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Control of Program Motion of Dynamic Systems Using Relative Motions

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In this paper, the problem of constructing one of the kind of program motion is discussed. The motion program is a set of conditions imposed on the behavior of phase trajectories of the particles of the system, by which the positions and velocities of the system under consideration cannot be arbitrary. In other words, the system of program motion belongs to the systems with constraints. This means that it is possible to treat the program as a set of constraints that restrict the motion of the system, in the sense of analytical mechanics. In this paper, these cases are discussed. The passive control system is designed using only relative motion. For better results, an active control system can be used. However an instability of system could appear. In such case some methods of a feedback controller system using sensors can be used to achieve a stable controlled system.

1 Introduction

In the development of today's technology, almost all of the engineering problems are related to program motion control. As it is commonly known, a dynamic system performs a program motion as it is exerted by appropriate forces from actuators which are driven by controllers. If a dynamic system receives external energy to power control actuators, it is called an active control system. Conversely, a passive control system varies its energy by changing its geometric properties such as the center of mass or the moment of inertia or its dissipation and elasticity parameters such as damping and spring coefficients. From the energy point of view, an active control system uses external forces to supplement additional energy to perform the prescribed program motion. A passive control system uses internal forces to vary its subsystem motion which causes to adjust its energy in order to carry out the desired program motion. The advantage of the active control approach is creating fast dynamic response and a direct solution of this problem (motions caused by forces). However, the supplementary energy may cause the system to become unstable. Moreover, to make actuators deliver the required forces sometimes can cause technically difficult problems (e.g. due to actuators' saturation) or can cause impulse motion (impact). In contrast, the advantage of the passive control approach is to make use of interactive motions among components of the system to adjust energy smoothly, and to eliminate impulse effects through dampers and springs. However, this approach hardly creates the required forces to meet the program motion due to inertial effects and the delay of dynamic responses. Recently, there appears the hybrid approach, a semi-active control method, which tries to combine the benefits of the two above approaches.

In the paper, the passive control method is used to force the dynamic system to follow the program motion through the relative motion control of a subsystem. In other words, the subsystem is used as a controller to force the mother system to perform the desired program motion. The motion of the subsystem indirectly changes the motion of the mother system. This kind of problem inherently embodies the synthesis property.

Let us consider a program of motion as a relation between the time, coordinates, velocities, and accelerations, which implies requirements on the behavior of the solutions of the equations of motion of the considered systems. A set of mathematical expressions describing a program is called a manifold. Mathematically, manifolds are similar to mechanic constraints. However, mechanical constraints are created by physical interactions among objects, whereas motion programs are simply imaginary restrictions on dynamic motion behavior of the system. Commonly, mechanical constraints are known as physical constraints to distinguish from motion programs.

2 The Construction of Program Motion Through Relative Motions

Let's consider a dynamic system. Its position is located by the generalized coordinates q_i ($i=\overline{1,m}$), subject to r ideal constraints in the forms

$$\mathbf{f}\ddot{\mathbf{q}} + \mathbf{f_o} = 0 , \tag{2-1}$$

where $\mathbf{f}$, $\ddot{\mathbf{q}}$ and $\mathbf{f_o}$ are matrices, the sizes of which are *r*x*m*, *m*x*1*, and *r*x*1*, respectively.

The stationary holonomic constraints with redundant coordinates and stationary linear nonholonomic constraints are written in the above mentioned form, where the elements of matrix $\mathbf{f}$ are functions of $\mathbf{q}$ and the elements of matrix $\mathbf{f_o}$ are functions of $\mathbf{q}$ and $\dot{\mathbf{q}}$. In the case of unstationary nonlinear nonhohonomic constraints of first order these elements depend on coordinates, velocities and time.

For the aim of simplicity we will only consider the system with the stationary constraints.

The purpose of this paper is to design a subsystem mounting on the original dynamic system such that the motion of the original system can be controlled by the motion of the subsystem to perform the program motion in the form

$$\mathbf{g}_\alpha(t,\mathbf{q}) = 0 \quad ; \quad \alpha = \overline{1,s} \tag{2-2}$$

The original system and the subsystem are named as the mother system and the child system, respectively. For simplicity, the child system is holonomic and its position is determined by holonomic generalized coordinates $u_\alpha (\alpha = \overline{1,s})$, which are independent to each other and to the generalized coordinates $q_i (i = \overline{1,m})$. Let's denote the control forces subject to the subsystem as U_γ ($\gamma = \overline{1,p}$), where p depends on the requirements of the control problem.

