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Free Vibration Analysis of Stiffened Laminated Plates Using a New Stiffened Element

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A new 9-noded rectangular stiffened plate element for the vibration analysis of laminated stiffened plates based on Mindlin's deformation plate theory has been developed. The stiffened plate element is a combination of basic rectangular element and beam bending component. The stiffened plate element has been developed to include the effects of transverse shear. The element can accommodate any number of arbitrarily oriented stiffeners and obviates the use of mesh lines along the stiffeners. Free vibration analyses of stiffened laminated plates have been carried out with this element and the results are compared with those published. The finite element results show very good matching with the experimental ones.

1 Introduction

Stiffened composite plates are used extensively in many industrial structures: Aerospace structures, ship hulls, bottom shell-machine girder plate of composite boat etc. So determination of the natural frequencies of the stiffened composite plates is a practical demand, in order to solve the anti-vibration problems when the structures are subjected to periodic exciting loads.

A number of analytical and numerical models for the analysis of stiffened laminated plates have been proposed in the literature such as Kirk (1961), Satsangi (1987), Mukhopadhyay (1989), Kolli (1996), Satish Kumar (2000), Gangadhara Prusty (2003) etc. Among all the numerical methods, the finite element method (FEM) has been found to be reasonably accurate with less complexity to model stiffened plates. A more accurate model is achieved by representing the plate and stiffeners separately and maintaining compatibility between them. Thomson (1988) et al. and Satsangi (1987) used 8-noded rectangular plate elements and assumed that the stiffeners follow the same displacement field as that of the plating. The FE model of Kolli (1996) consists of the 9-noded rectangular plate element and 3-noded beam element, the beams are placed along the plate nodal lines. Edward et al. (2000) used a stiffened plate element that is composed of a rectangular 9-noded rectangular plate element and a number of 3-noded stiffener elements placed within the plate element and parallel to the element edges. Gangadhara Prusty (2003) studied linear static analysis of composite hat-stiffened laminated shells using 8-noded rectangular plate element and 3-noded beam element. The existing finite element techniques stimulate the stiffener to pass along the plate nodal lines. In these studies, no discussion has been made for the stiffeners of various shapes and having arbitrary orientation in the plate. To overcome this problem, Satish Kumar and Mukhopadhyay (2000) have developed a stiffened triangular plate element for the analysis of laminated stiffened plates. This basic plate element is a combination of Allman's plane stress triangular element and discrete Kirchhoff-Mindlin plate bending element. However, the interpolation of displacements in their model is very complex and the natural frequencies obtained are less accurate than those published.

In an attempt to efficiently solve this problem, we developed a new 9-noded stiffened rectangular laminated element. This element can accommodate any number of arbitrarily oriented stiffeners and is completely free from the usual constraints on the mesh division of the stiffened plates.

2 Stiffness Matrix of the Stiffened Plate Element

The displacement field based on the first-order shear deformation plate theory is given by

$$\begin{aligned} u(x,y,z,t) &= u^0(x,y,t) + z\theta_x(x,y,t) \\ v(x,y,z,t) &= v^0(x,y,t) + z\theta_y(x,y,t) \\ w(x,y,z,t) &= w^0(x,y,t) \end{aligned} \tag{1a}$$

The displacement field of stiffener

$$\begin{aligned} u(\overline{x},z,t) &= u^0(\overline{x},t) + z\theta_{\overline{x}}(\overline{x},t) \\ w(\overline{x},t) &= w^0(\overline{x},t) \end{aligned} \tag{1b}$$

here, $\overline{x}$ is x-axis of stiffener. In general, $\angle(x,\overline{x}) = \varphi$ as shown in Figure 1.

