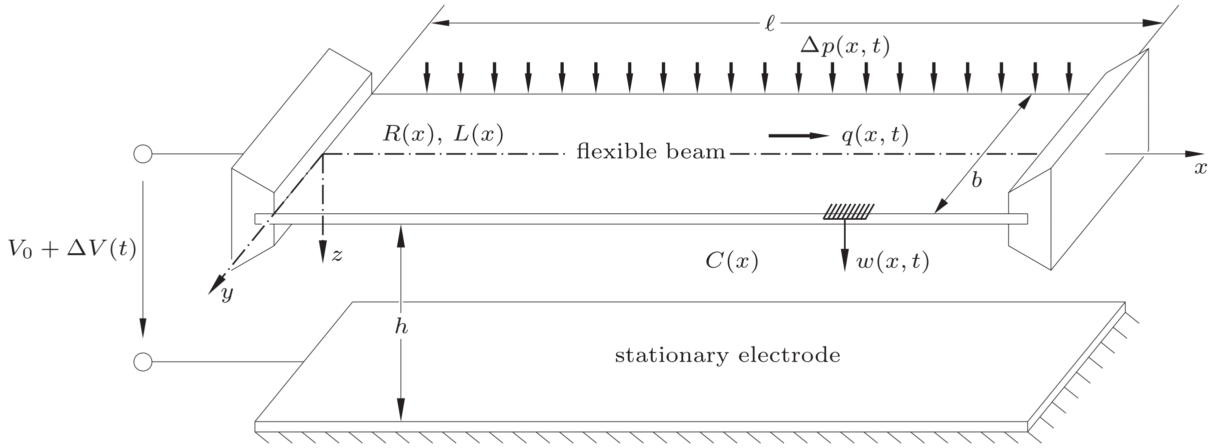


Figure 1. Phenomenological transducer model

sectional area A, and bending stiffness EI) of constant cross-sectional properties, and a uniform parallel electrode in a distance h when no mechanical and electrical input signals are acting. The originally straight beam is clamped across its long ends and free across its width. Dependencies on the coordinate y or other plate effects characterized by POISSON's ratio are not taken into account here. The system composes a capacitor of capacitance per unit length C (with dielectric constant ε of the gap medium where its mechanical properties as a fluid will be excluded). On the other hand, all electrical effects will be included, i.e., it is assumed that beside the capacitance C influenced by the deflection w, there are distributed inductance $L(x)$ and resistance $R(x)$ where for both the case of locally concentrated values by introducing appropriate DIRAC functions is included. As electric variable the charge density $q(x,t)$ is used related to the electric current density j by $q = \int j\mathrm{d}t$ which for the problem in hand – following Crandall et al. (1968) and formulations leading to the so-called telegraph equation – seems to be appropriate. As mechanical input, a small space- and time-dependent distributed load per unit length $\Delta p(x,t)$ is acting, and there is also an electrical input in form of a voltage source generating a DC voltage V_0 and a superimposed AC part $\Delta V(t)$ to be sufficiently small compared to V_0.

3 Formulation

The basis for a synthetic derivation are MAXWELL's equations and the balance of momentum with corresponding boundary conditions. For an analytic approach, a generalized form of HAMILTON's principle

$$\delta \int_{t_0}^{t_1} (T^* - U + W_\mathrm{m}^* - W_\mathrm{e})\mathrm{d}t + \int_{t_0}^{t_1} W_\mathrm{virt}\mathrm{d}t = 0 \tag{1}$$

is applied where for a 1-dimensional electroconductive structural member, a straightforward formulation can be given. Partial derivatives with respect to time t and position coordinate x are denoted by $(\,.\,)_{,t}$ and $(\,.\,)_{,x}$, respectively.