As mentioned above, the mechanical constrains are assumed to be stationary, the kinetic energy of the system is of the forms

$$T = \frac{1}{2}\sum_{i,j=1}^{m} a_{ij}\dot{q}_i\dot{q}_j + \frac{1}{2}\sum_{\alpha\beta}^{s} b_{\alpha\beta}\dot{u}_\alpha\dot{u}_\beta + \sum_{i}^{m}\sum_{\alpha}^{s} c_{i\alpha}\dot{q}_i\dot{u}_\alpha , \tag{2-3}$$

and we can write it in the matrix forms as

$$T = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{A}\dot{\mathbf{q}} + \frac{1}{2}\dot{\mathbf{u}}^{\mathbf{T}}\mathbf{B}\dot{\mathbf{u}} + \dot{\mathbf{q}}^{\mathbf{T}}\mathbf{C}\dot{\mathbf{u}} , \tag{2-4}$$

where $\mathbf{A}$ and $\mathbf{B}$ are symmetric quadratic matrices of dimension of m and s respectively, whose elements are functions of $\mathbf{q}$ and $\mathbf{u}$ only, $\mathbf{C}$ is a $m \times s$ matrix. By means of the Principle of Compatibility, the equations of motion of the whole system (the mother-child system) take the following forms

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = Q_{qi} + R_{qi} \qquad (i = \overline{1,m}) , \tag{2-5}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{u}_\alpha} - \frac{\partial T}{\partial u_\alpha} = Q_{u\alpha} + R_{u\alpha} \qquad (\alpha = 1,s) , \tag{2-6}$$

where

- R_{qi} and $R_{u\alpha}$ are generalized forces corresponding to the mechanical constraints.
- Q_{qi} and $Q_{u\alpha}$ ($i = \overline{1,m}; \alpha = \overline{1,s}$) are generalized forces of applied forces corresponding to generalized coordinates q_i and u_α.($i = \overline{1,m}; \alpha = \overline{1,s}$). Control forces U_γ ($\gamma = \overline{1,p}$) are included in $\mathbf{Q}_{u\alpha}$ ($\alpha = \overline{1,s}$).

Let's define the independent generalized coordinates of the mother system as $q_k, \left(k = \overline{1,n}\ ; n = m - r\right)$. To build the condition of ideality of the constraints, one first expresses the generalized accelerations $\ddot{q}_i$ ($i=\overline{1,m}$) in the terms of the independent generalized accelerations $\ddot{q}_k$ ($k = \overline{1,n}$) and the child system's generalized accelerations $\ddot{u}_\alpha$ ($\alpha=\overline{1,s}$) by the constraints (2-1). In such a way, we have

$$\ddot{q}_i = \sum_{k=1}^{n} d_{ik}\ddot{q}_k + ... , \tag{2-7}$$

where the unwritten terms do not include generalized accelerations The matrix $\mathbf{D}^{\mathrm{T}}$ of size (m+s) x (n+s) takes the forms quantities the

$$\mathbf{D}^T = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1,n+s} \\ d_{21} & d_{22} & \dots & d_{2,n+s} \\ \dots & \dots & \dots & \dots \\ d_{m+s,1} & d_{m+s,2} & \dots & d_{m+s,n+s} \end{bmatrix} \tag{2-8}$$

Notice that some elements in $\mathbf{D}^T$ take unit values. As is known, the condition of ideality of the constraints (2-1) can be expressed in the form

$$\mathbf{DR} = \mathbf{0} \tag{2-9}$$

Obviously, the generalized forces corresponding to the generalized coordinates absent in (2-7) are equal to zero. Accordingly, one easily notices that

$$\mathbf{R}_u = \mathbf{0}, \tag{2-10}$$

where the matrix $\mathbf{R}_u$ of size sx1 takes the forms

$$\mathbf{R}_u = \begin{bmatrix} R_1 & \dots & R_s \end{bmatrix}^T \tag{2-11}$$

The equations (2-5) and (2-6) can be written in the matrix forms as

$$\mathbf{A\ddot{q} + C\ddot{u} = Q^q + G^q + R^q} \tag{2-12}$$

$$\mathbf{C\ddot{q} + B\ddot{u} = Q^u + G^u}, \tag{2-13}$$

where

- $\mathbf{Q^q}$, and $\mathbf{Q^u}$ are the matrices of the generalized forces of the applied forces corresponding to the generalized coordinates **q** and **u**.
- $\mathbf{G^q}$, and $\mathbf{G^u}$ are the matrices built from the rest part of the equations.