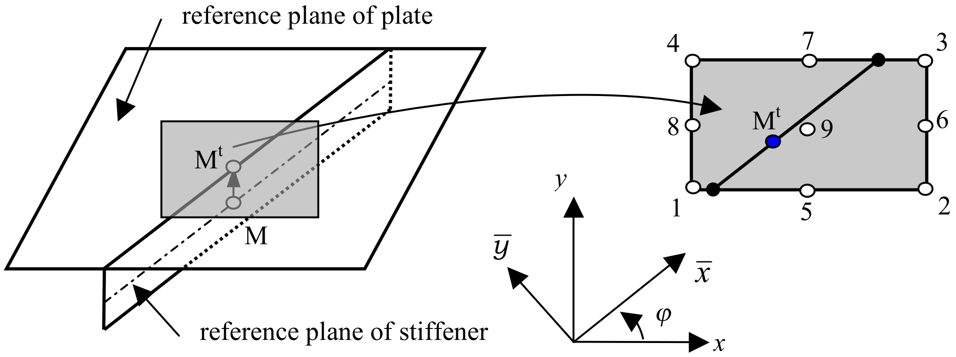

Figure 1. Nine-noded rectangular stiffened plate element

The element stiffness matrix of stiffened plate is determined by

$$\mathbf{K}_e = \mathbf{K}_e^p + \mathbf{K}_e^{st} \tag{2}$$

where $\mathbf{K}_e^p$ and $\mathbf{K}_e^{st}$ are the element stiffness of flat plate and stiffener respectively and given by Ngo (2007)

$$\begin{aligned} \mathbf{K}_e^p &= \int_{St} \left[\mathbf{B}_1^T \mathbf{A} \mathbf{B}_1 + \mathbf{B}_1^T \mathbf{B} \mathbf{B}_2 + \mathbf{B}_2^T \mathbf{B} \mathbf{B}_1 + \mathbf{B}_2^T \mathbf{D} \mathbf{B}_2 + \mathbf{B}_3^T \mathbf{A}' \mathbf{B}_3 \right] dS \\ \mathbf{K}_e^{st} &= b_{st} \int_{\ell g} \left[\mathbf{B}^{st}\right]^T \left[\mathbf{T}^{st}\right]^T \left[\mathbf{D}^{st}\right] \left[\mathbf{T}^{st}\right] \left[\mathbf{B}^{st}\right] d\overline{x} \end{aligned} \tag{3}$$

A, **B**, **D** and **A***'* are popular matrices (Tran, 1994); $\mathbf{D}^{st}$ is the rigidity matrix of the stiffener; b_{st} is the width of stiffener, B_i is the strain-displacement matrix of the plate (Ngo, 2007).

$$[\mathbf{B}_i] = \left[[L_i]N_1 \; [L_i]N_2 \; ....[L_i]N_9 \right] \quad i = (1, 2, 3)$$

N_i are quadratic shape functions; $\mathbf{B}^{st}$ is the strain-displacement matrix of the stiffener

$$\left[\mathbf{B}^{st}\right]=\begin{bmatrix} \frac{\partial}{\partial\overline{x}} & 0 & \frac{\partial}{\partial\overline{y}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial\overline{y}} & \frac{\partial}{\partial\overline{x}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial}{\partial\overline{y}} & \frac{\partial}{\partial\overline{x}} \\ 0 & 0 & 0 & \frac{\partial}{\partial\overline{x}} & 0 & 0 & \frac{\partial}{\partial\overline{y}} & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{\partial}{\partial\overline{y}} & \frac{\partial}{\partial\overline{x}} & 0 & 1 & 0 \end{bmatrix}\left[\overline{\mathbf{N}}\right] \tag{4}$$

$\mathbf{T}^{st}$ is the strain transformation matrix of the stiffener (Ngo, 2007)

$$\left[\mathbf{T}^{st}\right]=\begin{bmatrix} \cos^2\varphi & \sin^2\varphi & \frac{1}{2}\sin 2\varphi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos^2\varphi & \sin^2\varphi & \frac{1}{2}\sin 2\varphi & \frac{1}{2}\sin 2\varphi & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin\varphi & \cos\varphi \end{bmatrix} \tag{5}$$