The kinetic (co-)energy T^* and the potential energy U are given as

$$T^* = \frac{\rho A}{2}\int_0^\ell w_{,t}^2\mathrm{d}x, \quad U = \frac{EI}{2}\int_0^\ell w_{,xx}^2\mathrm{d}x + \int_0^\ell \frac{N}{2} w_{,x}^2\mathrm{d}x \tag{2}$$

where

$$N = N_0 + \frac{EA}{2\ell}\int_0^\ell w_{,x}^2\mathrm{d}x \tag{3}$$

represents a constant tensile (or compressive) axial force and the mid-plane stretching. The magnetic co-energy W_{m}^* and electric energy W_{e} are

$$W_{\mathrm{m}}^* = \int_0^\ell \frac{L}{2}\, q_{,t}^2 \mathrm{d}x, \quad W_{\mathrm{e}} = \int_0^\ell \frac{q_{,x}^2}{2C(w)} \mathrm{d}x \tag{4}$$

where

$$C(w) = \frac{\varepsilon b}{h - w(x,t)} \tag{5}$$

is a nonlinear function of the transverse displacement w to couple the mechanical displacement field $w(x,t)$ and the electric charge $q(x,t)$. The virtual work is given by the mechanical part

$$W_{\mathrm{virt1}} = \int_0^\ell \left[\Delta p\,(x,t) - k_{\mathrm{e}} \rho A w_{,t}\right] \delta w \mathrm{d}x \tag{6}$$

to take into consideration not only the mechanical input but also a velocity-dependent mass-proportional external damping (parameter k_{e}) as the simplest mechanical dissipation mechanism, and the electric part

$$W_{\mathrm{virt2}} = \int_0^\ell \left[(v_0(x) + \Delta v(x,t)) - R q_{,t}\right] \delta q \, \mathrm{d}x \tag{7}$$

characterizing the electric input and the electric dissipation due to the resistance R. To be more general, the voltage source $V_0 + \Delta V(t)$ is formulated as a distributed voltage density $v_0(x) + \Delta v(x,t)$ containing the locally concentrated electric input as a special case.

Evaluating HAMILTON's principle (1) using all energy and work contributions (2) – (7) leads to the governing boundary value problem composed of the two coupled non-linear field equations

$$\rho A w_{,tt} + k_{\mathrm{e}} \rho A w_{,t} + EI w_{,xxxx} - N w_{,xx} - \frac{q_{,x}^2}{2C_0 h} = \Delta p\,(x,t) \tag{8}$$

$$L q_{,tt} + R q_{,t} - \frac{1}{C(w)}\, q_{,xx} = v_0(x) + \Delta v(x,t) \tag{9}$$

where $C_0 = \varepsilon b/h$ and the corresponding boundary conditions

$$w(0,t) = 0,\ \ w_{,x}(0,t) = 0,\ \ w(\ell,t) = 0,\ \ w_{,x}(\ell,t) = 0 \tag{10}$$

$$q_{,x}(0,t) = 0,\ \ q_{,x}(\ell,t) = 0. \tag{11}$$

Obviously, there is a quadratic force law (see last term at the left-hand side of the mechanical field equation (8)) and a quasi-linear law of motion (see last term at the left-hand side of the second field equation (9)) which is typical for electrostatic transducers (see Crandall et al. (1968), for instance). To establish a transducer which can be used in both directions as a sensor as well an actuator, the presented 2-domain description is the minimal formulation to explain all functions within a closed concept.

For convenience, non-dimensional coordinates

$$\hat{x} = \frac{x}{\ell},\ \ \hat{t} = \sqrt{\frac{EI}{\rho A \ell^4}}\, t \tag{12}$$

and variables

$$\hat{w} = \frac{w}{h},\ \ \hat{q} = \frac{\ell}{h\sqrt{C_0 EI}}\, q \tag{13}$$

are introduced so that several characteristic non-dimensional key figures

$$d_{\mathrm{e}} = \sqrt{\frac{\rho A \ell^4}{EI}}\, k_{\mathrm{e}},\ \ n = \frac{\ell^2}{EI}\, N,\ \ l = \frac{C_0 EI}{\rho A \ell^2}\, L,\ \ r = C_0 \sqrt{\frac{EI}{\rho A}}\, R \tag{14}$$