Based on the condition of ideality of the constraints (2-1), the equations (2-12) and (2-13) turn into the forms

$$\mathbf{A^o\ddot{q} + C^o\ddot{u} = Q}^o \tag{2-14}$$

$$\mathbf{C\ddot{q} + B\ddot{u} = U^o}, \tag{2-15}$$

where

$$\mathbf{A^o = D\,A}\ ,\ \mathbf{Q}^o = \mathbf{D(Q + G^q)}\ ,\ \mathbf{U^o = (Q^u + G^u)}, \mathbf{C^o = DC}. \tag{2-16}$$

Notice that the system of equations (2-14), (2-1) and (2-2) is a complete one ($m+s$ equations with $m+s$ unknowns, q_i and u_α, $i = \overline{1,m}$; $\alpha = \overline{1,s}$). The control forces are included in the terms of $\mathbf{U^o}$.
Now solve the system of equations (2-1), (2-2) and (2-14) with the following initial conditions:

$$q(t_o) = q_o,\ \dot{q}(t_o) = \dot{q}^o, u(t_o) = u^o,\ \dot{u}(t_o) = \dot{u}^o\ , \tag{2-17}$$

we have

$$q_i = q_i(t),\ u_\alpha = u_\alpha(t),\ \dot{q}_i = \dot{q}(t),\ \text{and}\ \dot{u}_\alpha = \dot{u}(t) \tag{2-18}$$

The control forces $\boldsymbol{U}_\gamma$ ($\gamma = \overline{1,p}$) are determined by substituting expressions in (2-18) into equations (2-15). The problem is said to be complete if the number of control forces is equal to the number of motion programs. If the number of control forces is greater than that of motion programs, additional constraints should be supplemented into the mother system.

3 The Controller

If we use only relative motion *u(t)* as a control input, the control system is passive. For better performance, we can use active controllers. In this paper, two approaches are applied to design the controller for the system. The first one is an open loop controller. As the illustration in the next section, the control force *F*(*t*) exerts on the slider. By this approach, the relationship between input and the resultant state is the equations of motion of the system (including additional constraints). The desired state is obtained by solving directly the equations of motion (in DAE form). In contrast to the passive controller, the active controller can cause the system to become unstable. Hence a closed loop controller is recommended for getting stability or obtaining more accurate and more adaptive control. In this second case, both the control force *F(t)* and the moment *M*(*t*) vary. One of the nonlinear design methods is applied as Feedback Linearization (FBL) method. The control laws are assumed using full state feedback (no observers). The main idea of this method is trying to transform a nonlinear system into a linear system by coordinate transformation and applying linear control design methods to the new linear system (Jean- Jacques E. Slotine; Weiping Li, 1991). The likewise pseudo inverse method will be applied to find the control forces **U**. If the mass matrix **M** is invertible, we have:

$$\ddot{\mathbf{q}} = \mathbf{M}(\mathbf{q})^{-1}(\mathbf{f}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{BU}) \tag{3-1}$$

Linearize the system of equations (3-1) as follows:

$$\ddot{\mathbf{q}} = \mathbf{v}, \tag{3-2}$$

where $\mathbf{v} = \ddot{\mathbf{q}}_d - \mathbf{K}_1(\dot{\mathbf{q}} - \dot{\mathbf{q}}_\mathbf{d}) - \mathbf{K}_2(\mathbf{q} - \mathbf{q}_\mathbf{d})$. Here $\mathbf{K_1}$, and $\mathbf{K_2}$ are diagonal constant matrices that make the following system of equations stable:

$$\ddot{\mathbf{e}} + \mathbf{K_1}\dot{\mathbf{e}} + \mathbf{K_2}\mathbf{e} = \mathbf{0} \tag{3-3}$$

where **e** is the error vector, $\mathbf{e = q - q_d}$, **q** are state variables and $\mathbf{q_d}$ are desired state variables.
The control forces are calculated from equations (3-1) and (3-2) as follows:

$$\mathbf{U} = \mathbf{B}^+ \{\mathbf{Mv} - \mathbf{f}(\mathbf{q},\dot{\mathbf{q}})\} \tag{3-4}$$