The rigidity matrix of the stiffener due to parallel laminations: The rigidity matrix of the stiffener is expressed as

$$\left[\mathbf{D}^{st}\right]=\begin{bmatrix} A^{st} & B^{st} & 0 \\ B^{st} & D^{st} & 0 \\ 0 & 0 & A'^{st} \end{bmatrix} \tag{6}$$

where, the coefficients of rigidity matrix are calculated by ignoring the stresses in the width or $\overline{y}$ direction of stiffener $\left(\sigma_{\overline{y}}=\tau_{\overline{xy}}=\tau_{\overline{xz}}=0\right)$ but not the strains $\left(\varepsilon_{\overline{y}}\neq\gamma_{\overline{xy}}\neq\gamma_{\overline{yz}}\neq 0\right)$:

$$\begin{aligned} A^{st} &= \sum_{\overline{k}=1}^{n_{st}}\left[C'^{st}_{11}-C'^{st}_{12}\left(\overline{C}'^{st}_{22}C'^{st}_{12}+\overline{C}'^{st}_{26}C'^{st}_{16}\right)-C'^{st}_{12}\left(\overline{C}'^{st}_{26}C'^{st}_{12}+\overline{C}'^{st}_{66}C'^{st}_{16}\right)\right]_{\overline{k}}\left(z_{k+1}-z_k\right) \\ B^{st} &= \sum_{\overline{k}=1}^{n_{st}}\left[C'^{st}_{11}-C'^{st}_{12}\left(\overline{C}'^{st}_{22}C'^{st}_{12}+\overline{C}'^{st}_{26}C'^{st}_{16}\right)-C'^{st}_{12}\left(\overline{C}'^{st}_{26}C'^{st}_{12}+\overline{C}'^{st}_{66}C'^{st}_{16}\right)\right]_{\overline{k}}\left(z^2_{k+1}-z^2_k\right)/2 \\ D^{st} &= \sum_{\overline{k}=1}^{n_{st}}\left[C'^{st}_{11}-C'^{st}_{12}\left(\overline{C}'^{st}_{22}C'^{st}_{12}+\overline{C}'^{st}_{26}C'^{st}_{16}\right)-C'^{st}_{12}\left(\overline{C}'^{st}_{26}C'^{st}_{12}+\overline{C}'^{st}_{66}C'^{st}_{16}\right)\right]_{\overline{k}}\left(z^3_{k+1}-z^3_k\right)/3 \\ A'^{st} &= \sum_{\overline{k}=1}^{n_{st}}\left[C'^{st}_{55}-\frac{\left(C'^{st}_{45}\right)^2}{C'^{st}_{44}}\right]_k\left(z_{k+1}-z_k\right) \end{aligned} \tag{7}$$

C^{st}_{ij} (i, j = 1, 2, 3, 4, 5 and 6) are the stiffness coefficients of the stiffener material and

$$\begin{bmatrix} \overline{C}'^{st}_{22} & \overline{C}'^{st}_{26} \\ \overline{C}'^{st}_{26} & \overline{C}'^{st}_{66} \end{bmatrix}=\begin{bmatrix} C'^{st}_{22} & C'^{st}_{26} \\ C'^{st}_{26} & C'^{st}_{66} \end{bmatrix}^{-1} \tag{8}$$

The rigidity matrix of the stiffener due to perpendicular laminations: The rigidity matrix of the stiffener is expressed as

$$\left[\mathbf{D}^{st}\right]=\sum_{k=1}^{n}\begin{bmatrix}\left(C'^{st}_{11}\right)_k d_s & 0 & 0\\ 0 & \left(C'^{st}_{11}\right)_k \dfrac{d_s^{\,3}}{12} & 0\\ -2\left(C'^{st}_{16}\right)_k d_s & 0 & \left(C'^{st}_{66}\right)_k d_s\end{bmatrix} \tag{9}$$

where d_s is the height (depth) of the stiffener.