and input signals

$$\Delta\hat{p} = \frac{\ell^4}{hEI}\Delta p,\ \ \hat{v}_0 = \frac{C_0\ell^3}{h\sqrt{C_0EI}}\,v_0,\ \ \Delta\hat{v} = \frac{C_0\ell^3}{h\sqrt{C_0EI}}\,\Delta v \tag{15}$$

may be defined. Introducing all of them into the boundary value problem (8) – (11) yields

$$\ddot{\hat{w}} + d_{\rm e}\dot{\hat{w}} + \hat{w}'''' - n\hat{w}'' - \frac{1}{2}\,\hat{q}'^{\,2} \;=\; \Delta\hat{p}\,(\hat{x},\hat{t}\,) \tag{16}$$

$$l\ddot{\hat{q}} + r\dot{\hat{q}} - (1-\hat{w})\hat{q}'' \;=\; \hat{v}_0(\hat{x}) + \Delta\hat{v}(\hat{x},\hat{t}\,) \tag{17}$$

$$\hat{w}(0,\hat{t}\,) = 0,\ \ \hat{w}'(0,\hat{t}\,) = 0,\ \ \hat{w}(\hat{\ell},\hat{t}\,) = 0,\ \ \hat{w}'(\hat{\ell},\hat{t}\,) \;=\; 0 \tag{18}$$

$$\hat{q}'(0,\hat{t}\,) = 0,\ \ \hat{q}'(\hat{\ell},\hat{t}\,) \;=\; 0 \tag{19}$$

where $(\dot{\ \cdot\ }) = (\ .\ )_{,\hat{t}}$ and $(\ .\ )' = (\ .\ )_{,\hat{x}}$.

4 Discussion

A complete analytical solution of the governing non-linear boundary value problem (16) – (19) does not exist. The beam deflection $\hat{w}(\hat{x},\hat{t}\,)$ and the electric charge $\hat{q}(\hat{x},\hat{t}\,)$ are composed of time-independent components $\hat{w}_0(\hat{x})$ and $\hat{q}_0(\hat{x})$ due to the polarization DC voltage $\hat{v}_0(\hat{x})$ as the only input, and additional small dynamic components $\Delta\hat{w}(\hat{x},\hat{t}\,)$ and $\Delta\hat{q}(\hat{x},\hat{t}\,)$ due to the AC voltage $\Delta\hat{v}(\hat{x},\hat{t}\,)$ and the mechanical excitation $\Delta\hat{p}\,(\hat{x},\hat{t}\,)$:

$$\hat{w}(\hat{x},\hat{t}\,) \;=\; \hat{w}_0(\hat{x}) + \Delta\hat{w}(\hat{x},\hat{t}\,) \tag{20}$$

$$\hat{q}(\hat{x},\hat{t}\,) \;=\; \hat{q}_0(\hat{x}) + \Delta\hat{q}(\hat{x},\hat{t}\,). \tag{21}$$

To calculate the equilibrium state, the time derivatives and the fluctuating forcing terms are set equal to zero in eqs (16) – (19) to obtain the time-independent boundary value problem

$$\hat{w}_0'''' - n_0\hat{w}_0'' - \frac{1}{2}\hat{q}_0'^2 \;=\; 0 \tag{22}$$

$$-(1-\hat{w}_0)\hat{q}_0'' \;=\; \hat{v}_0(\hat{x}) \tag{23}$$

$$\hat{w}_0(0) = 0,\ \ \hat{w}_0'(0) = 0,\ \ \hat{w}_0(\hat{\ell}\,) = 0,\ \ \hat{w}_0'(\hat{\ell}\,) \;=\; 0 \tag{24}$$

$$\hat{q}_0'(0) = 0,\ \ \hat{q}_0'(\hat{\ell}\,) \;=\; 0 \tag{25}$$

where $n_0 = n(w_0)$. To generate the equations of motion describing the dynamic behavior of the transducer around the steady state, the ansatz (20), (21) is substituted into the original boundary value problem (16) – (19) dropping the terms representing the equilibrium state according to eqs (22) – (25). Since small oscillating input signals have been assumed, the equations of motion may additionally be linearized resulting in