where $\mathbf{B}^+$ is the pseudo inverse matrix of **B.**

4 An Illustrative Example

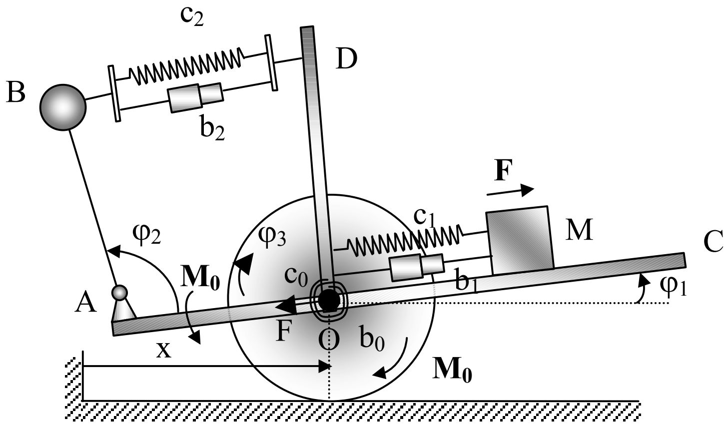

Figure 4.1. The model of the single-wheel vehicle

Consider a single-wheel vehicle moving along a straight horizontal road under acting the couple of the moment $M_0(t)$ exerted on the wheel of mass m_3 and radius r rolling without slip as sketched in Figure 4.1. The floorboard is subjected the couple of moment $M_0(t)$ in opposite direction. The mass of motor attached to the floorboard is neglected. The coefficient of rolling friction is a. A worker stands in the rear of the vehicle, holding a handle fixed at a height of L_2 from the floorboard. The worker-handle system is modeled as a massless rod AB of length L_2 rotated about joint A, a particle B of mass m_2 at the tip of the rod, and a system of a spring and viscous dashpot with spring constant c_2 and dashpot constant b_2 , respectively. The system AOC + OD + motor has the mass m_0 and the inertial moment J_0 about the common centre of mass of the system at point O. The system, supported by a bearing of dashpot constant b_0 and restrained by a torsional spring of constant c_0, is controlled by a subsystem in order to keep it in the horizontal balance. Note one of the ends of the torsional spring is connected to the center O of the wheel and the second one is connected to the floorboard AC. The subsystem consists of a slider M of mass m_1 and a spring of constant c_1 and a dashpot of constant b_1. The position of M is controlled by a controller device (not shown in the figure) that drives the force $F(t)$ acting on the slider on aligned direction with the floorboard AC and the force being equal in opposite to the $F(t)$ on the OD.
The generalized coordinates are x, φ_1, φ_2, φ_3 and u which respectively are the horizontal displacement of the vehicle, the angular displacement of the floorboard from the horizontal, the angular displacement of rod AB from the horizontal, the angular displacement of the wheel, and the displacement of the slider M with respect to the floorboard (the relative displacement). The slider M plays a part in the passive control. The control objectives are to keep the floor AC balanced ($\varphi_1 = 0, \dot{\varphi}_1 = 0, \text{and} \ddot{\varphi}_1 = 0$), the worker vibrates as little as possible ($\varphi_2 \cong \pi/2$), the variation of vehicle's velocity is little and the displacement u is in the valid range.
The following notations are used in the paper from now on
$\cos\varphi_i \equiv C_i$, $\sin\varphi_i \equiv S_i$, $\cos(\varphi_i + \varphi_j) \equiv C_{ij}$, $\sin(\varphi_i + \varphi_j) \equiv S_{ij}$
The kinetic energy and potential energy of the system are as follows

$$T = \frac{1}{2}m^o\dot{x}^2 + \frac{1}{2}[\mathrm{J}_0 + m_2(L_1^2 + L_2^2 - 2L_1L_2C_2) + m_o u^2]\dot{\varphi}_1^2 + \frac{1}{2}m_2L_2^2\dot{\varphi}_2^2 + \frac{1}{2}J_3\dot{\varphi}_3^2 + \frac{1}{2}m_o\dot{u}^2$$
$$+\frac{1}{2}[m_2(L_1S_1 - L_2S_{12}) - m_oS_1u]\dot{x}\dot{\varphi}_1 - m_2L_2S_{12}\dot{x}\dot{\varphi}_2 + m_oC_1\dot{x}\dot{u} + m_2(L_2^2 - L_1L_2C_2)\dot{\varphi}_1\dot{\varphi}_2 \tag{4-1}$$

$$\Pi = m_2 g(-L_1S_1 + L_2 S_{12}) + m_0 guS_1 + \frac{1}{2}c_1(u-u_o)^2 + \frac{1}{2}c_2L_2(\varphi_2 - \frac{\pi}{2})^2 + \frac{1}{2}c_0\left(\varphi_1 - \varphi_{10}\right)^2, \quad (4\text{-}2)$$

where $m^0 = m_0 + m_1 + m_2 + m_3$, $J_3 = m_3 r^2/2$ and the centre of mass of the body AODC coincides with the point O in assumption.