3 Mass Matrices

The kinetic energy of plate element with distributed mass is given by

$$T_e=\frac{1}{2}\int_{Ve}\dot{\mathbf{u}}^T\rho\ \dot{\mathbf{u}}dV \tag{10}$$

where

$$\dot{\mathbf{u}}^T=\left[\dot{u},\dot{v},\dot{w}\right]=\left[\dot{u}^0\quad \dot{v}^0\quad \dot{w}^0\quad \dot{\theta}_x\quad \dot{\theta}_y\right]\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\z&0&0\\0&z&0\end{bmatrix} \tag{11}$$

We can express **u** in terms of the nodal displacements, **a**, by using shape functions **N**.

$$T_e=\frac{1}{2}\int_{Se}\left\{\dot{\mathbf{d}}\right\}^T\left[\rho^*\right]\left\{\dot{\mathbf{d}}\right\}dS=\frac{1}{2}\int_{Se}\left\{\dot{\mathbf{a}}\right\}^T\left[\mathbf{N}\right]^T\left[\rho^*\right]\left[\mathbf{N}\right]\left\{\dot{\mathbf{a}}\right\}dS=\frac{1}{2}\left\{\dot{\mathbf{a}}\right\}^T\left[\mathbf{M}_e\right]\left\{\dot{\mathbf{a}}\right\} \tag{12}$$

where $\left[\mathbf{M}_e\right]$ is the mass matrix of plate element

$$\left[\mathbf{M}_e\right]=\int_{Se}\left[\mathbf{N}\right]^T\left[\rho^*\right]\left[\mathbf{N}\right]dS \tag{13}$$

and

$$\left[\rho\,*\right]=\begin{bmatrix}I_0&&&&\\0&I_0&&Sym&\\0&0&I_0&&\\I_1&0&0&I_2&\\0&I_1&0&0&I_2\end{bmatrix} \tag{14}$$

with

$$\left(I_0,\ I_1,\ I_2\right)=\sum_{k=1}^{n}\rho_k\int_{h_k}^{h_{k+1}}\left(1,\ z,\ z^2\right)dz \tag{15}$$

and

$$\mathbf{N}=\begin{bmatrix}N_1&0&0&0&0&N_2&0&0&0&0&\cdots&N_9&0&0&0&0\\0&N_1&0&0&0&0&N_2&0&0&0&\cdots&0&N_9&0&0&0\\0&0&N_1&0&0&0&0&N_2&0&0&\cdots&0&0&N_9&0&0\\0&0&0&N_1&0&0&0&0&N_2&0&\cdots&0&0&0&N_9&0\\0&0&0&0&N_1&0&0&0&0&N_2&\cdots&0&0&0&0&N_9\end{bmatrix} \tag{16}$$

The kinetic energy of stiffener member with distributed mass is given by

$$T_{est} = \frac{1}{2}\int_{Ve} \dot{\mathbf{u}}_{st}{}^{T} \rho \; \dot{\mathbf{u}}_{st} dV \tag{17}$$

$\dot{\mathbf{u}}_{st}$ can be expressed by

$$\dot{\mathbf{u}}_{st}{}^{T} = \left[\dot{\bar{u}}_{st}, \dot{\bar{w}}_{st}\right] = \left[\dot{\bar{u}}_{st}{}^{0} \quad \dot{\bar{w}}_{st}{}^{0} \quad \dot{\bar{\theta}}_{\bar{x}st}\right]\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ z & 0 \end{bmatrix} \tag{18}$$

These terms in plate element coordinates are expressed as

$$\begin{bmatrix} \bar{u}_{st} \\ \bar{w}_{st} \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 & z\cos\varphi & z\sin\varphi \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{st}{}^{0} \\ v_{st}{}^{0} \\ w_{st}{}^{0} \\ \theta_{x\,st} \\ \theta_{y\,st} \end{bmatrix} \tag{19}$$

The displacement compatibility between the stiffener and the plate is ensured by the beam element's displacement field which is interpolated from plate element's nodes

$$\begin{bmatrix} u_{st}{}^{0} \\ v_{st}{}^{0} \\ w_{st}{}^{0} \\ \theta_{x\,st} \\ \theta_{y\,st} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \delta & 0 \\ 0 & 1 & 0 & 0 & \delta \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^{0} \\ v^{0} \\ w^{0} \\ \theta_{x} \\ \theta_{y} \end{bmatrix} \tag{20}$$

where $\delta = \frac{d_s + h}{2}$ is eccentricity, d_s is depth of stiffener, and h is thickness of plate.