$$\Delta\ddot{\hat{w}} + d_{\rm e}\Delta\dot{\hat{w}} + \Delta\hat{w}'''' - n_0\Delta\hat{w}'' - \hat{q}_0'\Delta\hat{q}\,' \;=\; \Delta\hat{p}(\hat{x},\hat{t}\,) \tag{26}$$

$$l\Delta\ddot{\hat{q}} + r\Delta\dot{\hat{q}} - (1-\hat{w}_0)\Delta\hat{q}\,'' + \hat{q}_0''\Delta\hat{w} \;=\; \Delta\hat{v}(\hat{x},\hat{t}\,) \tag{27}$$

$$\Delta\hat{w}(0,\hat{t}\,) = 0,\ \ \Delta\hat{w}'(0,\hat{t}\,) = 0,\ \ \Delta\hat{w}(\hat{\ell},\hat{t}\,) = 0,\ \ \Delta\hat{w}'(\hat{\ell},\hat{t}\,) \;=\; 0 \tag{28}$$

$$\Delta\hat{q}\,'(0,\hat{t}\,) = 0,\ \ \Delta\hat{q}\,'(\hat{\ell},\hat{t}\,) \;=\; 0. \tag{29}$$

The present linear boundary value problem describes the classical transducer operation in two directions. To specify the transducer equations for sensor operations, the electric AC input has set to zero while the mechanical input contains the mechanical property to be measured which can be realized by measuring the potential drop at the resistor. On the other hand, if the transducer is used as an actuator, the mechanical input signal has set to zero while the AC voltage generates a force action due to the charge displacement.

It will be noticed that also non-linear effects of the dynamic behavior might be interesting. Then, the non-linear boundary value problem in the Δ-quantities has to be examined which within a simplified actuator description neglecting resistance and inductance effects is discussed by Younis and Nayfeh (2003), for instance.

To find approximate solutions for the equilibrium state but also for the linear transducer operation, the GALERKIN method, for example, can be applied by using a series expansion

$$\hat{w}_0(\hat{x}) = \sum_{i=1}^{N_1} a_i W_{0i}(\hat{x}), \quad \hat{q}_0(x) = \sum_{i=1}^{N_2} b_i Q_{0i}(\hat{x}) \tag{30}$$

and

$$\Delta\hat{w}(\hat{x},\hat{t}\,) = \sum_{i=1}^{M_1} y_i(\hat{t}\,) W_i(\hat{x}), \quad \Delta\hat{q}(\hat{x},\hat{t}\,) = \sum_{i=1}^{M_2} z_i(\hat{t}\,) Q_i(\hat{x}) \tag{31}$$

(where in general, N_1, N_2 and M_1, M_2 may be different) to be substituted into the weak formulation of the boundary value problems (22) – (25) and (26) – (29), respectively. The space-dependent shape functions $W_{i0}(\hat{x})$, $Q_{0i}(\hat{x})$ and $W_i(\hat{x})$, $Q_i(\hat{x})$ have to fulfill all boundary conditions, and the evaluation leads for a certain truncation to a corresponding system of coupled equations determining the coefficients a_i, b_i or time functions $y_i(\hat{t}\,), z_i(\hat{t}\,)$.

The roughest $N_1 = N_2 = 1$- and $M_1 = M_2 = 1$-term truncation yields in each case a set of 2 electromechanical equations, i. e., the (non-linear) algebraic equations

$$\alpha_{11}a_1 + \alpha_{12}b_1^2 = 0 \tag{32}$$
$$\alpha_{12}a_1b_1 + \alpha_{22}b_1 = u_{10} \tag{33}$$

to determine the equilibrium state, and the (linear) ordinary differential equations

$$\varepsilon_{11}\ddot{y}_1 + \gamma_{11}\dot{y}_1 + \beta_{11}y_1 + \beta_{12}z_1 = s_1(\hat{t}\,), \tag{34}$$
$$\varepsilon_{22}\ddot{z}_1 + \gamma_{22}\dot{z}_1 + \beta_{21}y_1 + \beta_{22}z_1 = t_1(\hat{t}\,) \tag{35}$$