The mother system is subject to a holonomic constraint as follows

$$\dot{x} + r\dot{\varphi}_3 = 0, \quad (4\text{-}3)$$

or in the form of (2-1) as $\ddot{x} + r\ddot{\varphi}_3 = 0$.

The generalized forces take the following forms

$$\begin{aligned}
&Q_x = 0,\\
&Q_1 = m_2 gL_1C_1 - m_2 gL_2C_{12} - m_o guC_1 + M_o - a\dot{\varphi}_3 - b_0\dot{\varphi}_1,\\
&Q_2 = -m_2 gL_2C_{12} - c_2L_2(\varphi_2 - \frac{\pi}{2}) - b_2\dot{\varphi}_2,\\
&Q_3 = -M_o + a\dot{\varphi}_3,\\
&Q_u = F(t) - m_1 gS_1 - c_1(u-u_o) - b_1\dot{u}.
\end{aligned} \quad (4\text{-}4)$$

The independent generalized coordinates are x, φ_1, φ_2, and u. The matrix **D** is in the form

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{r} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4\text{-}5)$$

In accordance with the equations (2-14) and (2-15), the equations of motion of the system are written as follows

$$\mathbf{M(q)\ddot{q}} = \mathbf{f(q,\dot{q})} + \mathbf{BU}, \quad (4\text{-}6)$$

where

$$\mathbf{M(q)} = \begin{bmatrix} m^o & m_2(L_1S_1 - L_2S_{12}) - m_oS_1u & -m_2L_2S_{12} & -\frac{J_3}{r} & m_oC_1 \\ m_2(L_1S_1 - L_2S_{12}) - m_oS_1u & \mathrm{J}_1 + m_2(L_1^2 + L_2^2 - 2L_1L_2C_2) + m_ou^2 & m_2(L_2^2 - L_1L_2C_2) & 0 & 0 \\ -m_2L_2S_{12} & m_2(\mathrm{L}_2^2 - L_1L_2C_2) & m_2L_2^2 & 0 & 0 \\ 1 & 0 & 0 & r & 0 \\ m_oC_1 & 0 & 0 & 0 & m_o \end{bmatrix},$$

$$\mathbf{f(q)} = \begin{bmatrix} -[m_2(L_1C_1 - L_2C_{12}) - m_ouC_1]\dot{\varphi}_1^2 + 2m_2L_2C_{12}\dot{\varphi}_1\dot{\varphi}_2 + 2m_oS_1\dot{u}\dot{\varphi}_1 + m_2L_2C_{12}\dot{\varphi}_2^2 + \frac{1}{r}a\dot{\varphi}_3) \\ -2m_2L_1L_2S_2\dot{\varphi}_1\dot{\varphi}_2 - 2m_ou\dot{u}\dot{\varphi}_1 - m_2L_1L_2S_2\dot{\varphi}_2^2 - a\dot{\varphi}_3 + m_2gL_1C_1 - m_2gL_2C_{12} - m_oguC_1 \\ m_2L_1L_2S_2\dot{\varphi}_1^2 - m_2gL_2C_{12} - c_2L_2(\varphi_2 - \frac{\pi}{2}) - b_2\dot{\varphi}_2 \\ 0 \\ -m_og\ S_1 - c_o(u-u_o) + m_ou\dot{\varphi}_1^2 - b_o\dot{u} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{r} & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{U(t)} = \begin{bmatrix} M_o(t) \\ F(t) \end{bmatrix}.$$

5 Simulation Results

The single wheeled vehicle's parameters used for simulation are as follows: m_0 = 30 kg, m_1 = 9 kg, m_2 = 60 kg, m_3 = 10 kg, L_1 = 0.5 m, L_2 = 1.5 m, J_0 = 0.75 kgm^2, c_0 = 200 Nm, c_1=3370 N/m, c_2=7616 N/m, b_0=200 Ns/m, b_1=900 Ns/m, b_2=600 Ns/m, r=0.1 m, and a=30 Ns/m. The vehicle's steady state speed is maintained at 2.77 m/s; the floorboard position is kept horizontally balanced; the rod AB is held vertically, and the slider is kept around 1 m from axis O. The simulation time is about 20 seconds. The initial conditions for the closed loop case are as follows