Finally

$$T_e^{st} = \frac{1}{2} b_{st} \int_{le} \{\dot{\mathbf{d}}\}^{T} [\bar{\rho}] \{\dot{\mathbf{d}}\} d\bar{x} = \frac{1}{2} b_{st} \int_{le} \{\dot{\mathbf{a}}\}^{T} \left[\bar{\mathbf{N}}\right]^{T} [\bar{\rho}] \left[\bar{\mathbf{N}}\right] \{\dot{\mathbf{a}}\} d\bar{x} = \frac{1}{2} \{\dot{\mathbf{a}}\}^{T} \left[\mathbf{M}_e\right]_{st} \{\dot{\mathbf{a}}\} \tag{20}$$

and $\left[\mathbf{M}_e\right]_{st}$ is the mass matrix of stiffener

$$\left[\mathbf{M}_e\right]_{st} = b_{st} l_{est} \left[\bar{\mathbf{N}}\right]^{T} [\bar{\rho}] \left[\bar{\mathbf{N}}\right] \tag{21}$$

where

$$[\bar{\rho}] = \sum_{k=1}^{n_{st}} (\rho_{st})_k \int_{h_{k\,st}}^{h_{k+1st}} \begin{bmatrix} \cos^2\varphi & \cos\varphi\sin\varphi & 0 & z\cos^2\varphi & z\cos\varphi\sin\varphi \\ & \sin^2\varphi & 0 & z\cos\varphi\sin\varphi & z\sin^2\varphi \\ & & 1 & 0 & 0 \\ & Sym & & z^2\cos^2\varphi & z^2\cos\varphi\sin\varphi \\ & & & & z^2\sin^2\varphi \end{bmatrix} dz \tag{22}$$

and

$$\left[\bar{\mathbf{N}}\right] = \begin{bmatrix} 1 & 0 & 0 & \delta & 0 \\ 0 & 1 & 0 & 0 & \delta \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} [\mathbf{N}] \tag{23}$$

4 Results and Discussions

4.1 Validation of the Model

Example 1

In order to check the reliability and accuracy of the present element, we consider the free vibration of a simply supported blade stiffened plates. The variation of natural frequencies with stiffener eccentricity are presented for specially orthotropic cross-ply laminates with three equally spaced stiffeners (Figure 2). The geometry of the stiffened laminates are $a \times b \times h = 400 \times 300 \times 3.4$ (mm^3), the lamination of plate and stiffeners is $(90^0/0^0/90^0)_T$. The ply properties are E_1 = 9.71 GPa; E_2 = 3.25 GPa; $G_{12} = G_{13}$ = 0.9025 GPa; G_{23} = 0.2356 GPa; ν_{12} = 0.29; ρ=1347kg/m^3 for both the plate and the stiffener. The width of the stiffener is 3 mm. The numerical results are compared with those of Biswal and Ghosh, (1994) (4-noded rectangular element with 7 d.o.f at each node) and Chao and Lee (1980). The variation of natural frequencies with stiffener half-depth (Figure. 3) shows better agreement with those of Biswal and Ghosh than those of Chao and Lee. This may be due to neglecting of the shear effects by Chao and Lee.