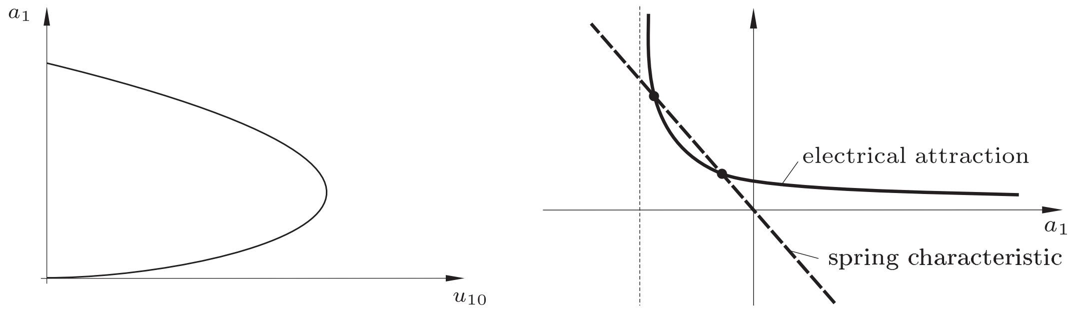
to describe the transducer operation which – neglecting inductance and resistance effects, i.e., $\varepsilon_{22}, \gamma_{22} = 0$ – is equivalent to the formulation of Crandall et al. (1968). The coefficients $\alpha_{ij}, \varepsilon_{ij}, \gamma_{ij}, \beta_{ij}$ are defined by the corresponding non-dimensional characteristic numbers d_e, n, l, r together with appropriate definite integrals over the shape functions:

$$\alpha_{11} = \int_0^1 (W_{01}'''' - n_0 W_{01}'')W_{01}\mathrm{d}\hat{x}, \;\; \alpha_{12} = -\frac{1}{2}\int_0^1 Q_{01}'^2 W_{01}\mathrm{d}\hat{x} \tag{36}$$
$$\alpha_{21} = \int_0^1 W_{01}Q_{01}''Q_{01}\mathrm{d}\hat{x}, \;\; \alpha_{22} = -\int_0^1 Q_{01}''Q_{01}\mathrm{d}\hat{x} \tag{37}$$
$$\varepsilon_{11} = \int_0^1 W_1^2\mathrm{d}\hat{x}, \;\; \gamma_{11} = d_\mathrm{e}\varepsilon_{11}, \;\; \beta_{11} = \int_0^1 (W_1'''' - n_0 W_1'')W_1\mathrm{d}\hat{x} \tag{38}$$
$$\beta_{12} = -\int_0^1 \hat{q}_0' Q_1' W_1\mathrm{d}\hat{x}, \;\; \beta_{21} = \int_0^1 \hat{q}_0'' W_1 Q_1\mathrm{d}\hat{x} \tag{39}$$
$$\varepsilon_{22} = l\int_0^1 Q_1^2\mathrm{d}\hat{x}, \;\; \gamma_{22} = \frac{r}{l}\varepsilon_{22}, \;\; \beta_{22} = -\int_0^1 (1-\hat{w}_0)Q_1''Q_1\mathrm{d}\hat{x}. \tag{40}$$

The right-hand side terms u_{10} and $s_1(\hat{t}\,), t_1(\hat{t}\,)$ are the weighted and spatially averaged DC voltage and time-dependent electrical as well mechanical transducer inputs, respectively:

$$u_{10} = \int_0^1 \hat{v}(\hat{x})Q_{01}\mathrm{d}\hat{x} \tag{41}$$
$$s_1(\hat{t}\,) = \int_0^1 \Delta\hat{p}(\hat{x},\hat{t}\,)W_1\mathrm{d}\hat{x}, \;\; t_1(\hat{t}\,) = \int_0^1 \Delta\hat{v}(\hat{x},\hat{t}\,)Q_1\mathrm{d}\hat{x}. \tag{42}$$

Discussing the equilibrium configuration, the corresponding non-linear boundary value problem (22) – (25) or truncated subsets, e. g., equation set (32),(33) as the roughest approximation, are extensively discussed by Abdel-Rahman et al. (2002); Younis and Nayfeh (2003), and others mentioned there. The result of main interest is shown in Fig. 2 where in Fig. 2a the deflection of the flexible beam at a certain location (corresponding to a_1



a.) Maximum deflection. b.) Balance of the characteristics.