$x(0)=0,\ \dot{x}(0)=0.2,\ \varphi_1(0)=\pi/4,\ \dot{\varphi}_1(0)=0,\ \varphi_2(0)=3\pi/4,\ \dot{\varphi}_2(0)=0,\ \varphi_3(0)=0,\ \dot{\varphi}_3(0)=-2,$

$u(0)=0.7,\ \dot{u}(0)=0$. The initial position of vehicle is not balanced. Under the effect of the controller, the vehicle would be gradually driven to the balanced status after a short time.

5.1 The Open Loop Approach

The simulation results are shown as follows

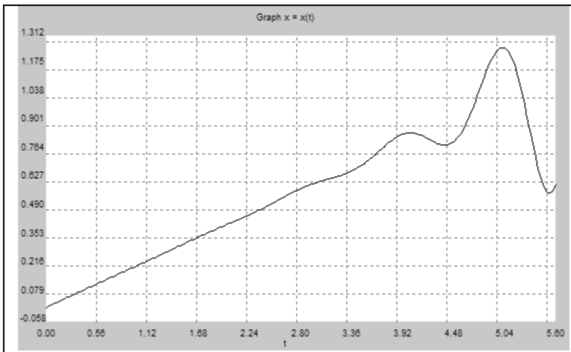

Figure 5.1. The time history of x

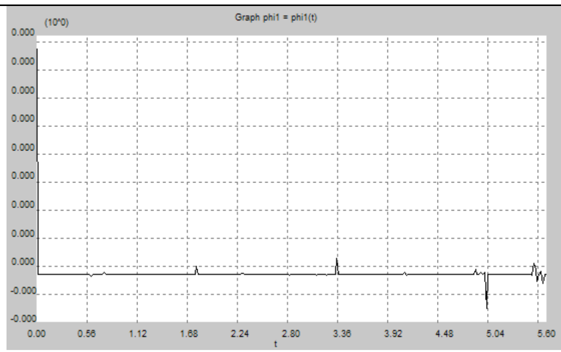

Figure 5.2. The time history of φ_1

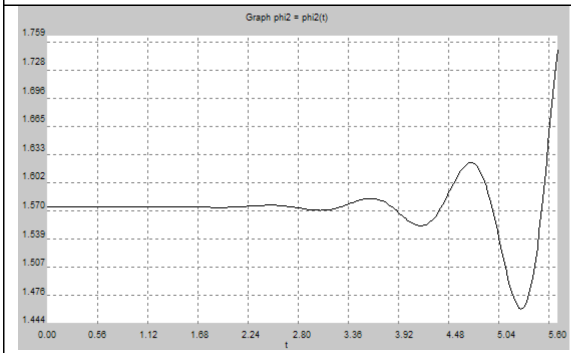

Figure 5.3. The time history of φ_2

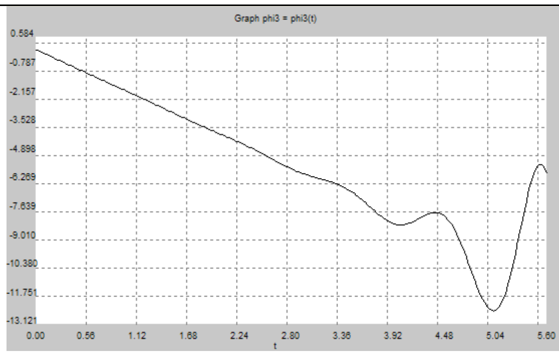

Figure 5.4. The time history of φ_3

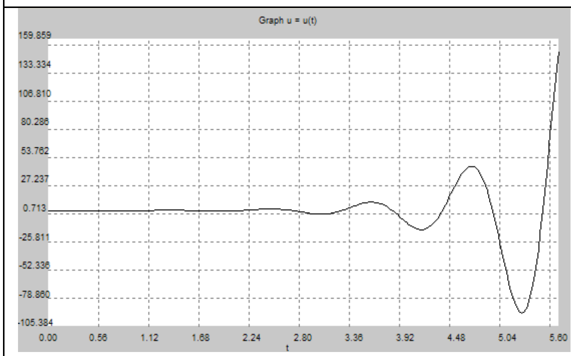

Figure 5.5. The time history of u

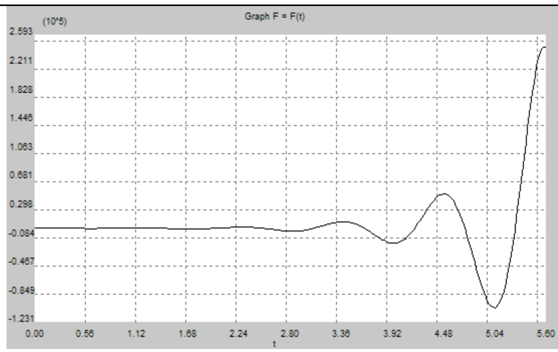

Figure 5.6. The time history of F

5.2 The Closed Loop Approach

The matrices $\mathbf{K}_1$, $\mathbf{K}_2$ are chosen as follows:

$$\mathbf{K}_1 = \begin{bmatrix} 18 & 0 & 0 & 0 \\ 0 & 18 & 0 & 0 \\ 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 18 \end{bmatrix}, \ \mathbf{K}_2 = \begin{bmatrix} 80 & 0 & 0 & 0 \\ 0 & 80 & 0 & 0 \\ 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 80 \end{bmatrix}$$