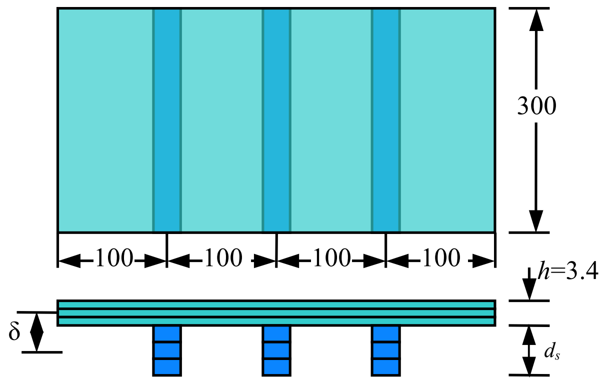

Figure 2. Laminated stiffened plates, eccentricity is variable

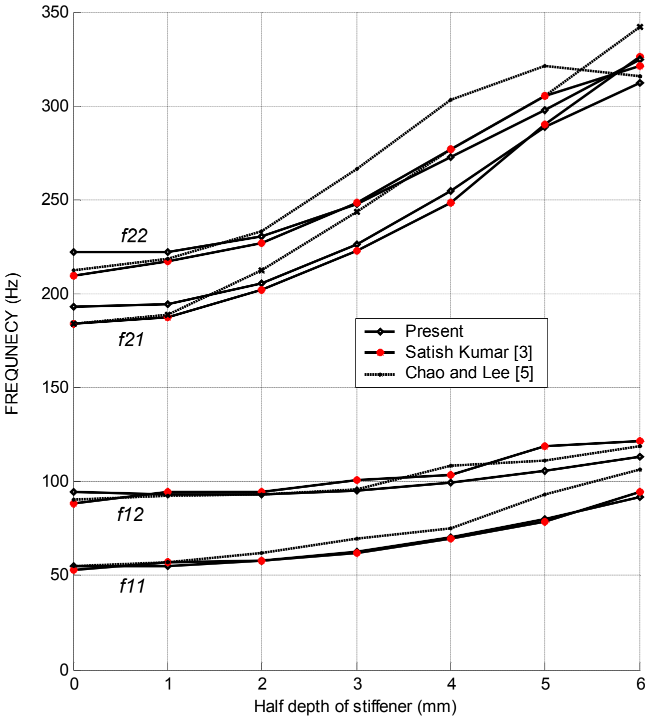

Figure 3. Frequency vs. half-depth of stiffeners

Example 2

In this example, we consider the free vibration of a cross-stiffened plate. The geometry of cross-stiffened plate is $a = b = 254$ mm, $h = 12.7$ mm, $d_{sx} = d_{sy} = 25.4$ mm, $b_{sx} = b_{sy} = 6.35$ mm. Where, d_{sx}, b_{sx} is depth and width of the x-direction stiffener; d_{sy}, b_{sy} is depth and width of the y-direction stiffener. The following ply properties of AS4/3501 graphite/epoxy composites are used: $E_1 = 144.8$ GPa; $E_2 = 9.67$ GPa; $G_{12} = G_{13} = 4.14$ GPa; $G_{23} = 3.45$ GPa; $\nu_{12} = 0.3$; $\rho = 1389.23$ kg/m^3. The stiffener laminations are parallel to the plate midplane. The frequencies of the first four modes of various boundary conditions of a (0/90) and (45°/-45°) cross-stiffened plate are presented in Table 1 and they are compared with those reported in (Satish Kumar et al., 2000), (Chandrasekhra et al., 1997) and (Ray, 1998), using 10×10 mesh for full plate. The fundamental frequencies shown good agreement with those of Satish Kumar et al.(2000), whereas the frequencies of higher modes compared excellently with those of Ray (1998). According to the analysis of Satish Kumar et al.(2000), Chandrasekhra et al. (1997) used the reduced stiffness coefficients in the rigidity matrix of the stiffener which reduced its stiffness. Moreover, they have ignored the coupling coefficients in the strain-energy of the stiffener element. Therefore, the fundamental frequencies with the present element are higher than that of Chandrasekhra et al.