Figure 2. Equilibrium configuration

within the 1-term approximation) is plotted qualitatively versus DC voltage (corresponding to u_{10} within the 1-term truncation). Clearly, the so-called pull-in instability is brought out, i. e., that value of voltage which produces a collapse of the system in such a form that the flexible strip snaps-through to contact the opposite electrode.

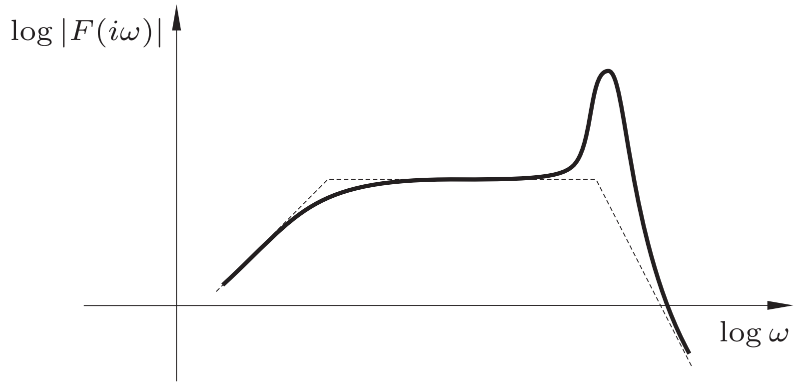


Figure 3. Frequency response of an acceleration sensor

Alternatively, see Crandall et al. (1968), it is possible to plot the spring characteristic and the electrical attraction versus the distance of the electrodes represented by the variable a_1 shown in Fig. 2b which can also explain these phenomena considering the stability of the two possible equilibrium states.

On the other hand, the linear transducer equations (26) – (29) or (34),(35) are the starting point to discuss the system as sensor or actuator for the case of periodically oscillating mechanical or electrical inputs. Fig. 3 shows a representation characterizing an acceleration sensor, for instance (taken from Crandall et al. (1968) describing the frequency response of a electrostatic microphone). In that case, the mechanical input has to be expressed by the acceleration variable to be measured, the corresponding frequency response can be checked such the voltage drop at the resistor leads to a true indication of the measuring instrument. Even the 1-term truncation verifies that for a true acceleration measurement the system has to be tuned in such a way that the electrical cut-off frequency has to be significantly lower and the mechanical eigenfrequency significantly higher than the frequency range of the signal to be measured.

While the 1-term approximation for displacement and charge (both for the steady state and the dynamics) can obviously be examined analytically, higher-order truncations can only be handled by iterative methods (statics) and computer simulations (vibrations).

5 Conclusion

To understand the 2-way interaction of a distributed electrostatic transducer and its specification as a sensor and an actuator, the simplest non-linear formulation for such a system has been presented.

The mechanical and electrical properties have been incorporated in such a way that a flexible beam as one of the electrodes of the resulting capacitor together with inductance and resistance effects characterizing the corresponding electric circuit have been assembled. The deflection of the beam and the charge within the gap medium together with a DC/AC voltage source and a corresponding time-dependent mechanical input may run the transducer in it two possible (direct and inverse) functions.

It is verified that the 1-dimensional distributed parameter system can describe not only the actuator specification discussed during the recent past by Abdel-Rahman et al. (2002); Batra et al. (2006, 2007); Najar et al. (2005); Younis and Nayfeh (2003) but is much more general to come to a comprehensive study of an electrostatic transducer operating as sensor or actuator supplementing the well-known study in Crandall et al. (1968) in a similar manner as some years ago in Wauer (1997) for piezoelectric transducers. Even for treating the very actual topic of energy harvesting (see, e. g., Ramlan et al. (2008)), the system description presented is an appropriate starting point.

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