The simulation results are shown as follows

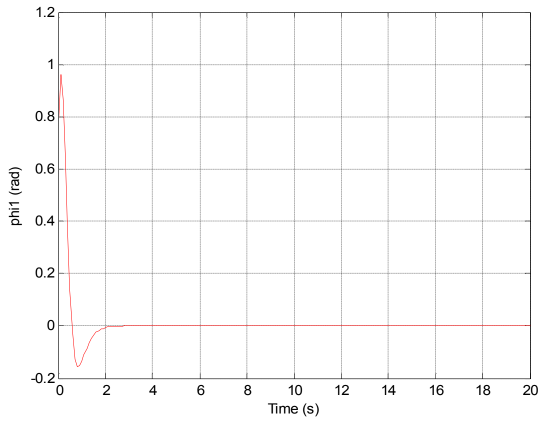

Figure 5.7. The angle φ_1

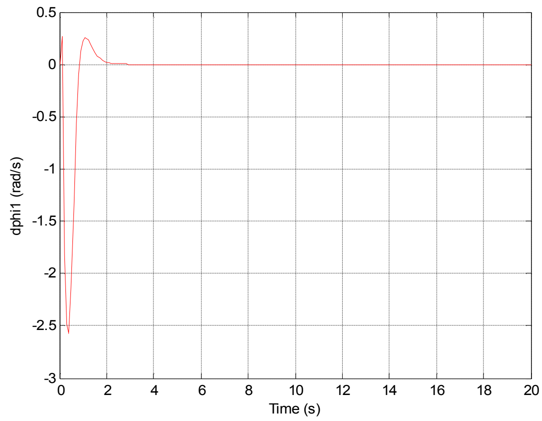

Figure 5.8. The angular velocity $\dot{\varphi}_1$

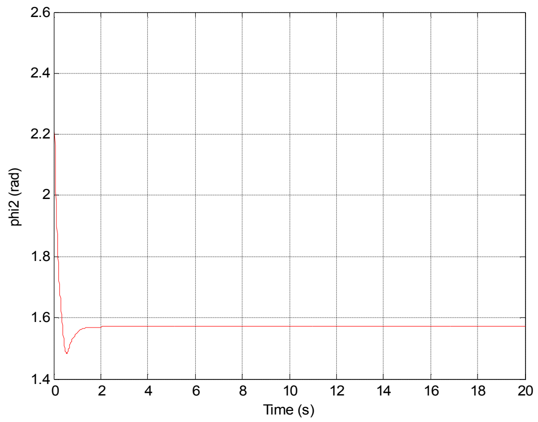

Figure 5.9. The angle φ_2

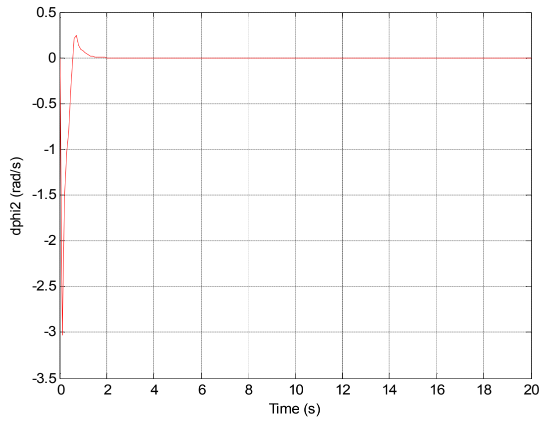

Figure 5.10. The angular velocity $\dot{\varphi}_2$

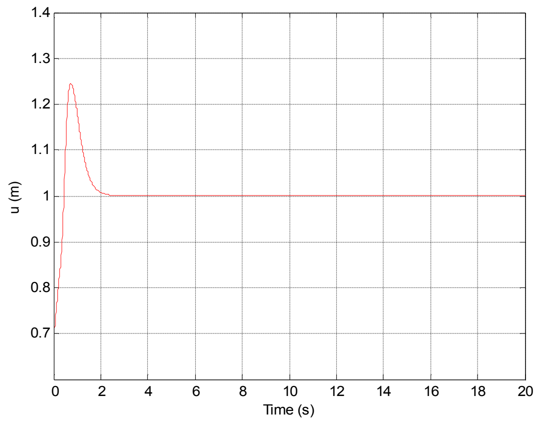

Figure 5.11. The relative displacement u

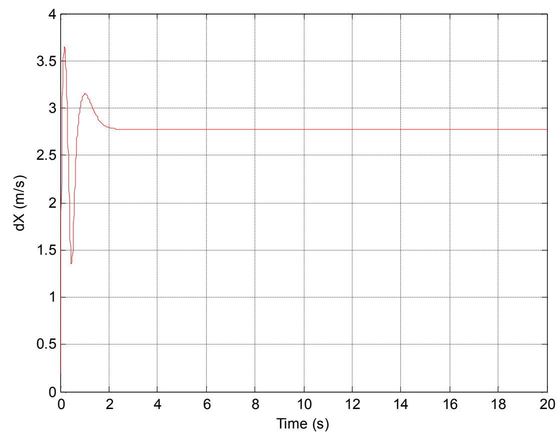

Figure 5.12. The velocity $\dot{x}$

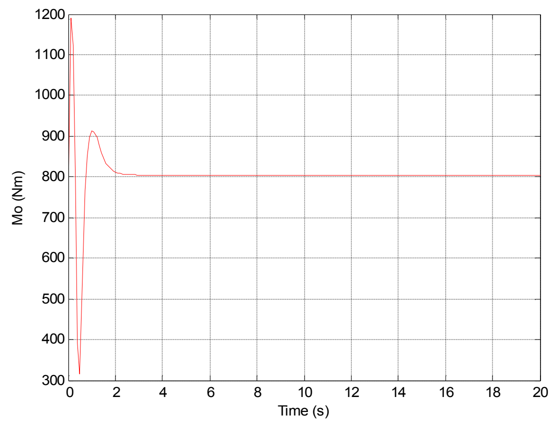

Figure 5.13. The moment $M_0(t)$

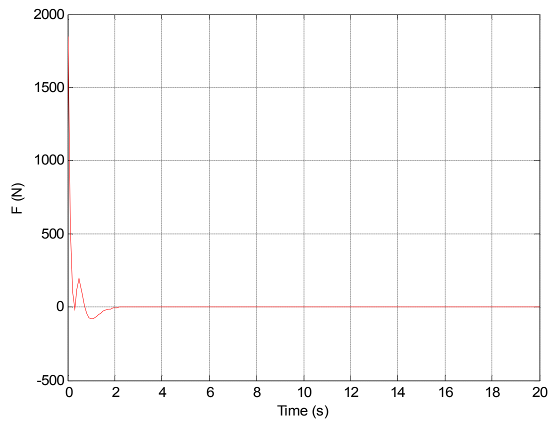

Figure 5.14. The force F(t)

6 Conclusion

In the development of today's technology, more and more engineering problems are involved in control of program motion using relative motion. In the paper, the single-wheel vehicle has been kept balanced by using two different control methods: one using only the subsystem and the other using both the subsystem and the mother system. From control point of view, the control system is active in both cases and the control system is passive in case to specify the relative motion. In summary, for unstable systems (like the single-wheel vehicle), the hybrid method that controls both the mother system (external force) and the subsystem (internal force) might achieve the best effect (results).

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