0°/90°						45°/-45°		
Boundary Conditions	Mode	Present	Ref. [3]	Ref. [13]	Ref. [14]	Present	Ref. [3]	Ref. [13]
SSSS	1	1014.0	1076.0	961.81	1092.64	1007.1	1005.7	870.94
	2	2139.5	2059.6	1954.41	1837.04	2284.7	2254.4	2164.51
	3	2397.9	2302.7	2325.41	2491.85	2434.5	2358.7	2470.08
	4	2683.0	2635.8	2641.18	2654.51	3208.3	3247.4	3863.65
CCCC	1	1542.1	1666.5	1583.50	1753.79	1573.8	1714.2	1465.37
	2	2848.3	2929.2	2831.53	2716.65	2909.4	3049.3	2918.90
	3	3041.4	3140.1	3165.27	3319.93	2967.7	3077.4	3178.11
	4	3653.5	3666.3	3634.62	3686.53	3896.3	3943.9	4813.52
CCSS	1	1333.7	1445.8	1342.1	1468.82	1304.0	1380.9	1191.43
	2	2259.6	2107.7	2101.6	2029.11	2483.6	2471.6	2508.92
	3	2929.5	3054.0	3024.58	3074.45	2835.2	2912.6	2803.99
	4	3221.7	3196.8	3211.27	3212.13	3557.0	3609.6	3992.08

Table 1. Effect of boundary conditions on natural frequencies (Hz) of cross-stiffened plates

4.2 Experimental Study

Let us consider a stiffened rectangular composite plate made of glass fiber/polyester 3210 with lamination [M300/WR800/M300/WR800/M450]. Where, M300 denotes glass fiber in Mat form with its weight per unit area is 300g/m^2; M450 denotes glass fiber in Mat form with its weight per unit area is 450g/m^2 and WR800 denotes glass fiber in WR form with its weight per unit area is 800g/m^2.The sides of plate are $a \times b = 800 \times 500$mm^2. The plate is reinforced by 6 longitudinal ($n_x = 6$) and 9 transverse ($n_y = 9$) closed section (hat) stiffeners (Figure 4). The stiffeners were made by glass fiber in Mat and have the same sizes as follows: $b_{st} \times h_{st} \times t_{st} = 10 \times 20 \times 1.8$ mm^3 (Nguyen, 2005).

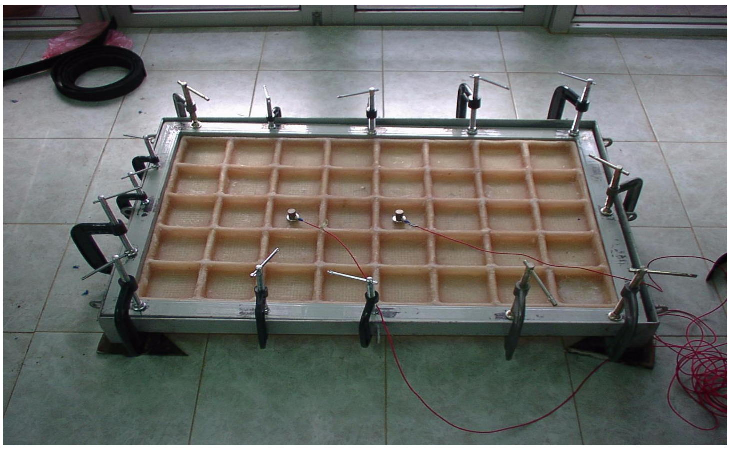

Figure 4. Experimental set-up and testing procedure for stiffened composite plate

Three first natural frequencies of unstiffened and stiffened plates subjected to various boundary conditions were measured by using a Multi-vibration measuring machine (DEWE BOOK-DASYLab 5.61.10) and are given in Table 2.

Plates	Mode	Clamped at 4 edges	CC at x-edges SS at y-edges	CC at y-edges SS at x-edges	Simply Supported at 4 edges
Unstiffened	1	36.430	34.595	22.620	18.310
	2	53.560	45.770	43.600	33.530
	3	82.320	68.510	61.030	59.380
Stiffened	1	147.660	139.280	82.070	65.460
	2	206.330	168.630	166.500	118.640
	3	322.370	259.810	242.080	225.135

Table 2. Experimental results on natural frequencies (Hz) of stiffened composite plate

4.3 Finite Element Results and Comparison

In this section, we calculate the natural frequencies for the above stiffened composite plate by our computer program. The first three natural frequencies will be compared with those of experimental ones.
The elastic constants of material used in the calculation were determined by our mechanical tests (Tran et al., 2005).

- For Mat layer: $E_{11} = E_{22} = 4.807$GPa; $G_{12} = 2.05$GPa; $\nu_{12} = 0.17$
- For WR layer (0° and 90°): $E_{11} = 10.58$GPa; $E_{22} = 2.64$GPa; $G_{12} = 1.02$GPa; $\nu_{12} = 0.17$
- Thickness of a M300 layer, $t_{M3} = 0.6$mm; thickness of a M450 and WR8 layer, $t_{M45} = t_{WR8} = 1$mm

The hat-stiffeners are modeled, as shown in Figure 5. The laminations of stiffeners are perpendicular and parallel to the plate midplane. The frequencies of the first three modes of various boundary conditions of a cross-stiffened plate using 12×12 mesh for full plate are presented in Table 3 and they are compared with those reported in (Nguyen, 2005). The fundamental frequency of clamped plate at 4 edges shown very good agreement with those of Nguyen (2005), whereas the frequencies of simply supported plate at 4 edges are higher than that of Nguyen (2005). This may be due to the boundary condition constructed in experimental study (not simply supported absolutely).

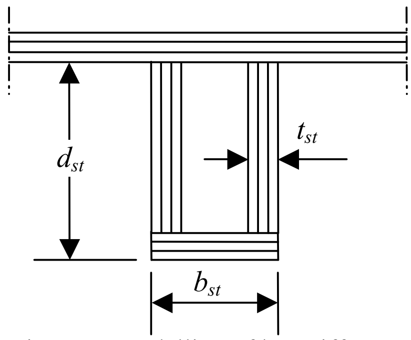

Figure 5. Modelling of hat-stiffener.

Plates	Mode	Clamped at 4 edges	CC at x-edges SS at y-edges	CC at y-edges SS at x-edges	Simply Supported at 4 edges
Unstiffened	1	38.141 (4.5%)	35.742 (3.2%)	23.219 (2.6%)	19.202 (4.6%)
	2	56.620 (5.4%)	47.591 (3.8%)	46.712 (6.6%)	35.500 (5.5%)
	3	90.764 (9.3%)	72.857 (5.9%)	64.335 (5.1%)	62.402 (4.8%)
Stiffened	1	152.804 (3.4%)	144.064 (3.3%)	90.196 (9,2%)	72.442 (9%)
	2	211.263 (2.3%)	178.788 (5.6%)	168.866 (1.4%)	125.446 (5.4%)
	3	324.447 (0.6%)	268.849 (3.3%)	259.238 (6.6%)	235.724 (4.5%)

Remark: (...%) denotes the error percent between FE results and experimental ones.

Table 3. Natural frequencies (Hz) of stiffened plates calculated by finite element program

5 Conclusions

In this paper, we have presented a new 9-noded stiffened rectangular plate element for the vibration analysis of a laminated composite plate with rectangular and hat laminated stiffeners. The stiffener is elegantly modeled and does not introduce any additional nodes. The plate element accommodates any number of arbitrary oriented stiffener elements and eliminates the usual constraints imposed on the mesh division of stiffened plates. The model is validated by comparing with existing results documented in the literature. Some problems on free vibration analyses of laminated stiffened plates made of graphite/epoxy and glass/polyester are analyzed with the present element. Moreover, the element has been very effective in analysis of both thin and moderately thick plates. The finite element results compare well with experimental ones. It is recommended that the present formulation can be used to determine the fundamental frequencies required in the design and analysis of eccentric composite stiffened plates.

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